Factor Price Equality and Biased Technical Change
in a Two-Cone Trade Model*

by

Daniel Becker and Erich Gundlach

Rostock University and Kiel Institute for the World Economy

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Daniel Becker
Department of Economics
Rostock University
Ulmenstr. 69,
18057 Rostock, Germany,
daniel.becker@uni-rostock.de

Erich Gundlach (Corresponding author)
Kiel Institute for the World Economy
P.O. Box 4309,
24100 Kiel, Germany
erich.gundlach@ifw-kiel.de

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ABSTRACT

We reconsider the effects of long-run economic growth on relative factor prices across cones of specialization. We model economic growth as exogenous technical change. Allowing for capital biased technical change with a sector bias and for endogenous commodity prices, we find that economic growth may increase or decrease factor price differences across cones. For a neutral demand side and capital biased growth in the most capital intensive sector, we find that economic growth encourages less factor price diversity across cones.

I. Introduction

The relationship between international trade and economic growth has always been of major interest in economic research. A recent paper by Deardorff (2001) addresses this topic by combining several neoclassical growth models with a two-cone Heckscher Ohlin (HO) model of international trade. Deardorff (2001) models growth as based on factor accumulation and thus considers which assumption about saving behavior might lead to factor price equalization (FPE) across countries with "large" initial factor endowment differences. He finds that neither an exogenous saving-income ratio (Solow), nor an exogenous saving-profit ratio (classical saving function), nor utility maximizing saving with infinite (Ramsey, Stiglitz) or finite life (Diamond) suggest that the capital stock in the labor abundant country would grow faster than in the capital abundant country. Hence alternative neoclassical growth models based on factor accumulation imply that factor endowments are unlikely to converge into a single cone with FPE.

We use a different starting point for modeling economic growth. We consider long-run growth as based on finite exogenous technical change, as in the seminal trade-and-growth paper by Findlay and Grubert (1959). Findlay and Grubert (1959) use the Lerner diagram to show how growth in the form of finite exogenous technical change affects relative factor prices in a one-cone model of international trade. The subsequent discussion in the literature has mainly proceeded either within growth models with two goods or within higher dimensional trade models without sector biased or factor biased growth.[1] More recently, Findlay and Jones (2000) have used finite technological improvements to model the effects of a major technological innovation on relative factor prices. Our contribution is to consider what a finite-size modeling of exogenous technical change implies for FPE across countries in a two-cone trade model once the impact of technical change on relative commodity prices is taken into account.

With our modeling of growth, we find that growth does not encourage FPE if technical change is Hicks neutral and simultaneously occurs in all sectors and if relative commodity prices remain unchanged. A more general result follows from our modeling of growth if we allow for capital biased technical change with a sector bias and for endogenous commodity

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[1] For surveys of this literature, see Findlay (1984) and Ethier (1984). For the effects of sector bias and factor bias of technical change on the interaction between trade and wages in the case of two goods, see Xu (2001).

prices. Depending on specific assumptions about technology and preferences, we then find that economic growth may increase or decrease factor price differences across cones of specialization.

We develop our argument in two steps. We first discuss how the two-cone equilibrium in the Lerner diagram is affected by capital biased and sector biased technical change. We then consider the general equilibrium effects of technical change on relative commodity prices when assessing the possibilities for FPE across cones, thereby following Krugman (2000). We demonstrate the qualitative robustness of our findings by using a numerical example for a given set of assumptions about technology and preferences. We conclude that a growth process which is driven by factor biased technical change in the most capital intensive sector may bring the countries of the world closer together in terms of their effective factor endowments to permit less global factor price diversity.

II. Two Cones in the Lerner Diagram

Figure 1 is a Lerner diagram, where the curves labeled X_i (i = 1,2,3) are the unit value isoquants of the three goods that are produced with two factors of production, capital K and labor L, at given commodity prices p_i. The three isoquants represent alternative technologies for producing one euro's worth of output. With $X_i = 1/p_i$, the relative position of an isoquant depends on the commodity prices and on the different techniques that are employed to produce the three goods.

Figure 1 illustrates a case where adding one good to the traditional HO setup with two goods and two factors of production leads to an equilibrium without FPE. Put differently, the world economy illustrated in Figure 1 cannot reproduce the equilibrium of a hypothetical integrated world economy (IWE), which would have perfect factor mobility.[2] As it is drawn, this outcome reflects a too diverse allocation of world factor endowments across countries for given levels of technology. More generally, the diversity in factor endowments that would be compatible with FPE was shown to be smaller for the case with three goods and two factors than in the traditional HO-model with two goods and two factors (Deardorff 1994).

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[2] The analytical concept of an integrated world economy (IWE) with perfect mobility of factors and goods is discussed in Dixit and Norman (1980), including the conditions for free trade in goods being sufficient to reproduce the IWE.

In Figure 1, there is a free-trade equilibrium with two cones of diversification and two different sets of factor prices, where w_j is the wage rate and r_j the rental rate of capital (j = 1, 2). Cone 1 includes countries with factor endowments between $\tilde{k}_1^1$ and $\tilde{k}_2^1$, and equilibrium factor prices w_1 and r_1. Cone 2 includes countries with factor endowments between $\tilde{k}_2^2$ and $\tilde{k}_2^3$, and equilibrium factor prices w_2 and r_2. Countries in cone 1 are labor-abundant and realize a lower wage relative to the countries in cone 2. Countries with factor endowments within cones produce two goods. Countries with capital intensities outside the cones specialize in the production of just one good. Factors are mobile between different sectors but they do not leave their country. Within cones, factors share their rewards because of trade, but factor prices differ across cones.[3] A major difference to a FPE equilibrium is that different techniques of production are used in different countries to produce the good with intermediate capital-intensity, X_2. In an integrated world economy, this would be inefficient. But the trading world economy here cannot reproduce the IWE just because the distribution of factor endowments is too unequal to allow for FPE for the given state of technology.

The question to be addressed is whether long-run economic growth would affect differences in factor prices across cones of diversification. If long-run growth is understood as a steady state process due to exogenous technical change, as in the Solow (1956) model, Figure 2 suggests a straightforward link between factor prices and growth: The position of all unit value isoquants in the extended Lerner diagram with two cones depends on a given state of technology in all sectors. If commodity prices can be treated as fixed – a crucial assumption that will be relaxed later on – technical progress that occurs in any sector shifts the corresponding unit value isoquant inward and factor prices change accordingly.

For instance, the difference in factor prices across the two cones will tend to decline if technical change is relatively stronger in the production of the labor-intensive good 1 or in the production of the capital-intensive good 3, the latter as shown in Figure 2 with a relative decline of the wage in cone 1. But the difference in factor prices across cones (countries) will increase if technical change is relatively stronger in the production of the intermediate good 2.

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[3] One could allow for several countries within a cone that differ in a Hicks neutral way and therefore do not share the absolute values of factor prices but only relative factor prices (see, e.g., Davis and Weinstein (2001)). But here we prefer to assume one country per cone (rich and poor), such that we have a world where both constituent countries are partially diversified in the production of two out of three goods.

By contrast, a change in the relative position of the factor price lines in Figures 1 or 2 cannot arise if technical change uniformly affects the production of all goods in the same way, given that the assumption of fixed commodity prices is maintained. Hence it may seem at first sight that the sector bias of technical change is all that matters for the question of factor price convergence across different cones of specialization.

However, the factor bias of technical change also matters. Findlay and Jones (2000) show that capital biased technical change can cause a decline of the wage even if it is concentrated in the relatively labor intensive sector, given that the country concerned has a factor endowment near the border of two cones and ends up in a different cone after the technological adjustment has taken place.[4] In addition, isolating the effects of technical change as in Figure 2 is based on the assumption that commodity prices do not react to technological shocks in a general equilibrium. This assumption implies that technical progress is limited to a small economy that cannot affect its terms of trade. Furthermore, it implies that the new technology is local in the sense that it is only available for firms in the same sector within the country, but not for firms located in a different country. In such a set up, the additional production possibilities generated by technical progress would not affect the goods market equilibrium of a trading world economy.

This restricted view of modeling technical change may sometimes by necessary to simplify the analysis, as in highly disaggregated trade models. But for a discussion of the effects of economic growth on relative factor prices across cones of specialization, modeling technical change without factor bias, without terms of trade effects, and as purely local appears to be a simplification of reality that may result in misleading thought experiments, as has been argued by Krugman (2000). Economic growth in the form of technical change has an impact on the goods market equilibrium in a trading world economy and this should not be assumed away when discussing the effect of growth on factor prices. Hence a less restricted view would hold that technical progress is likely to diffuse to other countries relatively quickly, not least through international trade in capital (intensive) goods. This is not to deny that there may remain barriers to instant international technology diffusion, but it nevertheless appears that

[4] Following Findlay and Jones (2000), we consider an exogenous factor bias of technical change. That is, we do not model how an endogenous factor bias of technical change might reflect the relative price of production factors, as in Samuelson (1965) and more recently in Acemoglu (2003).

the assumption of fixed commodity prices is not appropriate for discussing the question whether growth encourages FPE in the world economy.

III. Technical Change and Wages in General Equilibrium

From a standard 2x2 trade model with exogenous commodity prices, it follows that it is the sector bias of technical change that determines the effect on relative wages. Extending the standard model to the case with three goods and 2 factors, Findlay and Jones (2000) question that the factor bias of technical change is immaterial for the effects on relative factor prices. Krugman (2000) argues that the factor bias of technical change is only immaterial once such a change takes place in a small open economy as opposed to an economy that can affect world prices and where technical change occurs only in that economy rather than occurring simultaneously in other economies as well. We next define the factor bias of technical change and assess its effects in a 2x2 trade model with endogenous commodity prices. Introducing a third good into the model then allows us to study the conditions under which a two-cone free trade world economy without FPE evolves into one that resembles the hypothetical integrated world economy with FPE.

III.1 *Definition of Factor Bias*

For the definition of the factor bias of technical change, a concept of neutrality is needed. There are at least three different ways to define neutral technical change in terms of an exogenous shift of the production function (see, e.g., Barro and Sala-i-Martin (1995, p. 33)). In the neoclassical production function $F[A(t)K, B(t)L]$, t is an index of time, and $A(t)$ and $B(t)$ are augmenting factors that are evolving over time. These factors represent the shift parameters of the production function. According to the Harrod concept of technical change, $A(t)$ is a constant but $B(t)$ is increasing. According to the Solow concept of technical change, $A(t)$ is increasing but $B(t)$ is constant. According to the Hicks concept of technical change, which is most popular in trade theory, $A(t)$ and $B(t)$ increase with an identical rate so that the production function can be written as $C(t)F[K, L]$, with $C(t) = A(t) = B(t)$.

Given these formal definitions of the various concepts of technical change, each variant can be identified as a specific shift of the production function that leaves unchanged the factor shares of capital and labor. Hence Hicks neutral technical change is defined as a shift of the production function that leaves unchanged the factor price ratio for a constant capital intensity. Solow neutral technical change is defined as a shift of the production function that leaves unchanged the real wage for a constant labor output ratio, and Harrod neutrality is defined as a shift of the production function that leaves unchanged the rental rate of capital for a constant capital output ratio. Given these textbook definitions of neutral technical change, it follows that Harrod neutral technical change can be considered as an extreme form of capital biased technical change, which is the type of technical change considered in Figure 2.

When technical progress is no longer considered to be local but improvements are available worldwide, a trading world economy may be best modeled as a closed economy with endogenous prices. Our main interest is to clarify whether price reactions in the general equilibrium counterbalance the impact of technical improvements on relative factor prices. For example, the technical improvement in the most capital intensive sector in Figure 2 is likely to be followed by a decline in the price of that good. This would shift the unit value isoquant of good 3 back along a ray through the origin and the question is if there remains a net-inward shift due to technical change in order to maintain the tendency towards less factor price diversity across cones. We first discuss the effects of technical progress in a closed economy with two goods and will then reconsider the case with three goods and two cones of diversification.

III.2 *Biased Technical Change with Two Goods*

The Edgeworth-Bowley box diagram in Figure 3 can be used to calculate the net effect of technical change, including the price effect. We follow Krugman (2000) and assume Cobb-Douglas demand so that income is spent in fixed proportions on the different goods. This is a convenient border case that simplifies the demand side of the model. There is one capital intensive good, X_3, and one labor intensive good, X_2. The length and the height of the box in Figure 3 correspond to the economy's endowment with the two factors capital and labor. The use of capital and labor in the X_3 sector is measured from the origin 0_3, factors employed in the X_2-sector are measured from the origin 0_2.

The two solid curves X_3 and X_2 represent the initial situation. Their tangency point A gives the allocation of labor and capital in the X_3 sector and in the X_2 sector. The dashed line through A is the initial factor price line with slope -(w/r). The dotted lines indicate the initial K/L-ratios in the two sectors. The isoquants are no longer unit value isoquants, hence they do not represent the input requirements for producing one euro's worth of output. Instead, they now represent the input requirements for producing the worth of total output of good X_3 or X_2, for given commodity prices in the initial situation.

For a start, we consider capital biased technical progress in the capital intensive X_3 sector. The curve X_3' is the resulting isoquant with a strong bias towards K, representing a situation where the amount of labor as measured in effective units has increased.[5] The progress in technology is biased towards capital because for the initial factor price ratio w/r, an X_3-sector firm would use relatively more capital than before (not shown in the figure). As a result of technical progress, the initial allocation of capital and labor no longer represents an equilibrium. Given the new technological possibilities, a different factor intensity would be optimal if the old factor price ratio would prevail. Furthermore, the initial allocation of factors would create an excess supply of good X_3 that is not matched by additional demand when income is spent in fixed proportions on the two goods.

There are two ways two restore equilibrium when demand is characterized by constant expenditure shares, namely a change in commodity prices or a change in factor allocations. We first consider the price adjustment. We choose good 2 as numeraire and thus a decline in the price of good 3 is one way to a new equilibrium as it would alter the position of the isoquant X_3'. The isoquant X_2 will remain at its position as neither the price of good 2 nor the underlying technology has changed. Also, let the quantity of good 2 be fixed for a moment. If only a price adjustment is possible, point B would be the new equilibrium where the isoquants X_3'' and X_2 are tangent to each other. Note that whereas the initial shift of the value isoquant was assumed to be capital biased (or Harrod neutral as in an extreme form), the subsequent change of a commodity price moves a value isoquant in exactly the opposite way as Hicks-neutral technical progress – hence a decline in price shifts the value isoquant like Hicks neutral technical regress.

If the decline in the price of good 3 happens to be large enough, this would restore equilibrium where the expenditure shares are the same as before the technical improvement. But this would also mean that all the additional income that is generated by technical progress would be used exclusively to consume more of the progressing good 3. This is a very special

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[5] We consider Harrod-neutral technical change as a form of capital-biased technical change that would be consistent with steady state growth for a general functional form of the underlying production function.

situation that would require the assumption of quasi-linear utility. With Cobb-Douglas demand, additional income would also be used to consume more of good 2. This implies that the isoquant X_2 would also move, not because of a price change or a change in technology, but because of higher quantities demanded by consumers at the same price. In such a case, the new equilibrium would be somewhere inside the lens spanned by X_3' and X_2, which means that the additional higher consumption possibilities due to technical progress would result in a higher consumption of both goods.

What does this imply for the factor price ratio w/r ? If the new equilibrium would be in point B, the dashed factor price line w/r' would be flatter than the initial factor price line, indicating that the relative position of labor has been worsened by capital biased technical change and the subsequent change in the price of the progressing good. Capital biased technical change implies that capital becomes more productive and hence it receives a higher relative factor reward. The same net effect of capital biased technical change can be found when the new equilibrium results somewhere inside the lens spanned by X_3' and X_2. All other things constant, this result confirms that the effect of a capital biased (Harrod neutral) technical change in the X_3-sector is not completely counterbalanced by the resulting fall in the relative price of good 3 and, therefore, generates a tendency towards a flatter factor price line.

Further possibilities may be considered. If preferences (and demand) are still homothetic but the elasticity of substitution is lower than 1, the adjustment process in Figure 3 would be qualitatively the same, since the new equilibrium would still be inside the lens spanned by X_3' and X_2, but closer toward the X_3'-isoquant. With an income elasticity of demand for good 3 that is less than 1, the expenditure share of the numeraire good 2 increases with the rise in its (relative) price that is induced by technical progress in the X_3-sector. Producers of X_2 face additional demand and increase their output. Thus, in Figure 3, there would be an upward shift of X_3' as before but at the same time the then relevant X_2' isoquant (not shown) would be corresponding to a higher output level of good 2 than before.

In summary, we have four major effects of capital biased technical progress with homothetic preferences and a unit elasticity of substitution in a 2x2 trade model. First, there is a reallocation of labor away from X_3 and towards the production of X_2. Second, the capital intensity in the progressing sector increases. Third, the factor reward of capital rises relative to labor, as is represented by a flatter factor price line in the new equilibrium. The exact position of the new equilibrium point B depends – aside from the demand considerations just discussed – on the curvature of the isoquant X_3' and more generally on the elasticity of substitution between capital and labor in the production of both goods. Fourth, the *sector* bias of technical change does not matter for the change in relative factor prices, since technical progress biased towards capital in the production of good X_2 would have similar consequences. What matters in the present context is the *factor* bias of technical change, which changes factor intensities and relative commodity prices that cause changes in relative factor prices.

III.3 A Numerical Example for Leontief Technology and Cobb-Douglas Utility

The following numerical example solves the model considered so far for the case of a zero elasticity of substitution between capital and labor in the production of both goods. This further simplification helps to clarify in quantitative terms how the factor bias of technical change matters for absolute and relative factor prices. As before, we focus on the effects of technical progress in a closed economy (endogenous commodity prices) with two goods, X_3 and X_2. We now assume Leontief production functions and a fixed supply of the two factors of 100 units each, such that $K = 100$ and $L = 100$. We further assume symmetric initial factor requirements, such that $K = 100$ can be used to produce two units of X_3 and one unit of X_2, whereas $L = 100$ can be used to produce one unit of X_3 and two units of X_2, so the production of X_3 is relatively capital intensive. In combination with a Cobb-Douglas (or log-linear) utility function, this set of assumptions determines the *initial situation* as follows.

Solving the factor market restrictions for *K* and *L* for the two goods X_3 and X_2, it follows that initially both goods are produced in an equal number of units:

(1) $$\begin{bmatrix} X_3 \\ X_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 100 \\ 100 \end{bmatrix} = \begin{bmatrix} 33.3 \\ 33.3 \end{bmatrix} .$$

With a Cobb-Douglas utility function, $U = \ln X_3 + \ln X_2$, it follows that consumers spend their incomes in equal shares on the two goods, such that $p_{X_3} \cdot X_3 = p_{X_2} \cdot X_2$. With X_2 as the numeraire ($p_{X_2} = 1$), it follows that initially the commodity prices must be equal as well:

(2) $$p_{X_3} = \frac{X_2}{X_3} = \frac{33.3}{33.3} = 1 \quad .$$

The factor prices, *w* and *r*, which can be derived from the cost functions (with fixed input coefficients that are defined by the factor market restrictions), are initially also equal:

(3) $$\begin{bmatrix} r \\ w \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.333 \\ 0.333 \end{bmatrix} \quad .$$

By construction, total income equals total revenue, and the initial factor inputs required to produce one euro's worth of each good can be read from the columns of the inverted matrix of equation (1): one euro's worth of output of X_3 requires two units of capital and one unit of labor, and one euro's worth of output of X_2 requires one unit of capital and two units of labor.

We next consider the effects of *Hicks neutral technical change* in the relatively capital intensive sector, which we model as a proportional reduction by 25 percent of the factor requirements for producing one unit of X_3. Unsurprisingly, this implies that the production of X_3 will increase by 25 percent:

(4) $$\begin{bmatrix} X_3 \\ X_2 \end{bmatrix} = \begin{bmatrix} 1.5 & 1 \\ 0.75 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 100 \\ 100 \end{bmatrix} = \begin{bmatrix} 44.4 \\ 33.3 \end{bmatrix} \quad ,$$

whereas the production of X_2 is unaffected. With unchanged commodity prices ($p_{X_2} = p_{X_3} = 1$), the new factor requirements for producing one unit of X_3 imply new factor prices:

(5) $$\begin{bmatrix} r \\ w \end{bmatrix} = \begin{bmatrix} 1.5 & 0.75 \\ 1 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.555 \\ 0.222 \end{bmatrix} \quad .$$

Translating these results into a Lerner diagram (not shown; but see cone 2 in Figure 1 as an illustration), we would see that the slope of the relative factor price line (w/r) falls in absolute value from one to 0.4, and that the unit value isoquant for X_3 moves downwards along the initial ray with slope 2, since now it only takes 1.5 units of capital and 0.75 units of labor to produce one unit of X_3.

However, this situation is not an equilibrium for our closed economy because unaffected commodity prices are inconsistent with the assumption of a Cobb-Douglas utility function with equal expenditure shares. At the given commodity price vector there will be an excess supply of X_3. Hence the price of X_3 has to fall relative to the numeraire:

(6) $$p_{X_3} = \frac{X_2}{X_3} = \frac{33.3}{44.4} = 0.75 \quad .$$

At this new commodity price it takes $1.5/0.75 = 2$ units of capital and $0.75/0.75 = 1$ unit of labor to produce one euro's worth of X_3, which restores the initial relations. Hence the unit value isoquant for X_3 moves back to where it was once endogenous commodity prices are taken into account. Accordingly, factor prices also adjust to their initial level in response to the endogenous change of commodity prices:

(7) $$\begin{bmatrix} r \\ w \end{bmatrix} = \begin{bmatrix} 1.5 & 0.75 \\ 1 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0.75 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.333 \\ 0.333 \end{bmatrix} \quad .$$

So in this case, sector biased Hicks neutral technical change does not matter for the level and structure of factor prices.

Proposition 1. In case of Leontief production technology and Cobb-Douglas preference structure, Hicks neutral technical change in the relatively capital intensive sector leaves the unit value isoquant and the factor prices (in terms of the labor intensive good) unaffected and, consequently, has no effect on the slope of the relative factor price line.

Along the same lines, we consider the effect on relative factor prices of *capital biased technical change* in the relatively capital intensive sector, which we model as a reduction by

25 percent in the labor requirement for producing one unit of X_3.[6] In this case, the production of X_3 declines and the production of X_2 increases as compared to the initial situation (equation (1)), and the unit value isoquant moves leftward[7] because it now requires less labor to produce one euro's worth of X_3:

(8) $$\begin{bmatrix} X_3 \\ X_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0.75 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 100 \\ 100 \end{bmatrix} = \begin{bmatrix} 30.8 \\ 38.5 \end{bmatrix} \quad .$$

With unchanged commodity prices, the rental rate of capital rises and the wage rate falls, so in absolute value the factor price ratio w/r falls to 0.8:

(9) $$\begin{bmatrix} r \\ w \end{bmatrix} = \begin{bmatrix} 2 & 0.75 \\ 1 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.385 \\ 0.308 \end{bmatrix} \quad .$$

Yet for the same reason as before, commodity prices will not remain fixed but respond to changes in supply. Now the price of X_3 in terms of X_2 must increase to adjust demand to the reduced supply:

(10) $$p_{X_3} = \frac{X_2}{X_3} = \frac{38.5}{30.8} = 1.25 \quad .$$

At this price it takes $2/1.25 = 1.6$ units of capital and $0.75/1.25 = 0.6$ units of labor to produce one euro's worth of X_3. Hence the unit value isoquant for X_3 moves inward along a ray with slope 2.666, which is steeper than the initial ray with slope 2. The corresponding factor prices follow as:

(11) $$\begin{bmatrix} r \\ w \end{bmatrix} = \begin{bmatrix} 2 & 0.75 \\ 1 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1.25 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.538 \\ 0.231 \end{bmatrix} \quad .$$

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[6] For a definition of capital biased technical change, see section III.1. For a discussion of the effects of labor biased technical change, see the appendix.

[7] Not shown; see cone 2 in Figure 2 as an illustration.

So in this case, factor (capital) biased technical change in the relatively capital intensive sector does matters for the level and structure of factor prices, such that the rental rate of capital rises and the wage rate falls relative to the initial situation.

Proposition 2. In case of Leontief production technology and Cobb-Douglas preference structure, capital biased technical change in the relatively capital intensive sector causes a downward shift of the unit value isoquant and reduces the wage rate relative to the rental rate of capital (and more so than in case of unaffected commodity prices). The slope of the relative factor price line flattens, such that less factor price diversity across cones of specialization may result once more than one cone is considered.

III.4 Cone Convergence Because of Biased Technical Change

In a model with three goods and two cones of diversification, the above logic still applies. A diagram like Figure 1 can be used to sketch out the implications. We maintain the assumption that demand is Cobb-Douglas with fixed expenditure shares so that consumers want to spend additional real income in fixed proportions on all goods. As before, we consider capital biased technical progress in the most capital intensive sector. The direct effect of the technological improvement, namely an inward shift of the unit value isoquant of good 3, is the same as before. But for the price effect, namely the subsequent outward shift of the unit value isoquant of good 3, there is a slight difference. Technological improvements in the capital intensive cone 2 generate additional income that leads to additional demand not only for the goods 3 and 2, which are produced in that cone, but also for good 1, which is only produced in the labor intensive cone 1. Thus for the given assumptions about demand, biased technical progress in the X_3 sector and the subsequent decline in the price of good 3 relative to the price of the numeraire good 2 will cause a less pronounced increase in the output of good 2 if there are three goods in the model and not only two as in the previous subsections.[8]

The adjustment process in the labor intensive cone 1 provides a further important insight. Even if output of good 1 or good 2 increases due to the additional income generated by technical change in the production of good 3, the respective unit value isoquants will not change for *this* reason. Only technical change or changes in relative commodity prices can

[8] With three goods in the model rather than with two, our qualitative results for the change of relative factor prices tend to hold for a wider range of parameters for the elasticity of substitution in demand.

alter the position of the unit value isoquants in cone 1 in a Lerner diagram. Good 2 is chosen as numeraire and therefore the corresponding isoquant does not move when technical change occurs in the most capital-intensive sector 3. If we maintain our assumption that additional income in the capital intensive cone will lead to additional demand for good 1, it follows that this will in part lead to a higher output of good 1 and in part to a higher (relative) price of good 1. A higher price of good 1 shifts the unit value isoquant inward, because less capital and labor is then necessary to produce one euro's worth of output.[9] In a Lerner diagram like Figure 1, this would mean that the slope of the unit value isocost line in cone 1 becomes steeper because of a demand induced increase in the relative price of good 1 that causes an inward shift of the unit value isoquant X_1. All other things constant, this effect would imply that the factor price ratio in cone 1 becomes steeper and thus factor prices would become less diverse across cones.

The question remains how the adjustment process with endogenous commodity prices would look like in the capital intensive cone 2. In the last two subsections, we have discussed the reallocation of capital and labor for an integrated economy with 2 goods. We have seen that it is impossible to determine the exact position of the new allocation of capital and labor in the capital intensive cone without detailed knowledge of the parameters of utility if the adjustment process comes about both through a decline of the relative price of good 3 and increased production of good 2. Nevertheless, we have also seen that the new factor price line becomes flatter than before when technical progress in the production of the capital intensive good 3 is biased towards capital, at least for given assumptions about the demand side. Adopting this insight for a Lerner diagram like Figure 1, we conclude that the general equilibrium effect of capital biased technical change in the most capital intensive sector brings about a tendency towards less factor price diversity across cones.[10]

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[9] We assume throughout the paper that there is no situation where the endowments of the labor intensive cone are not sufficient to meet the demand for the most labor intensive good. Thus we assume that countries in cone 1 remain diversified in producing goods 2 and 1.

[10] Complete FPE is of course also a possibility. In the Lerner-Diagram, this occurs when all three isoquants, after the various shifts because of technical change and because of price movements, can be drawn along a single unit value cost line. The case of complete FPE could be illustrated by using the concept of a factor price lens (Deardorff 1994). It can be shown that technical change can bring countries closer to the situation where the factor lens condition is met (Xiang 2001).

This reasoning has an interesting implication for the volume of trade across cones. As long as the income generated by technical change is not solely used for more production and consumption of the progressing good 3, countries with factor endowments within the labor intensive cone would produce more of the most labor intensive good 1 and would exchange it for more of the most capital intensive good 3. Thus, if technical progress has a bias as we have assumed, we would expect to see increasing trade in goods that are produced with large differences in capital intensities, given that all participating countries have access to the same technology. In addition, there may be increased trade flows of the intermediate good 2 from the capital intensive cone to the labor intensive cone, again in exchange for the most labor intensive good 1. Hence the labor intensive cone would rely more than before on the production of the labor intensive good 1, but this would not lead to more factor price diversity but rather to less factor price diversity across cones.

Our conclusion that capital biased (or Harrod neutral) technical change helps to reduce factor price diversity across cones is of course subject to several qualifications. Identifying the exact effects of biased technical change on factor price diversity is complicated by the general equilibrium effects that are generated by alternative assumptions about demand. The fact that technical change in cone 2 creates additional income that is likely to lead to increased production and consumption of *all three goods* produces a spillover to the labor intensive cone 1 that further complicates the picture. One needs to know how strong the quantity adjustment (increased production of good 2 in the capital-intensive cone 2) is compared relative to the price adjustment if one wants to keep track of the changing capital intensities in all sectors and cones.

Obviously it is necessary to draw on a number of very specific assumptions in order to show that long-run economic growth in the form of technical change may in fact lead to less factor price diversity or even FPE. These assumptions establish specific configurations of factor bias, sector bias, demand conditions, and adjustment processes, and each assumption appears to be debatable. What we want to suggest with this note is that neoclassical models of trade and growth in principle allow for a rich set of possible outcomes with regard to the effects of growth on relative factor prices, with less factor price diversity across cones being one of them.

IV. Conclusion

Using a neoclassical two-cone trade model with three goods and two factors and specific assumptions about the nature of technology and preferences, we find as one of many possibilities that long-run economic growth based on technical change can encourage less factor price diversity across cones. The major difference to the paper by Deardorff (2001) is that we do not model growth as driven by factor accumulation but instead as driven by factor augmentation through exogenous capital biased technical change. Therefore, we need to take into account general equilibrium effects of biased technical change on relative commodity prices. Modeled this way, economic growth may result in a shift of cones of specialization, which in turn must have an impact on factor price diversity across cones, though not necessarily in the direction we have emphasized in the preceding sections.

We attempt to show how general equilibrium effects of factor- and sector biased technical change bring forward changes across cones in factor allocations and relative wages. What is missing from our analysis so far is to discuss in detail how technical change impacts on factor endowments and on the steady state. Technical change needs to be Harrod neutral to ensure the existence of a steady-state under rather general conditions. Since Harrod neutrality is an extreme form of a Hicks bias toward capital, this requirement as such does not seem to provide a major complication. However, so far we have not considered that Harrod neutral technical change would lead to endogenous capital accumulation, as in the Solow (1956) growth model. Taking this effect into account would mean that the size of the box diagram and the endowment points in the cone diagrams would have to change once technical progress is modeled as a shift of the (unit) value isoquant. Another extension of our analysis would be to consider the case of an infinite rate of substitution production function. We leave these additional aspects for further research.

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Appendix:
Labor Biased Technical Change with Leontief Technology and Cobb-Douglas Utility

We model the effect on relative factor prices of *labor biased technical change* in the relatively capital intensive sector as a reduction by 25 percent in the capital requirement for producing one unit of X_3.[11] In this case, the production of X_3 increases and the production of X_2 decreases as compared to the initial situation (equation (1)), and the unit value isoquant moves downward because it now requires less capital to produce one euro's worth of X_3:

(A1) $$\begin{bmatrix} X_3 \\ X_2 \end{bmatrix} = \begin{bmatrix} 1.75 & 1 \\ 1 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 100 \\ 100 \end{bmatrix} = \begin{bmatrix} 40.0 \\ 30.0 \end{bmatrix} \quad .$$

With unchanged commodity prices, the rental rate of capital rises and the wage rate falls, so in absolute value the factor price ratio w/r falls to 0.75:

(A2) $$\begin{bmatrix} r \\ w \end{bmatrix} = \begin{bmatrix} 1.75 & 1 \\ 1 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.40 \\ 0.30 \end{bmatrix} \quad .$$

Yet commodity prices will not remain fixed but respond to changes in supply. Now the price of X_3 in terms of X_2 must fall to adjust demand to the increased supply:

(A3) $$p_{X_3} = \frac{X_2}{X_3} = \frac{30.0}{40.0} = 0.75 \quad .$$

At this price it takes $1.75/0.75 = 2.333$ units of capital and $1/0.75 = 1.333$ units of labor to produce one euro's worth of X_3. Hence after the commodity price adjustment the unit value isoquant for X_3 moves upward along a ray with slope 1.75, which is flatter than the initial ray with slope 2. The corresponding factor prices follow as:

(A4) $$\begin{bmatrix} r \\ w \end{bmatrix} = \begin{bmatrix} 1.75 & 1 \\ 1 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0.75 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix} \quad ,$$

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[11] For a definition of labor biased technical change, see section III.1.

which implies that the slope of the relative factor price line increases to 2 in absolute value.

So in this case, factor (labor) biased technical change in the relatively capital intensive sector matters for the level and structure of factor prices, such that the rental rate of capital falls (despite the increase in the productivity of capital) and the wage rate rises relative to the initial situation.

Proposition A1. In case of Leontief production technology and Cobb-Douglas preference structure, labor biased technical change in the relatively capital intensive sector causes an upward shift of the unit value isoquant and increases the wage rate relative to the rental rate of capital. The rental rate of capital also falls in absolute terms.

Figure 1: The Two-Cone Equilibrium

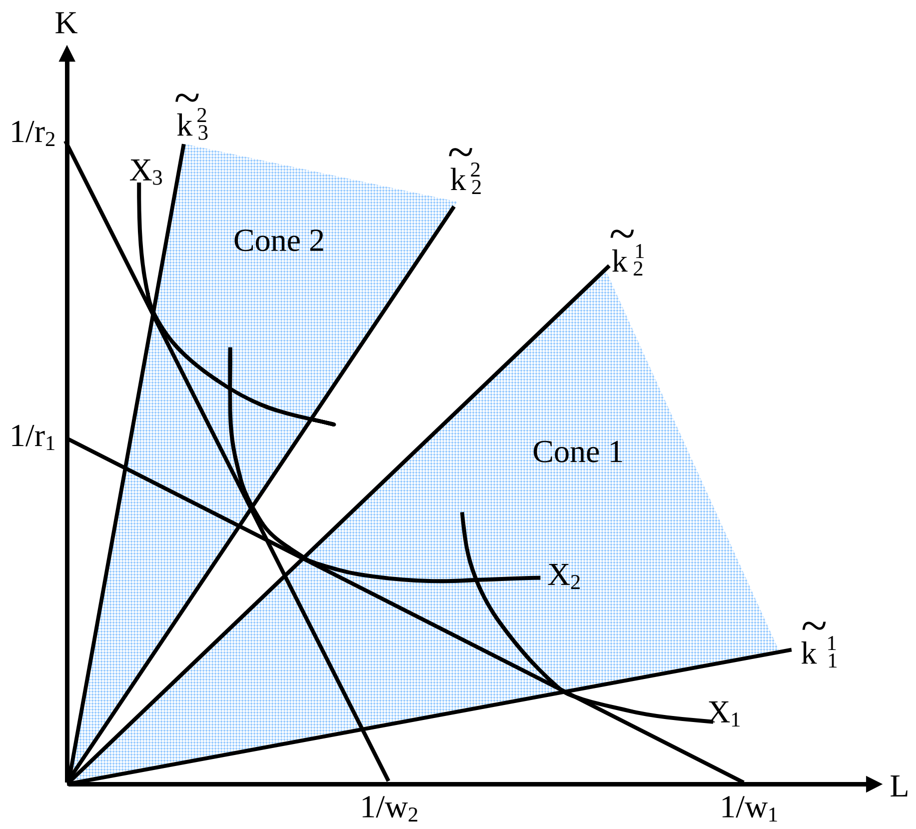

Figure 2: Capital-biased Technical Change and Relative Factor Prices with Constant Commodity Prices

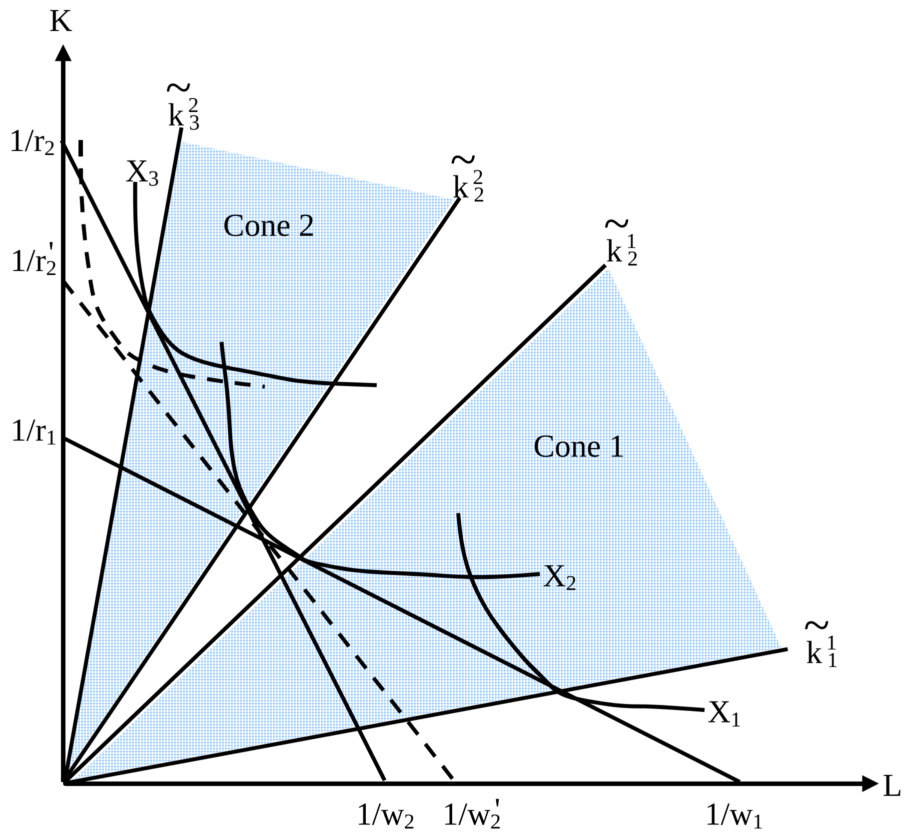

Figure 3: Capital-biased Technical Change with Endogenous Commodity Prices