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First and incomplete draft - comments welcome

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Abstract

This paper takes a first step in analysing how a monetary union performs in the presence of labour market asymmetries. Differences in wage flexibility, market power and country sizes are allowed for in a setting with both country-specific and aggregate shocks. The implications of asymmetries for both the overall performance of the monetary union and the country-specific situation are analysed. It is shown that asymmetries can have important effects, and that there are substantial spill-over effects. Among other things, it is found that aggregate output volatility is not strictly increasing in nominal rigidity but hump-shaped. A disproportionate share of the consequences of wage inflexibility may fall on small countries. In the case of country-specific shocks a country unambiguously benefits in terms of macroeconomic stability by becoming more flexible, but in general an inflexible country does not necessarily achieve more output stability by becoming more flexible. As this may be desirable for the monetary union as a whole, there is a risk of a 'reform deficit' in an asymmetric monetary union.

JEL classification: E30, E52, F41

Key words: wage formation, nominal wage rigidity, staggered contracts, monetary policy, monetary union, business cycles, shocks

1 Introduction

In the run up to the establishment of the European Monetary Union there was much focus on whether the potential member countries fulfilled the conditions for an optimal currency area. According to the traditional theory, participation in a currency union and the implied loss of autonomy in monetary policy require that labour markets are flexible in adjusting to country-specific shocks (see, e.g., de Grauwe(2005)). The flexibility can either be in terms of wage adjustment or labour mobility. In a second wave it was stressed that the conditions for an optimal currency area are endogenous since participation in a currency union affects market fundamentals (via further integration) and incentives in wage formation (see, e.g., Rose and Frankel (1998) and Calmfors (2001)). Whether the countries constitute an optimal currency union is an ex-post rather than an ex-ante question. The key issue is not labour market flexibility per se since the common monetary policy can react to average responses in the union, but rather asymmetries across member countries. The asymmetries include both asymmetric structures causing common shocks to be propagated differently as well as country-specific shocks interacting with country-specific labour market adjustment processes.

To assess the need for structural labour market changes, it is necessary to investigate how labour markets will perform in a monetary union. This is an intricate question when labour markets are asymmetric. The issue is not only how the overall performance of the monetary union is affected, but also how the country-specific situation is affected. This distinction is important since the latter has implications for the incentives to undertake structural reforms, and it may not be overlapping with the consequences at the aggregate level. Moreover, it is important whether there are spill-over effects in labour market structures. Is it necessarily the case that less flexible countries suffer from a disproportionate share of economic problems and so have the largest incentives to reform labour markets? Is it only asymmetries across large member countries that matter, or are the structural characteristics of smaller member countries also important? Surprisingly, these questions have not been much researched[1].

This paper takes a first step in analysing how a monetary union with a given monetary policy performs in the presence of labour market asymmetries across the participating countries. This has both a positive and a normative side. The positive side relates to the performance of a monetary union under labour market asymmetries, and the normative to how these asymmetries should affect the common monetary policy. The present paper deals with the positive side of these asymmetries, focusing on their implications for a given monetary policy. The aim is thus to study the role of labour market heterogeneities in a monetary union, rather than constructing a full empirical model for a monetary union.

It is well established that labour markets across European countries are fairly heterogeneous, reflecting different institutional, political and historical factors. These differences have been amply documented by, e.g., the OECD

[1] In Andersen (2004), these issues are analysed in a setting where the form of wage contracts is exogenously given.

(various issues) and the European Commission (2006). The present paper aims at analysing the consequences of labour market asymmetries, focusing on two key aspects, namely differences in wage responsiveness and in the degree of imperfect competition in the labour market. While other aspects of labour markets are also important, these two dimensions capture labour market mechanisms of importance for business cycle fluctuations both at the aggregate and the country-specific level. We consider both common (aggregate) and country-specific shocks, and how they are propagated across countries given labour market asymmetries, the common monetary policy and the trade links between countries.

Other forms of asymmetries or heterogeneities in a monetary union have been addressed in the literature. One important issue is the interdependency between monetary and fiscal policy when the former is centralised and the latter decentralised. This naturally leads to potential asymmetries in fiscal policy, which raises questions concerning both interdependencies between national fiscal policies as well as between the aggregate fiscal stance and the monetary policy (see, e.g., Lombardo and Sutherland (2004), Beetsma and Jensen (2005), and Andersen, (2005)). Also, Benigno (2002) analyses, from a normative point of view, how monetary policy should be designed when member countries have different degrees of nominal price rigidities, and it is shown that the central bank should attach more weight to inflation in countries characterised by more nominal inertia. Beetsma and Jensen (2004, 2005) allow for labour market asymmetries in their analysis of the interactions between monetary and fiscal policy in a monetary union. Dellas and Tavlas (2004) present a three-country model allowing for asymmetries in nominal wage flexibility, and find that countries with a high degree of nominal wage rigidity are better off in a monetary union.

The framework used in this paper is based on recent intertemporal approaches in both open economy macroeconomics and in the closed-economy new-Keynesian literature on monetary policy.[2] The monetary union is considered to be a closed area, while its member countries share a common monetary policy and engage in trade across national borders within the union. The model allows for country asymmetries in respect to wage adjustments, market power in labour markets and country sizes.

The paper is organised as follows. The model structure is laid out in section 2, and the equilibrium processes for output and inflation are determined in section 3. Section 4 considers the shock transmission in a symmetric baseline example. The implications of various forms of asymmetries are explored in section 5 both from a unionwide and country-specific perspective. Section 6 offers a few concluding comments.

[2] Seminal contributions are Obstfeld and Rogoff (1995) and Yun (1996), respectively. For monographic expositions, cf. Obstfeld and Rogoff (1996) and Woodford (2003).

2 A monetary union with heterogeneous labour markets

Consider a model economy with one central bank with the monetary authority over I separate and otherwise independent countries (or more generally regions) indexed by i. In particular, the central bank sets the nominal interest rate R_t earned on risk-free nominal bonds throughout this monetary union between periods t and $t+1$. The union is closed to the outside world.[3] Each country i is populated by a continuum of households $h \in [0,1]$ and has a continuum of firms $f \in [0,1]$. Countries may be of different sizes, where the relative size of country i is given by v_i so that $\sum_{i=1}^{I} v_i = 1$. All firms in a given country produce the same internationally traded consumption good, different from those produced in other countries (a specialised production structure). For simplicity, product markets are assumed to be perfectly competitive and prices to be flexible. Labour markets, in contrast, are imperfectly competitive and have nominal rigidities in the form of nominal wage contracts. Labour market structures are generally assumed to be different in that both degrees of market power in wage setting and degrees of nominal rigidity are different across countries. There is no mobility of labour across borders.

2.1 Firms

2.1.1 Labour demand

In each period t, each household h in country i supplies a differentiated labour service $N_{it}(h)$. The labour used in production in country i, L_{it}, is assumed to be an aggregate of the continuum of labour services supplied by the households:

$$L_{it} = \left[\int_0^1 N_{it}(h)^{\frac{\xi_i - 1}{\xi_i}} dh \right]^{\frac{\xi_i}{\xi_i - 1}} \tag{1}$$

where $\xi_i > 1$ is the elasticity of substitution between labour services.

Each household determines its wage rate, $\Upsilon_{it}(h)$, taking into account how firms' labour demand depends on the wage (a right-to-manage structure). That is, given wages, actual employment is determined by labour demand. The demand for household h's labour service is determined by the cost minimisation problems of the country's firms, which minimise costs taking households' wage rates, $\Upsilon_{it}(h)$, as given. The representative firm minimises

$$\int_0^1 \Upsilon_{it}(h) N_{it}(h)\, dh \tag{2}$$

with respect to $N_{it}(h)$ subject to (1). This leads to a demand for household h's

[3] This means that the model could also be interpreted as a closed-economy model of a single country with I sectors, which are potentially asymmetric in terms of structures and shocks.

labour service given by

$$N_{it}(h) = \left(\frac{\Upsilon_{it}(h)}{W_{it}}\right)^{-\xi_i} L_{it} \tag{3}$$

where W_{it} is the wage index defined by

$$W_{it} = \left[\int_0^1 \Upsilon_{it}(h)^{1-\xi_i}\, dh\right]^{\frac{1}{1-\xi_i}} \tag{4}$$

This wage index has the property that the minimum cost of L_{it} units of aggregate labour is given by $W_{it}L_{it}$. It follows that the demand for household h's labour service is a decreasing function of the household's relative wage, and the labour demand elasticity is ξ_i. Hence, the market power of wage setters is inversely related to ξ_i.

Integrating demand over firms shows that this is also the aggregate demand for household h's labour:

$$\int_0^1 N_{it}(h)\, df = N_{it}(h) \tag{5}$$

2.1.2 Profit maximisation

The representative firm in country i produces output Y_{it} according to the production function

$$Y_{it} = \frac{1}{\gamma} L_{it}^{\gamma} U_{it}^{1-\gamma} \tag{6}$$

where U_{it} is the stochastic period-t productivity of firms in country i, and $0 < \gamma < 1$ is the degree of returns to scale. Real capital is disregarded to simplify, but decreasing returns can be interpreted as arising from a second factor of production in fixed supply.

Product markets are perfectly competitive, and the representative firm in country i maximises profits, which it distributes to households. There are no nominal price rigidities, and the firm takes the price of its product, P_{it}, as given. The profit maximisation problem yields a demand for aggregated labour services given by

$$L_{it} = \left(\frac{W_{it}}{P_{it}}\right)^{-\frac{1}{1-\gamma}} U_{it} \tag{7}$$

Inserting in (6) gives the supply relation

$$Y_{it} = \frac{1}{\gamma}\left(\frac{W_{it}}{P_{it}}\right)^{\frac{\gamma}{\gamma-1}} U_{it} \tag{8}$$

2.2 Households

2.2.1 Consumption and bond holdings

Household h in country i has the utility function

$$E_t \sum_{\tau=0}^{\infty} \delta^{\tau} \left[\frac{\sigma}{\sigma-1} C_{it+\tau}(h)^{\frac{\sigma-1}{\sigma}} - \frac{1}{1+\mu} N_{it+\tau}(h)^{1+\mu} \right] \tag{9}$$

where E_t is an operator representing expectations over all states of the economy conditional on period-t information, $\delta \in (0,1)$ is the subjective discount factor, and $C_{it}(h)$ is a real consumption index. $\sigma > 0$ is the elasticity of intertemporal substitution of consumption and $\mu > 0$.[4]

The consumption index is defined over the differentiated commodities produced in the union's member countries. Specifically,

$$C_{it}(h) = \left[\sum_{j=1}^{I} v_j C_{ijt}(h)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{10}$$

where $\theta > 0$, v_j is the relative size of country j (as noted above), and $C_{ijt}(h)$ represents consumption of country j's commodity by household h in country i. In every period t, this household chooses $C_{ijt}(h)$ for a given level of real consumption by minimising

$$\sum_{j=1}^{I} P_{jt} C_{ijt}(h)$$

subject to (10). This yields a demand for country j's product by household h in country i given by

$$C_{ijt}(h) = \left(\frac{P_{jt}}{v_j P_t} \right)^{-\theta} C_{it}(h) \tag{11}$$

when P_t is the price index defined by

$$P_t = \left[\sum_{j=1}^{I} v_j \left(\frac{P_{jt}}{v_j} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{12}$$

This price index has the property that the minimum cost of C_{it} units of real consumption is given by $P_t C_{it}$. From (11) it follows that θ is the price elasticity of demand for C_{ijt}.

[4] Real money balances could be included in the utility function in order to analyse money demand. But the central bank's policy instrument is the interest rate, while it passively supplies the money demanded by households. Thus, as long as money enters additively separably in the utility function, nothing will change in what follows since the inclusion of money will only add a money demand relation recursively determining money demand as a function of the variables of interest. See, e.g., Woodford (2003) for a discussion.

Asset markets are assumed to be complete, i.e., available financial assets completely span the possible states of the economy. This assumption leads to the following period-t flow budget constraint for a household in country i:

$$E_t\left[Q_{t,t+1}B_{it}(h)\right]+P_tC_{it}(h)=B_{it-1}(h)+\Upsilon_{it}(h)N_{it}(h)+\Pi_{it}(h) \qquad (13)$$

The right-hand side gives available resources as the sum of initial financial wealth, $B_{it-1}(h)$, labour income, $\Upsilon_{it}(h)N_{it}(h)$, and nominal profit income, $\Pi_{it}(h)$. The left-hand side represents the allocation of resources to consumption, $P_tC_{it}(h)$, and bond-holdings, $E_t\left[Q_{t,t+1}B_{it}(h)\right]$, where $Q_{t,t+1}$ is the asset pricing kernel.[5]

Given existing wage contracts, the household maximises expected utility (9) subject to the sequence of budget constraints (13) and (implicitly) a solvency condition. Defining the net risk-free nominal interest rate R_t by the relation $(1+R_t)^{-1}=E\left[Q_{t,t+1}\right]$, the first-order conditions determining the optimal choice of consumption and bond-holdings can be combined to yield the Euler equation

$$C_{it}(h)^{-\frac{1}{\sigma}}=\delta(1+R_t)E_t\left(C_{it+1}(h)^{-\frac{1}{\sigma}}\frac{P_t}{P_{t+1}}\right) \qquad (14)$$

summarising the household's intertemporal consumption decisions.[6]

2.2.2 Wage setting

Wages are set by households in a staggered fashion with random duration of wage contracts analogous to the mechanism in Calvo (1983). In particular, in every period each household in country i is allowed to reset the wage rate it demands for its labour service with a fixed probability $(1-\alpha_i)$. Hence, the wage rate set by household h at time t, $\Upsilon^*_{it}(h)$, is the prevailing wage rate for the household at time $t+\tau$, i.e., $\Upsilon_{it+\tau}(h)=\Upsilon^*_{it}(h)$, with probability α_i^τ, while the expected duration of a contract is $(1-\alpha_i)^{-1}$.

When a household resets its wage, it does so to maximise expected utility (9) subject to the demand for its labour (3), its budget constraint (13) and the price setting mechanism just described. For a household changing its wage rate at time t, this is equivalent to maximizing the following function with respect to $\Upsilon^*_{it}(h)$ subject to (3) and (13):[7]

$$E_t\sum_{\tau=0}^{\infty}(\alpha_i\delta)^\tau\left[\frac{\sigma}{\sigma-1}C_{it+\tau}(h)^{\frac{\sigma-1}{\sigma}}-\frac{1}{1+\mu}N_{it+\tau}(h)^{1+\mu}\right] \qquad (15)$$

[5] The asset-pricing kernel is the period-t price of a claim to one unit of currency in state s^{t+1} in period $t+1$ divided by the probability of that state occurring conditional on period-t information, $\Pr_t\left(s^{t+1}\right)$. The bond B_{it} is a random variable paying $B_{it}\left(s^{t+1}\right)$ units of currency in state s^{t+1} in period $t+1$. At time t, the household chooses the complete specification of this random variable in all states s^{t+1}. It follows that $E_t\left[Q_{t,t+1}B_{it}\right]$ is the allocation of resources to a portfolio of bonds.

[6] Note that it is an implication of (14) that monetary policy affects aggregate demand in all countries symmetrically.

[7] This differs from (9) in that implicit terms representing states where the wage to be set is not the prevailing wage are excluded.

The first-order condition becomes

$$E_t \sum_{\tau=0}^{\infty} (\alpha_i \delta)^{\tau} \left[\left(\frac{\xi_i}{1-\xi_i} N_{it+\tau}(h)^{\mu} + C_{it+\tau}(h)^{-\frac{1}{\sigma}} \frac{\Upsilon_{it}^{*}(h)}{P_{t+\tau}} \right) N_{it+\tau}(h) \right] = 0 \quad (16)$$

It follows that the monopolistically competitive household sets its wage rate so that the marginal utility of income from an extra unit of labour effort is a constant mark-up over the marginal disutility in discounted expected value terms. This captures the standard result in wage bargaining models that the market power of wages setters depends on the elasticity of labour demand (see, e.g., Blanchard and Fisher, 1989).

In the special case with flexible wages where households are allowed to reset the wage each period, the first-order condition collapses to

$$\frac{\Upsilon(h)_{it}^{*}}{P_t} = \frac{\xi_i}{\xi_i - 1} \frac{N(h)_{it}^{\mu}}{C(h)_{it}^{-\frac{1}{\sigma}}} = \frac{\xi_i}{\xi_i - 1} MRS(h)_{it} \quad (17)$$

where MRS_{it} is the marginal rate of substitution between consumption and leisure.

2.2.3 Risk-sharing

Staggered wage setting implies that households in a given country are not identical, and there is no representative household. In the general case, individual decisions will depend on initial wealth, which implies that decisions will be path dependent, cf. Obstfeld and Rogoff (1995) and Corsetti and Pesenti (2001). This causes both substantial technical problems, and problems with multiplicity of equilibria. This has been overcome in the literature either by imposing assumptions precluding wealth transfers, or by assuming that risk-sharing arrangements are in place. We choose the latter approach here. By assuming that all households in the monetary union have entered the world with the same level of wealth, the complete-markets assumption implies that they will choose the same consumption levels, i.e., risk-sharing is complete both within and betrween the member countries of the monetary union.[8] While overcoming technical problems, this assumption also serves the purpose of focusing on the implications of supply side asymmetries across member countries in a monetary union without mixing them with demand and wealth effects. National risk-sharing implies that $C_{it}(h) = C_{it}$ for all h, and therefore the h index can be dropped in what follows.

Aggregate demand for good j can now be defined as the weighted sum over countries i of (11):

$$D_{jt} = \sum_{i=1}^{I} v_i C_{ijt} = \left(\frac{P_{jt}}{v_j P_t} \right)^{-\theta} C_t \quad (18)$$

[8] This follows from the first-order conditions of the utility-maximisation problems as all households face the same asset-pricing kernel.

where

$$C_t = \sum_{i=1}^{I} v_i C_{it} \tag{19}$$

is defined as aggregate unionwide consumption. International risk-sharing implies that $C_{it} = C_{jt}$ for all i, j. Hence $C_{it} = C_t$ for all i.

In addition, the complete-markets assumption implies that the fraction $(1 - \alpha_i)$ of households in country i changing their wage rates at time t choose the same rate Υ_{it}^*. The remaining fraction α_i of households continue with the wage rate prevailing at time $t-1$ where the distribution of wage rates is unchanged. Hence, the law of motion of the aggregate wage index in country i is given by

$$W_{it} = \left[\int_0^1 \Upsilon_{it}(h)^{1-\xi_i} dh\right]^{\frac{1}{1-\xi_i}} = \left[\alpha_i W_{it-1}^{1-\xi_i} + (1-\alpha)\left(\Upsilon_{it}^*\right)^{1-\xi_i}\right]^{\frac{1}{1-\xi_i}} \tag{20}$$

2.3 Log-linear representation

In order to solve the model, it is written in log-deviations from the non-stochastic steady state. Steady-state values are indicated by omission of time subscripts, and lower-case letters denote (log-)deviations from steady-state values of corresponding upper-case variables ($x_t \equiv dX_t/X \approx \ln(X_t/X)$). Throughout, aggregate log-variables are defined as weighted averages of country-specific log-variables, i.e., for any variable x

$$x_t = \sum_{i=1}^{I} v_i x_{it} \tag{21}$$

The steady state is symmetric so that $B_i = 0$, $C_i = C = Y_i = Y$, $R = \delta^{-1} - 1$, $P_i = P$ and $W_i = W$.[9]

The Euler equation (14) becomes

$$E_t c_{t+1} = c_t + \sigma\left(r_t - E_t \pi_{t+1}\right) \tag{22}$$

where $r_t \equiv \log(1 + R_t)$ and $\pi_{t+1} = p_{t+1} - p_t$ is inflation, and aggregate demand for commodity j (18)

$$d_{jt} = -\theta\left(p_{jt} - p_t\right) + c_t \tag{23}$$

As shown in appendix A, the first-order condition for the representative household's wage-setting problem (16), the household labour demand relation (3) and the law of motion of the wage index (20) can be used to derive the following wage-setting equation for country i:

$$\omega_{it} = \Lambda_i\left[mrs_{it} - (w_{it} - p_t)\right] + \delta E_t \omega_{it+1} \tag{24}$$

[9] In general, a log-linearisation around a steady-state of $X_t = \sum_{i=1}^{I} V_i X_{it}$ gives (21) where $v_i = \frac{V_j X_j}{X}$. Symmetry of the steady state implies $v_i = V_i$, and so the sum in (21) is well-defined. Note also that a linearisation around the symmetric steady-state of (12) gives $p_t = \sum_{j=1}^{I} v_j p_{jt}$.

where $\omega_{it} = w_{it} - w_{it-1}$ is wage inflation, $mrs_{it} = \mu l_{it} + \sigma^{-1} c_t$ is the marginal rate of substitution, and Λ_i is a decreasing function of the Calvo parameter α_i and of the elasticity of substitution between labour services ξ_i:

$$\Lambda_i = \frac{(1-\alpha_i)(1-\alpha_i \delta)}{\alpha_i (1+\mu \xi_i)} \tag{25}$$

For later reference, note that Λ_i depends on both the parameter characterising wage adjustment, α_i, and the parameter determining the mark-up or market power in wage formation, ξ_i.

Finally, the supply relation (8) becomes

$$y_{it} = \beta (p_{it} - w_{it}) + u_{it} \tag{26}$$

where $\beta \equiv \gamma / (1-\gamma)$.

The supply shock is assumed to be generated by the process

$$u_{it} = \rho u_{it-1} + \varepsilon_{it} \tag{27}$$

where $-1 < \rho < 1$ and $\varepsilon_{it} \tilde{} N(0, \sigma_i^2)$. With this specification, both aggregate and country-specific shocks can be considered. In case of an aggregate shock, all innovations are identical across regions, i.e., $\varepsilon_{it} = \varepsilon_t \quad \forall i$, which implies a correlation coefficient $Corr(\varepsilon_i, \varepsilon_j) = 1 \; \forall i, j$. In case of a country-specific shock, innovations are uncorrelated across regions, i.e., $Corr(\varepsilon_i, \varepsilon_j) = 0 \; \forall i, j$, $i \neq j$. In all cases, the innovations are iid $N(0, \sigma_i^2)$.

Market clearing requires that demand for each good equals its supply. That is, for all commodities $i = 1, 2, \ldots, I$ the equilibrium conditions read

$$d_{it} = y_{it} \tag{28}$$

This implies that there is no aggregate net wealth accumulation or decumulation:

$$c_t = y_t \tag{29}$$

Accordingly, the aggregate stock of bonds is zero in equilibrium.

2.4 Monetary policy

The aim here is to consider the implications of labour market asymmetries for a given monetary policy. Therefore, a standard monetary policy reaction function is specified, namely a so-called Taylor rule, cf. Taylor (1993). Specifically, it is asumed that the interest rate is determined by

$$r_t = k_\pi \pi_t + k_y \hat{y}_t \tag{30}$$

where $\hat{y}_t = y_t - \bar{y}_t$ is the output gap. The level of 'potential' output, $\bar{y}_t$, used in this definition is the level of output under flexible wages. As shown in appendix B, it is given by

$$\bar{y}_t = \Xi u_t \tag{31}$$

where

$$\Xi = \frac{1+\mu}{1+\mu(1+\beta)+\beta\sigma^{-1}} \tag{32}$$

Note that the level of output under flexible wages is independent of monetary policy.

3 Equilibrium inflation and output

The equilibrium process for the endogenous variables can be specified explicitly by solving the model by the undetermined coefficients method (see Appendix C).[10]

Output and inflation in country i are given by

$$y_{it} = \sum_{j=1}^{I} v_j b_0^{ij} u_{jt} + \sum_{j=1}^{I} v_j b_1^{ij} u_{jt-1} + \sum_{j=1}^{I} v_j b_2^{ij} y_{jt-1} \tag{33}$$

and

$$\pi_{it} = \sum_{j=1}^{J} v_j c_0^{ij} u_{jt} + \sum_{j=1}^{J} v_j c_1^{ij} u_{jt-1} + \sum_{j=1}^{J} v_j c_2^{ij} y_{jt-1} \tag{34}$$

respectively, where $\left(b_0^{ij}, b_1^{ij}, b_2^{ij}, c_0^{ij}, c_1^{ij}, c_2^{ij}\right)$ are $6I^2$ coefficients implicitly determined by $6I^2$ equations. It follows that aggregate output and inflation are given by

$$y_t = \sum_{i=1}^{I}\sum_{j=1}^{I} v_i v_j b_0^{ij} u_{jt} + \sum_{i=1}^{I}\sum_{j=1}^{I} v_i v_j b_1^{ij} u_{jt-1} + \sum_{i=1}^{I}\sum_{j=1}^{I} v_i v_j b_2^{ij} y_{jt-1} \tag{35}$$

and

$$\pi_t = \sum_{i=1}^{I}\sum_{j=1}^{I} v_i v_j c_0^{ij} u_{jt} + \sum_{i=1}^{I}\sum_{j=1}^{I} v_i v_j c_1^{ij} u_{jt-1} + \sum_{i=1}^{I}\sum_{j=1}^{I} v_i v_j c_2^{ij} y_{jt-1} \tag{36}$$

Note that the system is in $ARMA(1,1)$ form.

3.1 Numerical illustrations

To illustrate the model's properties we shall later present numerical simulations. They are made for a two-country version of the model, i.e., the case where $I = 2$ and $i \in \{1, 2\}$. Numerical results are particularly useful since the complexity of the model makes it difficult to extract analytical results. Restricting attention to the two-country case avoids unnecessary complications, while illustrating the main properties of the model.

[10] The minimal state representation of the equilibrium is followed, cf. McCallum (1983,1999).

The log-linear version of the model is solved numerically using algorithms from the 'Toolkit' by Uhlig (1999), which solve a system of linear expectational difference equations using the method of undetermined coefficients and subsequently a QZ decomposition to solve a quadratic matrix equation that results from equating coefficients.[11] The solution is used to calculate impulse response functions and to simulate time series so that moments of the variables of interest can be calculated. These moments are taken as measures of the performance of the macroeconomic variables. Simulations are performed for country-specific shocks, imperfectly correlated shocks and unionwide shocks, i.e., for the cases $Corr(\varepsilon_1, \varepsilon_2) = 0$, $Corr(\varepsilon_1, \varepsilon_2) = 0.5$ and $Corr(\varepsilon_1, \varepsilon_2) = 1$.[12]

4 Symmetry and shock transmission

To set the scene for the subsequent discussion of heterogeneities it is useful to consider the symmetric case, i.e., the case where all countries are of equal sizes ($v_i = v$ for all i) and have the same structural parameters ($\alpha_i = \alpha$ and $\xi_i = \xi$ for all i). However, shocks are allowed to differ. This structurally symmetric case serves the purpose of clarifying the basic mechanisms operating in the model.

4.1 Shock transmission

Consider the transmission of a productivity shock specific to country i. On impact, since prices are fully flexible, this shock will tend to both increase the output of commodity i, y_i, and to decrease it's price, p_i, cf. (26). Hence, the terms of trade changes to the disfavour of country i in the sense that the relative price of its export good decreases. This, however, works to shift demand from goods produced in other parts of the monetary union towards country i's commodity, cf. (23). In turn, this shift in demand cause the price of foreign products (p_j, $j \neq i$) to fall, but not by as much as the price of commodity i. Consequently, output levels in other countries decrease, i.e., a country-specific shock induces a negative correlation in country-specific outputs via the terms-of-trade effect. The higher the correlation of shocks, the smaller the terms-of-trade effect, and the more correlated national output levels. In the limiting case of perfectly correlated shocks (aggregate shocks), country-specific outputs are perfectly correlated. Country-specific prices (inflation) are positively correlated, and more so the larger the correlation in the shocks. Figure 1 shows these effects in terms of inpulse response functions for country-specific as well as aggregate

[11] The 'Toolkit' is a suite of Matlab programs, which can be found at http://www2.wiwi.hu-berlin.de/institute/wpol/html/toolkit.htm. Matlab is documented at http://www.mathworks.com/products/matlab/. For a general exposition of the QZ decomposition, see, e.g., Golub and van Loan (1996). For implementations of the QZ methods in solving systems of expectational difference equations, see Sims (1989, 2002), Klein (2000) who also provides a discussion, and Christiano (2002).

[12] The choice of the values for the remaining parameters is inspired by the calibration in Erceg, Henderson and Levin (2000). The baseline values are $\alpha_i = 0.75$, $\xi_i = 4$, $\sigma_i^2 = 1$, $\delta = 1$, $\sigma = \frac{2}{3}$, $\mu = 1.5$, $\theta = 4$, $\gamma = 0.7$, $\rho = 0.95$, $k_\pi = 1.5$, $k_y = 0.5$.

output and inflation for the symmetric two-country case with $\alpha_1 = \alpha_2 = 0.75$ and $\xi_1 = \xi_2 = 4.$[13]

4.2 Output and inflation volatility

As mentioned above, the implications of various structural factors are assessed in terms of the standard deviations and correlations of the two key variables output and inflation. Figure 2 shows the standard deviation for country-specific as well as aggregate output and inflation as a function of $\alpha = \alpha_1 = \alpha_2$.

First note a basic smoothing effect in the sense that aggregate output is less volatile than country specific outputs (except in the limiting case of perfectly correlated shocks where they are equal). This reflects the shock transmission and the implied negative correlation in output for the member countries just described. Since country-specific inflation rates are positively correlated, there is no smoothing effect on inflation at the aggregate level.

Second, considering the effects of changing the degree of nominal rigidity, α, it is found that country-specific output volatility is strictly increasing in the degree of nominal rigidity if shocks are not too highly correlated. However aggregate output variability is hump-shaped for all three types of shocks. Consequently, there is a critical level of nominal flexibility, α^*, where aggregate output variability is increasing in α for $\alpha < \alpha^*$, and decreasing in α for $\alpha > \alpha^*$. In the numerical illustration the critical value $\alpha^*$is close to one half corresponding to expected contract lengths of two periods.

To understand the mechanism generating this hump, note that since output is generated by an $ARMA(1,1)$ process the unconditional variance (in the case of common shocks) is given as

$$VAR\left(y_t\right) = \frac{b_0 + b_1^2 + 2b_0 b_1 b_2}{1 - b_2^2} \frac{1}{1 - \rho^2} \sigma_\varepsilon^2 \qquad (37)$$

where the coefficients are all functions of the nominal rigidity parameter, α, i.e., $b_0 = b_0(\alpha)$, $b_1 = b_1(\alpha)$ and $b_2 = b_2(\alpha)$. It is seen that the variance depends on the properties of the shock, but also on the endogenous responses captured by the impact effect (b_0) and the persistence-generating mechanisms (b_1,b_2). Note that b_0 is increasing in α, while b_1 and b_2 are decreasing in α. It follows that stronger nominal rigidities (higher α) tends to increase output variability by increasing the impact effects of shocks, while it tends to lower variability by reducing the persistence in the response to shocks. These two counteracting effects create the hump shaped relation, where output variability is at first increasing and then decreasing in the nominal rigidity (α).

Figure 2 also shows the volatility of country-specific and unionwide inflation. In both cases, the variability of the inflation rate increases when α is low, and at some level it remains almost invariant to changes in α. Hence, while inflation volatility is quite sensitive to changes in the degree of nominal wage rigidity

[13] It is seen that wage rigidity is not in itself sufficient to generate substantial persistence, a well-known result.

when wages are flexible (low α), while it is insensitive to changes in the degree of nominal wage rigidity when the starting point is one with rigid wages (high α).

Finally, considering the importance of imperfect competition (the ξ parameter) in figure 3, we also find that aggregate output is less volatile than country-specific output (and identical for perfect correlation of shocks). In this case, more competition (higher ξ) implies less volatility of both aggregate output and inflation, although the effect is moderate as soon as ξ reaches some critical level ξ^*. However, country-specific output is more volatile, the higher ξ, unless the shocks are highly correlated. This is due to the implied reduction in the weight, Λ_i, put on the current relation between the marginal rate of substitition and the real wage in wage setting, cf. (25). This reduction in Λ_i leads to a weaker instantaneous reaction of wages and hence, trough its effect on marginal costs, of output. This implies a larger adjustment burden on prices and thus the terms of trade.

5 Asymmetries in size and structure

We now turn to an analysis of the implications of labour market asymmetries between the member countries of the monetary union. To clarify the issues, this proceeds in three steps. First, we consider asymmetries arising from different country sizes ($v_1 \neq v_2$), maintaining symmetric labour market structures across countries. Specifically, we consider the case of a small and a large country, corresponding to a monetary union consisting of a large core and a small periphery. Second, we consider structural asymmetries, while maintaining identical country sizes, with respect to the degree of nominal wage rigidity ($\alpha_1 \neq \alpha_2$) and the degree of competition ($\xi_1 \neq \xi_2$). Finally, we intersect the two dimensions of asymmetry, size and structure.

5.1 Different country-sizes

Figure 4 shows the standard deviations of output and inflation as a function of $\alpha = \alpha_1 = \alpha_2$ in the case where one country is small relative to the other, i.e., $v_1 = 0.3$ and $v_2 = 0.7$. In this case, we find that the level of output volatility is generally higher than when countries are of equal sizes. Moreover, aggregate output volatility is hump-shaped in nominal rigidity (α), as in the case with symmetric country sizes. However, the volatilities of the country specific output levels differ. Output volatility is generally higher in the small than in the large country, and an increase in nominal wage rigidity (an increase in α) leads to larger increases in output volatility in the small country than in the large country (unless shocks are highly correlated). This suggests that nominal rigidities may be more problematic for small countries than for large countries.

For inflation we find that the level of volatility is generally higher due to asymmetric country sizes. Both aggregate and country-specific inflation tend to be increasing in nominal wage rigidity, but the effect levels off when nominal rigidities reaches a certain level.

Qualitatively, the effects of variations in the degree of imperfect competition are the same when countries have asymmetric sizes as when they are of equal sizes. But the level of volatility is generally larger with asymmetrically sized countries.

5.2 Asymmetric labour market structures

Assessing the implications of different structures is complicated by the fact that changing the structural parameters for one country has implications not only for the specific country but also at the aggregate level, and hence there is a risk of mixing up effects arising from asymmetries with effects arising from changing the aggregate properties of the currency union. To overcome this problem and to focus on the role of asymmetries for given aggregate structural characteristics of the monetary union, we keep aggregate variables unchanged in this subsection in the sense that the weighted average of the coefficients across countries is kept constant.[14]

Consider first the degree of wage flexibility. Figure 5 shows the behaviour of the standard deviations of output and inflation for different values of α_1 when the average degree of nominal wage rigidity is restricted to be 0.5, i.e.,

$$\bar{\alpha} = v_1\alpha_1 + v_2\alpha_2 = 0.5$$

For equally sized countries, i.e., for $v_1 = v_2 = 0.5$, this implies

$$\alpha_2 = 1 - \alpha_1$$

Hence, α_2 goes from 1 to 0 as α_1 goes from 0 to 1. In other words, the labour market in country 1 becomes relatively less flexible and the labour market in country 2 more flexible when moving from left to right on the x-axis.

We find that the volatility in aggregate output is somewhat lower with asymmetries in nominal rigidities than with symmetric labour market structures, i.e., asymmetries in nominal wage flexibility contribute to lowering aggregate output volatility.[15] Similarly for inflation, we find that its volatility is reduced due to asymmetries. Interestingly, the form of the volatility of country-specific output as a function of the degree of nominal rigidity now changes and becomes hump-shaped. This suggests interesting spill-over effects in the structural parameters between the two countries. For country-specific output we find that small asymmetries may imply a larger volatility in domestic output, while larger aymmetries may cause output volatility to be lower. This suggests that if countries

[14] In the next subsection we fix the structural parameters of country 2, while allowing those of country 1 to vary, in order to analyse the incentives for unilateral reform in country 1.

[15] At the unionwide level, the standard deviations are symmetric in the degree of nominal rigidity attaining a maximum at $\alpha_1 = \alpha_2 = 0.5$. This is the point where the two countries are identical. The symmetry, of course, arises because the countries have the same size so that the restriction $\bar{\alpha} = 0.5$ implies that $\alpha_2 = 1 - \alpha_1$.

have fairly asymmetric structures the direction in which there is an incentive to change structural characteristic via reform may be ambiguous.

Figure 6 shows the results of conducting a similar exercise for the parameters characterising the degree of monopolistic competition in labour markets, ξ_i for $i = 1, 2$. The average elasticity of substitution of labour services is restricted to be 4, i.e., $\overline{\xi} = 4$, and hence

$$\xi_2 = 8 - \xi_1$$

Figure 4 shows that this form of asymmetry has essentially no effect on the volatility of domestic and aggregate output as well as inflation.

5.3 Unilateral changes in structural characteristics

While the above sheds light on the implications of structural asymmetries, it does not directly clarify the incentives for structural reforms. Such reforms are unilateral and thus lead to changes in structural parameters in one country, leaving structural parameters in other countries unchanged. Assuming that the model's labour market parameters (α_i, ξ_i) can be affected through various structural policies, it is therefore of interest to determine the direction in which a country has an incentive to direct its reforms, when the country aims at stabilising its national output and inflation.

In figure 7 we present standard deviations as a function of α_1 when α_2 is fixed at 0.75. If shocks are uncorrelated, a lower value of α_1 decreases both country 1's output and inflation volatility. In this case therefore, a country has an incentive to implement reforms that make wages more flexible. Such reforms also tend to stabilise output in country 2, but not necessarily at the aggregate level. In addition, if α_1 is initially very high, the stabilisation of country 2's output may be at the expense of a higher volatility of the country's inflation.

The picture is less clear if shocks are correlated. In this case, the volatilities of output and inflation in country 1 are hump-shaped in nominal rigidity in country 1. Hence, it is not necessarily in the interest of country 1 to implement labour market reforms inducing more flexibility. Moreover, an intermediary level of nominal flexibility in country 1 may bring about the lowest output volatility, but the highest inflation volatility in country 2.

As suggested by the results in the previous subsection, we find that unilateral reforms aimed at changing the degree of monopolistic competition in labour markets have only very limited effect on the volatility of output and inflation. It follows that no incentives to change the degree of competition in labour markets follow from a concern over stabilization of output and inflation.

5.4 Different country-sizes and labour market structures

Finally we turn to the interaction between the various forms of asymmetries. The preceeding analysis has suggested that asymmetries in size and nominal flexibility are the more important both in respect to the aggregate and country-specific performance. In the following, the case is considered in which different country sizes (one country is small, the other large) interact with different degrees of nominal rigidities. The interaction between country sizes and degrees of imperfect competition, and the interaction between degrees of nominal rigidities and degrees of competition have also been considered. However, results are not reported as no new insights are gained by investigating these combinations of asymmetries. The former combination is close to the case with different country sizes only, and the latter to the case with asymmetries in nominal wage rigidity being the only deviation from the symmetric baseline example.

Consider the case where the weight of the small country is $v_1 = 0.3$ and aggregate nominal rigidity is $\overline{\alpha} = 0.75$ Figure 8 shows volatility of output and inflation. We find that more nominal rigiditity in the small country – in combination with less nominal rigitidy in the large country – has a hump-shaped effect on aggregate output. This is the same type of result as found above. However, while the volatility of country 2 output is strictly increasing in the nominal rigidity in country 1, the volatility of country 1's output is hump-shaped if shocks are sufficienly correlated. This suggest that there may be an important negative externality from the small to the large country in the sense that more relative nominal rigidity in the small country may increase output volatility in the large country and decrease it in the small country. For inflation – both country-specific and aggregate – the volatility is hump shaped. Hence, asymmetry lowers inflation variability in this case.

6 Concluding remarks

In this paper, we have analysed the consequences of labour market asymmetries in a monetary union with focus on degrees of nominal wage rigidity and of monopolistic competition in wage setting. We have considered both aggregate and country-specific shocks and how they are propagated across member countries that may not be equal in size. Moments of country-specific as well as union-wide output and inflation have been calculated. These moments are taken as measures of the macroeconomic performance in the monetary union.

First, our results indicate that asymmetry in the sizes of member countries may in itself be an impediment for macroeconomic stability in a monetary union. In particular, the level of output and inflation volatility is generally higher when countries are of different sizes then when they are equal in size. In a monetary union consisting of a large core and a small periphery, the output in the periphery is generally more volatile than in the core. In addition, the periphery is more sensitive to changes in the degree of nominal rigidities.

Second, asymmetry in the degrees of nominal rigidity may smooth aggregate output and inflation volatility, while asymmetry in the degree of monopolitic competition has essentially no effect on the volatility of the macroeconomic variables of interest. Thus, the present analysis suggests that structural asymmetries alone are no hindrance to macroeconomic stability at the unionwide level. At the country level, however, the picture is less clear; our results indicate that there are non-trivial spill-over effect from asymmetries in nominal rigidities.

Third, when shocks are country-specific, i.e., when there are no correlation between shocks hitting the countries in the monetary union, a country unambiguously benefits in terms of macroeconomic stability by persuing unilaterally structural labour market reforms that reduce wage rigidities. For aggregate shocks hitting the whole monetary union with the same force, however, results are ambiguous.

Forth, we find that structural labour market reforms have different effects on macroeconomic stability at the country level than at the aggregate level. Hence, there is risk of a 'reform deficit' from the unionwide perspective. An individual member country may not have an incentive to reform its labour market unilaterally, while such reforms may be beneficial for the monetary union as a whole.

Given that only a few unambiguous results can be established, the incentives for reform from the point of view of individual member countries, and the desirability of such reform from the point of view of the monetary union as a whole, depend crucially on the structural characteristics of national labour markets. An interesting topic for future research, then, is to estimate this model on data for a monetary union such as the EMU. As a final remark, we emphasise that, though we implicitly assume that macroeconomic stability is desirable, our statements about the volatility of macroeconomic variables cannot literally be interpreted as statements about welfare. Hence, in future research we hope to address this issue – along with normative issues concerning monetary policy responses to structural asymmetries – in more explicit terms.

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A Log-linearization

Write the first-order condition (16) as

$$\Upsilon_{it}^{*} E_t \sum_{\tau=0}^{\infty} (\alpha_i \delta)^{\tau} \frac{C_{it+\tau}^{-\frac{1}{\sigma}}}{P_{t+\tau}} N_{it+\tau} = E_t \sum_{\tau=0}^{\infty} (\alpha_i \delta)^{\tau} \frac{\xi}{(\xi - 1)} N_{it+\tau}^{1+\mu} \tag{38}$$

Taking the differential with respect to $\Upsilon_{it+\tau}^{*}$, $C_{it+\tau}$, $P_{t+\tau}$ and $N_{it+\tau}$, evaluating at the steady-state values – Υ, C, P and N respectively – dividing through by Υ

and rearranging gives the following log-linear approximation around the steady state:

$$\begin{aligned}
\upsilon_{it}^{*} &= (1-\alpha_i\delta)\, E_t \sum_{\tau=0}^{\infty} (\alpha_i\delta)^{\tau} \left(\mu n_{it+\tau} + \sigma^{-1} c_{it+\tau} + p_{t+\tau}\right) \qquad (39) \\
&= (1-\alpha_i\delta)\, E_t \sum_{\tau=0}^{\infty} (\alpha_i\delta)^{\tau} \left[\mu\left(l_{it+\tau} - \xi\left(\upsilon_{it}^{*} - w_{it+\tau}\right)\right) + \sigma^{-1} c_{it+\tau} + p_{t+\tau}\right]
\end{aligned}$$

where the second equality follows by using a log-linear version of (3) to replace $n_{it+\tau}$. Rearranging gives

$$\upsilon_{it}^{*} = \frac{(1-\alpha_i\delta)}{1+\mu\xi}\left(\mu l_{it} - \mu\xi w_{it} + \sigma^{-1} c_{it} + p_t\right) + (\alpha_i\delta)\, E_t \upsilon_{t+1}^{*} \qquad (40)$$

Similarly, a log-linear approximation to (20) is given by

$$w_{it} = \alpha_i w_{it-1} + (1-\alpha_i)\,\upsilon_{it}^{*} \qquad (41)$$

Subtracting w_{it} from both sides of (40) and using (41) to eliminate υ_{it}^{*} gives

$$\omega_{it} = \frac{(1-\alpha_i)(1-\alpha_i\delta)}{\alpha_i(1+\mu\xi)}\left[\mu l_{it} + \sigma^{-1} c_{it} - (w_{it} - p_t)\right] + \delta E_t \omega_{it+1} \qquad (42)$$

where $\omega_{it} = w_{it} - w_{it-1}$.

B Flexible wage equilibrium

Suppose wages are flexible as well as prices. In this case, the wage equation becomes

$$w_{it} = p_t + \mu l_{it} + \sigma^{-1} c_t \qquad (43)$$

Substituting out l_{it} by a linear version of the production function (6) gives

$$w_{it} = p_t + \mu\left(\frac{1}{\gamma} y_{it} - \frac{1-\gamma}{\gamma} u_{it}\right) + \sigma^{-1} c_t \qquad (44)$$

implying

$$w_t = p_t + \mu\left(\frac{1}{\gamma} y_t - \frac{1-\gamma}{\gamma} u_t\right) + \sigma^{-1} y_t \qquad (45)$$

Inserting this in aggregated supply

$$y_t = \beta\,(p_t - w_t) + u_t \qquad (46)$$

gives

$$\bar{y}_t = \beta\left(-\mu\left(\frac{1}{\gamma}\bar{y}_t - \frac{1-\gamma}{\gamma} u_t\right) - \sigma_t^{-1}\bar{y}\right) + u_t \qquad (47)$$

or

$$\bar{y}_t = \frac{1+\mu}{1+\mu(1+\beta)+\beta\sigma^{-1}} u_t \qquad (48)$$

C Sticky wage equilibrium

Imposing the equilibrium condition means $c_{it} = c_t = y_t$. Using this, the wage equation (24) and the aggregate supply relation (26) can be combined to give the 'AS' relation

$$\begin{aligned} & \pi_{it} + \beta^{-1}\left[(u_{it} - u_{it-1}) - (y_{it} - y_{it-1})\right] \\ = \; & \Lambda_i \left[\left(\mu + (1+\mu)\beta^{-1} + \theta^{-1}\right) y_{it} + \left(\sigma - \theta^{-1}\right) y_t - (1+\mu)\beta^{-1} u_{it}\right] \\ & + \delta\left(E_t \pi_{it+1} + \beta^{-1}\left[(\rho - 1) u_{it} - (E_t y_{it+1} - y_{it})\right]\right) \end{aligned} \tag{49}$$

where a log-linear version of (6) has been used to substitute out l_{it}. Similarly, combining the Euler equation (22), the Taylor rule (30) and the intratemporal demand function (23) gives the 'IS' relation

$$\begin{aligned} & y_{it} - y_{it-1} \\ = \; & -\theta(\pi_{it} - \pi_t) + E_t y_{t+1} \\ & -\sigma\left(k_\pi \pi_t + k_y (y_t - \Xi u_t) - E_t \pi_{t+1}\right) - y_{t-1} \end{aligned} \tag{50}$$

Hence, two equations summarise the dynamics of output and inflation for each country. Disturbances to the system follow from the stochastic process

$$u_{it} = \rho u_{it-1} + \varepsilon_{it} \tag{51}$$

We guess that output and inflation in country i take the forms:

$$y_{it} = \sum_j v_j b_0^{ij} u_{jt} + \sum_j v_j b_1^{ij} u_{jt-1} + \sum_j v_j b_2^{ij} y_{jt-1} \tag{52}$$

and

$$\pi_{it} = \sum_j v_j c_0^{ij} u_{jt} + \sum_j v_j c_1^{ij} u_{jt-1} + \sum_j v_j c_2^{ij} y_{jt-1} \tag{53}$$

These conjectures imply the following expressions for aggregate output and inflation:

$$y_t = \sum_i \sum_j v_i v_j b_0^{ij} u_{jt} + \sum_i \sum_j v_i v_j b_1^{ij} u_{jt-1} + \sum_i \sum_j v_i v_j b_2^{ij} y_{jt-1} \tag{54}$$

$$\pi_t = \sum_i \sum_j v_i v_j c_0^{ij} u_{jt} + \sum_i \sum_j v_i v_j c_1^{ij} u_{jt-1} + \sum_i \sum_j v_i v_j c_2^{ij} y_{jt-1} \tag{55}$$

In addition, expectations become

$$E_t y_{it+1} = \sum_j v_j \left(b_0^{ij}\rho + b_1^{ij}\right) u_{jt} + \sum_j v_j b_2^{ij} y_{jt} \tag{56}$$

$$E_t \pi_{it+1} = \sum_j v_j \left(c_0^{ij}\rho + c_1^{ij}\right) u_{jt} + \sum_j v_j c_2^{ij} y_{jt} \tag{57}$$

$$E_t y_{t+1} = \sum_i \sum_j v_i v_j \left(b_0^{ij}\rho + b_1^{ij}\right) u_{jt} + \sum_i \sum_j v_i v_j b_2^{ij} y_{jt} \tag{58}$$

$$E_t \pi_{t+1} = \sum_i \sum_j v_i v_j \left(c_0^{ij}\rho + c_1^{ij}\right) u_{jt} + \sum_i \sum_j v_i v_j c_2^{ij} y_{jt} \tag{59}$$

To verify our conjectures, we find values of the coefficients $\left(b_0^{ij}, b_1^{ij}, b_2^{ij}, c_0^{ij}, c_1^{ij}, c_2^{ij}\right)$ that satisfy the restrictions imposed by the log-linear model. Inserting the conjectures in (49) gives

$$\begin{aligned}
&\sum_j v_j c_0^{ij} u_{jt} + \sum_j v_j c_1^{ij} u_{jt-1} + \sum_j v_j c_2^{ij} y_{jt-1} \\
&+\beta^{-1}\left(u_{it} - u_{it-1}\right) - \beta^{-1}\left(\sum_j v_j b_0^{ij} u_{jt} + \sum_j v_j b_1^{ij} u_{jt-1} + \sum_j v_j b_2^{ij} y_{jt-1} - y_{it-1}\right) \\
= \; &\Lambda_i\left(\mu + (1+\mu)\beta^{-1} + \theta^{-1}\right)\left(\sum_j v_j b_0^{ij} u_{jt} + \sum_j v_j b_1^{ij} u_{jt-1} + \sum_j v_j b_2^{ij} y_{jt-1}\right) \\
&+\Lambda_i\left(\sigma - \theta^{-1}\right)\left(\sum_n \sum_j v_n v_j b_0^{nj} u_{jt} + \sum_n \sum_j v_n v_j b_1^{nj} u_{jt-1} + \sum_n \sum_j v_n v_j b_2^{nj} y_{jt-1}\right) \\
&-(1+\mu)\beta^{-1}\Lambda_i u_{it} + \delta \sum_j v_j \left(c_0^{ij}\rho + c_1^{ij}\right) u_{jt} \\
&+\delta \sum_n v_n c_2^{in}\left(\sum_j v_j b_0^{nj} u_{jt} + \sum_j v_j b_1^{nj} u_{jt-1} + \sum_j v_j b_2^{nj} y_{jt-1}\right) \\
&+\delta\beta^{-1}(\rho - 1) u_{it} - \delta\beta^{-1} \sum_j v_j \left(b_0^{ij}\rho + b_1^{ij}\right) u_{jt} \\
&-\delta\beta^{-1} \sum_j v_j b_2^{ij}\left(\sum_n v_n b_0^{jn} u_{nt} + \sum_n v_n b_1^{jn} u_{nt-1} + \sum_n v_n b_2^{jn} y_{nt-1}\right) \\
&+\delta\beta^{-1}\left(\sum_j v_j b_0^{ij} u_{jt} + \sum_j v_j b_1^{ij} u_{jt-1} + \sum_j v_j b_2^{ij} y_{jt-1}\right)
\end{aligned} \tag{60}$$

Equating coefficients on u_{it} gives the restriction

$$
\begin{aligned}
& v_i c_0^{ii} + \beta^{-1} - \beta^{-1} v_i b_0^{ii} \\
= \quad & \Lambda_i \left(\mu + (1+\mu)\beta^{-1} + \theta^{-1}\right) v_i b_0^{ii} + \Lambda_i \left(\sigma - \theta^{-1}\right) \sum_j v_j v_i b_0^{ji} \\
& - (1+\mu)\beta^{-1}\Lambda_i + \delta v_i \left(c_0^{ii}\rho + c_1^{ii}\right) + \delta \sum_j v_j v_i c_2^{ij} b_0^{ji} \\
& + \delta\beta^{-1}(\rho - 1) - \delta\beta^{-1} v_i \left(b_0^{ii}\rho + b_1^{ii}\right) \\
& - \delta\beta^{-1} \sum_j v_j v_i b_2^{ij} b_0^{ji} + \delta\beta^{-1} v_i b_0^{ii} \qquad (61)
\end{aligned}
$$

Equating coefficients on u_{jt} where $j \neq i$ gives

$$
\begin{aligned}
& c_0^{ij} - \beta^{-1} b_0^{ij} \\
= \quad & \Lambda_i \left(\mu + (1+\mu)\beta^{-1} + \theta^{-1}\right) b_0^{ij} + \Lambda_i \left(\sigma - \theta^{-1}\right) \sum_n v_n b_0^{nj} \\
& + \delta \left(c_0^{ij}\rho + c_1^{ij}\right) + \delta \sum_n v_n c_2^{in} b_0^{nj} \\
& - \delta\beta^{-1} \left(b_0^{ij}\rho + b_1^{ij}\right) - \delta\beta^{-1} \sum_n v_n b_2^{in} b_0^{nj} + \delta\beta^{-1} b_0^{ij} \qquad (62)
\end{aligned}
$$

Equating coefficients on u_{it-1}:

$$
\begin{aligned}
& v_i c_1^{ii} - \beta^{-1} - \beta^{-1} v_i b_1^{ii} \\
= \quad & \Lambda_i \left(\mu + (1+\mu)\beta^{-1} + \theta^{-1}\right) v_i b_1^{ii} \\
& + \Lambda_i \left(\sigma - \theta^{-1}\right) \sum_n v_n v_i b_1^{ni} + \delta \sum_n v_n v_i c_2^{in} b_1^{ni} \\
& - \delta\beta^{-1} \sum_n v_n v_i b_2^{in} b_1^{ni} + \delta\beta^{-1} v_i b_1^{ii} \qquad (63)
\end{aligned}
$$

Equating coefficients on $u_{jt-1}$where $j \neq i$:

$$
\begin{aligned}
& c_1^{ij} - \beta^{-1} b_1^{ij} \\
= \quad & \Lambda_i \left(\mu + (1+\mu)\beta^{-1} + \theta^{-1}\right) b_1^{ij} \\
& + \Lambda_i \left(\sigma - \theta^{-1}\right) \sum_n v_n b_1^{nj} + \delta \sum_n v_n c_2^{in} b_1^{nj} \\
& - \delta\beta^{-1} \sum_n v_n b_2^{in} b_1^{nj} + \delta\beta^{-1} b_1^{ij} \qquad (64)
\end{aligned}
$$

Equating coefficients on y_{it-1} gives

$$
\begin{aligned}
& v_i c_2^{ii} - \beta^{-1} \left(v_i b_2^{ii} - 1\right) \\
= \quad & \Lambda_i \left(\mu + (1+\mu)\beta^{-1} + \theta^{-1}\right) v_i b_2^{ii} + \Lambda_i \left(\sigma - \theta^{-1}\right) \sum_n v_n v_i b_2^{ni} \\
& + \delta \sum_n v_n v_i c_2^{in} b_2^{ni} - \delta\beta^{-1} \sum_n v_n v_i b_2^{in} b_2^{ni} + \delta\beta^{-1} v_i b_2^{ii} \qquad (65)
\end{aligned}
$$

and on $y_{jt-1}$where $j \neq i$:

$$\begin{aligned}
& v_j c_2^{ij} - \beta^{-1} v_j b_2^{ij} \\
= \quad & \Lambda_i \left(\mu + (1+\mu)\beta^{-1} + \theta^{-1}\right) v_j b_2^{ij} + \Lambda_i \left(\sigma - \theta^{-1}\right) \sum_n v_n v_j b_2^{nj} \\
& + \delta \sum_n v_n v_j c_2^{in} b_2^{nj} - \delta \beta^{-1} \sum_n v_n v_j b_2^{in} b_2^{nj} + \delta \beta^{-1} v_j b_2^{ij} \qquad (66)
\end{aligned}$$

Inserting conjectures in (50) gives

$$\begin{aligned}
& \sum_j v_j b_0^{ij} u_{jt} + \sum_j v_j b_1^{ij} u_{jt-1} + \sum_j v_j b_2^{ij} y_{jt-1} - y_{it-1} \\
= \quad & -\theta \left(\sum_j v_j c_0^{ij} u_{jt} + \sum_j v_j c_1^{ij} u_{jt-1} + \sum_j v_j c_2^{ij} y_{jt-1} \right) \\
& + (\theta - \sigma k_\pi) \left(\sum_n \sum_j v_n v_j c_0^{nj} u_{jt} + \sum_n \sum_j v_n v_j c_1^{nj} u_{jt-1} + \sum_n \sum_j v_n v_j c_2^{nj} y_{jt-1} \right) \\
& + \sum_n \sum_j v_n v_j \left(b_0^{nj} \rho + b_1^{nj} \right) u_{jt} \\
& + \sum_m \sum_n v_m v_n b_2^{mn} \left(\sum_j v_j b_0^{nj} u_{jt} + \sum_j v_j b_1^{nj} u_{jt-1} + \sum_j v_j b_2^{nj} y_{jt-1} \right) \\
& - \sigma k_y \left(\sum_n \sum_j v_n v_j b_0^{nj} u_{jt} + \sum_n \sum_j v_n v_j b_1^{nj} u_{jt-1} + \sum_n \sum_j v_n v_j b_2^{nj} y_{jt-1} \right) + \sigma k_y \Xi \sum_j v_j u_{jt} \\
& + \sigma \sum_n \sum_j v_n v_j \left(c_0^{nj} \rho + c_1^{nj} \right) u_{jt} \\
& + \sigma \sum_m \sum_n v_m v_n c_2^{mn} \left(\sum_j v_j b_0^{nj} u_{jt} + \sum_j v_j b_1^{nj} u_{jt-1} + \sum_j v_j b_2^{nj} y_{jt-1} \right) - \sum_j v_j y_{jt-1} \qquad (67)
\end{aligned}$$

Equating coefficients on u_{jt} gives

$$\begin{aligned}
& b_0^{ij} \\
= \quad & -\theta c_0^{ij} + (\theta - \sigma k_\pi) \sum_n v_n c_0^{nj} + \sum_n v_n \left(b_0^{nj} \rho + b_1^{nj} \right) \\
& + \sum_m \sum_n v_m v_n b_2^{mn} b_0^{nj} - \sigma k_y \sum_n v_n b_0^{nj} + \sigma k_y \Xi v_j + \sigma \sum_n v_n \left(c_0^{nj} \rho + c_1^{nj} \right) \\
& + \sigma \sum_m \sum_n v_m v_n c_2^{mn} b_0^{nj} \qquad (68)
\end{aligned}$$

On u_{jt-1}:

$$\begin{aligned} & b_1^{ij} \\ = \quad & -\theta c_1^{ij} + (\theta - \sigma k_\pi) \sum_n v_n c_1^{nj} + \sum_m \sum_n v_m v_n b_2^{mn} b_1^{nj} \\ & -\sigma k_y \sum_n v_n b_1^{nj} + \sigma \sum_m \sum_n v_m v_n c_2^{mn} b_1^{nj} \end{aligned} \tag{69}$$

y_{it-1}:

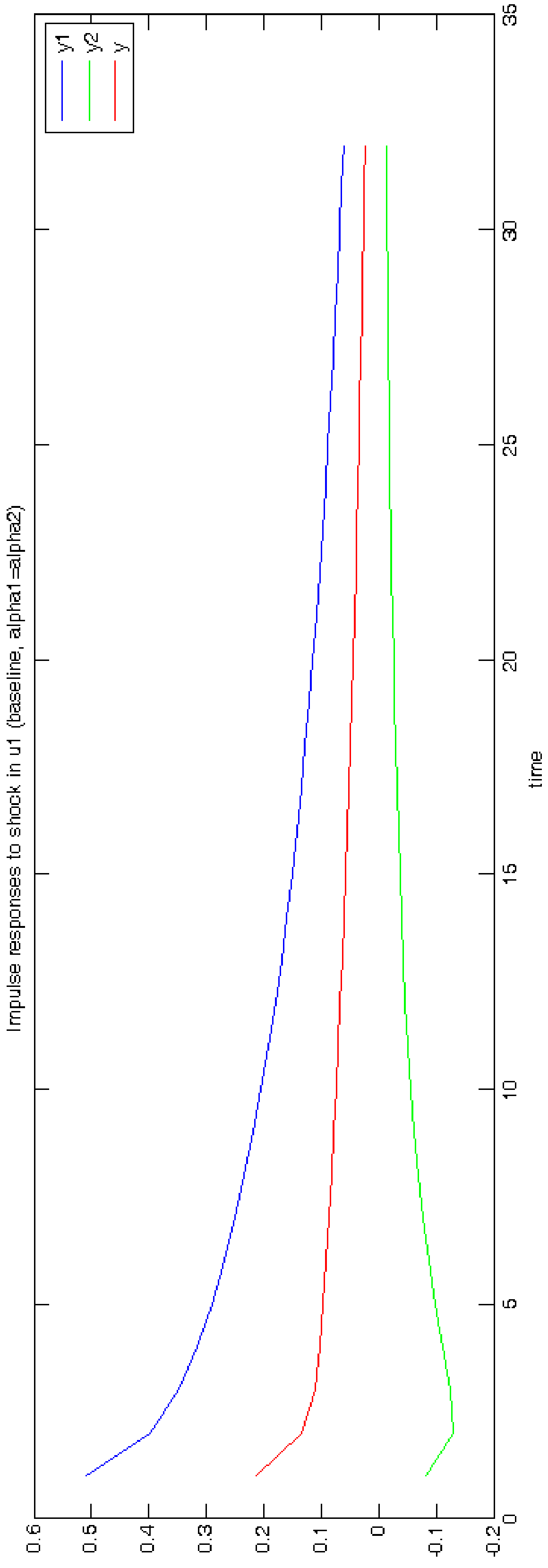
$$\begin{aligned} & v_i b_2^{ii} - 1 \\ = \quad & -\theta v_i c_2^{ii} + (\theta - \sigma k_\pi) \sum_n v_n v_i c_2^{ni} + \sum_m \sum_n v_m v_n b_2^{mn} v_i b_2^{ni} \\ & -\sigma k_y \sum_n v_n v_i b_2^{ni} + \sigma \sum_m \sum_n v_m v_n c_2^{mn} v_i b_2^{ni} - v_i \end{aligned} \tag{70}$$

and finally, y_{jt-1} where $j \neq i$:

$$\begin{aligned} & b_2^{ij} \\ = \quad & -\theta c_2^{ij} + (\theta - \sigma k_\pi) \sum_n v_n c_2^{nj} + \sum_m \sum_n v_m v_n b_2^{mn} b_2^{nj} \\ & -\sigma k_y \sum_n v_n b_2^{nj} + \sigma \sum_m \sum_n v_m v_n c_2^{mn} b_2^{nj} - 1 \end{aligned} \tag{71}$$

The restrictions (61)-(66) and (68)-(71) constitute a system of $6I^2$ equations determining the $6I^2$ coefficients in the conjectures. Indeed, this system is recursive. The $2I^2$ restrictions from equating coefficients on y_{it-1} may be combined to solve for $\{b_2^{ij}, c_2^{ij}\}_{ij}$, which may then be used in the $2I^2$ restrictions from u_{it-1} to solve for $\{b_1^{ij}, c_1^{ij}\}_{ij}$. Finally, these coefficients may be used in the restrictions from equation coefficients on u_{it} to find the remaining $2I^2$ coefficients $\{b_0^{ij}, c_0^{ij}\}_{ij}$.

Figure 1


Impulse responses to shock in u1 (baseline, alpha1=alpha2)
y1
y2
y
time
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
5
10
15
20
25
30
35

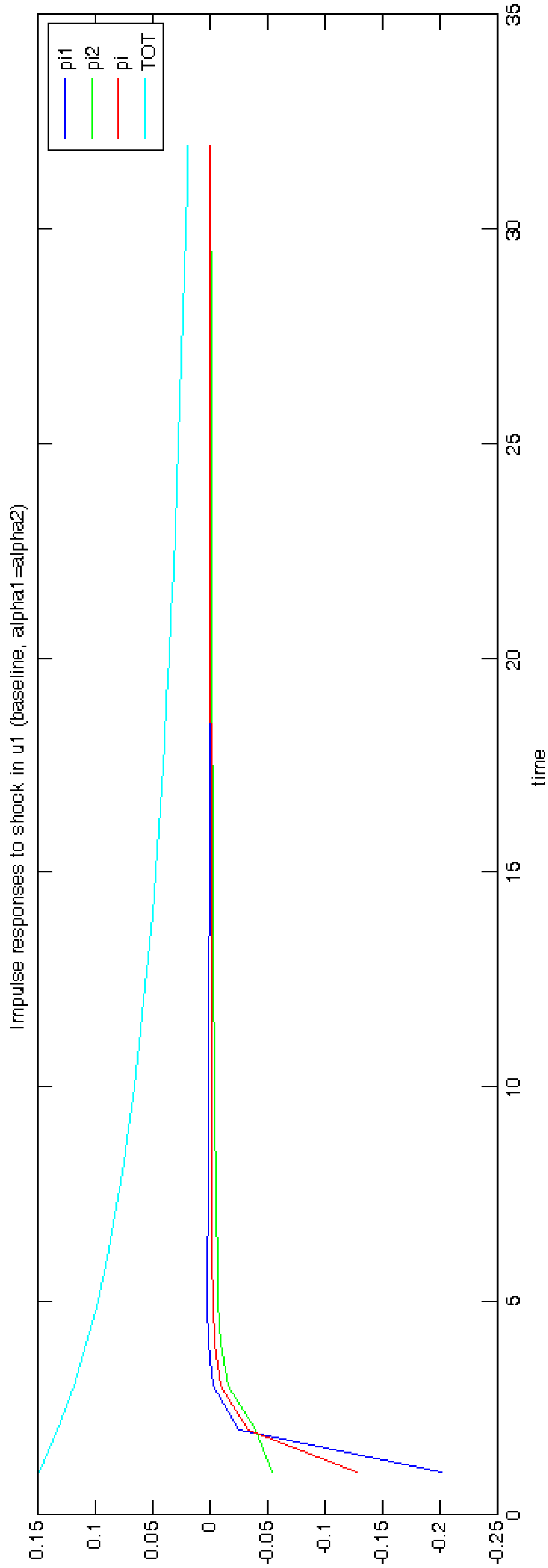

Impulse responses to shock in u1 (baseline, alpha1=alpha2)
pi1
pi2
pi
TOT
time
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
5
10
15
20
25
30
35

Figure 2

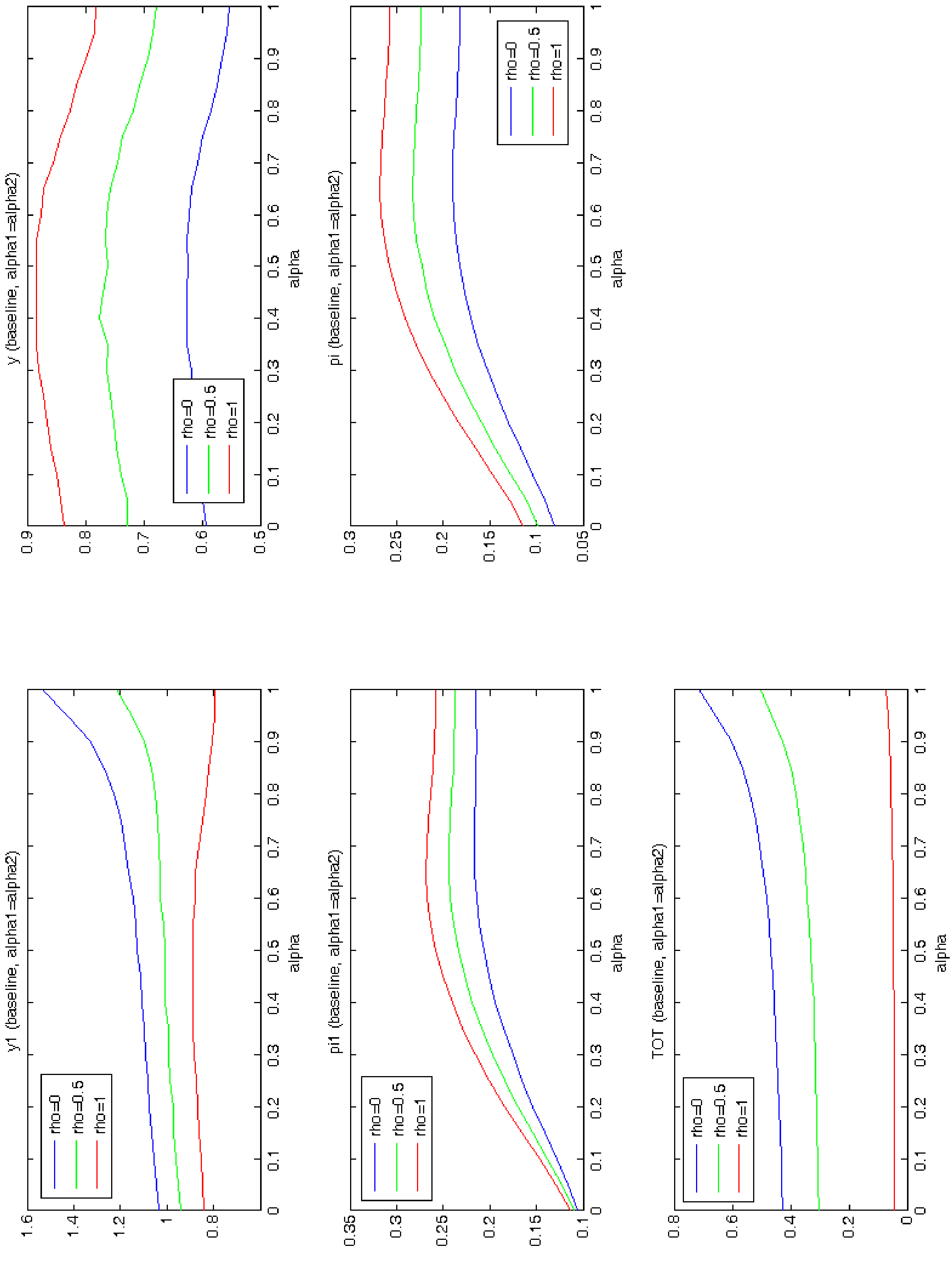

y (baseline, alpha1=alpha2)
pi (baseline, alpha1=alpha2)
y1 (baseline, alpha1=alpha2)
pi1 (baseline, alpha1=alpha2)
TOT (baseline, alpha1=alpha2)
alpha
rho=0
rho=0.5
rho=1

Figure 3

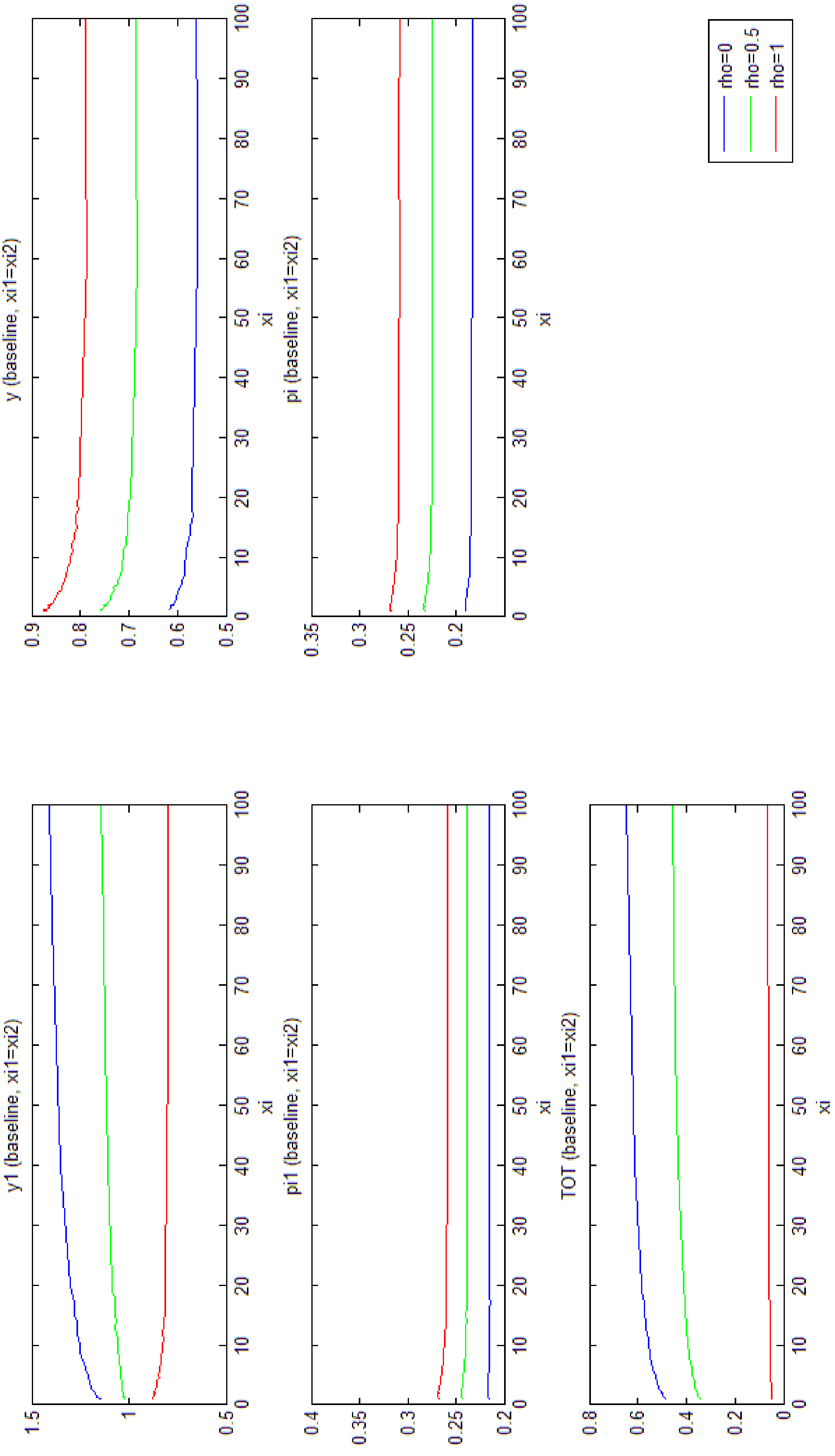

y (baseline, xi1=xi2)
pi (baseline, xi1=xi2)
y1 (baseline, xi1=xi2)
pi1 (baseline, xi1=xi2)
TOT (baseline, xi1=xi2)
xi
rho=0
rho=0.5
rho=1

Figure 4

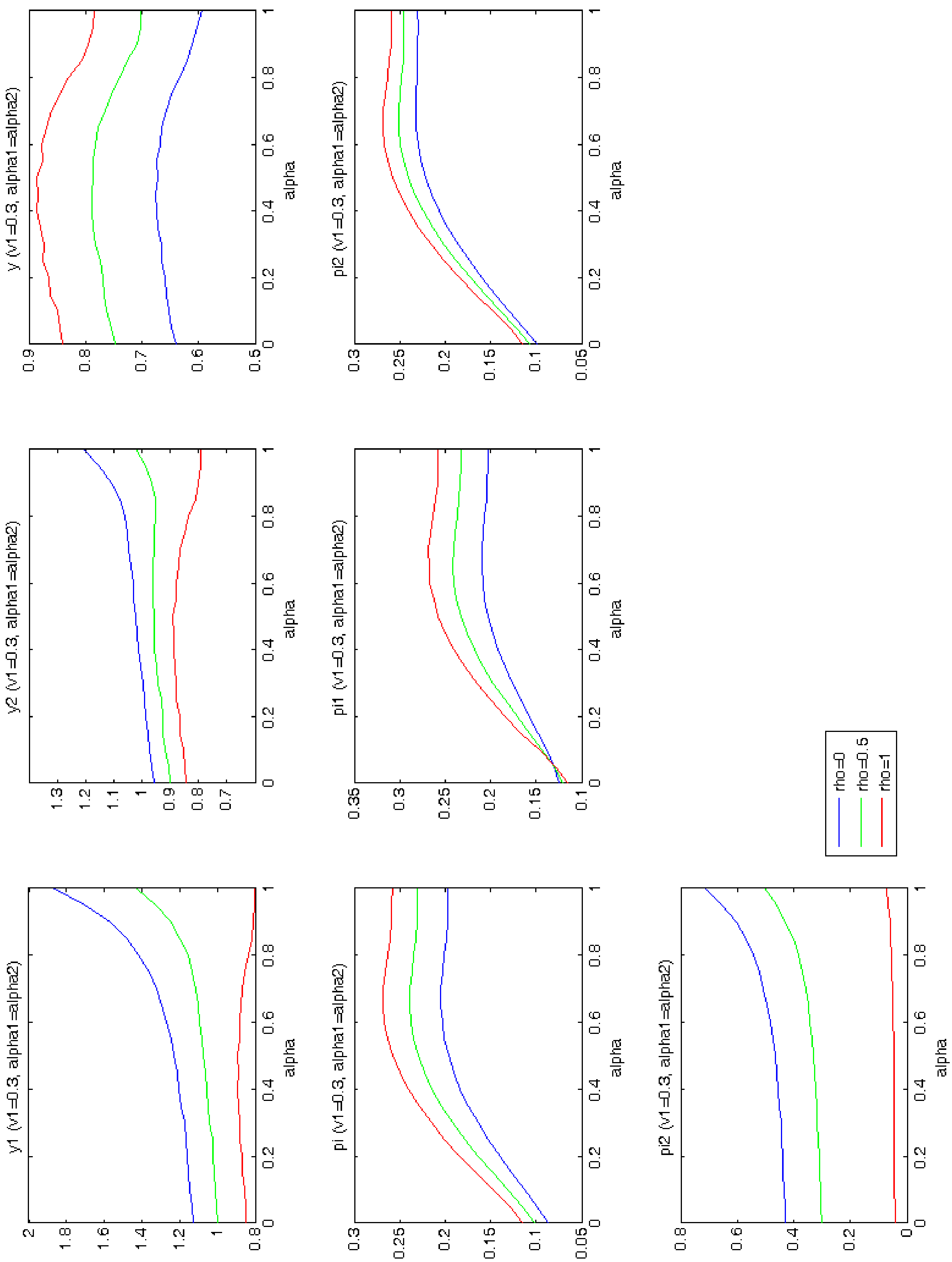
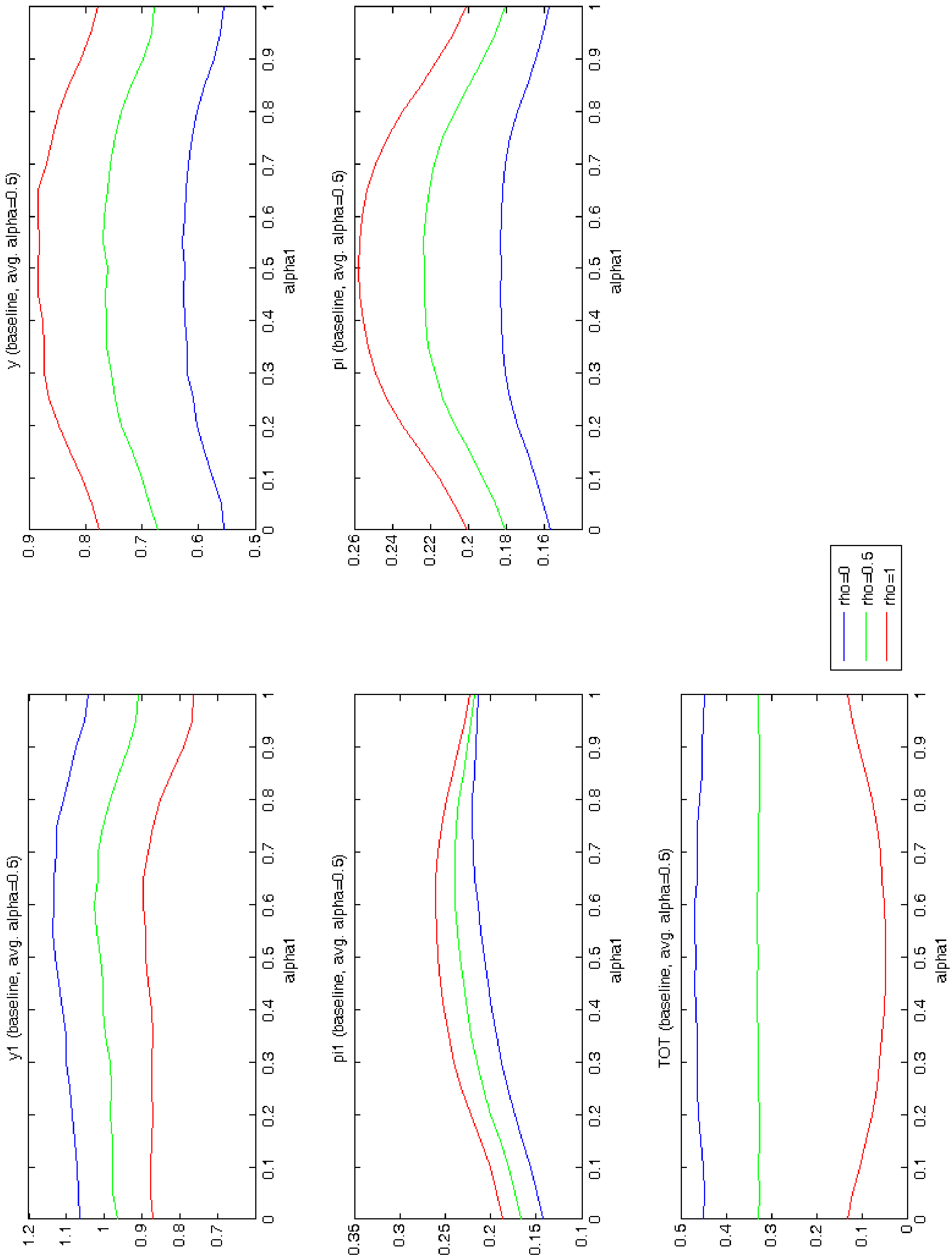
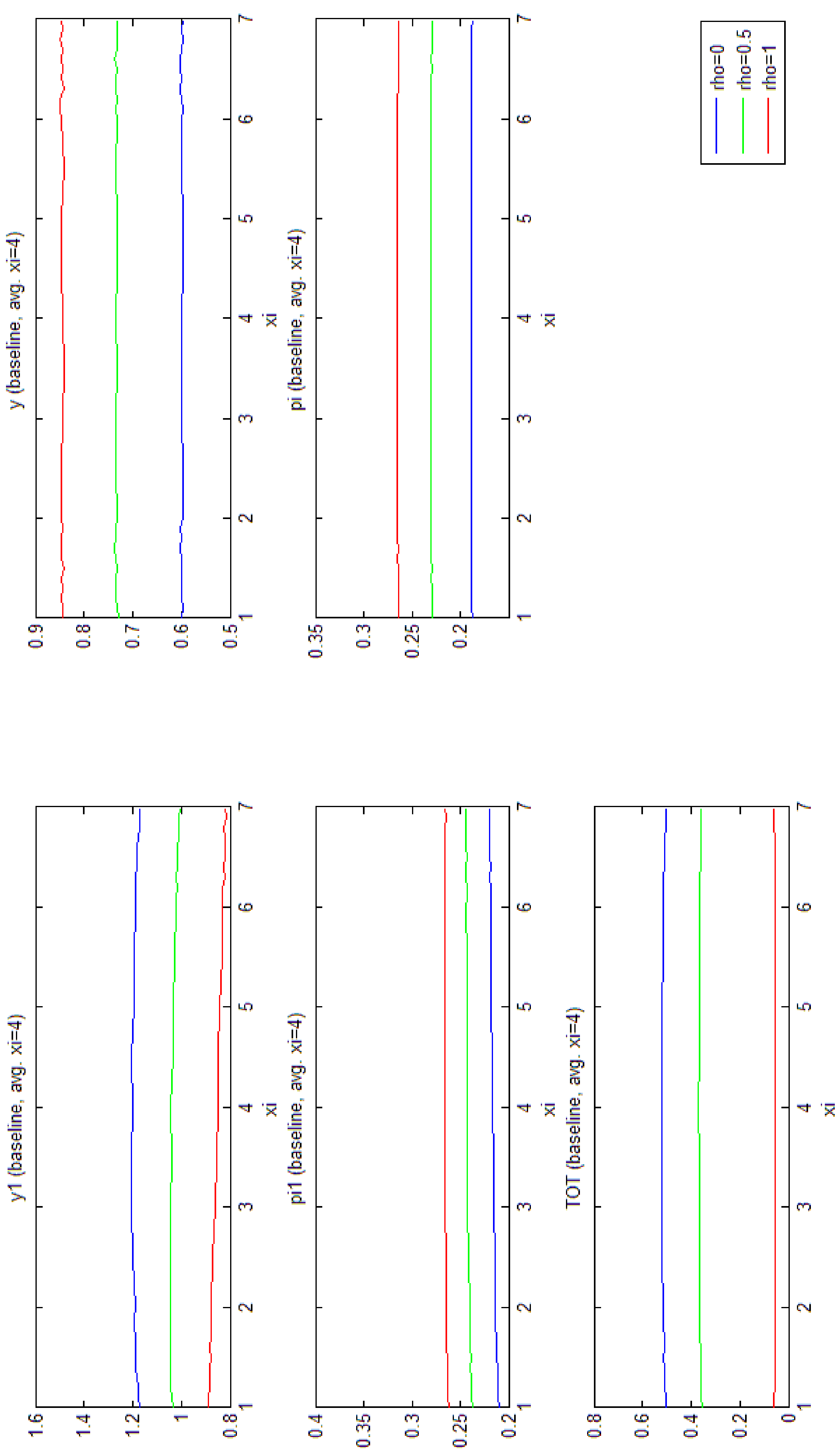

y1 (v1=0.3, alpha1=alpha2)
y2 (v1=0.3, alpha1=alpha2)
y (v1=0.3, alpha1=alpha2)
pi (v1=0.3, alpha1=alpha2)
pi1 (v1=0.3, alpha1=alpha2)
pi2 (v1=0.3, alpha1=alpha2)
pi2 (v1=0.3, alpha1=alpha2)
alpha
rho=0
rho=0.5
rho=1

Figure 5



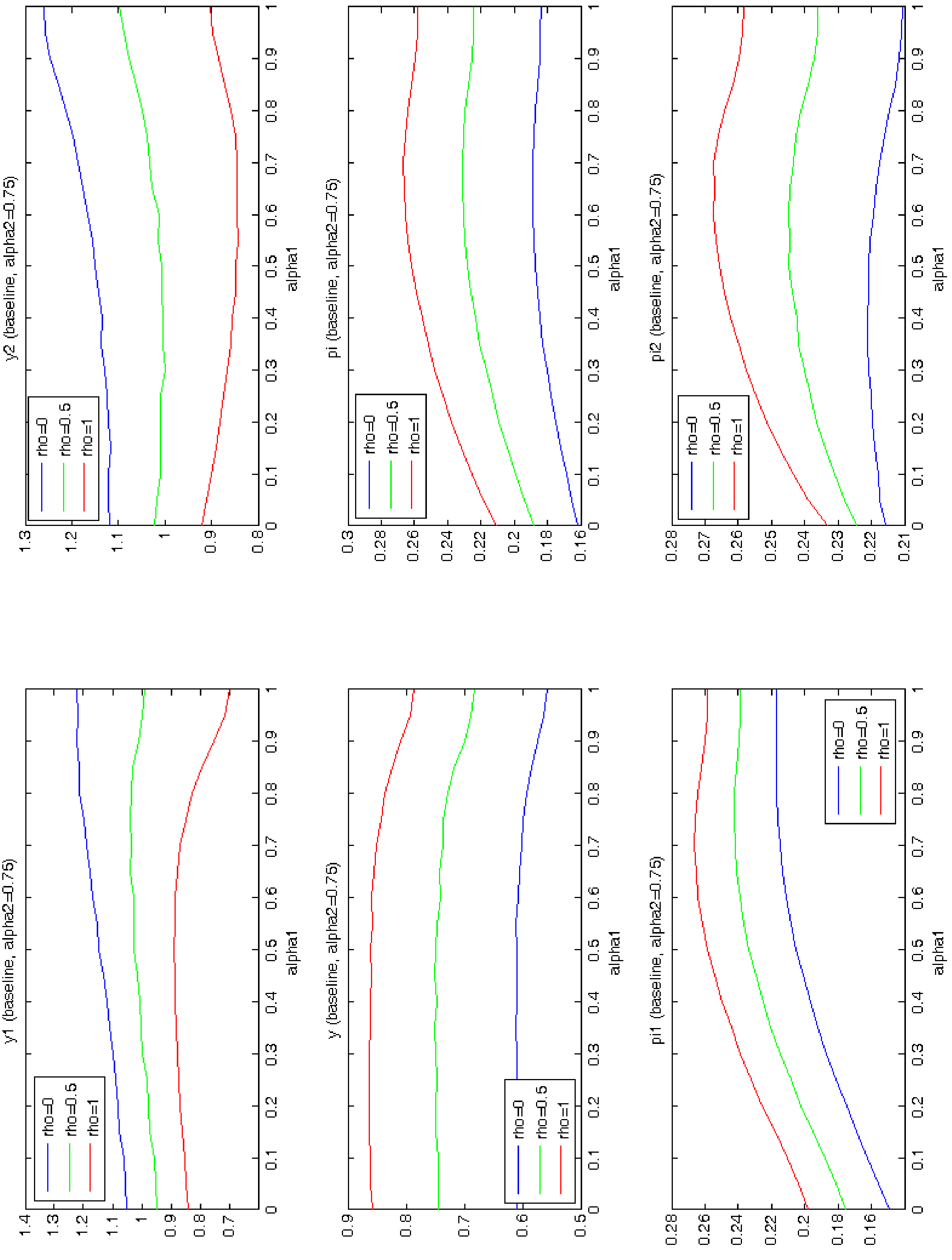
y (baseline, avg. alpha=0.5)
pi (baseline, avg. alpha=0.5)
y1 (baseline, avg. alpha=0.5)
pi1 (baseline, avg. alpha=0.5)
TOT (baseline, avg. alpha=0.5)
alpha1
rho=0
rho=0.5
rho=1

Figure 6



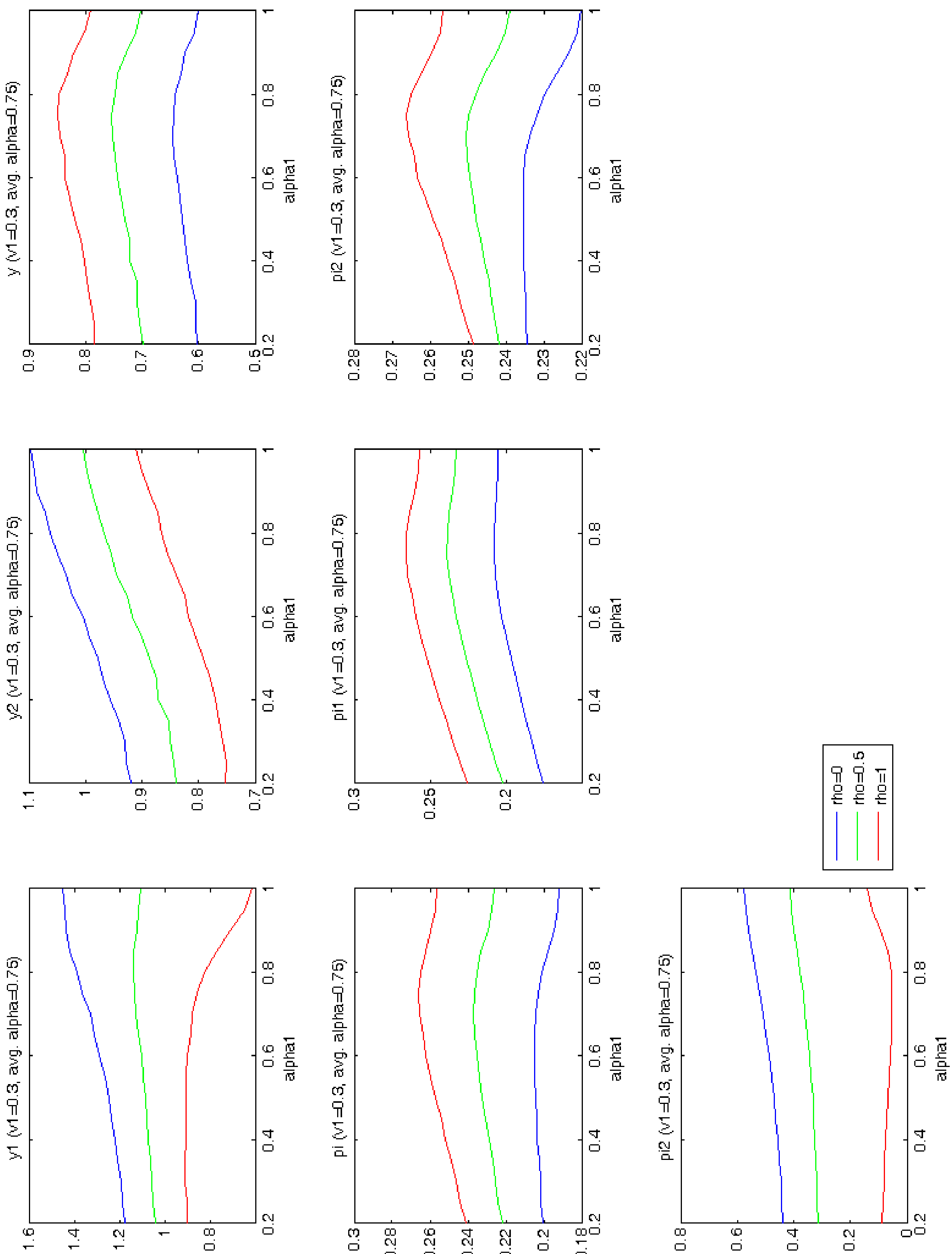
y (baseline, avg. xi=4)
pi (baseline, avg. xi=4)
y1 (baseline, avg. xi=4)
pi1 (baseline, avg. xi=4)
TOT (baseline, avg. xi=4)
xi
rho=0
rho=0.5
rho=1

Figure 7



y1 (baseline, alpha2=0.75)
y2 (baseline, alpha2=0.75)
y (baseline, alpha2=0.75)
pi (baseline, alpha2=0.75)
pi1 (baseline, alpha2=0.75)
pi2 (baseline, alpha2=0.75)
alpha1
rho=0
rho=0.5
rho=1

Figure 8



y1 (v1=0.3, avg. alpha=0.75)
y2 (v1=0.3, avg. alpha=0.75)
y (v1=0.3, avg. alpha=0.75)
pi (v1=0.3, avg. alpha=0.75)
pi1 (v1=0.3, avg. alpha=0.75)
pi2 (v1=0.3, avg. alpha=0.75)
pi2 (v1=0.3, avg. alpha=0.75)
alpha1
rho=0
rho=0.5
rho=1