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Is forward-looking inflation targeting destabilizing? The role of policy's response to current output under endogenous investment

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In sticky price models with endogenous investment, virtually all monetary policy rules that set a nominal interest rate in response solely to future inflation induce real indeterminacy of equilibrium. Applying the Samuelson-Farebrother conditions, we obtain a necessary and sufficient condition for local real determinacy, which reveals that increasing price stickiness or letting policy respond also to current output may help ensure a unique equilibrium. We find that the first channel by itself has a quantitatively negligible effect and almost all strict inflation-targeting rules lead to indeterminacy, whether with higher price stickiness or overall stickiness by incorporating firm-specific capital, sticky wages, or both. The effect of the second avenue depends on labor supply elasticity and stickiness. With high labor supply elasticity and price stickiness, indeterminacy is much less likely to occur as policy also responds to output. With estimated labor supply elasticity or empirically reasonable price stickiness, policy's response to output helps little in ensuring determinacy; even incorporating firm-specific capital makes only a marginal improvement. Incorporating sticky wages, on the other hand, greatly enhances the role of policy's response to output in ensuring determinacy. With both sticky wages and firm-specific capital incorporated, even a tiny response of policy to current output can render equilibrium determinate for a wide range of response of policy to future inflation.

JEL classification: E12, E31, E52.

Keywords: Forward-looking inflation targeting; Current output; Sticky prices; Sticky wages; Firm-specific capital; Endogenous investment; Indeterminacy; Samuelson-Farebrother conditions.

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1. Introduction

In recent years, explicit inflation-forecast targeting, which takes the form of forward-looking interest rate feedback rules that set a short-term nominal interest rate in response to the forecasted value of future inflation, has become a popular framework for conducting monetary policy at central banks around the world. This practice began in New Zealand in 1990 and, within a decade, spread to other industrial countries.[1] Since 1997, a number of emerging market and transition countries have adopted such a policy.[2] Many more are moving toward this direction.[3] In a sense, forward-looking inflation targeting has become a defining characteristic of monetary policymaking worldwide.

There are many reasons for adopting an inflation-targeting rule in monetary policymaking.[4] That it is the expected future inflation that needs to be targeted has been emphasized by both policymakers and researchers. Among other advantages, targeting the expected future inflation is essential for tackling the observed delay in the response of inflation and output to monetary policy actions, for anchoring private sector's inflation expectations, and for incorporating a wide variety of up-to-date information in policymaking. In this sense, forward-looking inflation targeting can be justified on the ground of both policy effectiveness and central bank accountability and credibility.[5] This is why many researchers recommend that central banks commit to forward-looking inflation targeting rules and why many policymakers follow suit.[6]

There is yet a pitfall of forward-looking inflation targeting: it is prone to real indeterminacy of equilibrium and therefore welfare-reducing fluctuations unrelated to economic fundamentals.[7]

[1]Among these countries are Canada, the United Kingdom, Australia, Finland, Sweden, Spain, Switzerland, Iceland, and Norway, all of which publish their inflation forecasts (Finland and Spain adopted the euro in 1999). The United States, the European Central Bank, and Japan are usually viewed as having followed some implicit inflation-forecast targeting procedures, where more explicit targeting has also received consideration recently. Leiderman and Svensson, eds. (1995), Bernanke and Mishkin (1997), and Bernanke, Laubach, Mishkin, and Posen (1999) provide some background information and analysis.

[2]These include Israel (now an industrial country), the Czech Republic, Korea, Poland, Brazil, Chile, Colombia, South Africa, Thailand, Mexico, Turkey, Hungary, Peru, and Philippines, all of which but Mexico publish their inflation forecasts. See Schaechter, Stone, and Zelmer (2000), Roger and Stone (2005), and Jonas and Mishkin (2005) for more details.

[3]Estimated forward-looking interest rate feedback rules explain well the behavior of interest rates in the United States, Germany, and Japan in the 1980s and 1990s. See, among others, Chinn and Dooley (1997), Clarida and Gertler (1997), Orphanides (1998), Clarida, Galí, and Gertler (1998), Orphanides and Williams (2003), and Carare and Stone (2005).

[4]See, among many others, Haldane, eds. (1995), Blinder, Goodhart, Hildebrand, Lipton, and Wyplosz (2001), Svensson (2001), Fracasso, Genberg, and Wyplosz (2003), and Leeper (2003).

[5]See, among others, Svensson (1997), Bernanke and Woodford (1997), Batini and Haldane (1999), Levin, Wieland, and Williams (2003), Orphanides and Williams (2003), and Bernanke and Woodford, eds. (2005).

[6]See, among others, Svensson (1997, 1999), Svensson and Woodford (1999), and Goodhart (2000).

[7]See, among others, Bernanke and Woodford (1997), Clarida et al. (1998, 2000), Woodford (2000, 2003), and

Some researchers argue that inflation-forecast targeting central banks can avoid such policy-induced instability by appealing to some flexible rules under which a nominal interest rate responds not only to expected future inflation but also to other endogenous variables such as current output.[8] Their studies, however, have all abstracted from investment activity. Carlstrom and Fuerst (2005) argue that the indeterminacy problem is more severe when investment activity is taken into account.[9] They show that essentially all strict inflation-forecast targeting rules induce real indeterminacy of equilibrium in a sticky price model with endogenous investment, and they suggest that letting the interest rate respond also to output would not help much in avoiding such indeterminacy.

The present paper takes up this issue. We start by considering a standard model of sticky prices with endogenous investment and show that virtually all monetary policy rules that set a nominal interest rate in response solely to future inflation are subject to real indeterminacy of equilibrium. We apply the celebrated Samuelson-Farebrother conditions for handling high order systems of linear difference equations to obtain the necessary and sufficient condition for local real determinacy for a baseline case of our model economy. This condition reveals that increasing the degree of price stickiness or letting policy respond also to current output may help ensure a unique equilibrium.[10]

We find that the first channel in itself has a quantitatively negligible effect. Once again, almost all strict forward-looking inflation-targeting rules that respond solely to future inflation lead to real indeterminacy of equilibrium, whether with higher price stickiness, or with higher overall stickiness through incorporating into the baseline model firm-specific capital, sticky wages, or both.

We find that the effect of the second avenue depends on the elasticity of labor supply and the degree of stickiness in the model. With high labor supply elasticity and price stickiness, such as those assumed in Carlstrom and Fuerst (2005), indeterminacy is much less likely to occur if policy responds also to current output. With estimated labor supply elasticity or empirically reasonable price stickiness, however, policy's response to current output helps little in ensuring determinacy in the baseline model. Even incorporating firm-specific capital can only make a marginal improvement.

Carlstrom and Fuerst (2000).

[8]See, for example, Clarida et al. (1998, 2000), Christiano and Gust (1999), Rotemberg and Woodford (1999), Woodford (1999, 2003), and Levin et al. (2003). In practice, almost all inflation targeting central banks follow such a flexible inflation-forecast targeting procedure.

[9]See Huang and Meng (2006) for a related study and Dupor (2001) for a continuous-time analysis.

[10]Under current-inflation targeting with endogenous capital accumulation, Sveen and Weinke (2005) find that indeterminacy is more likely to occur with a greater degree of stickiness while Sveen and Weinke (2005) and Benhabib and Eusepi (2005) show that letting policy respond also to current output can help avoid such indeterminacy.

Incorporating sticky wages, on the other hand, significantly enhances the role of policy's response to current output in ensuring determinacy of equilibrium. When both sticky wages and firm-specific capital are incorporated into the baseline model, even a tiny response of policy to current output can render equilibrium determinate for a wide range of response of policy to future inflation. This last result is important in light of the recent finding by Schmitt-Grohé and Uribe (2006) which suggests that interest rate policy rules that feature a large response to output can be a potential source of significant inefficiencies.

The remainder of the paper is organized as follows. Section 2 sets up a sticky price model with endogenous investment and a capital rental market, presents a necessary and sufficient condition for local real determinacy, describes model calibration and reports numerical results. Sections 3, 4, and 5 incorporate firm-specific capital, sticky wages, and both firm-specific capital and sticky wages, respectively, into the baseline model and describes the results. Section 6 concludes. The Appendix restates the Samuelson-Farebrother conditions and proves our proposition and corollary.

2. A baseline model with staggered prices

The model features a continuum of firms each of which produces a differentiated good indexed by $f \in [0,1]$. At each date t, a representative distributor combines all differentiated goods $\{Y_t(f)\}_{f\in[0,1]}$ into a composite good $Y_t = \left[\int_0^1 Y_t(f)^{(\epsilon_y-1)/\epsilon_y} df\right]^{\epsilon_y/(\epsilon_y-1)}$, where $\epsilon_y \in (1,\infty)$ is the elasticity of substitution between the individual goods. The distributor takes the prices $\{P_t(f)\}_{f\in[0,1]}$ of the differentiated goods as given and chooses the bundle of the individual goods to minimize the cost of fabricating a given quantity of the composite good, which it sells to a representative household at the unit fabricating cost $P_t = \left[\int_0^1 P_t(f)^{1-\epsilon_y} df\right]^{1/(1-\epsilon_y)}$, which is also the price level. The resultant demand for a type f good is

$$Y_t(f) = \left[\frac{P_t(f)}{P_t}\right]^{-\epsilon_y} Y_t. \tag{1}$$

Quantity of the composite good purchased by the household (Y_t), which corresponds to real output or real GDP, can be either consumed (C_t) or invested (I_t) to accumulate capital stock that the household rents to firms in a competitive capital market.

The production of a type f good uses capital and labor and a constant-return-to-scale technology

$$Y_t(f) = K_t(f)^{\alpha} N_t(f)^{1-\alpha}. \tag{2}$$

Firms are price takers in factor markets but monopolistic competitors in goods markets. With markup pricing, factor payments are distorted and α and $1-\alpha$ determine respectively the share of payments to capital and labor in value-added production cost rather than in gross output. Specifically, cost minimization by firms implies that nominal wage rate W_t and nominal marginal cost MC_t are linked as follows:

$$W_t = (1-\alpha)\left(\frac{N_t}{K_t}\right)^{-\alpha} MC_t, \tag{3}$$

and nominal capital rental rate is linked to nominal wage rate by $R_t^k = [\alpha/(1-\alpha)](N_t/K_t)W_t$, where $N_t = \int_0^1 N_t(f)df$ and $K_t = \int_0^1 K_t(f)df$, and we have used the fact that labor to capital ratio and marginal cost are identical across firms in equilibrium. We shall use lowercases w_t and mc_t to denote real wage and real marginal cost, respectively.

Firms set prices in a staggered fashion à là Calvo (1983). At each date, each firm receives a random signal with a constant probability θ_p which forbids it to reset price. The random signal is identically and independently distributed across firms and time. With the large number of firms which validates the law of large numbers, at each point in time there is fraction $(1-\theta_p)$ of randomly selected firms that can reset prices. At date t, if a firm f can reset its price, it chooses $P_t^*(f)$ to maximize the expected present value of its profits

$$\sum_{s=t}^{\infty} \theta_p^{s-t} R_{t,s-1}^{-1} \left[P_t(f) - MC_s\right] \left[\frac{P_t(f)}{P_s}\right]^{-\epsilon_y} Y_s,$$

where $R_{t,t-1} \equiv 1$ and $R_{t,s-1} = \prod_{\tau=t}^{s-1} R_\tau$ denotes a cumulative rate of return from rolling over a position on the nominal bond from t to $s > t$. The optimal pricing decision is

$$P_t^*(f) = \frac{\epsilon_y}{\epsilon_y - 1} \frac{\sum_{s=t}^{\infty} \theta_p^{s-t} R_{t,s-1}^{-1} P_s^{\epsilon_y} Y_s MC_s}{\sum_{s=t}^{\infty} \theta_p^{s-t} R_{t,s-1}^{-1} P_s^{\epsilon_y} Y_s}, \tag{4}$$

The optimal price is a markup over a weighted average of marginal costs in the current and future periods during which the firm is expected not to have another chance to reset price.

The representative household's lifetime utility is given by

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \psi \, \frac{N_t^{1+\eta}}{1+\eta} \right),$$

where $\beta \in (0,1)$ is a subjective discount factor, N_t denotes the household's labor in period t, and σ and η denote its relative risk aversion in consumption and in labor hours, respectively. The household's budget constraint in period t requires that its expenditures on consumption, investment, and asset accumulation do not exceed its total income earned in the same period,

$$P_t(C_t + I_t) + B_t - R_{t-1}B_{t-1} \leq R_t^k K_t + W_t N_t + \Pi_t,$$

where B_{t-1} is the household's holding of a one-period nominal bond acquired in period $t-1$, R_{t-1} is the gross nominal rate of return on holding the bond from $t-1$ to t, and Π_t is the household's claim to firms' profits in period t. The household maximizes its utility subject to the budget constraint, a convex capital adjustment cost

$$\frac{I_t}{K_t} = I\left(\frac{K_{t+1}}{K_t}\right), \tag{5}$$

where $\delta \equiv I(1) \in [0,1]$ is the steady-state capital depreciation rate, $I'(1) = 1$, and $\epsilon_q \equiv I''(1)$ denotes the steady-state elasticity of investment to capital ratio with respect to Tobin's q, and a borrowing constraint $B_t \geq -B$, for some large positive number B, which serves to prevent the household from playing Ponzi schemes without bound. The household takes its initial capital stock K_0, bond holding B_{-1}, and all prices, capital rental rate, and wage rate as given in solving the utility-maximization problem.[11] The optimality conditions include an intertemporal consumption Euler equation

$$\frac{P_t R_t}{P_{t+1}} = \frac{1}{\beta}\left(\frac{C_{t+1}}{C_t}\right)^{\sigma}, \tag{6}$$

an intratemporal consumption-labor relation

$$w_t = \psi C_t^{\sigma} N_t^{\eta}, \tag{7}$$

[11] The assumption that at any date t the stock of capital K_t is predetermined implies that capital available for firms to rent at any given date is accumulated by the household during the previous period. In other words, additional capital resulting from the household's investment decision becomes productive with a one-period lag.

and a capital Euler equation

$$\frac{P_t R_t}{P_{t+1}} \frac{\partial I_t}{\partial K_{t+1}} = \frac{\alpha}{1-\alpha} \frac{N_{t+1}}{K_{t+1}} w_{t+1} - \frac{\partial I_{t+1}}{\partial K_{t+1}}. \tag{8}$$

A monetary authority is able to commit to a inflation-forecast targeting rule under which the nominal interest rate responds to the inflation forecast and current output,

$$R_t = R^{ss} \left(\frac{P_{t+1}}{P_t} \right)^{\tau_\pi} \left(\frac{Y_t}{Y^{ss}} \right)^{\tau_y}, \tag{9}$$

where R^{ss} and Y^{ss} denote respectively the steady-state values of the nominal interest rate and real output, and $\tau_\pi \geq 0$ and $\tau_y \geq 0$ measure respectively the degree of responsiveness of the nominal interest rate to the deviation of the expected future inflation from an inflation target (which is set to zero) and output around the steady state. With a zero steady-state inflation rate, we have $R^{ss} = 1/\beta$, as implied by the steady-state version of (6).

Equations (1)-(9), and those defining the composite good and price level, together with factor market clearing conditions, $N_t = \int_0^1 N_t(f)df$ and $K_t = \int_0^1 K_t(f)df$, and the market clearing condition for the composite good, $Y_t = C_t + I_t$, characterize an equilibrium.

2.1. Some log-linearized equilibrium conditions

For local determinacy analysis, we examine a log-linearized system of equilibrium conditions around a steady state with zero inflation. Throughout the rest of the paper, a variable with a hat denotes the percentage deviation of the variable in level from its steady-state value. Note that, with a constant steady-state price level, $\widehat{\pi}_{p,t} \equiv \log(P_t/P_{t-1})$ is both the actual period-t price inflation and the percentage deviation of the rate of price inflation in period t from its steady-state value.

The log-linearized versions of the consumption Euler equation (6), the policy rule (9), the aggregated version of the production function (2), the good market clearing condition, the factor market relation (3), and the capital Euler equation (8) are, respectively,

$$\widehat{R}_t - \widehat{\pi}_{p,t+1} = \sigma \left(\widehat{C}_{t+1} - \widehat{C}_t \right), \tag{10}$$

$$\widehat{R}_t = \tau_\pi \widehat{\pi}_{p,t+1} + \tau_y \widehat{Y}_t, \tag{11}$$

$$\widehat{Y}_t = \alpha \widehat{K}_t + (1-\alpha)\widehat{N}_t, \tag{12}$$

$$\widehat{Y}_t = (1-\delta k_y)\widehat{C}_t + k_y \left[\widehat{K}_{t+1} - (1-\delta)\widehat{K}_t\right], \tag{13}$$

$$\widehat{w}_t = \widehat{mc}_t - \alpha\left(\widehat{N}_t - \widehat{K}_t\right), \tag{14}$$

$$\tilde{\delta}\left(\widehat{w}_{t+1} + \widehat{N}_{t+1} - \widehat{K}_{t+1}\right) - \sigma\left(\widehat{C}_{t+1} - \widehat{C}_t\right) = \epsilon_q\left[\left(\widehat{K}_{t+1} - \widehat{K}_t\right) - \beta\left(\widehat{K}_{t+2} - \widehat{K}_{t+1}\right)\right], \tag{15}$$

where $\tilde{\delta} \equiv 1 - \beta(1-\delta)$ and $k_y \equiv [(\epsilon_y - 1)\alpha\beta]/(\epsilon_y \tilde{\delta})$, and we have used (10) in rewriting (15). Conditions (10)-(14) will stay invariant to all modifications to the baseline model that we will make in the subsequent sections. Note that in the case with no capital adjustment cost, the right-hand-side of (15) reduces to 0.

The log-linearized version of (7) takes the following form:

$$\widehat{w}_t = \sigma\widehat{C}_t + \eta\widehat{N}_t. \tag{16}$$

This condition will be replaced with a wage inflation equation in the subsequent sections where we incorporate staggered wage-setting into the baseline model.

Approximating and combining the price-setting equation (4) and the equation defining the price level, we can derive a log-linearized New Phillips curve

$$\widehat{\pi}_{p,t} = \beta\widehat{\pi}_{p,t+1} + \lambda_p \widehat{mc}_t, \tag{17}$$

where

$$\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}.$$

While the price inflation equation will all take the form in (17), the coefficient λ_p in front of real marginal cost will need to be modified when we consider firm-specific capital.

2.2. An analytical result

We first consider a version of our model that is essentially the baseline model of Carlstrom and Fuerst (2005). This is a case with labor indivisibility and no capital adjustment cost. We present here a necessary and sufficient condition for local real determinacy for this version of the model with complete depreciation of capital. Our analytical result is summarized in the following proposition.

Proposition 1. *The necessary and sufficient condition for local real determinacy for the case with labor indivisibility and no capital adjustment cost and complete depreciation of capital is*

$$-\frac{(1-\beta)\mathcal{T}_y}{(1-\alpha)(b-1)} < \mathcal{T}_\pi < \min\left\{\frac{(1+\beta)\mathcal{T}_y}{(1+\alpha)(1+b)} + \frac{2(1+\frac{1}{\beta})}{1+\alpha},\ U,\ \frac{3\mathcal{T}_y + \frac{3b}{\beta} - \frac{1}{\beta} - 1 - b}{3\alpha b - 1}\right\} \tag{18}$$

where

$$b \equiv \frac{\epsilon_y}{(\epsilon_y - 1)\alpha\beta}, \quad \mathcal{T}_y \equiv \left(1 - \alpha + \frac{b-1}{\sigma}\right)\frac{\tau_y}{\beta}, \quad \mathcal{T}_\pi \equiv \frac{\lambda_p(\tau_\pi - 1)}{\beta},$$

$$\Delta \equiv \left\{\mathcal{T}_y - \left[\alpha(b-1)^2 + (1-\alpha)\left(\frac{1}{\beta} - 1\right)b\right]\right\}^2 + 4\mathcal{T}_y(1-\beta\alpha)b\left(\alpha b - 1 + \frac{1-\alpha}{\beta}\right),$$

$$U \equiv \frac{(2\alpha b - 1)\mathcal{T}_y + \alpha(b-1)^2 + (\frac{1}{\beta} - 1)b(2\alpha b - 1 - \alpha) - \sqrt{\Delta}}{2\alpha b(\alpha b - 1)}.$$

Otherwise, there is a continuum of equilibria.

Carlstrom and Fuerst (2005) show that a strict forward-looking inflation-targeting rule that sets a nominal interest rate in response solely to expected future inflation, that is, setting $\tau_y = 0$ in (9), renders equilibrium almost always indeterminate in their baseline model. Proposition 1 serves to illustrate two points among other things. First, an increase in the degree of price rigidity tends to help remedy the indeterminacy problem. Second, an increase in the degree of policy's response to output can enlarge the determinacy region. The first point is rather transparent, as in the determinacy condition characterized by (18), the price stickiness parameter θ_p affects only $\mathcal{T}_\pi$ and the relationship is negative — $\mathcal{T}_\pi$ is proportional to λ_p which decreases with θ_p. The following corollary helps make the second point more transparent.

Corollary 1. *The lower bound in* (18) *is negative and strictly decreasing in* τ_y *and each of the three upper bounds in* (18) *is strictly positive and strictly increasing in* τ_y*.*

These implications serve as a guidance for our subsequent analysis. With these points in mind, we turn now to derive numerical results for the calibrated model.

2.3. Model calibration and numerical results

We can derive from the log-linearized equilibrium conditions (10)-(17) a self-closed system of four first-order linear difference equations. To begin, first substitute (11) and (12) into (10) to obtain

$$\sigma\widehat{C}_{t+1} - (\tau_\pi - 1)\widehat{\pi}_{p,t+1} = \sigma\widehat{C}_t + \tau_y(1-\alpha)\widehat{N}_t + \tau_y\alpha\widehat{K}_t. \tag{19}$$

Second, substitute (12) into (13) to get

$$k_y\widehat{K}_{t+1} = (\delta k_y - 1)\widehat{C}_t + (1-\alpha)\widehat{N}_t + [(1-\delta)k_y + \alpha]\widehat{K}_t. \tag{20}$$

Next, substitute (14) and (16) into (17) to get

$$\beta\widehat{\pi}_{p,t+1} = -\lambda_p\sigma\widehat{C}_t + \widehat{\pi}_{p,t} - \lambda_p(\eta+\alpha)\widehat{N}_t + \lambda_p\alpha\widehat{K}_t. \tag{21}$$

Finally, rolling (16) and (20) one period forward and substituting both of them into (15), and manipulating, we obtain

$$\gamma_c\widehat{C}_{t+1} + \gamma_n\widehat{N}_{t+1} + \gamma_k\widehat{K}_{t+1} = -\sigma\widehat{C}_t - \epsilon_q\widehat{K}_t, \tag{22}$$

where

$$\gamma_c \equiv (\tilde{\delta} - 1)\sigma + \epsilon_q\beta\left(\delta - \frac{1}{k_y}\right),$$

$$\gamma_n \equiv \tilde{\delta}(\eta + 1) + \epsilon_q\beta\left(\frac{1-\alpha}{k_y}\right),$$

$$\gamma_k \equiv -\tilde{\delta} + \epsilon_q\beta\left(\frac{\alpha}{k_y} - \delta\right) - \epsilon_q.$$

This is a system of four first-order linear difference equations in three jump variables, $\widehat{C}_t$, $\widehat{\pi}_{p,t}$, and $\widehat{N}_t$, and one predetermined variable, $\widehat{K}_t$. Thus determinacy requires three explosive roots and one stable root.

For our baseline calibration, we set α to 0.33 so that the share of payment to capital in value-added productive factors is equal to one third, as in the National Income and Product Account. Given that one period in our model corresponds to one quarter of a year, we set $\beta = 0.99$ to be consistent with a steady-state annualized real interest rate of 4 percent, and we set δ to 0.02 to match the steady-state annual capital depreciation rate of 8 percent. These are standard parameter values used in the literature. While some studies in the literature suggest that σ can be as low as 0 or as high as 30, the general consensus is that it lies between 1 and 10 (e.g., Kocherlakota, 1996; Vissing-Jorgensen, 2002). Our results are quantitatively invariant to the choice of σ in its empirically reasonable range. We therefore fix the value of σ at 2.

Our baseline value of η is 10, corresponding to an intertemporal hours-worked elasticity of 10%, which lies in the middle of the empirical estimates reported in Pencavel (1986), Altonji (1986), Ball (1990), and Card (1994) based on micro data, while we examine the cases with $\eta = 5$ and 20 as well, which roughly covers both the range of these empirical estimates and the values used in many studies (e.g., Ball and Romer, 1990; Reis, 2006). As for analytical convenience many papers in determinacy analysis assumes $\eta = 0$ (e.g., Carlstrom and Fuerst, 2005; Benhabib and Eusepi, 2005), we also examine our results for the this case, as well as the case with $\eta = 1$, as a unitary labor supply elasticity is sometimes assumed as well.

We set our baseline value of θ_p to 0.33, so that the duration of a newly set price for a given firm (as well as the average frequency of price adjustment and the cross-sectional average age and average lifetime of posted prices) is about four and half months, which lies somewhat near the upper end of the recent empirical estimates by Bils, Klenow, and Kryvtsov (2003), Bils and Klenow (2004), and Klenow and Krystov (2005) based on micro data. We also examine our results for the case with $\theta_p = 0.25$, which corresponds roughly to the lower end of these empirical estimates and is used in some papers in determinacy analysis (e.g., Weder, 2006), as well as the case with $\theta_p = 0.57$, which is in line with the values used by Carlstrom and Fuerst (2005) and others.

We consider a value of ϵ_y equal to 11, as in many studies featuring monopolistic competition, such as Chari, Kehoe, and McGrattan (2000) and Sveen and Weinke (2005, 2006). We also examine our results for the case with $\epsilon_y = 4$, which is in line with the values used by Erceg, Henderson, and Levin (2000) and Benhabib and Eusepi (2005), among others. The values used in many other papers fall in between these two cases (e.g., Ball and Romer, 1990; Reis, 2006). These values cover the range of the empirical estimates reported in Domowitz, Hubbard, and Petersen (1986), Shapiro (1987), Basu (1996), Basu and Kimball (1997), Basu and Fernald (1994, 1995, 1997), Rotemberg and Woodford (1997), and Linnemann (1999).

We consider $\epsilon_q = 3$ for the case with capital adjustment cost, as in Woodford (2003) and Sveen and Weinke (2005, 2006), as well as $\epsilon_q = 0$ for the case that abstracts from capital adjustment cost, as in many papers in determinacy analysis.

These calibrated parameter values are summarized in the upper panel of Table 1. With these values of the fundamental parameters at hand, we can start to search for ranges of the two policy parameters, τ_π and τ_y, that ensure a unique equilibrium.

It turns out that the determinacy region is characterized by an upper bound and a lower bound for policy's response to future inflation τ_π as a function of policy's response to current output τ_y, just as suggested by Proposition 1. Figures 1-15 display such upper and lower bounds—the horizontal axis measures τ_y and the vertical axis measures τ_π—for different models under the various parameter values (the models incorporating firm-specific capital, sticky wages, and both sticky wages and firm-specific capital, as well as the calibration of additional parameters for the models incorporating sticky wages are to be described below in detail). As is clear from these figures, if policy's response to output is muted, then varying the degree of stickiness in the model or other parameter values has a quantitatively negligible effect on the determinacy region. To be specific, if $\tau_y = 0$, then virtually no value of τ_π can render equilibrium determinate. Whether with higher price stickiness, or with higher overall stickiness through incorporating firm-specific capital, sticky wages, or both into the baseline model, the upper bound and the lower always intercept the vertical axis at essentially the same point.

While this robust failure highlights the potential importance of policy's response to output, the effect of this avenue depends on the elasticity of labor supply and the degree of price stickiness. As is apparent from the figures, the tension in most cases is on the upper bound, so we will focus our subsequent discussions on the upper bound as well. The first line (an infinite labor supply elasticity) of Figure 1 (the price stickiness parameter $\theta_p = 0.57$) corresponds to the labor supply elasticity and price stickiness used in Carlstrom and Fuerst (2005): here, the upper bound for τ_π increases fairly rapidly with τ_y and thus indeterminacy is much less likely to occur as policy's response to current output increases. With the calibrated labor supply elasticity (the fourth line in Figure 1), however, the upper bound for τ_π increases very slowly with τ_y and thus increasing policy's response to current output increases the determinacy region only marginally. As Figure 2 illustrates, when the price stickiness parameter θ_p takes on its calibrated value of 0.33, the role of policy's response to current output in helping ensure determinacy is much weakened even with high labor supply elasticity (the first two lines in Figure 2). With the empirically estimated values of labor supply elasticity (the last three lines in Figure 2), the upper bound for τ_π is almost always overlapped with the lower bound regardless of the value of τ_y and thus policy's response to current output helps very little in ensuring determinacy. With even lower but still empirically justifiable price stickiness (Figure 3), the results are even more pessimistic.

We therefore conclude that letting policy respond to current output helps little in ensuring determinacy in our calibrated baseline model with staggered prices and a capital rental market.

3. Incorporating firm-specific capital

This section abandons the assumption of a capital rental market made in the baseline model and assumes instead firm-specific capital, as in Sveen and Weinke (2005).

At each date, the representative distributor sells the composite good that it fabricates from the individually differentiated goods to the household and firms at the unit fabricating cost (which equals the price level of the economy). Thus it is assumed that the distributor cannot discriminate its selling price between the household and the firms, or across different firms. Quantity of the composite good purchased by the household is consumed entirely in the same period and the household faces a simple budget constraint

$$P_t C_t + B_t - R_{t-1} B_{t-1} \leq W_t N_t + \Pi_t,$$

while its optimal choice of consumption, bond, and labor still implies (6) and (7).

Quantity of the composite good $I_t(f)$ purchased by a firm f at date t is invested to accumulate its own capital stock from $K_t(f)$ at t to $K_{t+1}(f)$ at $t+1$ subject to a convex adjustment cost

$$\frac{I_t(f)}{K_t(f)} = I\left(\frac{K_{t+1}(f)}{K_t(f)}\right), \tag{23}$$

where, as before, $I(1) = \delta$, $I'(1) = 1$, and $I''(1) = \epsilon_q$. Thus, both the one period to build and the convex adjustment cost for capital occur at the individual firm level. As a consequence, nominal marginal cost and labor to capital ratio are firm-specific and are linked to the economy-wide nominal wage rate as follows:

$$W_t = (1-\alpha)\left[\frac{N_t(f)}{K_t(f)}\right]^{-\alpha} MC_t(f). \tag{24}$$

At any date t, a firm f's capital stock $K_t(f)$ is given. If the firm is a price adjuster at t, it chooses the sequence $\{P_s^*(f), K_{s+1}(f), N_s(f)\}_{s\geq t}$, taking the price level, wage index, and aggregate demand for the composite good in all corresponding periods, $\{P_s, W_s, Y_s\}_{s\geq t}$, as given, to maximize

$$\sum_{s=t}^{\infty} R_{t,s-1}^{-1}\left[P_s(f)Y_s(f) - W_s N_s(f) - P_s I_s(f)\right]$$

subject to (1), (2), (23), and

$$P_{s+1}(f) = \begin{cases} P_{s+1}^*(f) & \text{with probability } 1-\theta_p, \\ P_s(f) & \text{with probability } \theta_p. \end{cases} \tag{25}$$

If the firm cannot adjust its price at t, it solves the same problem while taking its own price at t, $P_t(f)$, as given as well. The resultant optimal pricing decision is

$$P_t^*(f) = \frac{\epsilon_y}{\epsilon_y - 1}\frac{\sum_{s=t}^{\infty}\theta_p^{s-t}R_{t,s-1}^{-1}P_s^{\epsilon_y}Y_s MC_s(f)}{\sum_{s=t}^{\infty}\theta_p^{s-t}R_{t,s-1}^{-1}P_s^{\epsilon_y}Y_s}, \tag{26}$$

Thus the optimal price is a markup over a weighted average of the firm-specific marginal costs in the current and future periods in which the firm is expected not to have another chance to reset its price. The firm-specific capital Euler equation is

$$\frac{P_t R_t}{P_{t+1}}\frac{\partial I_t(f)}{\partial K_{t+1}(f)} = \frac{\alpha}{1-\alpha}\frac{N_{t+1}(f)}{K_{t+1}(f)}w_{t+1} - \frac{\partial I_{t+1}(f)}{\partial K_{t+1}(f)}. \tag{27}$$

It can be shown through proper aggregation that equations (10)-(17) still approximates up to a first order the true equilibrium conditions, with the only modification being that λ_p in (17) is now approximated by

$$\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_y}.$$

For our local determinacy analysis, we can thus still analyze the system of the four first-order linear difference equations (19)-(22) with λ_p modified as above.

Figures 4-6 display the determinacy region for the model incorporating firm-specific capital under the various parameter values. As a comparison between these figures and Figures 1-3 reveals, although replacing a capital rental market with firm-specific capital enlarges the determinacy region in every case, the improvement is only marginal, especially for the cases with empirically reasonable price stickiness (Figures 5 and 6) and estimated labor supply elasticity (the last three lines in the figures). We thus conclude that incorporating firm-specific capital enhances only marginally the role of policy's response to current output in helping avoid indeterminacy.

4. Incorporating staggered wages

This section abandons the assumption of a homogenous labor skill and a competitive labor market made in the baseline model while maintaining that of a capital rental market. We assume there is a continuum of households, each endowed with a differentiated labor skill indexed by $h \in [0, 1]$, who set nominal wages for their labor services in a staggered fashion.

At each date t, all differentiated skills $\{N_t(h)\}_{h\in[0,1]}$ are aggregated into a composite skill $N_t = \left[\int_0^1 N_t(h)^{(\epsilon_n-1)/\epsilon_n} dh\right]^{\epsilon_n/(\epsilon_n-1)}$, where $\epsilon_n \in (1, \infty)$ is the elasticity of substitution between the differentiated skills. The aggregation activity is assumed to be perfectly competitive. The resultant demand for a type h skill is

$$N_t(h) = \left[\frac{W_t(h)}{W_t}\right]^{-\epsilon_n} N_t, \tag{28}$$

where the wage rate W_t for the composite skill and the wage rates $\{W_t(h)\}_{h\in[0,1]}$ for the differentiated skills are linked by $W_t = \left[\int_0^1 W_t(h)^{1-\epsilon_n} dh\right]^{1/(1-\epsilon_n)}$.

For each firm f, the labor input in the production function (2) is in terms of the composite labor, and (3) and (4) hold exactly as in the baseline model.

All households are price takers in good, bond, and capital rental markets and monopolistic competitors in the labor market, where they set nominal wages for their differentiated labor skills in a staggered fashion à là Calvo (1983). At each date, each household receives a random signal with a constant probability θ_w which forbids it to reset its nominal wage. The random signal is identically and independently distributed across households and time. With the large number of households which validates the law of large numbers, at each point in time there is fraction $(1-\theta_w)$ of randomly selected households that can reset wages.

At any date t, a household h's capital stock $K_t(h)$ and bond holding $B_{t-1}(h)$ are given. If the household is a wage setter at t, it chooses the sequence $\{W_s^*(h), C_s(h), K_{s+1}(h), B_s(h)\}_{s \geq t}$, taking the price level, wage index, rate of return on capital and bond, and aggregate demand for the composite good and labor in all corresponding periods, $\left\{P_s, W_s, R_s^k, R_{s-1}, Y_s, N_s\right\}_{s \geq t}$, as given, to maximize

$$\sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s(h)^{1-\sigma} - 1}{1-\sigma} - \psi \, \frac{N_s(h)^{1+\eta}}{1+\eta} \right],$$

subject to

$$P_s \left[C_s(h) + I_s(h)\right] + B_s(h) - R_{s-1} B_{s-1}(h) \leq R_s^k K_s(h) + W_s(h) N_s(h) + \Pi_s(h),$$

$$\frac{I_s(h)}{K_s(h)} = I \left(\frac{K_{s+1}(h)}{K_s(h)} \right), \tag{29}$$

a borrowing constraint $B_s(h) \geq -B$, for some large positive number B, the demand schedule for its labor skill (28), and

$$W_{s+1}(h) = \begin{cases} W_{s+1}^*(h) & \text{with probability } 1 - \theta_w, \\ W_s(h) & \text{with probability } \theta_w. \end{cases} \tag{30}$$

If the household cannot adjust its wage at t, it solves the same problem while taking its own wage at t, $W_t(h)$, as given as well.

As is standard in the literature on staggered wage-setting, we suppose that there are (implicit) financial arrangements that make it possible to insure each household against any idiosyncratic income risk that may arise from the asynchronized wage adjustments so that equilibrium consumption and investment are identical across households, although nominal wages and hours worked may differ (e.g., Rotemberg and Woodford, 1997; Erceg et al., 2000; Christiano, Eichenbaum, and Evans, 2005).[12] As such, (6) and (8) continue to hold for aggregate consumption and capital, just as in the baseline model with a homogenous labor skill and a competitive labor market, while (7)

[12] As Huang, Liu, and Phaneuf (2004) show, this assumption is made mainly for analytical convenience and an alternative interpretation of the model can produce identical equilibrium dynamics without requiring such implicit financial arrangements for the purpose of aggregation.

is replaced with the following optimal wage-setting decision

$$W_t^*(h) = \left[\frac{\psi \epsilon_n}{\epsilon_n - 1} \frac{\sum_{s=t}^{\infty} \theta_w^{s-t} R_{t,s-1}^{-1} W_s^{(\eta+1)\epsilon_n} N_s^{\eta+1} P_s C_s^{\sigma}}{\sum_{s=t}^{\infty} \theta_w^{s-t} R_{t,s-1}^{-1} W_s^{\epsilon_n} N_s} \right]^{\frac{1}{1+\eta\epsilon_n}}. \tag{31}$$

The log-linearized equilibrium conditions are given by (10)-(15) and (17), along with a wage inflation equation,

$$\widehat{\pi}_{w,t} = \beta \widehat{\pi}_{w,t+1} + \lambda_w \left(\sigma \widehat{C}_t + \eta \widehat{N}_t - \widehat{w}_t \right), \tag{32}$$

which is obtained by approximating and combining the wage-setting equation (31) and the equation defining the wage index, where

$$\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w} \frac{1}{1+\eta\epsilon_n}.$$

Note that $\widehat{\pi}_{w,t} \equiv \log(W_t/W_{t-1})$ is both the actual period-t wage inflation and the percentage deviation of the rate of wage inflation in period t from its steady-state value.

For our local determinacy analysis, we can derive from these log-linearized equilibrium conditions a self-closed system of six first-order linear difference equations. The first two are the same as (19) and (20). The next two are modified versions of (21) and (22), given by

$$\beta \widehat{\pi}_{p,t+1} = \widehat{\pi}_{p,t} - \lambda_p \alpha \widehat{N}_t + \lambda_p \alpha \widehat{K}_t - \lambda_p \widehat{w}_t, \tag{33}$$

$$(\gamma_c - \tilde{\delta}\sigma)\widehat{C}_{t+1} + (\gamma_n - \tilde{\delta}\eta)\widehat{N}_{t+1} + \gamma_k \widehat{K}_{t+1} + \tilde{\delta}\widehat{w}_{t+1} = -\sigma \widehat{C}_t - \epsilon_q \widehat{K}_t. \tag{34}$$

The last two are obtained by using the identity $\widehat{w}_t = \widehat{w}_{t-1} + \widehat{\pi}_{w,t} - \widehat{\pi}_{p,t}$ to rewrite (32) as

$$\beta \widehat{\pi}_{p,t+1} + \beta \widehat{w}_{t+1} = -\lambda_w \sigma \widehat{C}_t + \widehat{\pi}_{p,t} - \lambda_w \eta \widehat{N}_t + (1 + \lambda_w + \beta)\widehat{w}_t - \widehat{z}_t, \tag{35}$$

$$\widehat{z}_{t+1} = \widehat{w}_t, \tag{36}$$

where this last one is a definition equation. This is a system of six first-order linear difference equations in four jump variables, $\widehat{C}_t$, $\widehat{\pi}_{p,t}$, $\widehat{N}_t$, and $\widehat{w}_t$, and two predetermined variable, $\widehat{K}_t$ and $\widehat{z}_t$. Thus determinacy requires four explosive roots and two stable root. Note that λ_p here is as specified in Section 2.

With staggered wages, we need to assign values for the two additional parameters, the elasticity of substitution of differentiated skills, ϵ_n, and the probability of non-adjustment in wage, θ_w. We set ϵ_n to 4, the mid point of the empirical estimates by Griffin (1992, 1996) based on micro data, which is from 2 to 6. Our results are in fact quantitatively invariant to the choice of ϵ_n in its empirically reasonable range. We set θ_w to 0.75, in the light of the empirical evidence in Taylor (1999), Smets and Wouters (2003), Levin, Onatski, Williams, and Williams (2005), and Christiano et al. (2005). These values are roughly in line with those used in the literature on staggered wage-setting (e.g., Erceg et al., 2000; Sveen and Weinke, 2006). These two parameter values are reported in the lower panel of Table 1.

Figures 7-9 display the determinacy region for the model incorporating sticky wages under the various parameter values. Notice the contrast between these figures and Figures 1-3, especially for the cases with a less than unit η (the last four lines in the figures), where the role of policy's response to current output in ensuring determinacy of equilibrium is significantly enhanced by the presence of sticky wages. In almost all cases, the upper bound for τ_π increases fairly quickly with τ_y and thus indeterminacy is much less likely to occur as policy's response to current output increases. For moderate values of τ_y, the lower bound for τ_π can go much below 1, especially for the cases with the estimated values of η (the last three lines in the figures), implying that even very passive response of policy to future inflation can render equilibrium determinate.

It is also worth noting that, with sticky wages incorporated, the determinacy region is much less sensitive to the magnitude of η. Nevertheless, for empirically reasonable θ_p (Figures 8 and 9), it calls for a moderately large response of policy to current output in order to ensure determinacy for a large range of response of policy to future inflation.

5. Incorporating both staggered wages and firm-specific capital

This section abandons both the assumption of a capital rental market and the assumption of a homogenous labor skill with a competitive labor market, and incorporates firm-specific capital and staggered wage-setting into the baseline model. The details are already spelled out in Sections 3 and 4 above. Local determinacy analysis involves examining the system of (19) and (20), and

(33)-(36), while λ_p here is as specified in Section 3.

Figures 10-12 display the determinacy region for the model incorporating both sticky wages and firm-specific capital under the various parameter values. Notice the sharp contrast of these figures with Figures 1-3, Figures 4-6, and even Figures 7-9. In all cases incorporating both sticky wages and firm-specific capital enlarges the determinacy region drastically and even a tiny response of policy to current output can render equilibrium determinate for a wide range of response of policy to future inflation even under the baseline calibration.

To help make that contrast and this last point more transparent, we plot the determinacy regions for the four models with baseline calibration in the same figure, with a finer scale across a smaller horizon for the horizontal axis that measures policy's response to current output τ_y. As Figures 13-15 illustrate, the determinacy region for the model featuring sticky prices (SP), sticky wages (SW), and firm-specific capital (FSC) is significantly wider than the determinacy region for any of the other three models and for all the three values of the price stickiness parameter θ_p.

For $\theta_p = 0.57$ (Figure 13), most values of τ_π that satisfy the Taylor principle (i.e., $\tau_\pi > 1$) can ensure determinacy in the SP&SW&FSC model even for τ_y as small as 0.05, while any τ_π greater than 5 would induce indeterminacy in the SP&SW model if τ_y is no greater than 0.05, and virtually no value of τ_π can ensure determinacy in the SP or the SP&FSC model even for τ_y as big as 0.4. When θ_p takes on its baseline value of 0.33 (Figure 14), all τ_π between 0.98 and 13.8 still ensure determinacy for τ_y as small as 0.1, while any τ_π greater than 3.4 would lead to indeterminacy in the SP&SW model if τ_y is no greater than 0.1. Even for θ_p as small as 0.25 (Figure 15), all τ_π between 0.99 and 9 would ensure determinacy for τ_y as small as 0.1, while any τ_π greater than 2.4 would lead to indeterminacy in the SP&SW model if τ_y is no greater than 0.1.

The contrasts among the four different models illustrated by Figures 13-15, and as we discussed above, reveal a nontrivial interaction between sticky wages and firm-specific capital that is crucial for enhancing the role of policy's response to current output in helping avoid indeterminacy that could potentially be caused by forward-looking inflation targeting. The joint presence of sticky wages and firm-specific capital in the sticky price model with endogenous investment empower a tiny response of policy to output to ensure determinacy for a wide range of the policy's response to inflation. This is important given the recent finding by Schmitt-Grohé and Uribe (2006) which suggests that interest rate policy rules that feature a large response to output can lead to significant welfare losses.

6. Conclusion

We have explored the role of policy's response to current output activity in maintaining macroeconomic stability in a sticky price model with endogenous investment in which a central bank systematically adjusts a short-term nominal interest rate to changes in expected future inflation. We found that virtually all interest rate policy rules that feature a muted response to output would lead to real indeterminacy of equilibrium, regardless of price stickiness or overall stickiness embodied in the model by incorporating firm-specific capital, sticky wages, or both.

This reveals the potential importance of policy's response to current output in helping avoid macroeconomic instability that could be caused by forward-looking inflation targeting. We have found, however, letting policy respond to output would help little in our calibrated model with sticky prices and a capital rental market while incorporating firm-specific capital would only make a marginal improvement; in either case, only a narrow range of response of policy to future inflation could ensure determinacy even with a moderately large response of the policy to current output. We have shown that incorporating sticky wages could make a significant improvement; nevertheless, it could still call for a moderately large response of policy to output in order to ensure determinacy for a large range of the policy's response to inflation.

We have illustrated a nontrivial interaction between sticky wages and firm-specific capital that is crucial for enhancing the role of policy's response to output in helping avoid indeterminacy. We showed that in our full-blown model with endogenous investment that features sticky prices, sticky wages, and firm-specific capital, a tiny response of policy to current output is sufficient to ensure macroeconomic stability for a wide range of the policy's response to future inflation. The fact that the required output response is tiny is important in light of the recent finding by Schmitt-Grohé and Uribe (2006) which suggests that a large response of policy to output can be a potential source of significant inefficiencies.

Appendix

To prove Proposition 1, we use the celebrated Samuelson-Farebrother conditions for handling high order polynomial equations. We restate these conditions here for convenience.

Theorem A [Samuelson (1947)–Farebrother (1973)]. *A cubic equation,*

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0,$$

where a_1, a_2, and a_3 are real numbers, has three stable roots if and only if

$$\begin{aligned} 1 + a_1 + a_2 + a_3 &> 0, \\ 1 - a_1 + a_2 - a_3 &> 0, \\ 1 - a_2 + a_1 a_3 - a_3^2 &> 0, \\ a_2 &< 3. \end{aligned}$$

These results can be found in Samuelson (1947, p. 436) and Farebrother (1973).

Proof of Proposition 1: The log-linearized equilibrium conditions (10)-(15) for the case stated in the proposition can be combined into a system of four first-order linear difference equations,

$$\begin{bmatrix} \widehat{C}_{t+1} \\ \widehat{mc}_{t+1} \\ \widehat{\pi}_{t+1} \\ \widehat{K}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 - \frac{(1-\alpha)\tau_y}{\alpha} & \frac{1}{\sigma}\left[\frac{(1-\alpha)\tau_y}{\alpha} - \frac{\lambda_p(\tau_\pi - 1)}{\beta}\right] & \frac{\tau_\pi - 1}{\sigma\beta} & \frac{\tau_y}{\sigma} \\ (1-\alpha)\sigma\left(1 - \frac{\tau_y}{\alpha}\right) & \frac{(1-\alpha)\tau_y}{\alpha} - \frac{\lambda_p(\tau_\pi - 1)}{\beta} & \frac{\tau_\pi - 1}{\beta} & \tau_y \\ 0 & -\frac{\lambda_p}{\beta} & \frac{1}{\beta} & 0 \\ -\frac{(1-\alpha)\sigma}{\alpha} - \left[1 + \frac{(1-\alpha)\sigma}{\alpha}\right](b-1) & \frac{(1-\alpha)b}{\alpha} & 0 & b \end{bmatrix} \begin{bmatrix} \widehat{C}_t \\ \widehat{mc}_t \\ \widehat{\pi}_t \\ \widehat{K}_t \end{bmatrix},$$

in three jump variables, C, mc, and π, and one predetermined variable, K. Therefore, determinacy requires three explosive roots and one stable root. With some algebra, it can be shown that the four eigenvalues of the above 4×4 matrix can be obtained by solving for the four roots of the following fourth-order polynomial equation in λ,

$$\lambda\,(\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3) = 0,$$

where

$$\begin{aligned} b_1 &= -\left(\frac{1}{\beta} + 1 + b - \mathcal{T}_\pi\right), \\ b_2 &= \frac{1}{\beta} - \alpha\mathcal{T}_\pi + b\left(\frac{1}{\beta} + 1 - \mathcal{T}_\pi\right) + \beta\mathcal{T}_y, \\ b_3 &= -\left[b\left(\frac{1}{\beta} - \alpha\mathcal{T}_\pi\right) + \mathcal{T}_y\right]. \end{aligned} \tag{37}$$

Thus determinacy requires that the equation

$$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0, \tag{38}$$

has three explosive roots. Clearly, a necessary condition for this to be the case is $|b_3| > 1$. It follows that (38) can be rewritten as

$$\mu^3 + (b_2/b_3)\mu^2 + (b_1/b_3)\mu + (1/b_3) = 0, \tag{39}$$

where $\mu = 1/\lambda$. Thus (38) has three explosive roots for λ if and only if (39) has three stable roots for μ. Applying the Samuelson-Farebrother conditions presented in Theorem A, (39) has three stable roots for μ if and only if

$$\frac{b_3 + b_2 + b_1 + 1}{b_3} > 0,$$

$$\frac{b_3 - b_2 + b_1 - 1}{b_3} > 0,$$

$$\frac{b_3^2 - b_1 b_3 + b_2 - 1}{b_3^2} > 0,$$

$$\frac{b_1}{b_3} < 3.$$

We claim that determinacy requires $b_3 < 0$. To show this, we can substitute (37) into the numerator of the first two of the above inequalities to get

$$\frac{-(1-\alpha)(b-1)\mathcal{T}_\pi - (1-\beta)\mathcal{T}_y}{b_3} > 0, \tag{40}$$

$$\frac{(1+\alpha)(1+b)\mathcal{T}_\pi - 2(\frac{1}{\beta}+1)(1+b) - (1+\beta)\mathcal{T}_y}{b_3} > 0. \tag{41}$$

If $b_3 > 0$, then (40) requires $\mathcal{T}_\pi < 0$ while (41) requires $\mathcal{T}_\pi > 0$, a contradiction. This is to say, only when $b_3 < 0$ may (40) and (41) hold simultaneously. This proves our claim. Combining this with the requirement $|b_3| > 1$, we can summarize the necessary and sufficient condition for determinacy by the following inequalities:

$$b_3 < -1,$$

$$b_3 + b_2 + b_1 + 1 < 0,$$

$$b_3 - b_2 + b_1 - 1 < 0,$$

$$b_3^2 - b_1 b_3 + b_2 - 1 > 0,$$

$$b_1 > 3b_3.$$

Using (37), we can prove that the above inequalities are equivalent to

$$\mathcal{T}_\pi < \frac{\mathcal{T}_y}{\alpha b} + \frac{1}{\alpha\beta} - \frac{1}{\alpha b} \equiv U_1, \tag{42}$$

$$\mathcal{T}_\pi > -\frac{(1-\beta)\mathcal{T}_y}{(1-\alpha)(b-1)} \equiv L_1, \tag{43}$$

$$\mathcal{T}_\pi < \frac{(1+\beta)\mathcal{T}_y}{(1+\alpha)(1+b)} + \frac{2(1+\frac{1}{\beta})}{1+\alpha} \equiv U_2, \tag{44}$$

$$\begin{aligned} & \mathcal{T}_\pi < \frac{(2\alpha b-1)\mathcal{T}_y + \alpha(b-1)^2 + (\frac{1}{\beta}-1)b(2\alpha b-1-\alpha) - \sqrt{\Delta}}{2\alpha b(\alpha b-1)} \equiv U_3 \\ \text{OR} \quad & \mathcal{T}_\pi > \frac{(2\alpha b-1)\mathcal{T}_y + \alpha(b-1)^2 + (\frac{1}{\beta}-1)b(2\alpha b-1-\alpha) + \sqrt{\Delta}}{2\alpha b(\alpha b-1)} \equiv L_2, \end{aligned} \tag{45}$$

$$\mathcal{T}_\pi < \frac{3\mathcal{T}_y + \frac{3b}{\beta} - \frac{1}{\beta} - 1 - b}{3\alpha b - 1} \equiv U_4. \tag{46}$$

The proofs of (42)-(44) and (46) are straightforward, noting that $\alpha b > 1$. To prove (45), we can show, with some manipulations, that the original nonlinear inequality is equivalent to

$$\begin{aligned} G(\mathcal{T}_\pi) \equiv \quad & [\alpha b(\alpha b - 1)]\, \mathcal{T}_\pi^2 \\ - \quad & \left[(2\alpha b - 1)\mathcal{T}_y + \alpha(b-1)^2 + \left(\frac{1}{\beta} - 1\right) b(2\alpha b - 1 - \alpha)\right] \mathcal{T}_\pi \\ + \quad & \left[\mathcal{T}_y^2 + \left(\frac{2b-1}{\beta} + \beta - 1 - b\right) \mathcal{T}_y + (b-1)\left(\frac{b}{\beta^2} - \frac{b+1}{\beta} + 1\right)\right] > 0. \end{aligned} \tag{47}$$

It can be shown that the two roots to the equation $G(\mathcal{T}_\pi) = 0$ are given by U_3 and L_2. Since $\alpha b > 1$ and $\mathcal{T}_y \geq 0$, we have $\Delta > 0$, and thus U_3 and L_2 are two distinct real roots, with $U_3 < L_2$. The fact that $\alpha b > 1$ also implies that $G(\mathcal{T}_\pi)$ is a convex function of $\mathcal{T}_\pi$. This proves that (45) is equivalent to the original nonlinear inequality.

To sum up inequalities (42)-(46), there is a determinant equilibrium if and only if

$$L_1 < \mathcal{T}_\pi < \min\{U_1, U_2, U_3, U_4\} \qquad \text{OR} \qquad \max\{L_1, L_2\} < \mathcal{T}_\pi < \min\{U_1, U_2, U_4\}. \tag{48}$$

Note that $L_1 < 0 < L_2$. In fact, with some manipulations, we can show that

$$L_2 - U_1 = \frac{\sqrt{\Delta} + \mathcal{T}_y - [\alpha(b-1)^2 + (\frac{1}{\beta} - 1)b(1-\alpha)] + 2(\frac{b}{\beta} - 1)(1-\alpha)}{2\alpha b(\alpha b - 1)}.$$

Denote

$$A_1 \equiv \mathcal{T}_y - \left[\alpha(b-1)^2 + \left(\frac{1}{\beta} - 1\right) b(1-\alpha)\right],$$

$$A_2 \equiv 4\mathcal{T}_y(1 - \beta\alpha)b\left(\alpha b - 1 + \frac{1-\alpha}{\beta}\right) \geq 0,$$

then

$$\Delta = A_1^2 + A_2.$$

It follows that

$$L_2 - U_1 = \frac{\sqrt{A_1^2 + A_2} + A_1 + 2(\frac{b}{\beta} - 1)(1-\alpha)}{2\alpha b(\alpha b - 1)} \geq \frac{2(\frac{b}{\beta} - 1)(1-\alpha)}{2\alpha b(\alpha b - 1)} > 0.$$

Therefore, (48) reduces to

$$L_1 < \mathcal{T}_\pi < \min\{U_1, U_2, U_3, U_4\}. \tag{49}$$

We claim that $U_3 < U_1$. To prove our claim, first note that

$$\begin{aligned} U_3 - U_1 &= \frac{-\sqrt{\Delta} + \mathcal{T}_y - [\alpha(b-1)^2 + (\frac{1}{\beta} - 1)b(1-\alpha)] + 2(\frac{b}{\beta} - 1)(1-\alpha)}{2\alpha b(\alpha b - 1)} \\ &= \frac{-\sqrt{A_1^2 + A_2} + A_1 + 2(\frac{b}{\beta} - 1)(1-\alpha)}{2\alpha b(\alpha b - 1)}. \end{aligned}$$

Thus, our claim holds true if and only if

$$A_1 + 2\left(\frac{b}{\beta} - 1\right)(1-\alpha) < \sqrt{A_1^2 + A_2}.$$

Suppose that this is not the case and instead

$$A_1 + 2\left(\frac{b}{\beta} - 1\right)(1-\alpha) \geq \sqrt{A_1^2 + A_2}.$$

Then

$$A_1^2 + 4\left(\frac{b}{\beta} - 1\right)(1-\alpha)A_1 + 4\left(\frac{b}{\beta} - 1\right)^2(1-\alpha)^2 \geq A_1^2 + A_2,$$

(with a precondition that $A_1 + 2(b/\beta - 1)(1-\alpha) \geq 0$), which leads to

$$[(1-\alpha) - (1-\beta\alpha)b]\mathcal{T}_y \geq \left(\frac{b}{\beta} - 1\right)(1-\alpha)(b-1),$$

which is impossible since the left-hand side is negative while the right-hand side is strictly positive. Thus our claim holds true and (49) reduces to

$$L_1 < \mathcal{T}_\pi < \min\{U_2, U_3, U_4\}. \tag{50}$$

It is easy to construct examples to show that U_2, U_3, and U_4 can alternate in the order of their magnitudes, depending on the values of the fundamental parameters. Therefore, (50) is the most compact necessary and sufficient condition for local real determinacy. Substituting into (50) the values for L_1, U_2, U_3, and U_4 establishes the proposition. **Q.E.D.**

Proof of Corollary 1: The only nontrivial proof is for the case of the second upper bound. To prove that U is strictly increasing in τ_y, we rewrite Δ as

$$\Delta = \left\{\mathcal{T}_y + \left[\alpha(b-1)^2 + (1-\alpha)\left(\frac{1}{\beta}-1\right)b\right]\right\}^2 - 4\mathcal{T}_y\alpha(\alpha b-1)(\beta b-1) \equiv x_1^2 - 2\mathcal{T}_y x_2,$$

and use it to show that $\partial U/\partial\tau_y > 0$ if and only if

$$2\alpha b - 1 > \frac{x_1 - x_2}{\sqrt{\Delta}}.$$

A sufficient condition for the above inequality to hold is that

$$x_1^2 - 2\mathcal{T}_y x_2 > \left(\frac{x_1 - x_2}{2\alpha b - 1}\right)^2,$$

which can be shown, using $x_1^2 - 2\mathcal{T}_y x_2 = (x_1 - x_2)^2 - x_2(x_2 - 2x_1 + 2\mathcal{T}_y)$, to be equivalent to

$$\left(\frac{x_1 - x_2}{2\alpha b - 1}\right)^2 > -(\beta b - 1)(1 - \beta\alpha)\left(\alpha b - 1 + \frac{1-\alpha}{\beta}\right),$$

which clearly always holds. Thus U is strictly increasing in τ_y. This combined with the fact that U simplifies to

$$\frac{1}{\alpha}\left(\frac{1}{\beta} - 1\right) > 0$$

when τ_y is 0 implies that U is strictly positive for all $\tau_y \geq 0$ (this can also be checked by verifying that $L_2U_3 > 0$ and $L_2 > 0$, where the notations are as defined in the proof of Proposition 1 above).

Q.E.D.

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TABLE 1—PARAMETER VALUES

Parameters for All Models	Values
Share of payment to capital in total value added (α)	0.33
Subjective quarterly discount factor (β)	0.99
Quarterly depreciation rate of capital (δ)	0.02
Relative risk aversion in consumption (σ)	2
Relative risk aversion in labor hours (η)	$\{0, 1, 5, 10^{*}, 20\}$
Probability of non-adjustment in price (θ_p)	$\{0.25, 0.33^{*}, 0.57\}$
Elasticity of substitution of differentiated goods (ϵ_y)	$\{4, 11^{*}\}$
Elasticity of investment to capital ratio w.r.t. Tobin's q (ϵ_q)	$\{0, 3^{*}\}$

Additional Parameters for Models with Staggered Wages	Values
Elasticity of substitution of differentiated skills (ϵ_n)	4
Probability of non-adjustment in wage (θ_w)	0.75

Note: For multiple values the one with an asterion denotes the baseline calibration

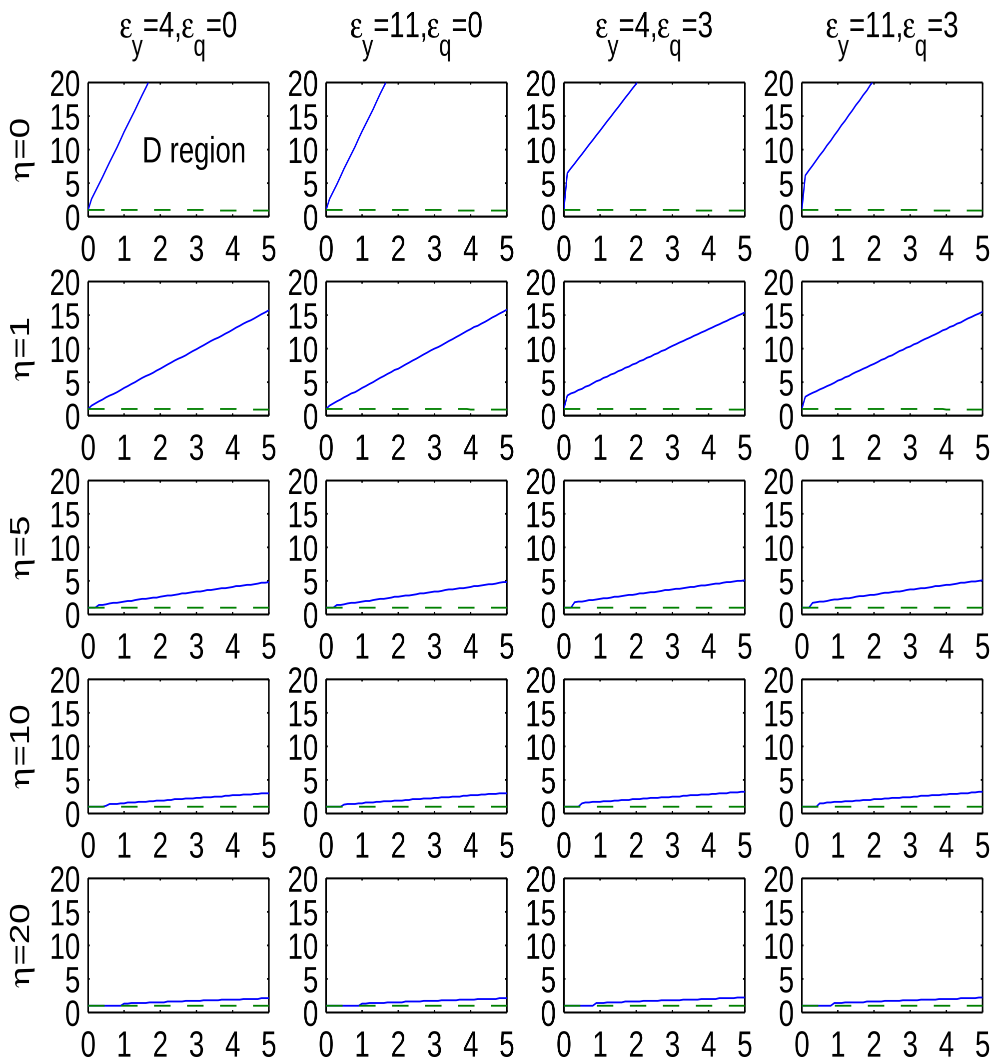

Fig. 1. Determinacy region with staggered prices and a capital rental market ($\theta_p = 0.57$): Upper bound (solid line) and lower bound (broken line) for policy's response to future inflation τ_π (vertical axis) as a function of policy's response to current output τ_y (horizontal axis)

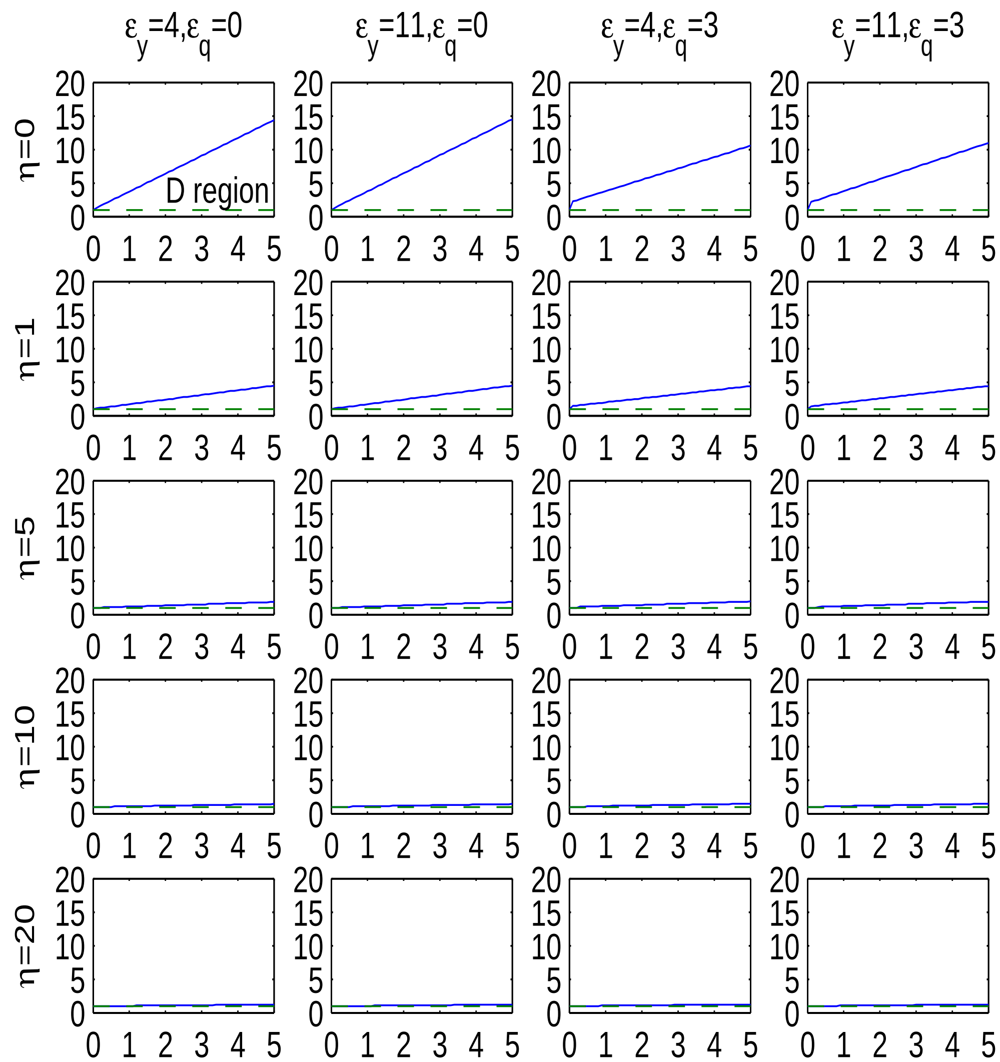

Fig. 2. Determinacy region with staggered prices and a capital rental market ($\theta_p = 0.33$): Upper bound (solid line) and lower bound (broken line) for policy's response to future inflation τ_π (vertical axis) as a function of policy's response to current output τ_y (horizontal axis)

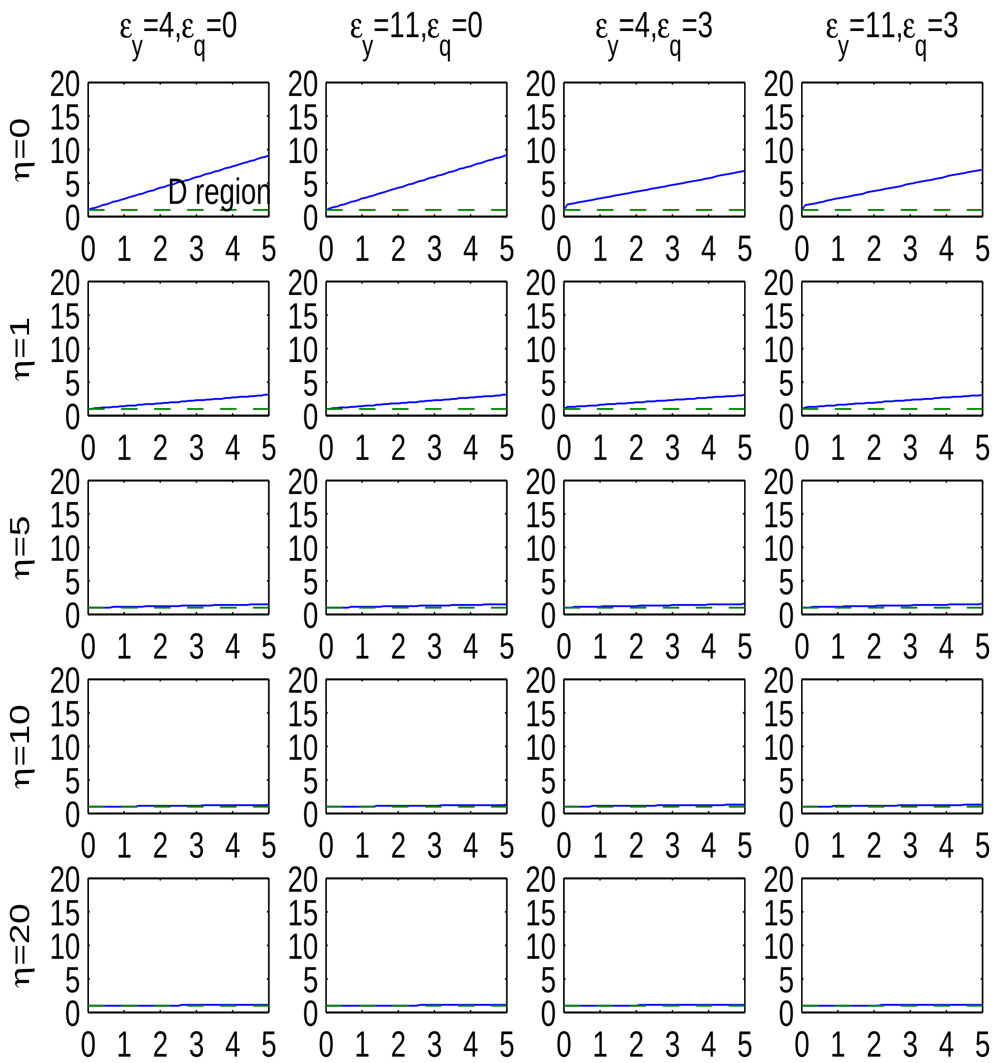

Fig. 3. Determinacy region with staggered prices and a capital rental market ($\theta_p = 0.25$): Upper bound (solid line) and lower bound (broken line) for policy's response to future inflation τ_π (vertical axis) as a function of policy's response to current output τ_y (horizontal axis)

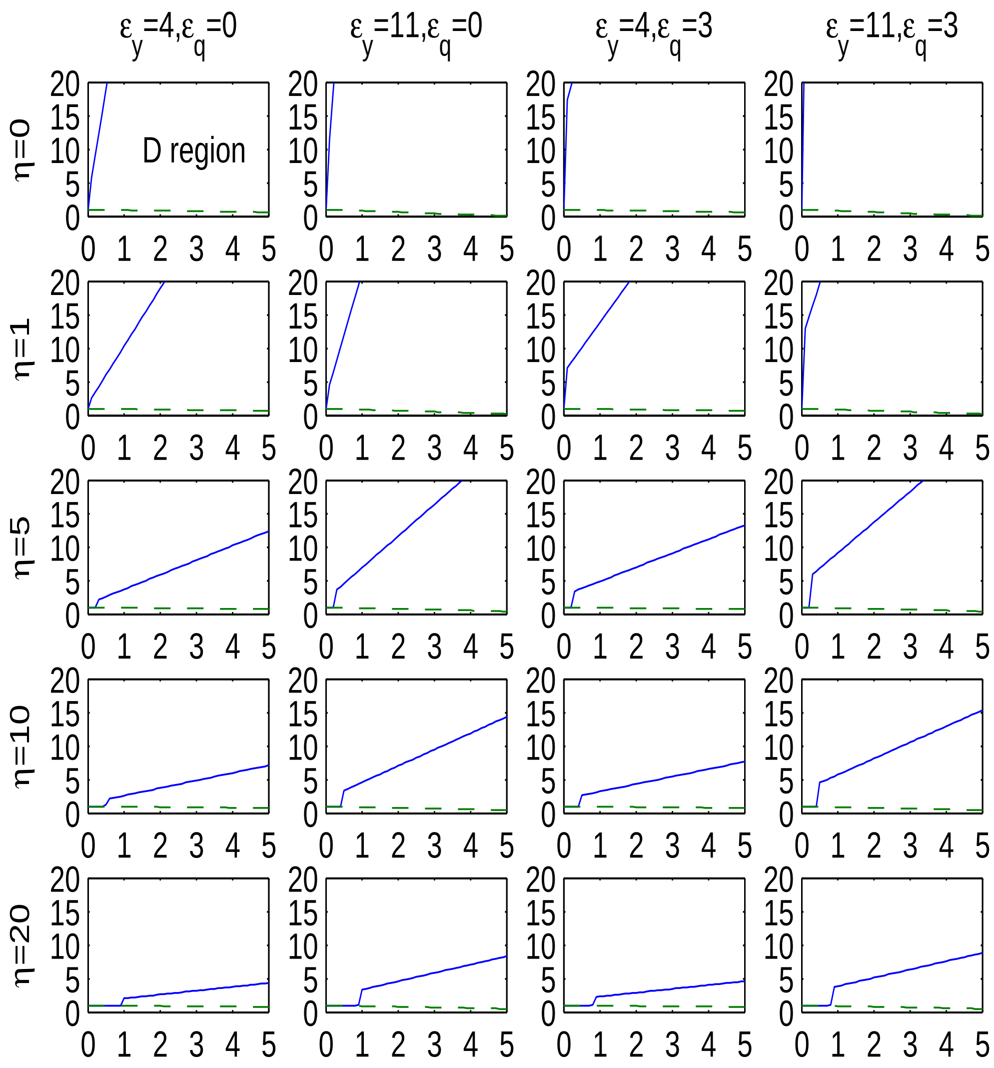

Fig. 4. Determinacy region with staggered prices and firm-specific capital ($\theta_p = 0.57$): Upper bound (solid line) and lower bound (broken line) for policy's response to future inflation τ_π (vertical axis) as a function of policy's response to current output τ_y (horizontal axis)

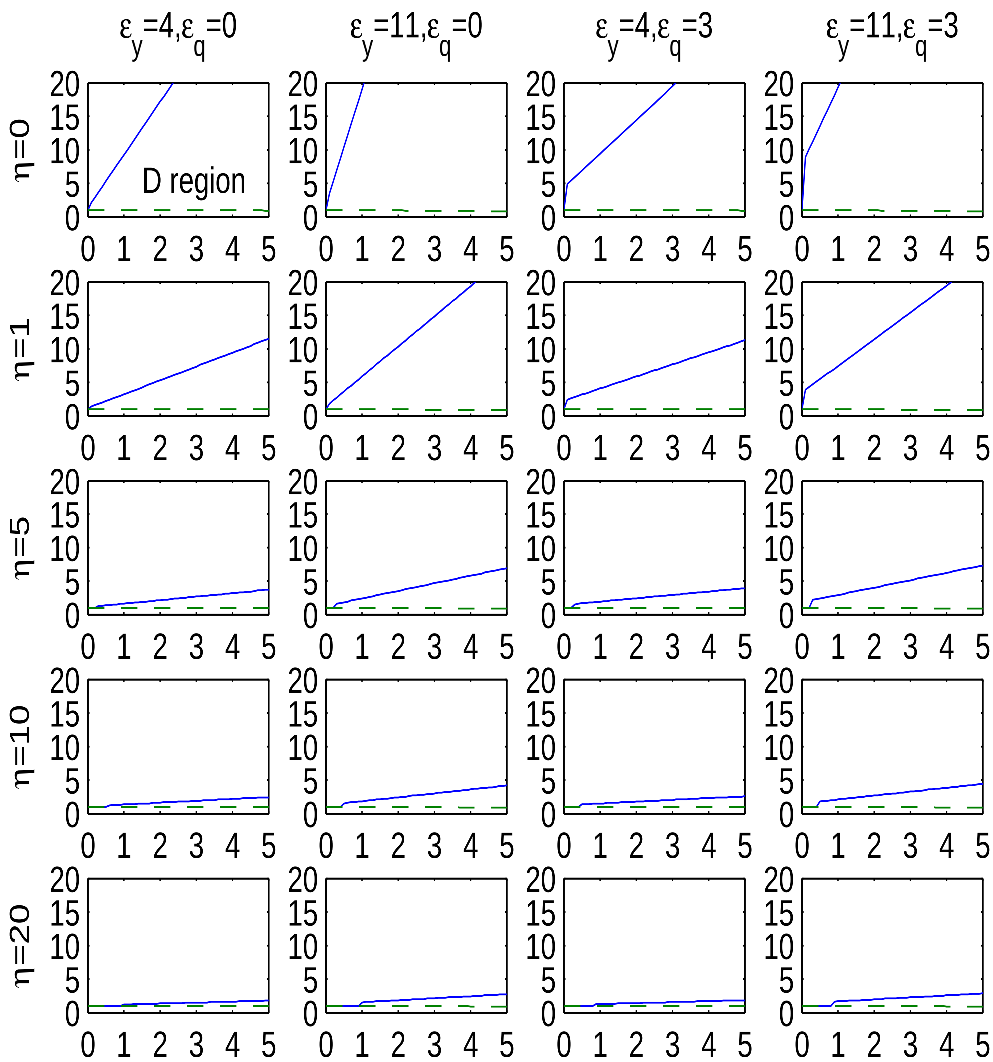

Fig. 5. Determinacy region with staggered prices and firm-specific capital ($\theta_p = 0.33$): Upper bound (solid line) and lower bound (broken line) for policy's response to future inflation τ_π (vertical axis) as a function of policy's response to current output τ_y (horizontal axis)

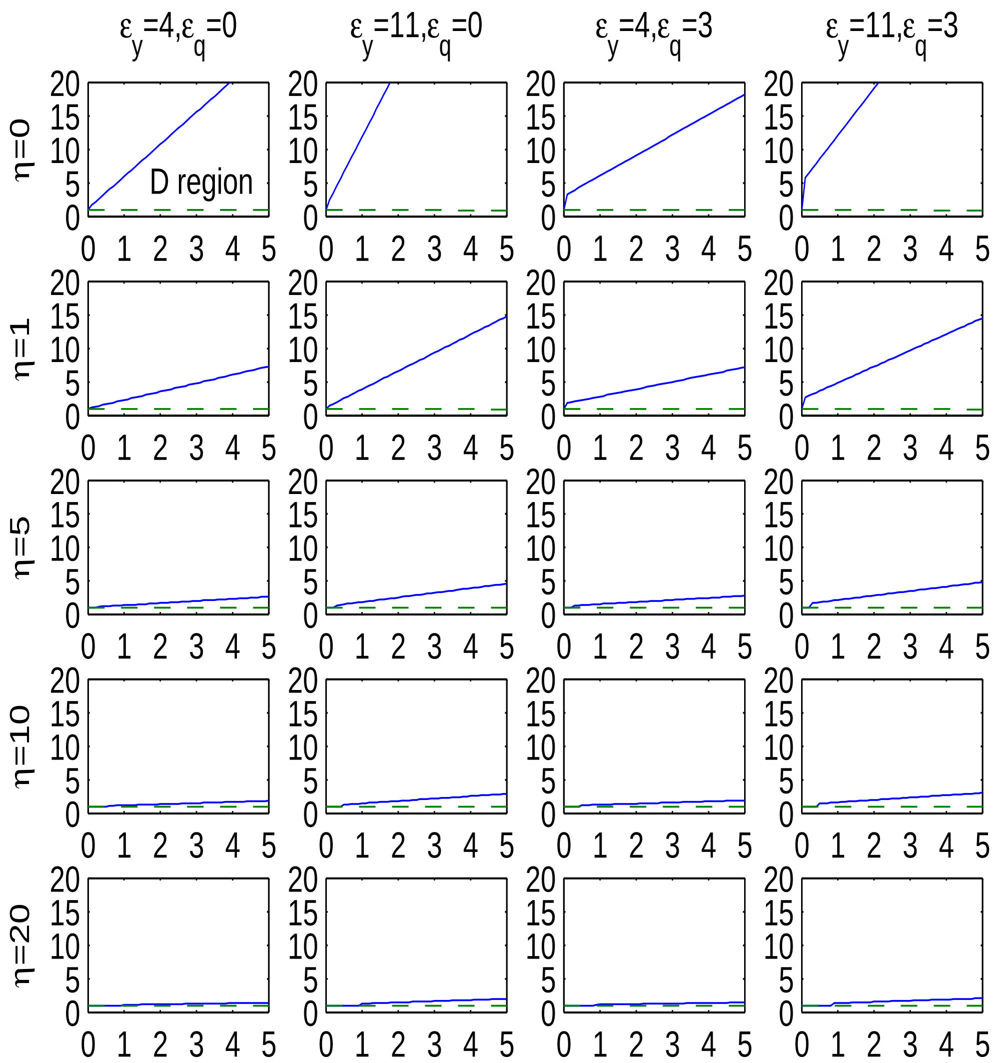

Fig. 6. Determinacy region with staggered prices and firm-specific capital ($\theta_p = 0.25$): Upper bound (solid line) and lower bound (broken line) for policy's response to future inflation τ_π (vertical axis) as a function of policy's response to current output τ_y (horizontal axis)

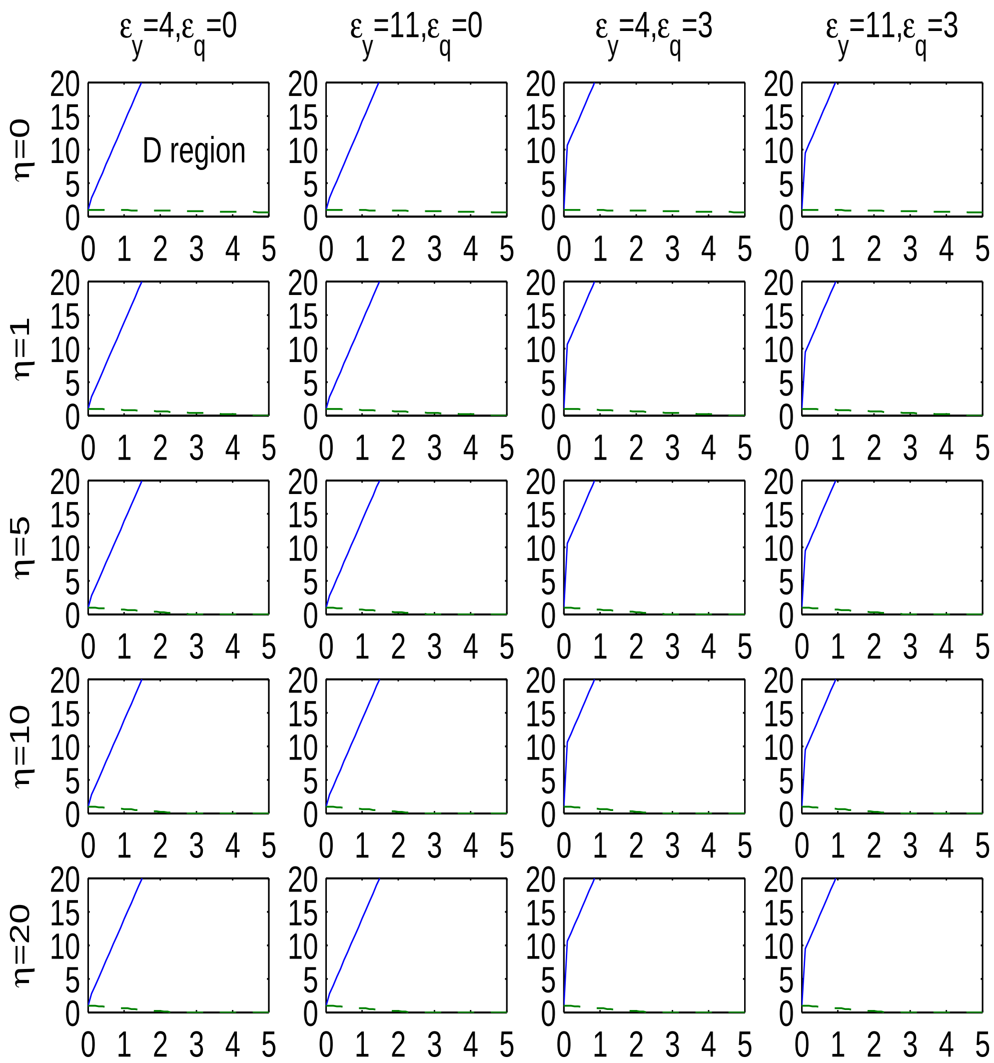

Fig. 7. Determinacy region with staggered prices, staggered wages, and a capital rental market ($\theta_p = 0.57$): Upper bound (solid line) and lower bound (broken line) for policy's response to future inflation τ_π (vertical axis) as a function of policy's response to current output τ_y (horizontal axis)

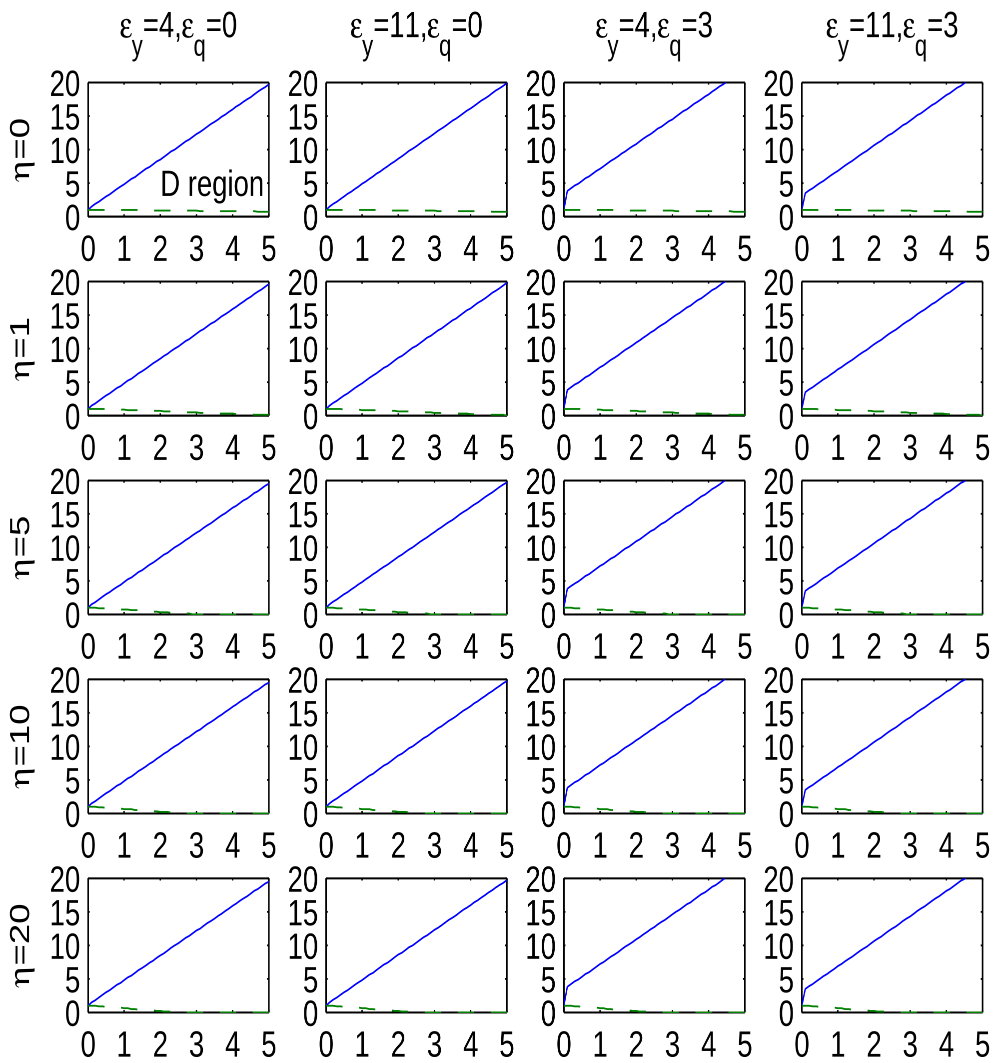

Fig. 8. Determinacy region with staggered prices, staggered wages, and a capital rental market ($\theta_p = 0.33$): Upper bound (solid line) and lower bound (broken line) for policy's response to future inflation τ_π (vertical axis) as a function of policy's response to current output τ_y (horizontal axis)

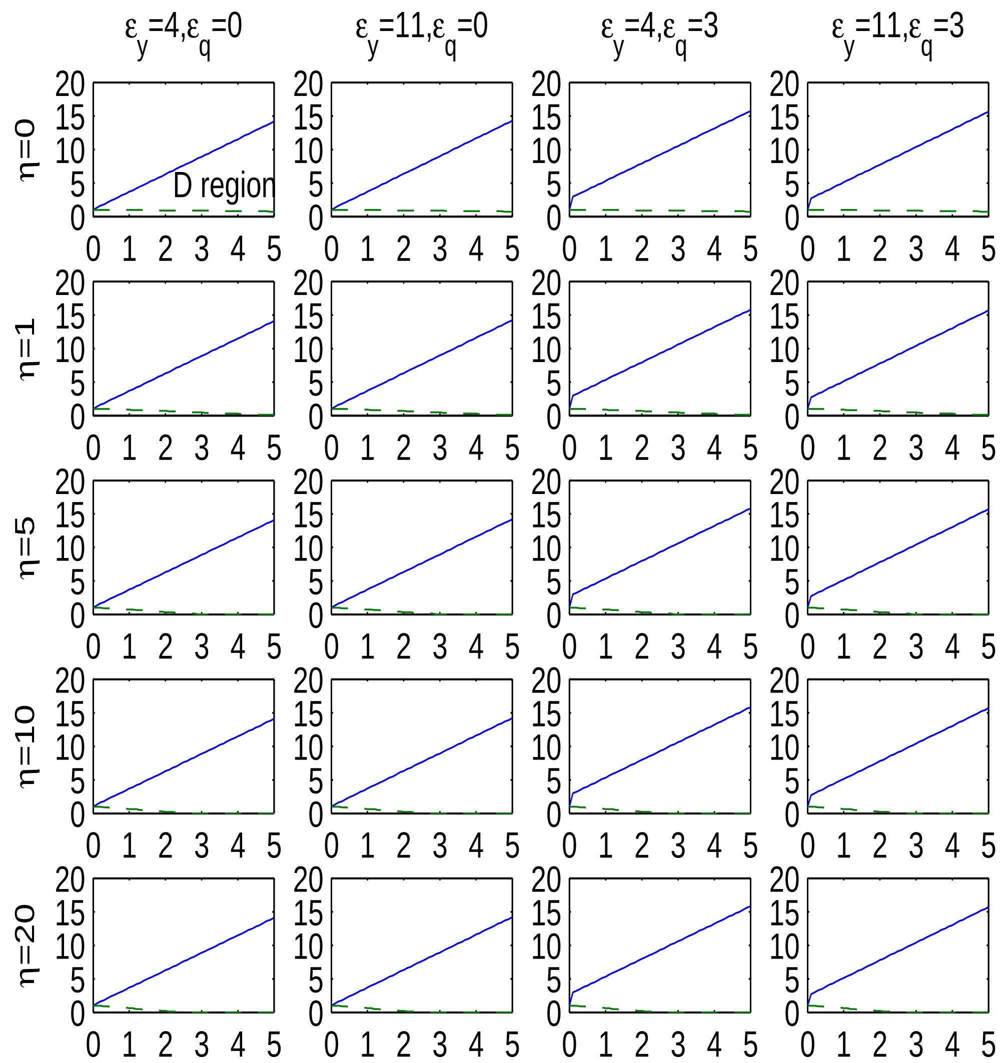

Fig. 9. Determinacy region with staggered prices, staggered wages, and a capital rental market ($\theta_p = 0.25$): Upper bound (solid line) and lower bound (broken line) for policy's response to future inflation τ_π (vertical axis) as a function of policy's response to current output τ_y (horizontal axis)

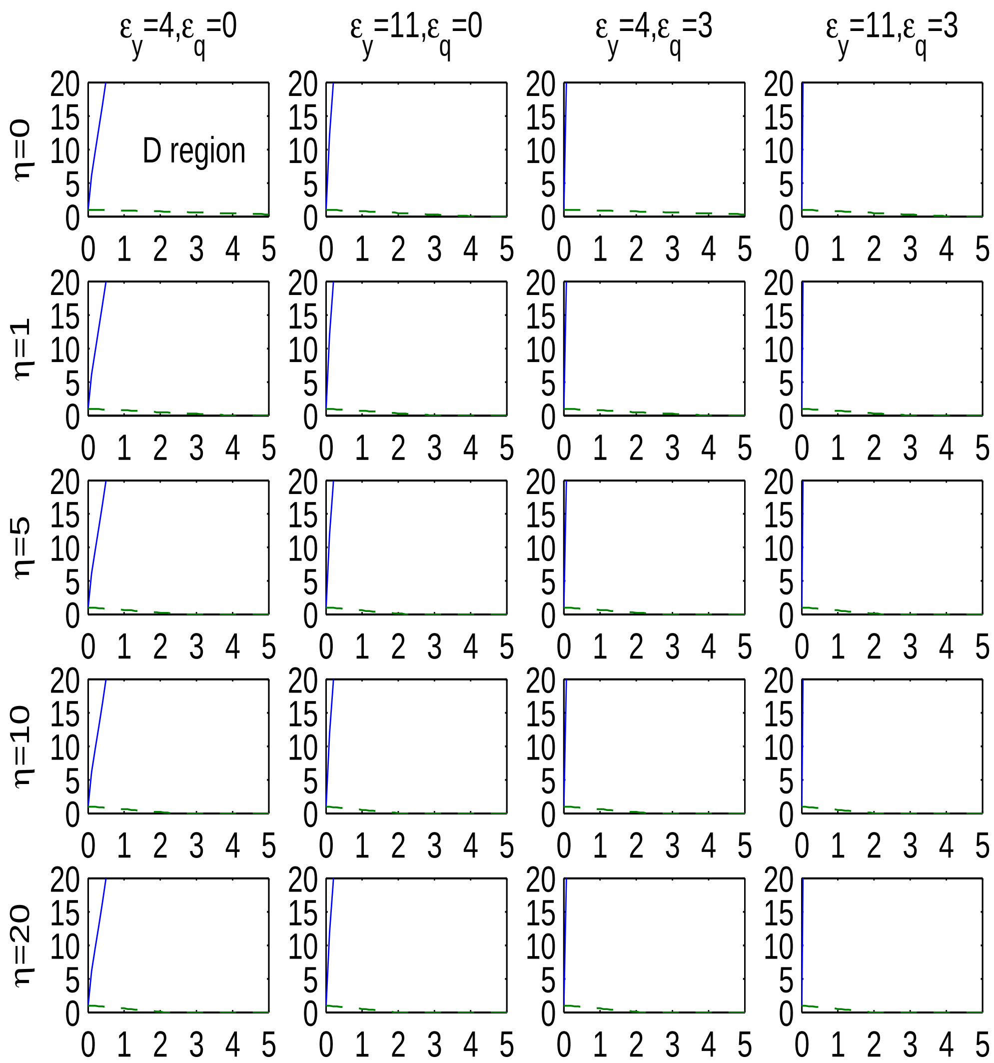

Fig. 10. Determinacy region with staggered prices, staggered wages, and firm-specific capital ($\theta_p = 0.57$): Upper bound (solid line) and lower bound (broken line) for policy's response to future inflation τ_π (vertical axis) as a function of policy's response to current output τ_y (horizontal axis)

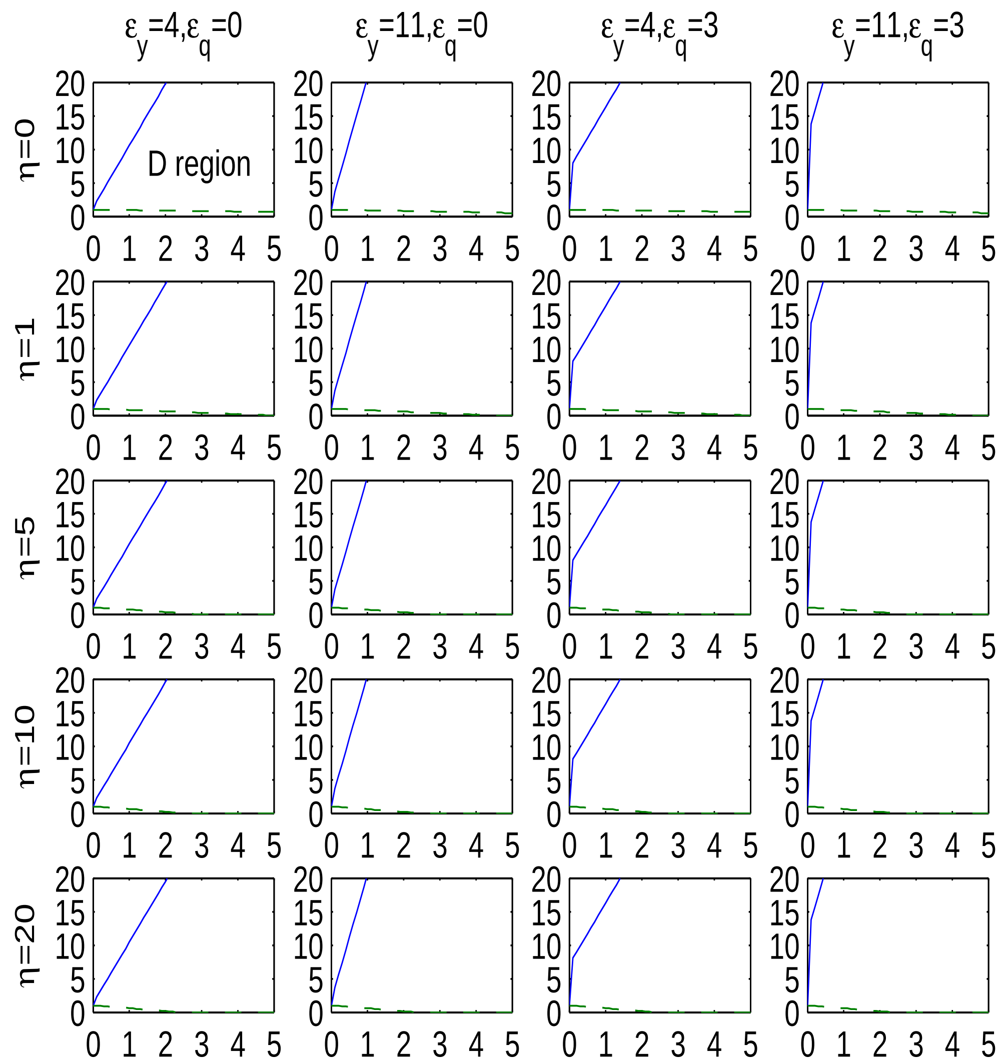

Fig. 11. Determinacy region with staggered prices, staggered wages, and firm-specific capital ($\theta_p = 0.33$): Upper bound (solid line) and lower bound (broken line) for policy's response to future inflation τ_π (vertical axis) as a function of policy's response to current output τ_y (horizontal axis)

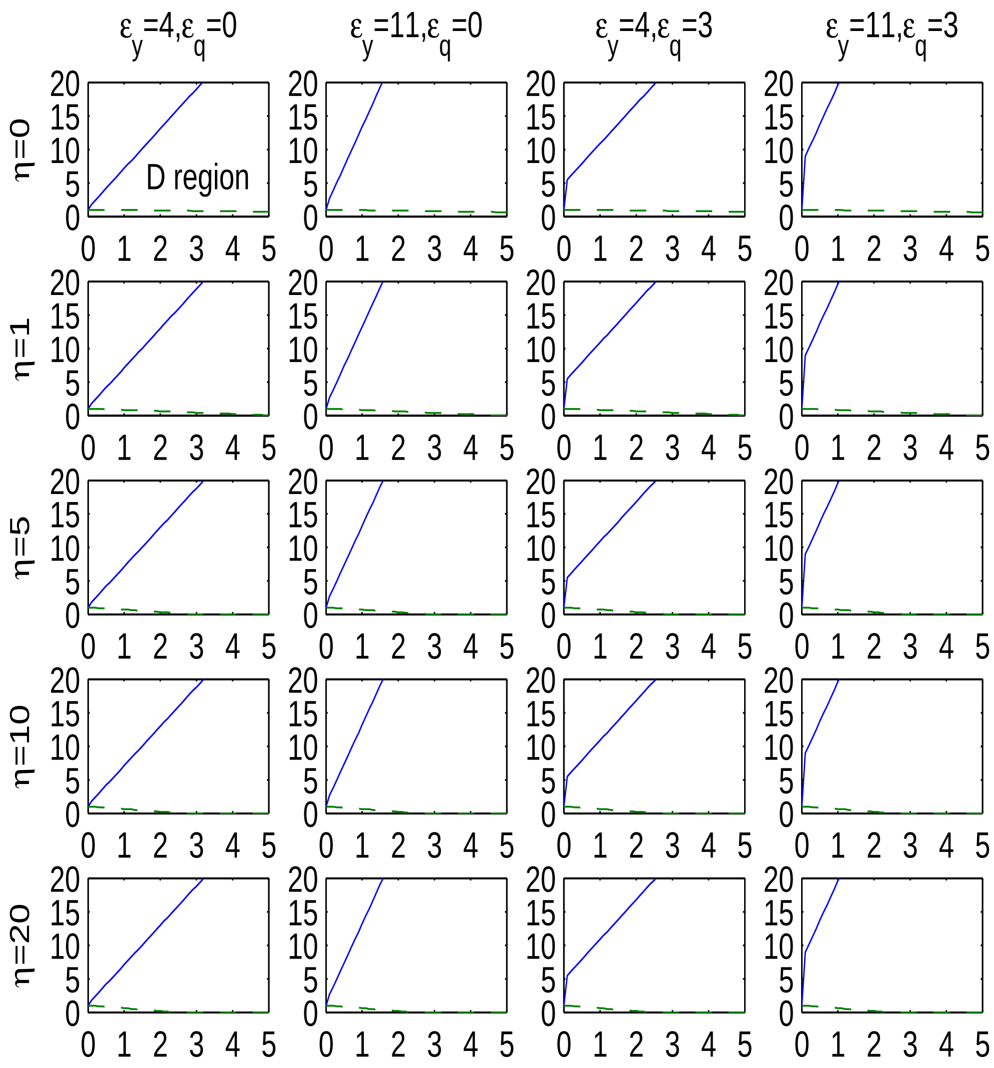

Fig. 12. Determinacy region with staggered prices, staggered wages, and firm-specific capital ($\theta_p = 0.25$): Upper bound (solid line) and lower bound (broken line) for policy's response to future inflation τ_π (vertical axis) as a function of policy's response to current output τ_y (horizontal axis)

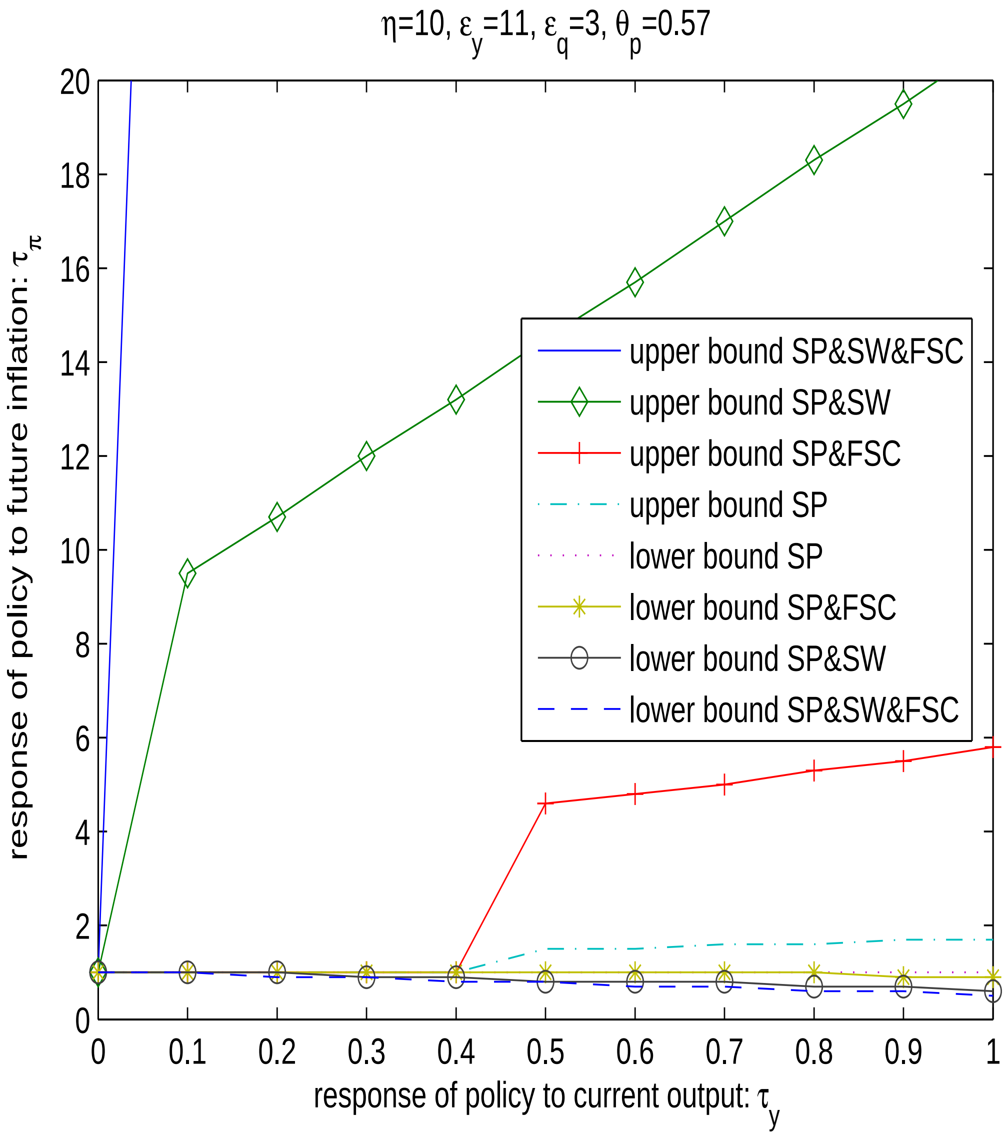

Fig. 13. Determinacy region for different models with baseline calibration

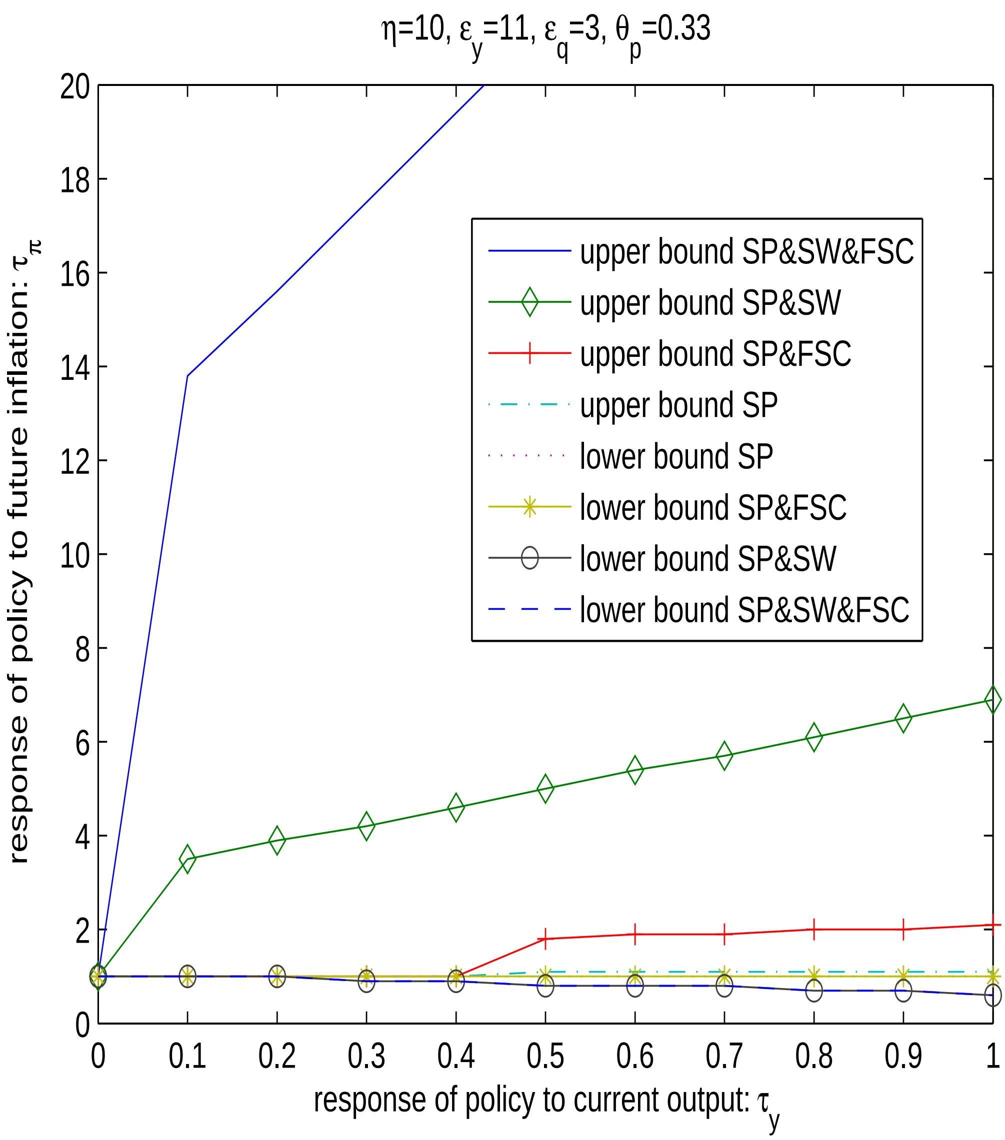

Fig. 14. Determinacy region for different models with baseline calibration

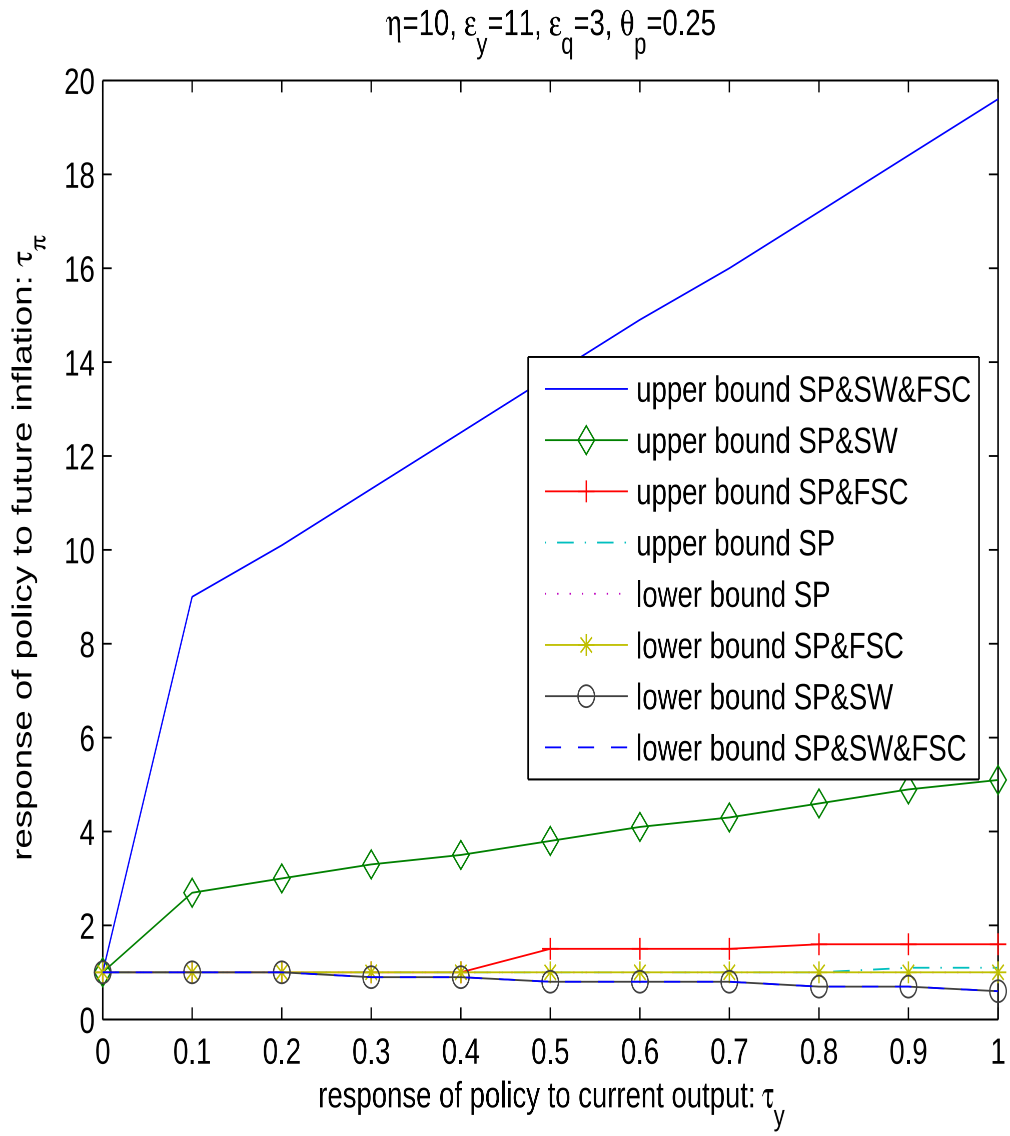

Fig. 15. Determinacy region for different models with baseline calibration