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Ray C. Fair

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Institut für Weltwirtschaft an der Universität Kiel
Kiel Institute for the World Economy

Kiel Institute

Kiel Institute for World Economics
Duesternbrooker Weg 120
24105 Kiel (Germany)

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by

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Abstract

How inflation and unemployment are related in both the short run and long run is perhaps the key question in macroeconomics. This paper tests various price equations using quarterly U.S. data from 1952 to the present. Issues treated are the following. 1) Estimating price and wage equations in which wages affect prices and vice versa versus estimating "reduced form" price equations with no wage explanatory variables. 2) Estimating price equations in (log) level terms, first difference (i.e., inflation) terms, and second difference (i.e., change in inflation) terms. 3) The treatment of expectations. 4) The choice and functional form of the demand variable. 5) The choice of the cost-shock variable. The results reject the use of rational expectations and suggest that the best specification is a price equation in level terms imbedded in a price-wage model, where the wage equation is also in level terms. The best cost-shock variable is the import price deflator, and the best demand variable is the unemployment rate. There is some evidence of a nonlinear effect of the unemployment rate on the price level at low values of the unemployment rate. Many of the results in this paper are contrary to common views in the literature, but the empirical support for them is strong.

*Cowles Foundation and International Center for Finance, Yale University, New Haven, CT 06520-8281. Voice: 203-432-3715; Fax: 203-432-6167; email: ray.fair@yale.edu; website: http://fairmodel.econ.yale.edu. For presentation at "The Phillips Curve and the Natural Rate of Unemployment," June 3–4, 2007, Kiel Institute for World Economics, Kiel, Germany.

1 Introduction

How inflation and unemployment are related in both the short run and long run—i.e., the specification of price equations—is perhaps the key question in macroeconomics. As a recent review of price equations by Rudd and Whelan (2007) shows, there is certainly no consensus view of the best explanation of inflation. This paper tests various price equations using quarterly U.S. data from 1952 to the present. The aim is to see which price equation best explains the historical data. Some of the questions considered are the following. 1) Is it better to estimate price and wage equations in which wages affect prices and vice versa or to estimate "reduced form" price equations with no wage explanatory variables? 2) Is it best to estimate price equations in (log) level terms, in first difference (i.e., inflation) terms, or in second difference (i.e., change in inflation) terms? 3) Is it better to take expectations to be rational or not? 4) What is the best choice and functional form for the demand variable? 5) What is the best choice for the cost-shock variable?

The results in this paper reject the use of rational expectations and suggest that the best specification is a price equation in level terms imbedded in a price-wage model, where the wage equation is also in level terms. The best cost-shock variable is the import price deflator, and the best demand variable is the unemployment rate. The functional form for the unemployment rate is likely to be nonlinear at very low values of the unemployment rate, but the functional form is hard to estimate because there are so few observations with very low values. However, some evidence of a nonlinear functional form has been found. Many of the results in this paper are contrary to common views in the literature, but it will be seen that the empirical support for them is strong.

This paper begins with the new-Keynesian Phillips curve (NKPC) in Section 2. This section shows that FIML estimation of this equation, which is the obvious procedure to use if expectations are rational, leads to an insignificant coefficient estimate of the expected future inflation variable. Section 3 examines the reduced-form price equation in which inflation depends on past inflation, demand, and cost shocks. This model is sometimes called the "triangle" model because of the three main effects on inflation. The results in this section show that the dynamics of this equation are rejected. Section 4 then presents a price-wage model, with a price equation and a wage equation, which is shown to be more accurate than a price equation alone. Section 4 also examines the sensitivity of the results to different measures of demand and cost shocks and to different functional forms of the demand variable.

The main conclusion of this paper is that the best explanation of inflation in terms of explaining the U.S. historical data is a model in which the log of the price level depends on the lagged log price level, the log of the wage rate, the log of the price of imports, the unemployment rate, a constant term, and a time trend and in which the log of the wage rate depends on the lagged log wage rate, the log of the price level, the lagged log of the price level, a constant term, and a time trend. This specification is not new. It is currently part of my macroeconometric model—Fair (2004)— and it is close to the specification that existed in the version of the model 23 years ago—Fair (1984), U.S. equations 10 and 16.

2 The New-Keynesian Phillips Curve

A typical version of the NKPC is:

$$\pi_t = \beta_1 E_t \pi_{t+1} + \beta_2 y_t + \epsilon_t \tag{1}$$

where π is the inflation rate and y is the output gap. E_t denotes expectations based on information at time t. Expectations are assumed to be rational. This equation is part of the new-Keynesian (NK) model, and there is a huge literature on it. There is also a "hybrid" version of this equation that is popular, which adds the lagged inflation rate to the equation:

$$\pi_t = \beta_1 E_t \pi_{t+1} + \beta_2 y_t + \beta_3 \pi_{t-1} + \epsilon_t \tag{2}$$

where $\beta_1 + \beta_3$ is usually constrained to be one.

Rudd and Whelan (2005, 2006, 2007) in a series of papers have shown that $E_t \pi_{t+1}$ does not have an important influence on current inflation given the model and the assumption that expectations are rational. Their results are robust to different measures of the output gap, including the use of the labor share, as Galí and Gertler (1999) advocate.

Rudd and Whelan's tests do not involve the full information maximum likelihood (FIML) estimation of equation (2). In fact, there has been little attempt in the literature to use FIML in this context. Most of the direct estimates of equation (2) involve GMM. For example, Blanchard and Galí (2005) estimate equation (2) with the change in the PPI raw materials index relative to the GDP deflator added, and they use as instruments four lags each of inflation, the demand variable (the unemployment rate), and the raw materials index variable. The problem with this choice of instruments is that except for inflation lagged once, the lagged values are not part of the model and so theoretically are not appropriate to use. To use these lags, one has to argue that the equation is part of a larger model in which the lags appear, but this is not very satisfying. A stronger test of the equation would be to specify the rest of the model and estimate the model by FIML, where the assumption that $E_t\pi_{t+1}$ is model consistent—i.e., that expectations are rational—is imposed.

Lindé (2005) reports estimating by FIML a version of equation (2) in the context of the NK model. He has added some lags to the model that are not strictly part of the NK model, but the model is a complete system that can be estimated by FIML. The three-equation model is:

$$\pi_t = \beta_1 E_t\pi_{t+1} + (1-\beta_1)\pi_{t-1} + \beta_2 y_t + \epsilon_t \tag{3}$$

$$y_t = \alpha_1 E_t y_{t+1} + (1-\alpha_1)\sum_{i=1}^{4}\delta_i y_{t-i} + \alpha_2(r_t - E_t\pi_{t+1}) + v_t \tag{4}$$

$$r_t = (1-\sum_{i=1}^{3}\rho_i)(\theta_1\pi_t + \theta_2 y_t) + \sum_{i=1}^{3}\rho_i r_{t-i} + w_t \tag{5}$$

where r is the nominal interest rate and $\sum_{i=1}^{4}\delta_i = 1$. The data are quarterly U.S. data for the 1960:1–1997:4 period. The important question for present purposes is whether β_1 is different from zero.

This model is linear, and so it can be estimated by linear techniques, which Lindé reports doing. It can also be estimated by the method in Fair and Taylor (1990) (FT), which can handle nonlinear models as well. For linear models like equations (3)–(5) the FT method is much more computational intensive, by orders of magnitude, than are linear methods, but it is still well within the computationally feasible range of modern computers. I wanted to see if I could duplicate Lindé's results using the FT method, and I asked him for the data. Unfortunately, he lost the data in a computer crash but said he got the data from Jeremy Rudd at the Federal Reserve Board. Rudd sent me the data that he thought he sent to Lindé, but I was unable to duplicate Lindé's results. In fact, using his estimates as a starting point, I was unable even to solve the model, which is the first thing the FT method needs to do to get going. I then sent Lindé the data that Rudd sent me asking whether he can duplicate his results using his method and these data. As far as I know, after a number of email exchanges, he has not duplicated the results. So although Lindé's results show a significant estimate of β_1, this has not been duplicated, and it should be interpreted with considerable caution.

An earlier and alternative approach using FIML, taken by Fuhrer (1997), is to estimate a version of (2) with fewer restrictions on the rest of the model. Fuhrer estimates

$$\pi_t = \beta_1[(\pi_{t-1}+\pi_{t-2}+\pi_{t-3})/3]+(1-\beta_1)[E_t(\pi_{t+1}+\pi_{t+2}+\pi_{t+3})/3]+\beta_2 y_t+\epsilon_t \quad (6)$$

plus two vector autoregressive equations, one for y_t and one for the federal funds rate. Each of the two vector autoregressive equations includes a constant term and four lagged values each of inflation, the output gap, and the federal funds rate. The sample period is 1966:1–1994:1. Fuhrer estimated the two vector autoregressive equations by ordinary least squares first and then took these as fixed for purposes of estimating equation (6). He used a linear method for the FIML estimation, but again the FT method can be used.

I also wanted to duplicate Fuhrer's results using the FT method, but he had also lost the original data. He sent me a later data set (but the same sample period) and kindly reestimated his model using his method and this data set. I was able to duplicate these new results using the FT method! The original estimate of β_1 was 0.80 and it is now 0.95, and the original estimate of β_2 was 0.12 and it is now 0.13. Fuhrer used the BHHH algorithm for computing the variance-covariance matrix of the coefficient estimates, and I used the inverse of the second derivatives (computed numerically) of the log of the likelihood function. This makes some difference in the estimated standard errors. I get a standard error for the estimate of β_1 of 0.42 and Fuhrer gets 0.11. For the estimate of β_2 the estimates are 0.12 and 0.03. In either case, however, the estimate of β_1 is not significantly different from 1.0 and thus the expected future inflation variable is not significant.

This FIML result is consistent with the results of Rudd and Whelan in that there is no evidence that expectations of future inflation affect current inflation conditional on expectations being rational. An interesting question for future research is whether versions of the NK model can be developed in which model-consistent expected future inflation rates are significant in the inflation equation under FIML estimation. In future work of this kind, one need not be limited to linear models if the FT method is used. If such a model is developed, it can be added to the models used in Sections 3 and 4 to see if it explains the data well relative to the other models. For now, however, little appears to be lost by assuming that expectations are not rational, which is done in the rest of this paper.

3 The Triangle Model[1]

Prior to the NKPC equation, the standard inflation equation took inflation to depend on past inflation, demand—usually the unemployment rate—and cost shocks. This is sometimes called the triangle model because of the three basic determinants of inflation. A simple version of the equation is

$$\pi_t - \pi_{t-1} = \beta(u_t - u^*) + \gamma s_t + \epsilon_t, \quad \beta < 0, \quad \gamma > 0, \tag{7}$$

where u is the unemployment rate and s is a cost shock variable. u^*, sometimes call the natural rate or the NAIRU, is the unemployment rate at which the inflation rate does not change aside from changes in s_t and ϵ_t. A more general version of equation (7) is

$$\pi_t = \alpha + \sum_{i=1}^{n} \delta_i \pi_{t-i} + \sum_{i=0}^{m} \beta_i u_{t-i} + \sum_{i=0}^{q} \gamma_i s_{t-i} + \epsilon_t, \quad \sum_{i=1}^{n} \delta_i = 1. \tag{8}$$

For this specification the NAIRU is $-\alpha / \sum_{i=0}^{m} \beta_i$.

The two dynamic restrictions in equation (8) are that 1) the δ_i coefficients sum to one (or in equation (7) that the coefficient of π_{t-1} is one) and 2) the (log) price level never appears as a separate variable but only in change form as the rate of inflation. It is straightforward to test these two restrictions. Let p_t denote the log of the price level, where $\pi_t = p_t - p_{t-1}$. Using this notation, equations (7) and (8) can be written in terms of p rather than π. Equation (7), for example, becomes

$$p_t = 2p_{t-1} - p_{t-2} + \beta(u_t - u^*) + \gamma s_t + \epsilon_t. \tag{9}$$

[1] The results in this section are updates of those in Fair (2004, Chapter 4).

In other words, equation (7) can be written in terms of the current and past two price levels,[2] with restrictions on the coefficients of the past two price levels. Similarly, if in equation (8) n is, say, 4, the equation can be written in terms of the current and past five price levels, with two restrictions on the coefficients of the five past price levels. (Denoting the coefficients on the past five price levels as a_1 through a_5, the two restrictions are $a_4 = 5 - 4a_1 - 3a_2 - 2a_3$ and $a_5 = -4 + 3a_1 + 2a_2 + a_3$.) These restrictions can be tested by simply adding p_{t-1} and p_{t-2} to the equation and testing whether they are jointly significant. An equivalent test is to add π_{t-1} (i.e., $p_{t-1} - p_{t-2}$) and p_{t-1}. Adding π_{t-1} breaks the restriction that the δ_i coefficients sum to one, and adding both π_{t-1} and p_{t-1} breaks the summation restriction and the restriction that each price level is subtracted from the previous price level before entering the equation.

Equation (8) was used for the tests, where s_t in the equation is postulated to be $pm_t - \tau_0 - \tau_1 t$, the deviation of pm from a trend line. pm is the log of the price of imports, which is taken here to be the cost shock variable. The estimation period is 1955:3–2006:2, 204 observations. All the data used in this paper are discussed in the appendix. The price variable is the private nonfarm deflator. n is taken to be 12 and m and q are taken to be 2. This fairly general specification regarding the number of lagged values is used to lessen the chances of the results being due to a particular choice of lags.

Equation (8) was estimated in the following form:

$$\Delta\pi_t = \lambda_0 + \lambda_1 t + \sum_{i=1}^{n-1} \theta_i \Delta\pi_{t-i} + \sum_{i=0}^{m} \beta_i u_{t-i} + \sum_{i=0}^{q} \gamma_i pm_{t-i} + \epsilon_t, \qquad (10)$$

[2] "Price level" in this paper always refers to the log of the price level.

where $\lambda_0 = \alpha + (\gamma_0 + \gamma_1 + \gamma_2)\tau_0 + (\gamma_0 + 2\gamma_1 + 3\gamma_2)\tau_1$ and $\lambda_1 = (\gamma_0 + \gamma_1 + \gamma_2)\tau_1$. α and τ_0 are not identified in equation (10), but for purposes of the tests this does not matter. For reference it will be useful to write equation (10) with π_{t-1} and p_{t-1} added:

$$\begin{aligned}\Delta\pi_t = \lambda_0 + \lambda_1 t + \textstyle\sum_{i=1}^{n-1} \theta_i \Delta\pi_{t-i} + \sum_{i=0}^{m} \beta_i u_{t-i} + \sum_{i=0}^{q} \gamma_i pm_{t-i} \\ +\phi_1 \pi_{t-1} + \phi_2 p_{t-1} + \epsilon_t.\end{aligned} \tag{11}$$

The results of estimating equations (10) and (11) are presented in Table 1. They show that when π_{t-1} and p_{t-1} are added, the standard error of the equation falls from .00356 to .00329. The t-statistics for the two variables are -5.88 and -5.04, respectively, and the χ^2 value for the hypothesis that the coefficients of both variables are zero is 31.26.[3]

The 5 percent critical χ^2 value for two degrees of freedom is 5.99 and the 1 percent critical value is 9.21. If the χ^2 distribution is a good approximation to the actual distribution of the "χ^2" values, the two variables are highly significant and thus the dynamics of equation (10) are strongly rejected. If, however, equation (10) is in fact the way the price data are generated, the χ^2 distribution may not be a good approximation for the test.[4] To check this, the actual distribution was computed using the following procedure.

First, estimate equation (10), and record the coefficient estimates and the estimated variance of the error term. Call this the "base" equation. Assume that

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[3]Note that there is a large change in the estimate of the coefficient of the time trend when π_{t-1} and p_{t-1} are added. The time trend is serving a similar role in equation (11) as the constant term is in equation (10).

[4]If the χ^2 distribution is not a good approximation, then the t-distribution will not be either, and so standard tests using the t-statistics in Table 1 will not be reliable. The following analysis focuses on correcting the χ^2 critical values, and no use of the t-statistics is made.

Table 1
Estimates of Equations (10) and (11)
The Left Hand Side Variable is $\Delta\pi_t$

	Equation (10)		Equation (11)	
Variable	Estimate	t-stat.	Estimate	t-stat.
cnst	.0048	1.22	-.0378	-3.83
t	.0000002	0.01	.0002233	4.45
u_t	-.312	-3.01	-.227	-2.34
u_{t-1}	.096	0.53	.053	0.31
u_{t-2}	.120	1.11	.023	0.23
pm_t	.039	2.36	.050	3.22
pm_{t-1}	.032	1.02	.025	0.86
pm_{t-2}	-.070	-3.94	-.043	-2.32
$\Delta\pi_{t-1}$	-.872	-12.97	-.367	-3.46
$\Delta\pi_{t-2}$	-.678	-8.39	-.304	-3.05
$\Delta\pi_{t-3}$	-.505	-5.95	-.250	-2.70
$\Delta\pi_{t-4}$	-.326	-3.75	-.153	-1.71
$\Delta\pi_{t-5}$	-.359	-4.28	-.231	-2.70
$\Delta\pi_{t-6}$	-.287	-3.50	-.185	-2.23
$\Delta\pi_{t-7}$	-.209	-2.73	-.134	-1.73
$\Delta\pi_{t-8}$	-.098	-1.30	-.045	-0.60
$\Delta\pi_{t-9}$	-.120	-1.62	-.069	-0.95
$\Delta\pi_{t-10}$	-.229	-3.34	-.181	-2.74
$\Delta\pi_{t-11}$	-.093	-1.70	-.070	-1.36
π_{t-1}			-.621	-5.88
p_{t-1}			-.054	-5.04
SE	.00356		.00329	
χ^2			31.26	

- p_t = log of price level, $\pi_t = p_t - p_{t-1}$, u_t = unemployment rate, pm_t = log of the price of imports.
- Estimation method: ordinary least squares.
- Estimation period: 1955:3–2006:2 (204 obs.).
- When p_{t-1} and p_{t-2} are added in place of π_{t-1} and p_{t-1}, the respective coefficient estimates are $-.675$ and .621 with t-statistics of -5.90 and 5.88. All else is the same.
- Five percent χ^2 critical value = 5.99; one percent χ^2 critical value = 9.21.

the error term is normally distributed with mean zero and variance equal to the estimated variance. Then:

1. Draw a value of the error term for each quarter. Add these error terms to the base equation and solve it dynamically to generate new data for p. Given the new data for p and the data for u and pm (which have not changed), compute the χ^2 value as in Table 1. Record this value.

2. Do step 1 1000 times, which gives 1000 χ^2 values. This gives a distribution of 1000 values.

3. Sort the χ^2 values by size, choose the value above which 5 percent of the values lie and the value above which 1 percent of the values lie. These are the 5 percent and 1 percent critical values, respectively.

These calculations were done, and the 5 percent critical value was 18.88 and the 1 percent critical value was 24.30. These values are considerably larger than the critical values from the actual χ^2 distribution (5.99 and 9.21), but they are still smaller than the computed value of 31.26. The two price variables are thus significant at the 99 percent confidence level even using the alternative critical values.

The above procedure treats u and pm as exogenous, and it may be that the estimated critical values are sensitive to this treatment. To check for this, the following two equations were postulated for u and pm:

$$pm_t = a_1 + a_2 t + a_3 pm_{t-1} + a_4 pm_{t-2} + a_5 pm_{t-3} + a_6 pm_{t-4} + \nu_t, \qquad (12)$$

$$\begin{aligned} u_t = b_1 + b_2 t + b_3 u_{t-1} + b_4 u_{t-2} + b_5 u_{t-3} + b_6 u_{t-4} + b_7 pm_{t-1} \\ + b_8 pm_{t-2} + b_9 pm_{t-3} + b_{10} pm_{t-4} + \eta_t. \end{aligned} \qquad (13)$$

These two equations along with equation (10) were taken to be the "model," and they were estimated by ordinary least squares along with equation (10) to get the "base" model. The error terms ϵ_t, ν_t, and η_t were then assumed to be multivariate normal with mean zero and covariance matrix equal to the estimated covariance matrix (obtained from the estimated residuals). Each trial then consisted of draws of the three error terms for each quarter and a dynamic simulation of the model to generate new data for p, pm, and u, from which the χ^2 value was computed. The computed critical values were not very sensitive to this treatment of pm and u, and they actually fell slightly. The 5 percent value was 12.30 compared to 18.88 above, and the 1 percent value was 17.48 compared to 24.30 above. The U.S. data thus reject the dynamics implied by equation (10): π_{t-1} and p_{t-1} are significant when added to equation (10).

The dynamics of equation (10) can be further examined by adding only π_{t-1}. This breaks the summation restriction but not the other (price level) restriction. Although not reported in Table 1, when this is done, the sum of the δ_i coefficient estimates is .806, which is considerably less than one. The sum is not, however, significantly less than one if the computed critical χ^2 values are used. The χ^2 value when π_{t-1} is added is 7.55, and the computed 5 and 1 percent critical values are 9.00 and 13.28, respectively.[5] π_{t-1} is thus not significant at even the 5 percent level when added to equation (10). A further test is to add p_{t-1} to equation (10) with π_{t-1} already added. When this is done the χ^2 value is 22.82 with computed 5 and 1 percent critical values of 10.34 and 18.20, respectively.[6] p_{t-1} is thus significant when added to the equation with π_{t-1} already added.

Another way to examine equations (10) and (11) is to consider how well they

[5]These critical values and the ones cited next were computed using equations (12) and (13).

[6]In this case the base equation for the computation of the critical values is equation (10) with π_{t-1} added.

predict outside sample. Focusing on outside-sample predictions reduces the chance of spurious results due to data mining. To examine these predictions, the following root mean squared error (RMSE) test was performed. Each equation was first estimated for the period ending in 1969:4 (all estimation periods begin in 1955:3), and a dynamic eight-quarter-ahead prediction was made beginning in 1970:1. The predicted values were recorded. The equation was then estimated through 1970:1, and a dynamic eight-quarter-ahead prediction was made beginning in 1970:2. This process was repeated through the estimation period ending in 2006:1. Since observations were available through 2006:2, this procedure generated 146 one-quarter-ahead predictions, 145 two-quarter-ahead predictions, through 139 eight-quarter-ahead predictions, where all the predictions are outside sample. RMSEs were computed using these predictions and the actual values.

The actual values of u and pm were used for all these predictions, which would not have been known at the time of the predictions. The aim here is not to generate predictions that could have in principle been made in real time, but to see how good the dynamic predictions from each equation are conditional on the actual values of u and pm.

The RMSEs are presented in the first two rows of Table 2 for the four- and eight-quarter-ahead predictions for p, π, and $\Delta\pi$. (Ignore the third and fourth rows for now.) Comparing the two rows (equation (10) versus (11)), the RMSEs for $\Delta\pi$ are similar, but they are much smaller for p and π for equation (11). The restrictions clearly lead to a loss of predictive power for the price level and the rate of inflation. It is thus the case that the addition of π_{t-1} and p_{t-1} to equation (10) has considerably increased the accuracy of the predictions, and so these variables

Table 2
Outside-Sample RMSEs

	p		π		$\Delta\pi$	
	Quarters Ahead					
	4	8	4	8	4	8
Eq. (10)	1.93	4.62	2.73	3.49	2.17	2.16
Eq. (11)	1.74	3.66	2.41	2.61	2.16	2.21
Eqs. (14) & (15)	1.15	2.20	1.73	1.73	1.90	1.89
Eqs. (17) & (15)	1.12	2.17	1.68	1.77	1.89	1.88

- p = log of the price level, $\pi = \Delta p$.
- Prediction period: 1970:1–2006:2.
- Errors are in percentage points and are at annual rates for π and $\Delta\pi$.

are not only statistically significant but also important in a predictive sense.

4 A Price and Wage Equation Specification

Prior to the triangle model, separate price and wage equations were sometimes estimated in which prices affect wages and vice versa. Led by Gordon (1980), this specification gradually got replaced by looking only at the reduced form equation for prices. In this section I examine the price and wage equations in my US macroeconometric model. This specification is not new—the original specifications goes back to Fair (1984)—but, as will be seen, it appears to dominate the price equations discussed in the previous section. The two equations are:

$$p_t = \beta_0 + \beta_1 p_{t-1} + \beta_2(w_t + d_t) + \beta_3 pm_t + \beta_4 u_t + \beta_5 t + \epsilon_t, \qquad (14)$$

$$w_t = \gamma_0 + \gamma_1 w_{t-1} + \gamma_2 p_t + \gamma_3 p_{t-1} + \gamma_4 t + \mu_t, \qquad (15)$$

where

$$\gamma_3 = [\beta_1/(1-\beta_2)](1-\gamma_2) - \gamma_1. \qquad (16)$$

w_t is the log of (W_t/λ_t), where W_t is the nominal wage rate and λ_t is trend productivity (output per worker hour). d_t is the log of $(1 + D_t)$, where D_t is the employer social security tax rate. It is possible that u_t should be in the wage equation, but it will be seen below that it is not significant. Even if u_t is in the wage equation, the two equations are identified because pm_t is excluded from the wage equation and w_{t-1} is excluded from the price equation.

The theory behind equations (14) and (15) is presented in Fair (2004), and a briefer discussion is in Section 2 in Fair (2007). This discussion will not be repeated here. Firms are assumed to set prices and wages in a monopolistic competitive setting. Regarding the use of levels versus changes, the decision variables of a firm in the model are its price and wage levels. For example, the market share equations in the theoretical model have a firm's market share as a function of the ratio of the firm's price to the average price of other firms. These are price levels, and the objective of the firm is to choose the price level path (along with the paths of the other decision variables) that maximizes a multiperiod objective function. A firm decides what its price level should be relative to the price levels of other firms. A similar argument holds for the wage decision. This theory thus argues for the specification of price and wage equations in levels, which is what is done in equations (14) and (15).

The time trend in equation (14) is meant to pick up any trend effects on the price level not captured by the other variables. Adding the time trend to an equation like (14) is similar to adding the constant term to an equation specified in terms of changes rather than levels. The time trend will also pick up any trend mistakes made in constructing λ_t. If, for example, $\lambda_t = \lambda_t^a + \alpha_1 t$, where λ_t^a is the correct measure, then the time trend will absorb this error. A similar argument holds for the time trend in the wage equation (15). The coefficient restriction (16) insures that the real wage, $w_t - p_t$, depends on $w_{t-1} - p_{t-1}$, but not on w_{t-1} or p_{t-1} separately. It is not sensible that in the long run the real wage depends on the level of prices or wages.

In the US model equations (14) and (15) are estimated by two stage least squares (2SLS). Equation (14) is first estimated, and then the coefficient estimates of β_1 and β_2 are used in the constraint (16) to estimate (15). The estimation period for the present results is 1954:1–2006:2. The variables treated as endogenous in the estimation are p_t, w_t, pm_t, and u_t. The first stage regressors are predetermined variables in the US model and are available on the website mentioned in the introductory footnote. The results of estimating equation (14) and (15) are presented in Tables 3 and 4 respectively.

The coefficient estimates of the two equations are significant in Tables 3 and 4. The first test of the wage equation in Table 4 is of the restriction (16). The improvement in the fit when the restriction is relaxed is negligible, with a χ^2 value of only 0.002. The restriction is thus not rejected. The second test is to add u_t to the wage equation. For this test only the coefficient estimate of u_t is presented. The coefficient estimate is not significant, with a t-statistic of -0.43. If the unemployment rate is not in the wage equation, as this result suggests, then the price and wage equations together imply that a demand change first affects prices,

Table 3
Estimates of Equation (14)
Left Hand Side Variable is p_t

Variable	Estimate	t-stat.
cnst	-.0395	-3.66
p_{t-1}	.8845	98.55
$w_t + d_t$	.0350	3.20
pm_t	.0507	22.42
u_t	-.1756	-7.35
t	.0003	9.82
SE	.00343	
Tests:		
u_t	-.1494	-5.58
$Y_t/YTREND_t$	.0180	2.05
u_t	-.1890	-6.19
$(YS_t - Y_t)/YS_t$	.0144	0.85
u_t	-.2088	-6.77
$(YSS_t - Y_t)/YSS_t$	.0291	1.82
u_t	-.2088	-6.77
$(YSS_t - Y_t)/YSS_t$	.0291	1.82
u_t	-.1744	-7.23
$lshare_t$	.0315	1.65
pm_t	.0526	17.45
$pcrudm_t$	-.0030	-0.95
pm_t	.0510	20.69
$pcrudo_t$	-.0038	-0.35

• p_t = log of price level, w_t =log of the nominal wage divided by trend productivity, d_t =log of 1 plus the employer social security tax rate, u_t = unemployment rate, pm_t = log of the price of imports, Y =output of the firm sector, $YTREND$ =trend output of the firm sector, YS =potential output of the firm sector in the US model, YSS =potential output of the firm sector from peak-to-peak interpolations, $lshare$ =log of the labor share of the nonfarm business sector, $pcrudm_t$ = log of the crude materials producer price index, $pcrudo_t$ = log of the crude petroleum producer price index.
• Estimation method: two stage least squares.
• Estimation period: 1954:1–2006:2 (210 obs.).

Table 4
Estimates of Equation (15)
Left Hand Side Variable is w_t

Variable	Estimate	t-stat.
cnst	-.0624	-4.01
w_{t-1}	.9292	44.03
p_t	.7959	13.43
p_{t-1}	-.7421	a
t	.0001	2.93
SE	.00843	
Tests:		
Restriction:	$\chi^2 = 0.002$	p-value = .9689
u_t	-.0224	-0.43

a : coefficient restricted.
• See notes to Table 3 for notation.
• Estimation method: two stage least squares.
• Estimation period: 1954:1–2006:2 (210 obs.).

which then affect wages. Likewise, a change in the price of imports first affects prices and then wages.

The tests in the second half of Table 3 are of alternative measures of demand and alternative measures of cost shocks. For each test a new variable was added and the equation reestimated. For the demand-variable tests the coefficient estimates of u_t and the new demand variable are presented, and for the cost-shock tests the coefficient estimates of pm_t and the new cost-shock variable are presented. The unemployment rate is always highly significant in the demand-variable tests. The only other variable that is significant is the ratio of output to trend output, which has a t-statistic of 2.05. The labor share variable stressed by Galí and Gertler (1999) is not significant. The unemployment rate thus clearly dominates the output and labor share variables. The two other measures of cost shocks, the crude materials producer price index and the crude petroleum producer price index, are completely dominated by the import price index, as can be seen at the bottom of Table 3.[7]

Equations (14) and (15) can be compared to equations (10) and (11) as to how well they explain the data by computing outside-sample RMSEs. For these calculations the beginning estimation quarter was 1954.1, and the first end estimation quarter was 1969.4. Each of the 146 sets of estimates used the 2SLS technique with the coefficient restriction imposed, where the values used for β_1 and β_2 in the restriction were the estimated values from equation (14). The same first stage regressors were used for these estimates as were used in the basic estimation of the equations. The predictions of p and w from equations (14) and (15) were generated using the actual values of u and pm, just as was done for equations (10) and (11).

The RMSEs are presented in the third row in Table 2.[8] The results show that the RMSEs using equations (14) and (15) are noticeably smaller than those using even equation (11). For the eight-quarter-ahead prediction, for example, the RMSE for p is 2.20 versus 3.66 for equation (11), and the RMSE for π is 1.73 versus 2.61 for equation (11). Even for $\Delta\pi$ the RMSE using equations (14) and (15) is smaller: 1.88 versus 2.21 for equation (11). The structural price and wage equations clearly do better than even the price equation with the NAIRU restrictions relaxed. These results thus call into question the movement away

[7]Two other tests were performed for equation (14). First, the lagged values p_{t-1}, $w_{t-1} + d_{t-1}$, pm_{t-1}, and u_{t-1} were added to the equation, and they were not jointly significant (p-value of .272). Second, the equation was estimated under the assumption of fourth order serial correlation of the error term, and the four serial correlation coefficient estimates were not jointly significant (p-value of .121). These two tests are rather strong tests of the dynamic specification of equation (14). Two other tests were also performed for equation (15). First, $w_{t-2} - p_{t-2}$ was added, and it was not significant (p-value of .228). Second, the equation was estimated under the assumption of fourth order serial correlation of the error term, and the four serial correlation coefficient estimates were not jointly significant (p-value of .282).

[8]Although predictions of both p and w are generated from this procedure, only the results for p are presented in Table 2.

from the estimation of structural price and wage equations to the estimation of reduced-form price equations. Considerable predictive accuracy is lost when this is done.

Functional Form for the Unemployment Rate

The specification that the unemployment rate enters linearly in equation (14) is not likely to be sensible at low values of the unemployment rate. It is difficult to test for nonlinear effects because there are very few observations of low unemployment rate values. Nevertheless, I did try various functional forms for the unemployment rate, like $\log u_t$, $1/u_t$, $\log(u_t - \beta_6)$, and $1/(u_t - \beta_6)$, where β_6 was taken to be values between 0.01 and 0.03. Better results were obtained using the log form than the reciprocal form. In particular, $\log(u_t - \beta_6)$ seemed to work well for different values of β_6. To test this further, the following equation was estimated by nonlinear 2SLS:

$$p_t = \beta_0 + \beta_1 p_{t-1} + \beta_2(w_t + d_t) + \beta_3 pm_t + \beta_4 \log(u_t - \beta_6) + \beta_5 t + \epsilon_t, \quad (17)$$

where β_6 is also estimated. The estimates are presented in Table 5. The estimate of β_4, the coefficient multiplying $\log(u_t - \beta_6)$, and the estimate of β_6 are highly correlated, and so the precision of the two estimates is not high. Table 5 shows that the t-statistic for $\log(u_t - \beta_6)$ is only -1.13 and the t-statistic for β_6 is only 0.52. However, the estimate of β_6 of .0163 is sensible, and the fit of this equation is better than that in Table 3.

To test this nonlinear specification further, β_6 was restricted to be .0163, and the equation was estimated with both $\log(u_t - .0163)$ and u_t added. These results

Table 5
Estimates of Equation (17)
Left Hand Side Variable is p_t

Variable	Estimate	t-stat.
Nonlinear 2SLS		
cnst	-.0683	-3.26
p_{t-1}	.8807	95.50
$w_t + d_t$	.0410	3.51
pm_t	.0500	21.06
$\log(u_t - \beta_6)$	-.0073	-1.13
β_6	.0163	0.52
t	.0003	9.35
SE	.00339	
Linear 2SLS		
cnst	-.0600	-3.49
p_{t-1}	.8816	97.54
$w_t + d_t$	.0394	3.55
pm_t	.0503	22.45
$\log(u_t - .0163)$	-.0053	-1.55
t	.0003	9.62
u_t	-.0507	-0.61
SE	.00339	

- See notes to Table 3 for notation.
- Estimation period: 1954:1–2006:2 (210 obs.).

are presented in the second half of Table 5. The t-statistic for $\log(u_t - .0163)$ is -1.55, and the t-statistic for u_t is -0.61. This is thus slight evidence in favor of the nonlinear specification. As a final test, outside-sample RMSEs were computed using equations (17) and (15), and these are reported in Table 2. The RMSEs for this specification are very close to those for equations (14) and (15), and it is clear that these results cannot discriminate between the two specifications.

Given that there is at least slight support for the nonlinear specification, it is interesting to examine the dynamic properties of equations (17) and (15) for different values of the unemployment rate. For this specification the effects on prices and wages from a change in the unemployment rate will depend on the initial level of the unemployment rate. Table 6 reports results of permanently decreasing the unemployment rate by .005 percentage points from various levels. Take the first column, where the initial level of u is .08. Equations (17) and (15) were first solved dynamically using $u_t = .080$ for all periods, and then they were solved using $u_t = .075$ for all periods. The values in Table 6 are the differences in the two solution values for each period. (These differences don't depend on the values of the other variables in the equations as long as the values don't differ between the two solutions.)

The effects of the change on the price level and the inflation rate are presented in Table 6 for the first 12 quarters after the change. At the initial level of $u_t = .050$ the price level is higher after a year by 0.43 percentage points. This increases to 0.63 for an initial level of .040, to 1.20 for an initial level of .030, and to 2.28 for an initial level of .025. These values suggest that at least at around $u_t = .040$ a change of this sort would likely trigger a substantial Fed reaction to cool off the economy, which is, of course, why there are so few very low values of the unemployment rate.

The long run effects of the change in the unemployment rate in Table 6 are for the price level to be permanently higher and for the inflation rate to go back to the rate that existed in the base case, i.e., no permanent change in the inflation rate. This is a consequence of the specification of equations (17) and (15) in price and wage levels. The empirical support for the use of levels over changes is the significance of p_{t-1} and p_{t-2} when added to equation (10). On the theoretical side, if a firm is setting its price level in response to demand and its expectations of other

Table 6
Effects of a Permanent Change in u of .005
Equations (17) and (15) Used

	$u = .080$		$u = .050$		$u = .040$		$u = .030$		$u = .025$	
Quar.	a	b	a	b	a	b	a	b	a	b
1	1.0006	0.25	1.0012	0.49	1.0018	0.72	1.0034	1.37	1.0065	2.58
2	1.0012	0.23	1.0023	0.44	1.0034	0.65	1.0066	1.25	1.0124	2.35
3	1.0017	0.20	1.0033	0.40	1.0049	0.59	1.0094	1.14	1.0178	2.14
4	1.0022	0.19	1.0043	0.37	1.0063	0.54	1.0120	1.03	1.0228	1.94
5	1.0026	0.17	1.0051	0.33	1.0075	0.49	1.0144	0.94	1.0273	1.77
6	1.0030	0.15	1.0058	0.30	1.0086	0.45	1.0166	0.86	1.0314	1.61
7	1.0033	0.14	1.0065	0.28	1.0097	0.41	1.0186	0.78	1.0352	1.46
8	1.0036	0.13	1.0072	0.25	1.0106	0.37	1.0204	0.71	1.0387	1.33
9	1.0039	0.12	1.0077	0.23	1.0114	0.34	1.0220	0.64	1.0418	1.21
10	1.0042	0.11	1.0083	0.21	1.0122	0.31	1.0235	0.59	1.0447	1.10
11	1.0044	0.10	1.0087	0.19	1.0129	0.28	1.0249	0.53	1.0473	1.00
12	1.0047	0.09	1.0092	0.17	1.0136	0.25	1.0261	0.48	1.0497	0.91

a = New predicted price level divided by base predicted price level.
b = New predicted inflation rate minus base predicted inflation rate (annual rate).

firms' price levels, a shift in demand may simply lead in the long run to a permanently higher price level of the firm but not to a permanent increase in how much it raises its price level each period. Again, this story may break down at very high demand periods, but the Fed rarely allows this to happen.

5 Conclusion

As noted in Section 2, it will be interesting to see in future work if new-Keynesian Phillips curves can be developed that when embedded in a NK model yield significant FIML coefficient estimates of the expected future inflation variable under the assumption of rational expectations. These models need not be linear if the FT method is used. The results so far, however, are not encouraging, which argues for examining models not based on the assumption of rational expectations.

An alternative to the NKPC specification is the triangle model, but the results in Section 3 reject the dynamics implied by this model. When the price levels lagged once and lagged twice are added to the equation—equation (10)—they are highly significant. Also the outside-sample RMSEs are much better with the two variables added.

The price-wage model in Section 4 makes two basic changes from the triangle model: it adds a wage equation, and the specification is in level terms. In terms of outside sample RMSEs, this model is better than both equation (10) and equation (10) with the two lagged price levels added. The results in this section show that the unemployment rate dominates other measures of demand and that the price of imports dominates other measures of cost shocks. There is also some evidence of nonlinear effects of the unemployment rate on the price level at low values of the unemployment rate.

Finally, a comment on equations (14) (or (17)) and (15) and the Lucas (1976) critique. These two equations are considered to be estimated decision equations of the firm sector. They are obviously only approximations, but they do not suffer from the Lucas critique if expectations are not rational. To the extent that expectations of future prices and wages affect current decisions, agents are assumed to form these expectations on the basis of past values, where the parameters multiplying these values are constant. Expectations are backward looking in this sense. The parameters in the expectation equations are assumed not to depend on the parameters in the model: expectations are not model consistent (rational). In the specification of a decision equation to estimate, if expected future values influence the current decision (which is usually the case), these values are substituted out by replacing them with the lagged values upon which they are assumed to depend. The decision equation is then estimated with these values included. If the parameters in the expectation equations are constant, then this substitution does not introduce non constant parameters in the decision equation. It is usually not the case that one can back out from the estimated decision equation the parameters of the expectations equations, but there is usually no need to do so. Under the above assumptions, expectations have been properly accounted for in the decision equation.

This treatment of expectations does not mean that policy changes have no effect on behavior. Say that the Fed announces a new policy regime, one in which it is going to weight inflation more than it has done in the past. If expectations are rational, this announcement will immediately affect them and thus immediately affect current decisions. Current decisions can be affected even before the Fed has actually changed the interest rate. In the treatment here expectations and thus decisions will be affected only after the interest rate has been changed. For example, an interest rate change affects demand, which affects output, which affects unemployment, which affects prices and wages. In this treatment decisions respond to policy changes, but only in response to actual changes in the policy variables. Announcements of new policy rules and the like have no effect on decisions because agents don't know the model and thus don't use it to form their expectations.

Data Appendix

Most of the data used in this paper are part of the US model in Fair (2004) and are available on the website mentioned in the introductory footnote. These data are listed in Table 7. The other variables used are the following. $\log YTREND$ was

Table 7
The Variables Used

Paper	US Model—Fair (2004)
$p = \log PF$	PF = Private nonfarm price deflator.
$w = \log WF$	WF = Average hourly earnings excluding overtime of workers in the firm sector. Includes supplements to wages and salaries except employer contributions for social insurance.
$d = \log(1 + D5G)$	$D5G$ = Employer social security tax rate.
$pm = \log PIM$	PIM = Import price deflator.
$u = UR$	UR = Civilian unemployment rate.
$Y = Y$	Y = Output of the firm sector.
$YS = YS$	YS = Potential output of the firm sector.
$\lambda = LAM$	LAM = Amount of output capable of being produced per worker hour.

obtained from a regression of $\log Y$ on a constant and t for the 1952:1–2006:2 period. $YTREND$ is then $\exp(\log YTREND)$. $\log YSS$ was obtained from a peak-to-peak interpolation of $\log Y$, where the peaks are 1953:2, 1962:2, 1966:2, 1973:2, 1989:2, 2000:2, and 2006:1. YSS is then $\exp(\log YSS)$. $pcrudm$ is the log of the crude materials producer price index, which is U.S. Bureau of Labor Statistics (BLS) series id WPUSOP1000, and $pcrudo$ is the log of the crude petroleum producer price index, which is BLS series id WPU0561. $lshare$ is the log of the labor share, which is also based on BLS data. The labor share is compensation divided by nominal output in the nonfarm business sector, where compensation equals compensation per hour (COMPNFB) times hours of all persons (HOANBS) and nominal output equals output (OUTNFB) times the implicit price deflator (IPDNBS).

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