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The Dynamic Effects of Public Capital: VAR Evidence for 22 OECD Countries

by

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Abstract

The issue of whether government capital is productive has received a great deal of recent attention. Yet, empirical analyses of public capital productivity have been limited to a small sample of countries for which official capital stock estimates are available. Building on a new database that provides internationally comparable capital stock estimates, this paper estimates the dynamic effects of public capital using the vector autoregressive (VAR) methodology for a large set of OECD countries. The empirical results suggest that there is evidence for positive output effects of public capital in OECD countries, but hardly any evidence for positive employment effects.

Keywords: Public capital; VAR model; Cointegration; OECD countries

JEL classification: C320; E600; H540

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1 Introduction

The issue of whether government capital is productive has received a great deal of recent attention. Early empirical studies investigating public capital productivity in general followed structural approaches such as the so-called production function approach pioneered by Aschauer (1989).[1] In most recent years, however, many researchers have estimated vector autoregressive (VAR) models that place less restrictions on the interaction among the model variables. So far, these studies have been limited to a small number of countries because of a lack of capital stock data. Building on a new database that provides internationally comparable capital stock estimates for a large set of OECD countries, this paper attempts to fill this gap, estimating the dynamic effects of public capital using the VAR methodology.

The VAR approach has a number of advantages over structural approaches such as the production function approach: (i) Whereas the production function approach assumes a causal relationship running from the three inputs to output, the VAR approach does not impose any causal links between the variables a priori. Rather, VAR models allow to test whether the causal relationship implied by the production function approach is valid or whether there are feedback effects from output to the inputs. (ii) Unlike the production function approach, the VAR approach allows for indirect links between the model variables. In the production function approach, the long-run output effect of public capital is given by the elasticity of output with respect to capital. In contrast, in the VAR approach, the long-run output effect of a change in public capital results from the interaction of the model variables. For example, it is conceivable that public capital does not directly affect output but that a change in public capital has an impact on output only indirectly via its effects on the private factors of production. The VAR approach allows to capture such indirect effects. (iii) Unlike the production function approach, the VAR approach does not assume that there is at most one long-run (cointegration) relationship among the four model variables. The Johansen (1988, 1991) methodology described in Section 3 allows to explicitly test for the cointegration rank (the number of long-run relationships) and to impose it in the estimation of the VAR model.

Estimation of VAR models is based on a reduced form. Without the prior solution of an identification problem, the VAR estimates cannot be given a structural interpretation and can in general not be used for policy analysis. In this paper, we consider the solution to the identification problem known as the recursive approach, that was introduced by Sims (1980)

[1] For a review of this literature see Sturm et al. (1998).

and is standard in the related literature. This approach is applied in Section 4, presenting empirical results on the dynamic effects of public capital for 22 OECD countries.

The paper is organized as follows. Section 2 briefly reviews recent studies that have applied the VAR approach to study the effects of public capital. Section 3 describes the econometric methodology underlying our empirical application. Section 4 presents new empirical evidence on the dynamic effects of public capital for 22 OECD countries building on capital stock estimates provided by Kamps (2004a). Section 5 discusses the robustness of the empirical results. The last section summarizes the main findings.

2 A Short Survey of the Literature

This section briefly reviews the empirical literature having applied the VAR approach to study the dynamic effects of public capital. The only survey of the VAR approach so far, Sturm et al. (1998), traced merely four studies. Instead, Table 1 summarizes information on twenty VAR studies, witnessing the increased popularity of this approach in the very recent past. A number of interesting findings with respect to the object of investigation and model specification emerge from the table: (i) Nearly half of the considered VAR studies have investigated the effects of public capital for the United States. Moreover, only two studies, Mittnik and Neumann (2001) as well as Pereira (2001b), have extended the analysis to a group of OECD countries. (ii) The vast majority of studies has relied on annual data, due to the restriction that capital stock data are not available at higher frequency. (iii) The majority of studies has considered a model in the four variables public capital, private capital, employment and output. In the remaining cases, in general either investment has been substituted for capital or additional variables have been included in the model. (iv) There is a wide variety of model specifications as regards the (non-)consideration of cointegration. Some studies, such as Cullison (1993), specify VAR models in first differences without testing for cointegration. This way of proceeding seems dubious since it neglects potential long-run relationships between the levels. Other studies, such as Ligthart (2002), specify VAR models in levels based on the result of Sims et al. (1990) that ordinary least squares estimates of VAR coefficients are consistent even if the variables are non-stationary and possibly cointegrated. Unfortunately, the consistency of VAR coefficient estimates does not carry over to estimates of impulse response functions as discussed in the next section. Finally, some studies, such as Pereira (2000), test for cointegration using the Engle-Granger (1987) approach, thus neglecting the possibility that there may be more than one cointegration relationship in higher-dimensional systems.

The last column of Table 1 reports the main conclusions of the considered studies regarding the long-run output effects of public capital.[2] As can be seen in the majority of studies the long-run response of output to a shock to public capital is positive. In general, the effects are considerably smaller than those reported in the literature applying the production function approach (see, e.g., Pereira (2000)). However, almost all of these studies fail to provide any measure of the uncertainty surrounding the impulse response estimates so that it is impossible to judge the statistical significance of the results. For those studies for which such measures are provided, the long-run output effect is in general insignificant. Another important result emerging from this literature is that many studies find evidence for reverse causation, i.e., feedback from output to public capital and vice versa (see, e.g., Batina (1998)). This suggests that it is indeed important to treat public capital as endogenous variable.

Our study can be viewed as both a reassessment of and an addition to the existing empirical literature: (i) We reassess the empirical literature by carefully addressing the important issue of cointegration and by providing confidence intervals measuring the uncertainty surrounding the point estimates of the impulses responses. (ii) We add to the empirical literature by presenting results for a large sample of OECD countries for many of which there is no VAR evidence so far.[3]

3 Econometric Methodology

The Unrestricted VAR Model

A p-th order vector autoregressive model, denoted VAR(p), can be expressed as[4]

$$X_t = A_1 X_{t-1} + A_2 X_{t-2} + \ldots + A_p X_{t-p} + \Phi D_t + \varepsilon_t, \qquad (1)$$

[2] Some of the studies listed in Table 1 do not perform a policy analysis. In these cases, the last column of the table has an “n.a.” (not available) entry.

[3] Note that the two studies that come closest to ours in scope, Mittnik and Neumann (2001) and Pereira (2001b), both use public investment as model variable whereas we use public capital.

[4] This section builds on the assumption of a known lag order p. In the empirical application, the optimal lag order is explicitly tested for.

where $X_t \equiv [x_{1t}, \ldots, x_{kt}]'$ is a set of variables collected in a $(k \times 1)$ vector, A_j denotes a $k \times k$ matrix of autoregressive coefficients for $j = 1, 2, \ldots, p$, and Φ denotes a $k \times d$ matrix of coefficients on deterministic terms collected in the $d \times 1$ vector D_1. The vector $\varepsilon_t \equiv [\varepsilon_{1t}, \ldots, \varepsilon_{kt}]'$ is a k-dimensional white noise process, i.e., $E[\varepsilon_t] = 0$, $E[\varepsilon_t \varepsilon_t'] = \Omega$, and $E[\varepsilon_t \varepsilon_s'] = 0$ for $s \neq t$, with Ω a $(k \times k)$ symmetric positive definite matrix.

Estimation of the unrestricted VAR model is particularly easy. Conditioning on the first p observations $\left(\text{denoted } X_{-p+1}, X_{-p+2}, \ldots, X_0\right)$ and basing estimation on the sample $X_1, X_2, \ldots, X_T$, the k equations of the VAR can be estimated separately by ordinary least squares (OLS). Under general conditions, the OLS estimator of $A \equiv [A_1, \ldots, A_p]$ is consistent and asymptotically normally distributed. Remarkably, this result not only holds in the case of stationary variables, but also in the case in which some variables are integrated and possibly cointegrated (Sims et al. (1990)). Based on this result many researchers have ignored nonstationarity issues and estimated unrestricted VAR models in levels. A drawback of this approach is that, while the autoregressive coefficients in Equation (1) are estimated consistently, this may not be true for other quantities derived from these estimates. In particular, Phillips (1998) showed that impulse responses and forecast error variance decompositions based on the estimation of unrestricted VAR models are inconsistent at long horizons in the presence of non-stationary variables. In contrast, vector error correction models (VECMs) produce consistent estimates of impulse responses and of forecast error variance decompositions if the number of cointegration relations is estimated consistently. As impulse response analysis is one of the main tools for policy analysis based on VAR models, a careful investigation of the cointegration properties of the VAR system is warranted.

The Cointegrated VAR Model

The starting point of the cointegration analysis is that any VAR(p) model (1) can always be written in equivalent VECM form

$$\Delta X_t = \Pi X_{t-1} + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \ldots + \Gamma_{p-1} \Delta X_{t-p+1} + \Phi D_t + \varepsilon_t, \tag{2}$$

where $\Pi \equiv -I + \sum_{i=1}^{p} A_i$ and $\Gamma_j \equiv -\sum_{i=j+1}^{p} A_i$ $(j = 1, 2, \ldots, p-1)$ denote $(k \times k)$ matrices of coefficients, respectively.

Three interesting cases can be distinguished: (i) If the cointegration rank $r = 0$, then $\operatorname{rank}(\Pi) = 0$ and the variables collected in X_t are not cointegrated. In this case, there are k independent stochastic trends in the system and it is appropriate to estimate the VAR model in first differences, dropping X_{t-1} as regressor in Equation (2). (ii) At the other extreme, if $r = k$, then $\operatorname{rank}(\Pi) = k$ and each variable in X_t taken individually must be stationary. Or, in other words, the number of stochastic trends, given by $k - r$, is equal to zero. In this case, the system can be estimated by applying OLS either to the unrestricted VAR in levels (Equation (1)) or to its equivalent representation given by (2). (iii) In the intermediate case, $0 < r < k$, the variables in X_t are driven by $0 < k - r < k$ common stochastic trends and $\operatorname{rank}(\Pi) = r < k$. In this case, estimating the system given by (2) by OLS is not appropriate since cross-equation restrictions have to be imposed on the matrix Π. Instead, the maximum likelihood approach developed by Johansen (1988, 1991) can be applied in order to estimate the space spanned by the cointegrating vectors. An additional asset of Johansen's approach is that it enables us to test for the number of cointegrating relations, which in many applications is unknown a priori.

The specification of the deterministic terms D_t in Equation (2) plays an important role in the analysis because the asymptotic distributions of the test statistics used for the determination of the number of cointegrating vectors depends on the assumptions made on these terms (see Johansen (1995: 156–157)). Johansen (1995) distinguishes five alternative models, corresponding to alternative sets of restrictions on the deterministic terms. In the following, we concentrate on the model which seems to be the most relevant for our problem: the constant is left unrestricted and the trend is restricted to the cointegrating space.[5] This specification eliminates the potential for quadratic trends in X_t, while allowing for linear trends in X_t and for trend-stationary cointegrating relations. The latter may be justified on the grounds that the cointegrating space might contain a production function as one cointegrating vector (see, e.g., Sturm and De Haan (1995)).

The Structural VAR Model

The previous two sub-sections have described how the VAR model can be estimated for alternative assumptions on the cointegrating rank. As these models are reduced-form models,

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[5] Pesaran and Smith (1998: 483) argue that the case analyzed here is one of two cases particularly relevant in practice, the other one being that of a restricted constant and no linear trend.

little can be learned about the underlying economic structure unless identifying restrictions are imposed. This sub-section shows how to give VAR models a structural interpretation and, in particular, shows how to derive impulse response functions from the reduced-form parameter estimates. Impulse responses give an insight into the reaction of key macroeconomic variables to an unexpected change in one variable (here, e.g., public capital).

The subsequent analysis is based on the unrestricted VAR model given by Equation (1). This model can serve in the structural analysis irrespective of whether the variables in X_t are non-stationary or not.[6] Pre-multiplying Equation (1) by the $(k \times k)$ matrix A_0 gives the structural form

$$A_0 X_t = A_1^* X_{t-1} + A_2^* X_{t-2} + \ldots + A_p^* X_{t-p} + A_0 \Phi D_t + B e_t, \tag{3}$$

where $A_i^* \equiv A_0 A_i$ for $i = 1, \ldots, p,$ and $Be_t = A_0 \varepsilon_t$ describes the relation between the structural disturbances e_t and the reduced-form disturbances ε_t. In the following, it is assumed that the structural disturbances e_t are white noise and uncorrelated with each other, i.e. the variance-covariance matrix of the structural disturbances, denoted Σ, is diagonal. The matrix A_0 describes the contemporaneous relation among the variables collected in the vector X_t. Without restrictions on the parameters $A_{0,}$ A_i^* and B, model (3) is not identified. In the empirical literature, a large number of alternative identification procedures have been applied. In the empirical application we use the recursive approach originally proposed by Sims (1980) that restricts *B* to a *k*-dimensional identity matrix and A_0 to a lower triangular matrix.

The solution to the identification problem given by the recursive VAR approach implies that $\Omega = PP'$, where $P \equiv A_0^{-1} \Sigma^{1/2}$ and A_0 is lower triangular. This, in turn, implies that *P* is a lower triangular matrix with the standard deviations of the structural disturbances on its principal diagonal. Moreover, it can be shown that *P* is the (unique) Cholesky factor of the symmetric positive definite matrix Ω (Hamilton (1994: 91-92)). Note, however, that while *P* is unique for a given ordering of the variables in X_t, there are *k*! possible orderings in total. Hence, it is important to check how sensitive the dynamic properties of the model are to alternative orderings of the variables.

Once the identification problem has been solved, the model dynamics can be analyzed by impulse response functions. Let Θ_n for n = 1,2,… denote the matrix holding the impulse responses at horizon *n*. Then the row *i*, column *k* element of Θ_n gives the response of variable *i* to an one-standard-deviation increase in the *k*th variable, *n* periods ago. As the impulse responses are random variables it is useful to provide confidence intervals in order to measure the uncertainty surrounding the estimated impulse responses. In the empirical application, we report confidence intervals based on the bootstrap methodology. The simple bootstrap algorithm can be summarized as follows:

1. Estimate the parameters of the model (1) by the appropriate method.
2. Generate bootstrap residuals $\varepsilon_1^*, \ldots, \varepsilon_T^*$ by randomly drawing with replacement from the set of estimated residuals $\hat{\varepsilon}_1, \ldots, \hat{\varepsilon}_T$.
3. Condition on the pre-sample values $(X_{t-p+1}^*,\ldots,X_0^*) = (X_{t-p+1},\ldots,X_0)$ and construct bootstrap time series X_t^* recursively using Equation (1),

   $$X_t^* = \hat{A}_1 X_{t-1}^* + \ldots + \hat{A}_p X_{t-p}^* + \hat{\Phi} D_t + \varepsilon_t^*, \qquad t = 1, \ldots, T .$$
4. Re-estimate the parameters $A_1, \ldots, A_p, \mu_0$ and μ_1 from the generated data and calculate the impulse response functions $\hat{\Theta}_n^*, n = 1,,2, \ldots$.
5. Repeat steps 2–4 a large number of times (in the empirical application: 1000) and calculate the α and $1-\alpha$ percentile interval endpoints of the distribution of the individual elements of $\hat{\Theta}_n^*, n = 1,,2, \ldots$. In the empirical application, we set $\alpha = 0.16$ and accordingly report 68% confidence intervals.[7]

[6] While in the estimation of the VAR parameters it is crucial to distinguish the three cases analyzed in the previous section, the analysis can proceed based on the representation (1) once the estimation stage has been completed. All that is necessary is to map the parameters Π and Γ_i from the VECM (2) to the A_i matrices.

[7] In the empirical VAR literature, typically either 68% or 95% confidence intervals are reported. Sims (1987: 443) argues against the use of 95% confidence intervals in VAR studies on the grounds that "there is no scientific justification for testing hypotheses at the 5 % significance level in every application". He suggests to treat the statistical significance of impulse responses derived from VAR coefficient estimates differently from that of coefficient estimates in standard econometric models. It is inherent in VAR models that most of the parameter estimates are insignificantly different from zero when tested at the 5% level, and this translates into relatively large confidence intervals for impulse responses. Still, estimates from unconstrained VAR models are widely thought to provide a useful data summary. Against this background, Sims and Zha (1999: 1118) recommend the use of 68% confidence intervals for estimated impulse responses. In the empirical application,

4 Empirical Results

This section presents empirical evidence on the dynamic effects of public capital for 22 OECD countries based on VAR models. Section 4.1 deals with model selection and determination of cointegration rank. Section 4.2 presents the results of an impulse response analysis based on a set of benchmark identifying assumptions.

4.1 Model Specification and Estimation

Data

The countries considered in this paper are the same as those considered in Kamps (2004a).[8] With a few exceptions, the sample periods cover the years 1960-2001.[9] For each country, we specify a four-variable VAR model including the public net capital stock, K^G, the private net capital stock, K^P, the number of employed persons, N, and real GDP, Y.[10] Expressing all variables in natural logarithms multiplied by 100 and denoting the transformed variables by lower-case letters, the vector of endogenous variables X_t can be expressed as $X_t \equiv [k_t^G, k_t^P, n_t, y_t]'$.[11] The evidence reported in Kamps (2004a: 17) suggests that the variables are integrated of order one.

VAR Order Selection

The exposition of the VAR methodology in Section 3 was based on the implicit assumption of a known lag order *p*. In empirical applications, however, the lag order is typically unknown. In the econometric literature, a number of selection criteria have been proposed that can be used to determine the optimal lag order. The selection criteria considered here are (i)

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we follow this advice, yet we refrain from drawing strong conclusions about the statistical significance of the estimated impulse responses.

[8] Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and the United States

[9] Austria 1965–2001; France 1965–2001; Greece 1961–2001; Iceland 1967–2001; Netherlands 1969–2001; New Zealand 1962–2001; Spain 1961–2001.

[10] Real GDP and employment are drawn from the OECD Analytical Database, Version June 2002. The capital stock estimates are taken from Kamps (2004a) and available at http://www.ifw-kiel.de/staff/kamps_netcap.xls.

[11] Multiplying the variables in logarithms by 100 facilitates the interpretation of the estimated impulse responses. In this case, the impulse responses give the percentage change in the level of the respective variable.

the Akaike (1974) information criterion (AIC), (ii) the Schwarz (1978) information criterion (SC), and (iii) the Hannan and Quinn (1979) information criterion (HQ).

The first three columns of Table 2 give the optimal lag order selected by the three criteria for each of the 22 OECD countries considered. Whereas the AIC selects a lag order of 4 for most countries, the HQ and SC criteria select a lag order of 2 in most cases. Given the small sample size, we are interested in a parsimonious specification of the model. Thus, we choose the lag order selected by the SC criterion in general. Yet, we also perform specification tests that check whether for the lag length selected by the SC criterion the residuals are free from first-order autocorrelation, homoscedastic and normally distributed. Since the trace test for cointegration is robust to deviations from the normality assumption (see Cheung and Lai (1993: 324)) and since the asymptotic properties of the VAR parameter estimators do not depend on the normality assumption (see Lütkepohl (1991: 359)), we do not dismiss the specification chosen by the SC criterion if the normality test indicates that the residuals are non-normal. However, if the autocorrelation test indicates that the residuals are autocorrelated, we increase the lag order compared to the one selected by the SC criterion until the autocorrelation test does not reject the null hypothesis anymore.[12] The last three columns of Table 2 show the results of the three specification tests for the chosen lag order for each of the 22 OECD countries considered. The results show that at the 5% significance level there are no signs of residual autocorrelation and in general no signs of heteroscedastic residuals.[13] The following steps of the empirical analysis are, thus, based on the lag orders displayed in the middle column of Table 2.

Determination of Cointegration Rank

Neoclassical growth theory suggests that along the balanced growth path (steady state) the so-called great ratios are constant, i.e., variables such as output, capital, consumption and investment grow at the same constant rate. King et al. (1991) first investigated the

[12] In the case of Denmark, a dummy variable (set to 1 in 1973, -1 in 1974 and 0 otherwise) was included because without the dummy variable the null hypothesis of no serial correlation had to be rejected at the 5% significance level for all lag orders between 0 and 4. In the case of Germany, a dummy variable (set to 1 in 1991 and 0 otherwise) was included in order to account for the level shift in the variables due to German Reunification.

[13] Exceptions are Ireland and Italy for which the heteroscedasticity test statistic is significant at the 5% level. In both cases, increasing the lag length to 4, as suggested by the AIC, worsened the performance of the model with respect to residual autocorrelation. As autocorrelation is more detrimental than heteroscedasticity, we choose the shorter lag length in both cases.

cointegration implications of neoclassical growth theory. They showed that the constancy of the great ratios implies that if the individual variables are non-stationary they must be driven by a single common stochastic trend. Translated to our problem this implies that the public capital to output ratio and the private capital to output ratio are potential cointegrating relations. In addition, a third potential cointegrating relation might be given by a production function of the type considered, e.g., by Aschauer (1989). Yet, this critically hinges on the nature of technology. If technology is modeled as a trend-stationary process (see, e.g., Sturm and De Haan (1995)), then the production function could be a cointegrating relation.[14] However, if technology is a non-stationary process (see, e.g., Crowder and Himarios (1997)) then the production function will not describe a stationary relation between the variables collected in the vector $\widetilde{X}_t \equiv [k_t^G, k_t^P, n_t, y_t, t]'$. To sum up, based on economic theory we expect to find at most three cointegrating relations.

We test for the number of cointegrating relations using Johansen's (1988, 1991) trace test.[15] The testing sequence can be expressed as follows (Lütkepohl (2001)):

$$H_0(r_0):\ rank(\Pi) = r_0 \quad \text{versus} \quad H_1(r_0):\ rank(\Pi) = k\,, \qquad r_0 = 0,1,\ldots,3. \tag{4}$$

The testing sequence starts with the null hypothesis that the cointegration rank is zero. If this hypothesis cannot be rejected, then the testing sequence terminates and a VAR model in first differences is the appropriate model. At the other extreme, if all null hypotheses have to be rejected, then the variables can be regarded as (trend-)stationary in levels.

Table 3 displays the test results for each of the 22 countries considered here. The results show that for a large majority of countries the number of cointegrating relations is either two or three. For the remaining countries, the cointegration rank is lower; for two countries, New Zealand and Portugal, it is even zero. As a consequence, for these two countries we estimate a VAR model for the variables in first differences. For the other countries, we estimate a VECM imposing the appropriate rank restriction.

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[14] This, of course, raises the question of where the stochastic trends in the data come from. Technology is widely viewed to be the prime candidate for a stochastic trend.

[15] We use the critical values tabulated by MacKinnon et al. (1999). These critical values are also used in the case of Denmark and Germany. The empirical models for these two countries include dummy variables. It is well known that dummy variables may affect the asymptotic distribution of the trace test statistic. This is particularly true for step dummies that give rise to broken linear trends in the levels of the variables. The dummy variables considered here, instead, are asymptotically negligible.

4.2 Impulse Response Analysis

This section analyzes the dynamic properties of the estimated VAR models for the 22 OECD countries considered in this study with the help of impulse response functions. As was discussed in Section 3, there is a need to identify VAR models in order to be able to give the impulse response functions a structural interpretation. In this section, we identify the VAR models for the individual countries by assuming that the relation between the reduced-form disturbances ε_t and the structural disturbances e_t takes the following form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 \\ a_{41} & a_{42} & a_{43} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_t^{k^G} \\ \varepsilon_t^{k^P} \\ \varepsilon_t^{n} \\ \varepsilon_t^{y} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_t^{k^G} \\ e_t^{k^P} \\ e_t^{n} \\ e_t^{y} \end{bmatrix}, \qquad (5)$$

There are six unknown parameters in Equation (5) as well as four unknown parameters in the diagonal covariance matrix of the structural disturbances, Σ. Since there are ten distinct elements in the covariance matrix of the reduced-form residuals, $\hat{\Omega}$, the model is just identified. This set of identifying assumptions is an example for the recursive approach originally proposed by Sims (1980); it has been widely applied in related literature (see Section 2). As the ordering of variables in the recursive approach may affect the results, the robustness of the results to alternative orderings of the variables is explored in Section 5.

The particular ordering of variables resulting from the benchmark identification scheme has the following implications: (i) Public capital does not react contemporaneously to shocks to the other variables in the system, (ii) private capital does not react contemporaneously to shocks to employment and real GDP, but is affected contemporaneously by shocks to public capital, (iii) employment does not react contemporaneously to shocks to real GDP, but is affected contemporaneously by shocks to both private and public capital, and, (iv) real GDP is affected contemporaneously by shocks to all other variables in the system. Note that after the initial period the variables in the system are allowed to interact freely, i.e., for example, shocks to real GDP can affect public capital in all periods after the one in which the shock occurs.

The assumptions on the contemporaneous relations between the variables can be justified as follows: Movements in government spending, unlike movements in taxes, are largely unrelated to the business cycle. In particular, government spending on capital items involves large decision and implementation lags. Therefore, it seems sensible to assume that public capital is not affected contemporaneously by shocks originating in the private sector. In a similar vein, private capital is largely unrelated to the business cycle.[16] Employment, while being strongly pro-cyclical, in general lags the business cycle.[17] Thus, it seems appropriate to assume that employment is unaffected contemporaneously by output shocks. Ordering output last can be justified by, e.g., a production function which shows that the three inputs affect output contemporaneously.

The Dynamic Effects of Public Capital

Figure 1 shows the effects of a shock to public capital for the 22 OECD countries considered here for a horizon of 25 years. Each subplot in the figure displays a point estimate of the impulse responses as well as a 68% confidence interval computed with the bootstrap procedure described in Section 3. The shocks to public capital have size one standard deviation for each country. While this precludes a quantitative comparison of the effects across countries, shocks of such size have the attractive feature that they can be viewed as representative for typical shocks that occurred during the sample period in the individual countries. A quantitative comparison of the long-run effects across countries is given at the end of this section.

The subplots in Figure 1 show that in general the output effect of a shock to public capital is positive. For most countries, the output response is positive at all plotted horizons up to the endpoint of 25 years. The figure also reveals that the impulse responses are estimated quite imprecisely, as witnessed by large confidence intervals for some countries. Judged by the 68% confidence intervals, the output responses are statistically significant in about half of all cases. Apart from the general pattern described above, two other interesting patterns can be observed. First, there are a few countries for which the output response is negative at all plotted horizons (Ireland, Japan, Portugal). Second, there are some countries for which the short-run output response is negative, while the medium-run response is positive (Canada, Iceland, Norway, Spain, United Kingdom).

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[16] See, e.g., King and Rebelo (1999: 938) for evidence for the United States.

[17] See, e.g., Stock and Watson (1999: 41) for evidence for the United States. Note, however, that their results are computed for quarterly data. It is, thus, unclear whether the finding that employment lags output also applies on an annual basis. We follow the literature and order employment before output in the structural VAR model.

Given these result patterns, it is interesting to investigate whether they can be traced back to the responses of the other three variables. If a neoclassical production function was a valid description of the relation between the four endogenous variables, then the impulse responses of public capital, private capital and employment taken together should enable us to explain the observed patterns of output responses. As regards the impulse responses of public capital to a shock to public capital – not plotted here[18] –, the point estimates of the responses are positive for all countries. For the majority of countries, the point estimates are positive for all horizons, and, judged by the 68% confidence intervals, the responses are statistically significant in most cases. Thus, the responses of public capital are consistent with the general pattern observed for the output responses. As regards the two other patterns of the output responses, in most cases they can not be explained by the pattern of the public capital responses alone. In particular, negative output responses as observed for some countries are not easily reconciled with positive public capital responses unless public capital is conceived to have a negative marginal productivity. Among those countries with a negative output response, this is only conceivable in the case of Japan. As shown in Kamps (2004a), Japan exhibits by far the largest public capital to output ratio among the OECD countries in our sample. It is, thus, conceivable that the public capital to output ratio in Japan is beyond its optimal level so that additional public capital has a negative effect on output. While this might be an explanation for the negative output response in the case of Japan, it is implausible for the other countries exhibiting negative output responses. In particular, Portugal exhibits the lowest public capital stock per head among the OECD countries in our sample (see Table 3 in Kamps (2004a)). Against this background, it is hardly imaginable that the marginal productivity of public capital is negative in Portugal.

Another possible explanation is that public capital crowds out private capital and employment. The impulse responses of private capital to a shock to public capital – not plotted here[19] – show that in the vast majority of countries private capital and public capital are complements in the medium run.[20] Interestingly, however, in almost half of the countries private capital and public capital are substitutes in the short run. Among these countries are Canada and Spain, two of those countries for which a negative short-run but a positive medium-run output response was observed. Thus, for these two countries the responses of

[18] The figure holding the responses of public capital to shocks to public capital is available upon request.

[19] The figure holding the responses of private capital to shocks to public capital is available upon request.

[20] This is true for Portugal, implying that crowding out of private capital does not seem to be the reason for the negative output response observed for this country.

private capital may explain the pattern of the output responses. The general equilibrium analysis performed by Baxter and King (1993) suggests the following explanation for the sign switch of the private capital responses: There are two opposing forces determining the response of private capital. One of these forces is the resource cost associated with financing an additional unit of public capital. This cost reduces the resources available to the private sector and all other things being equal induces a fall in private investment. The other force is the positive effect of an increase in public capital on the marginal productivity of private capital, all other things being equal inducing a rise in private investment. If public capital accumulates gradually, then the first force will dominate the second in the short run, whereas in the medium to long run the second force will dominate.

As regards the impulse responses of employment to a shock to public capital – not plotted here[21] –, they do not show a general pattern. For roughly one third of the countries the responses are negative – implying that employment and public capital are substitutes –, while for the other countries the responses are either positive – implying that employment and public capital are complements – or not significantly different from zero, judged by the 68% confidence interval.[22] The lack of clear-cut results for employment is deplorable also from a theoretical perspective because the responses of employment can be very useful in order to test competing theoretical models. For example, traditional Keynesian models predict that employment will rise in response to an increase in government spending, which is a testable hypothesis. Issues are more complicated when it comes to neoclassical models such as the general equilibrium model such as the one considered by Baxter and King (1993). The policy experiments performed with this model suggest a possible explanation for the inconclusive evidence on the employment response. An increase in public capital exerts two opposing wealth effects and – depending on the way additional public capital is financed – possibly also a substitution effect. For example, if public capital is financed by non-distortionary taxes and is only mildly productive, then employment will rise in response to a shock to public

[21] The figure holding the responses of employment to shocks to public capital is available upon request.

[22] The employment responses of Portugal are not significantly different from zero, implying that the employment responses – like the public and private capital responses – cannot rationalize the negative output response observed for this country. Note that Portugal is one of only two countries for which both the Engle-Granger test and the Johansen test fail to reject the null hypothesis of no cointegration. Possibly, the empirical model is misspecified even though the specification tests reported in Table 2 suggest otherwise. Alternatively, data quality might be an issue in the case of Portugal. For example, whereas the real GDP series for Portugal contained in the OECD Analytical Database starts in 1960, the Portuguese statistics office INE and Eurostat currently publish GDP data according to ESA 1995 starting only in 1987 and 1995, respectively. As we do not know the underlying cause for the puzzling results of the impulse response analysis, we choose to treat Portugal as an outlier in the following.

capital. However, if public capital is financed by distortionary taxes and is only mildly productive, then employment will fall in response to such a shock. The empirical model is silent on these issues because it does not include any government revenue variables. The reason is, of course, that including all variables in the VAR model that are interesting in this respect (non-distortionary taxes, distortionary taxes, government debt and government consumption) would quickly exhaust the available degrees of freedom.

Table 4 displays summary information about the long-run effects of public capital for the 22 OECD countries in our sample. The table displays long-run elasticities of private capital, employment and real GDP with respect to public capital, respectively. These long-run elasticities are special in that they capture the dynamic feedback between the four variables in the system.[23] Therefore, they can be viewed as the empirical counterpart of the general equilibrium effects typically considered in theoretical models. The long-run elasticities considered here are conceptually different from the elasticities of a production function. Whereas for a production function, e.g., the elasticity of output with respect to public capital gives the percentage change in output per exogenous one-percent change in public capital holding fixed the private inputs and excluding feedback effects, e.g., from output to public capital, the long-run elasticities with respect to public capital reported here account for the dynamic interaction between the variables in the system.[24]

The results reported in Table 4 show that for most countries the long-run elasticity of output with respect to public capital is positive, giving support to the hypothesis that public capital is productive.[25] Judged by the 68% confidence intervals, this long-run elasticity is statistically significant in the majority of countries. In most of these countries, the long-run elasticity is smaller than 1, i.e., a one-percent long-run increase in public capital is associated with a less than proportional increase in output. The long-run elasticity of private capital with respect to public capital is positive for most countries, indicating that private capital and public capital are complements in the long run. As is the case for output, this elasticity – again judged by the

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[23] See, e.g., Pereira (2001b) for another study using this concept.

[24] Baxter and King (1993: 330)), e.g., make a similar distinction in their quantitative analysis of a dynamic general equilibrium model.

[25] There are two exceptions to this general finding: Japan and Portugal. As was mentioned above, the estimate for Portugal is difficult to rationalize, therefore we treat it as outlier. As regards Japan, the long-run elasticity taken on its own seems to suggest a very strong negative output effect of public capital. However, none of the three elasticities reported for Japan is statistically significant judged by the 68% confidence interval. Moreover, the long-run response of public capital to a shock to public capital is almost zero. While the long-run responses of the other three variables are also close to zero, they are larger in absolute value than the long-run response of public capital. This translates into misleadingly large long-run elasticities of private capital, employment and output, respectively. It is more likely that the true long-run elasticities are zero in the case of Japan.

68% confidence intervals – is statistically significant and smaller than 1 in the majority of countries. As already noted in the interpretation of the impulse responses, the results for employment are less conclusive. In all countries except four, the long-run elasticity of employment with respect to public capital is not statistically significant. In those countries in which it is significant, this elasticity is either positive or negative. Taken together, the results for employment seem to suggest that public capital and employment are neither complements nor substitutes in the long run, but rather that they are unrelated in the long run. To sum up, an increase in public capital in OECD countries on average can be expected to lead to an increase in output in the long run, but there is little evidence that it is the appropriate policy measure if the aim is to increase employment in the long run.

Is There Evidence for Reverse Causation?

In Kamps (2004a), estimation of a production function was based on the assumption that public capital, private capital and employment are exogenous with respect to output. This implied that feedback from output to the inputs was excluded by assumption. The VAR model, instead, allows for such feedback by treating all variables as endogenous. Whether it is important to do so in an empirical application, can be clarified with the help of a causality analysis. While it is possible to formally test for causality in the sense of Granger (1969), the impulse response analysis can also be regarded as a type of causality analysis (Lütkepohl 2001)).[26] In our context, it is most interesting to investigate whether there is feedback from output to public capital, i.e., whether the impulse responses of public capital to an output shock are significantly different from zero at some point in the response horizon.

Figure 2 depicts the impulse responses of public capital to a shock to real GDP. Note that our identifying assumptions restrict the impact response of public capital to be zero. The impulse responses show that in the vast majority of countries public capital increases after a positive output shock. In most cases, these responses are statistically significant, judged by the 68% confidence intervals. These results suggest that it is indeed important to treat public capital as endogenous variable in empirical investigations.[27] The general result that public

[26] The methodology for testing Granger causality in higher dimensional systems is developed in Dufour and Renault (1998). These authors show that impulse responses do not summarize all information about causal links in higher dimensional systems (Dufour and Renault (1998: 1113)). Impulse responses not significantly different from zero are not sufficient to rule out causality. Yet, if the impulse responses are significantly different from zero, then this is a clear indication of causality. As is shown below, this is the case for the vast majority of countries in our sample.

[27] The same is true for private capital and employment which both also show responses to an output shock significantly different from zero. Detailed results are available upon request.

capital positively reacts to output shocks has a straightforward interpretation: An unanticipated increase in output will in general entail an increase in government revenue so that the resources available for public investment increase. Likewise, an unanticipated decline in output will lead to a deterioration of public finances. The historical record suggests that governments in OECD countries in the 1970s and 1980s tended to react to high budget deficits – that arose at a time when trend growth in output declined – by cutting public investment (see De Haan et al. (1996: 71)).

5 Sensitivity Analysis: Alternative Identification Assumptions

As was mentioned in Section 3, the results of the impulse response analysis may be sensitive to the ordering of variables in the recursive VAR approach. In the benchmark VAR model analyzed in Section 4, the variables were ordered as follows: $X_t \equiv [k_t^G, k_t^P, n_t, y_t]'$. All in all, there are $4! = 24$ possible orderings of the variables. As an analysis of all possible orderings would be extremely arduous in the present context, we consider a single alternative ordering that places public capital last in the list of variables: $X_t \equiv [k_t^P, n_t, y_t, k_t^G]'$. This implies that public capital is affected contemporaneously by shocks to all other variables, but that the other variables are unaffected contemporaneously by shocks to public capital. This can be regarded as an extreme departure from the benchmark case in which public capital was unaffected contemporaneously by shocks to private capital, employment and output. While the benchmark ordering of variables seems more plausible given the decision and implementation lags involved in fiscal policy, it would be reassuring if the results obtained for this alternative ordering were similar to those obtained in the benchmark case.

Figure 3 displays the impulse responses of GDP to a shock to public capital for this alternative ordering of variables. The figure shows that, with a few exceptions, the results are qualitatively very similar to those obtained for the benchmark ordering of variables (see Figure 1). The main exceptions are Finland and New Zealand for which the impulse responses switch signs. Quantitatively, the impulse responses are in general somewhat smaller in absolute value than in the benchmark case. All in all, Figure 3 suggests that the output effects of public capital – which are the focus of interest – are not very sensitive to alternative orderings of the model variables.

6 Conclusion

This paper has provided new evidence on the dynamic effects of public capital in OECD countries based on the VAR methodology. In contrast to the production function approach routinely applied in the literature, this methodology treats all variables as endogenous and, thus, allows for feedback effects from output to the three input variables. Moreover, application of the Johansen (1988,1991) method has shown that it is important to account for the possibility of multiple cointegrating vectors among the model variables. The main results of the analysis can be summarized as follows: (i) For the majority of countries in our sample, shocks to public capital tend to have significant positive output effects. (ii) In contrast to the results documented in the literature for the production function approach, there is little evidence for "supernormal" returns to public capital. The results presented in this paper suggest that one reason for the extremely high returns to public capital obtained for some countries for the production function approach might be that the latter approach ignores feedback effects from output to public capital. (iii) For the vast majority of countries in our sample, public capital and private capital are long-run complements. As regards the short-run relation between these variables, two groups of countries can be distinguished: a first group for which public capital and private capital are short-run substitutes, i.e., private capital declines in response to a shock to public capital, and, a second group for which public capital and private capital are short-run complements. (iv) For the vast majority of countries, the long-run response of employment to a shock to public capital is statistically insignificant. In other words, there is little evidence for the hypothesis that employment can be fostered by additional public capital.

7 References

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Table 1: Studies Using the VAR Approach

Study	Country	Sample	Model	Variables	Output effect of public capital[a]
Cullison (1993)	United States	1955–1992 (A)	VAR (FD)	I^G, G^D, B^G, Y, M	insignificant
McMillin & Smyth (1994)	United States	1952–1990 (A)	VAR (L, FD)	$E, \pi, K^G/K^P, N/K^P, Y/K^P$	insignificant
Crowder and Himarios (1997)	United States	1947–1989 (A)	VECM	K^G, K^P, Y, N, E	n.a.
Batina (1998)	United States	1948–1993 (A)	VECM, VAR (L)	K^G, Y, N, K^P	positive[b]
Pereira & Flores de Frutos (1999)	United States	1956–1989 (A)	VAR (FD)	K^G, K^P, N, Y	positive[b]
Pereira (2000)	United States	1956–1997 (A)	VAR (FD)	I^G, I^P, N, Y	positive[b]
Pereira (2001a)	United States	1956–1997 (A)	VAR (FD)	I^G, I^P, N, Y	n.a.
Pereira & Andraz (2001)	United States	1956–1997 (A)	VAR (FD)	I^G, I^P, N, Y	positive[b]
Flores de Frutos et al. (1998)	Spain	1964–1992 (A)	VARMA (L)	K^G, K^P, N, Y	positive[b]
Pereira & Roca Sagales (1999)	Spain	1970–1989 (A)	VAR (FD)	K^G, K^P, N, Y	positive[b]
Pereira & Roca Sagales (2001)	Spain	1970–1993 (A)	VAR (FD)	K^G, K^P, N, Y	positive[b]
Pereira & Roca Sagales (2003)	Spain	1970–1995 (A)	VAR (FD)	K^G, K^P, N, Y	positive[b]
Otto and Voss (1996)	Australia	1959–1992 (Q)	VAR (L)	K^G, K^P, N, Y	insignificant[c]
Everaert (2003)	Belgium	1953–1996 (A)	VECM	K^G, K^P, Y	n.a.
Mamatzakis (1999)	Greece	1959–1993 (A)	VECM	K^G, K^P, N, Y	n.a.
Sturm et al. (1999)	Netherlands	1853–1913 (A)	VAR (L)	I^G, I^P, Y	insignificant[c]
Ligthart (2002)	Portugal	1965–1995 (A)	VAR (L)	K^G, K^P, N, Y	insignificant
Voss (2002)	United States, Canada	1947–1996 (Q)	VAR (FD)	$Y, p^G, p^P, r, I^G/Y, I^P/Y$	n.a.
Mittnik & Neumann (2001)	6 OECD countries	1955–1994 (Q)	VAR (FD), VECM	I^G, C^G, I^P, Y	insignificant[c] / positive
Pereira (2001b)	12 OECD countries	1960–1990 (A)	VAR (FD), VECM	I^G, I^P, N, Y	positive[b]

Notes: A = annual data. Q = quarterly data. VAR = vector autoregression. VECM = vector error correction model. VARMA = vector autoregressive moving average model. FD = model in (log) first differences. L = model in (log) levels. Y = output. N = employment. K^P = private capital. K^G = public capital. I^P = private investment. I^G = public investment. C^G = public consumption. G^D = government defense spending. B^G = government debt. M = money supply. E = energy price. π = inflation. p^G = relative price of public investment. p^P = relative price of private investment. r = real interest rate.

[a] Long-run output effect of public capital (public investment), measured by the impulse responses of output to a shock to public capital (public investment). – [b] Study does not report any measure of the statistical significance of the estimated effect. – [c] Positive and statistically significant short-run effect.

Table 2: Specification of VAR Orders

Country	VAR order minimizing			Chosen VAR order[d]	Specification tests (p-values)[e]		
	AIC[a]	SC[b]	HQ[c]		Autocorre-lation[f]	Hetero-scedasticity[g]	Normality[h]
Australia	2	2	2	2	0.604	0.336	0.073
Austria	4	1	2	2	0.587	0.209	0.185
Belgium	2	2	2	2	0.654	0.083	0.188
Canada	4	2	2	2	0.745	0.902	0.059
Denmark	4	3	3	3	0.110	0.344	0.001*
Finland	4	1	1	2	0.680	0.118	0.354
France	4	2	4	2	0.515	0.291	0.154
Germany	4	2	2	2	0.577	0.275	0.211
Greece	3	2	2	3	0.657	0.672	0.031*
Iceland	4	2	4	2	0.276	0.496	0.063
Ireland	4	2	2	2	0.524	0.042*	0.144
Italy	4	2	2	2	0.400	0.025*	0.445
Japan	4	2	2	3	0.182	0.188	0.003*
Netherlands	4	1	4	2	0.168	0.061	0.022*
New Zealand	4	1	3	2	0.054	0.142	0.062
Norway	4	2	2	2	0.370	0.757	0.136
Portugal	4	2	2	2	0.355	0.292	0.322
Spain	4	2	4	4	0.118	0.343	0.000*
Sweden	4	2	4	2	0.101	0.157	0.032*
Switzerland	4	2	2	2	0.238	0.139	0.077
United Kingdom	2	2	2	2	0.562	0.145	0.078
United States	4	2	4	2	0.054	0.546	0.115

Notes: The maximum order considered is equal to 4. The underlying VAR model contains constants and linear time trends. In the case of Germany, the VAR model also contains a dummy variable (set to 1 in 1991 and 0 otherwise) as well as its lagged value. In the case of Denmark, the VAR model also contains a dummy variable (set to 1 in 1973, –1 in 1974 and 0 otherwise) as well as its lagged value.

[a]Akaike information criterion (Akaike (1974)). – [b]Schwarz information criterion (Schwarz (1978)). – [c]Hannan-Quinn information criterion (Hannan and Quinn (1979)). – [d]The VAR order is chosen on the basis of the information criteria and on the basis of specification tests. – [e]The specifications tests are based on the residuals from the estimation of an unrestricted VAR (p), where p is the integer reported in the column "Chosen VAR order". * denotes statistical significance at the 5 percent level. – [f]Multivariate autocorrelation LM test (Johansen (1995: 22)). Under the null hypothesis of no serial correlation of order h (here: $h=1$) the test statistic is asymptotically distributed χ^2 with 16 degrees of freedom. – [g]Multivariate extension of White's (1980) heteroscedasticity test (Doornik (1996)). Under the null hypothesis of homoscedastic residuals the test statistic is asymptotically distributed χ^2 with $10(8p+2)$ degrees of freedom, where p is the chosen VAR order. – [h]Multivariate residual normality test (Lütkepohl (1991: 155–158)). Under the null hypothesis of normally distributed residuals the test statistic is asymptotically distributed χ^2 with 8 degrees of freedom.

Table 3: Johansen (1988, 1991) Cointegration Test

Country	VAR order	Trace statistic				Cointegration rank[a]
		H_0: $r = 0$	H_0: $r = 1$	H_0: $r = 2$	H_0: $r = 3$	
Australia	2	97.41	57.73	32.46	11.34	3
Austria	2	89.55	50.24	25.82	6.02	2
Belgium	2	63.88	34.59	17.24	5.72	1
Canada	2	81.59	52.03	27.48	12.73	3[c]
Denmark	3	104.79	58.79	30.06	8.49	3
Finland	2	70.25	38.76	17.37	8.43	1
France	2	80.34	45.24	21.89	10.56	2
Germany	2	72.53	38.35	16.13	0.96	1
Greece	3	107.93	46.95	23.53	7.58	2
Iceland	2	73.03	43.10	19.13	6.44	2
Ireland	2	79.85	47.98	23.15	10.02	2
Italy	2	98.94	57.27	31.98	12.56	3[c]
Japan	3	100.85	46.34	19.66	8.82	2
Netherlands	2	69.85	40.66	20.41	8.58	1
New Zealand	2	58.06	34.98	15.08	5.56	0
Norway	2	84.45	49.09	27.67	10.09	3
Portugal	2	58.10	38.03	22.70	10.57	0
Spain	4	121.30	65.80	31.82	9.21	3
Sweden	2	86.35	54.62	30.96	10.87	3
Switzerland	2	71.84	40.01	20.66	5.55	1
United Kingdom	2	89.08	56.76	27.10	8.64	3
United States	2	120.30	59.18	26.71	11.89	3
Critical values[b]		63.87	42.92	25.86	12.52	

Notes: The underlying VAR model contains unrestricted intercepts and restricted trend coefficients and is of order *p*, where *p* is the integer reported in the column "VAR order". In the case of Germany, the VAR model also contains a dummy variable (set to 1 in 1991 and 0 otherwise) as well as its lagged value. In the case of Denmark, the VAR model also contains a dummy variable (set to 1 in 1973, –1 in 1974 and 0 otherwise) as well as its lagged value.

[a]The test decision is based on the asymptotic critical values reported in the bottom row of the table. – [b]The asymptotic critical values for a 5% significance level for Johansen's log-likelihood based trace statistic are taken from MacKinnon et al. (1999), Table V. – [c]In the cases of Canada and Italy, the test results suggest that the model variables are stationary (r=4). However, recursively calculated eigenvalues and trace statistics (see Hansen and Juselius (1995: 50-63) for details) suggest that for both countries the fourth eigenvalue is not significantly different from zero. Against this background, we choose r=3 for both countries.

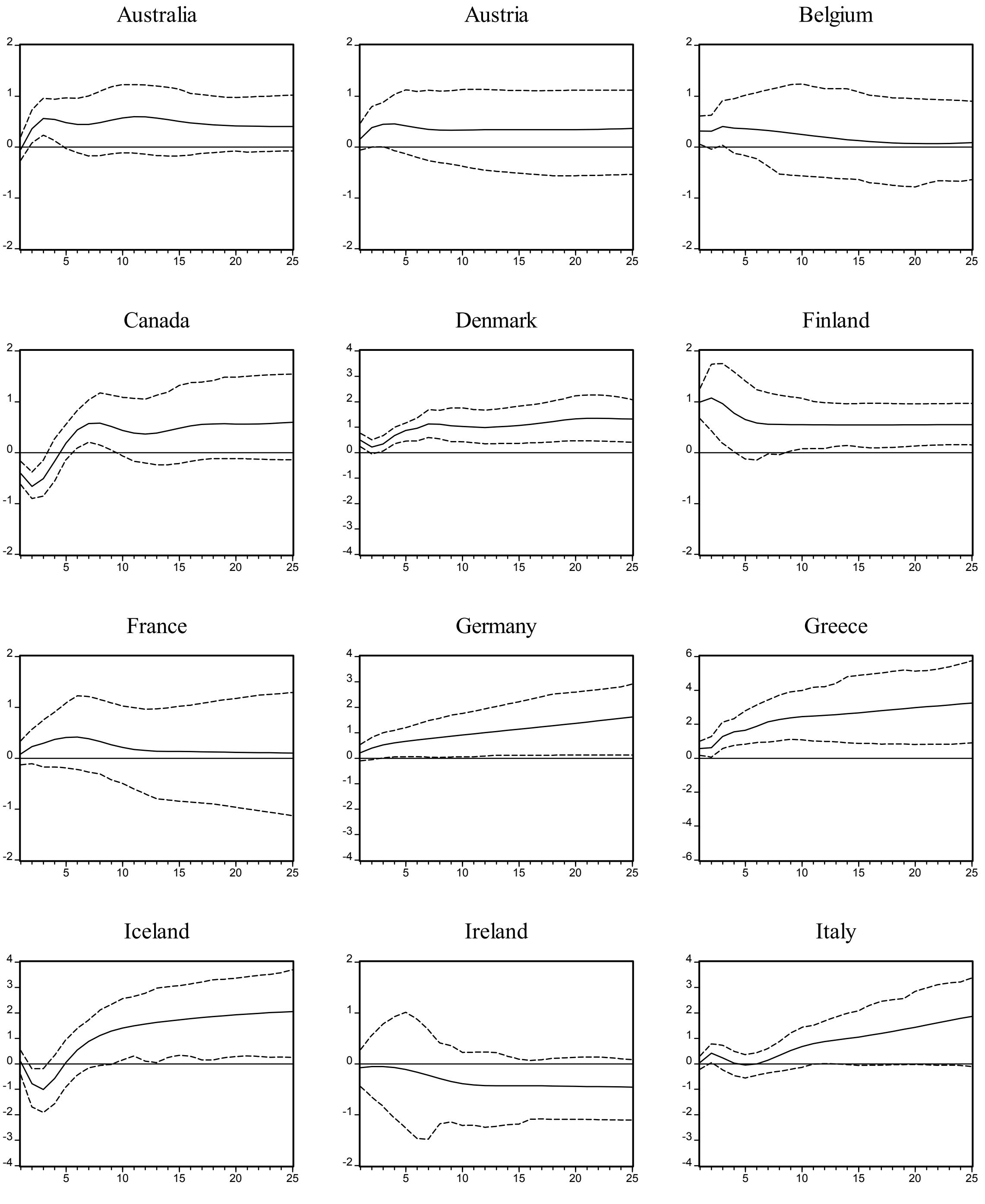
Table 4: Long-Run Effects of Public Capital

Country	Long-run elasticity of … with respect to public capital[a]		
	Private capital	Employment	Real GDP
Australia	0.33*	–0.24	0.29*
Austria	0.22*	0.12	0.07
Belgium	–0.18	0.06	0.15
Canada	1.54	0.85	1.25*
Denmark	0.63**	0.03	0.41**
Finland	0.68*	0.50	0.72*
France	1.44*	–0.48	0.84*
Germany	0.22	–0.12	0.53*
Greece	1.32*	–0.32*	1.77**
Iceland	0.64*	0.31	0.78*
Ireland	0.58*	–0.36	0.01
Italy	1.25*	–0.38*	1.73
Japan	–11.14	–5.81	–8.58
Netherlands	0.24	–0.24	0.52
New Zealand	0.15	0.06	0.11
Norway	1.46*	0.11	0.41*
Portugal	0.30	–0.33	–0.77*
Spain	1.13	–0.27	1.09*
Sweden	0.84*	0.55*	0.55*
Switzerland	0.38**	–0.05	0.41*
United Kingdom	0.40**	–0.27*	0.08
United States	–0.71*	–0.48	0.33

Notes: * (**) denotes that the 68% (95%) confidence interval does not include zero. The confidence intervals for the individual countries are computed using the bootstrap procedure described in Section 3.

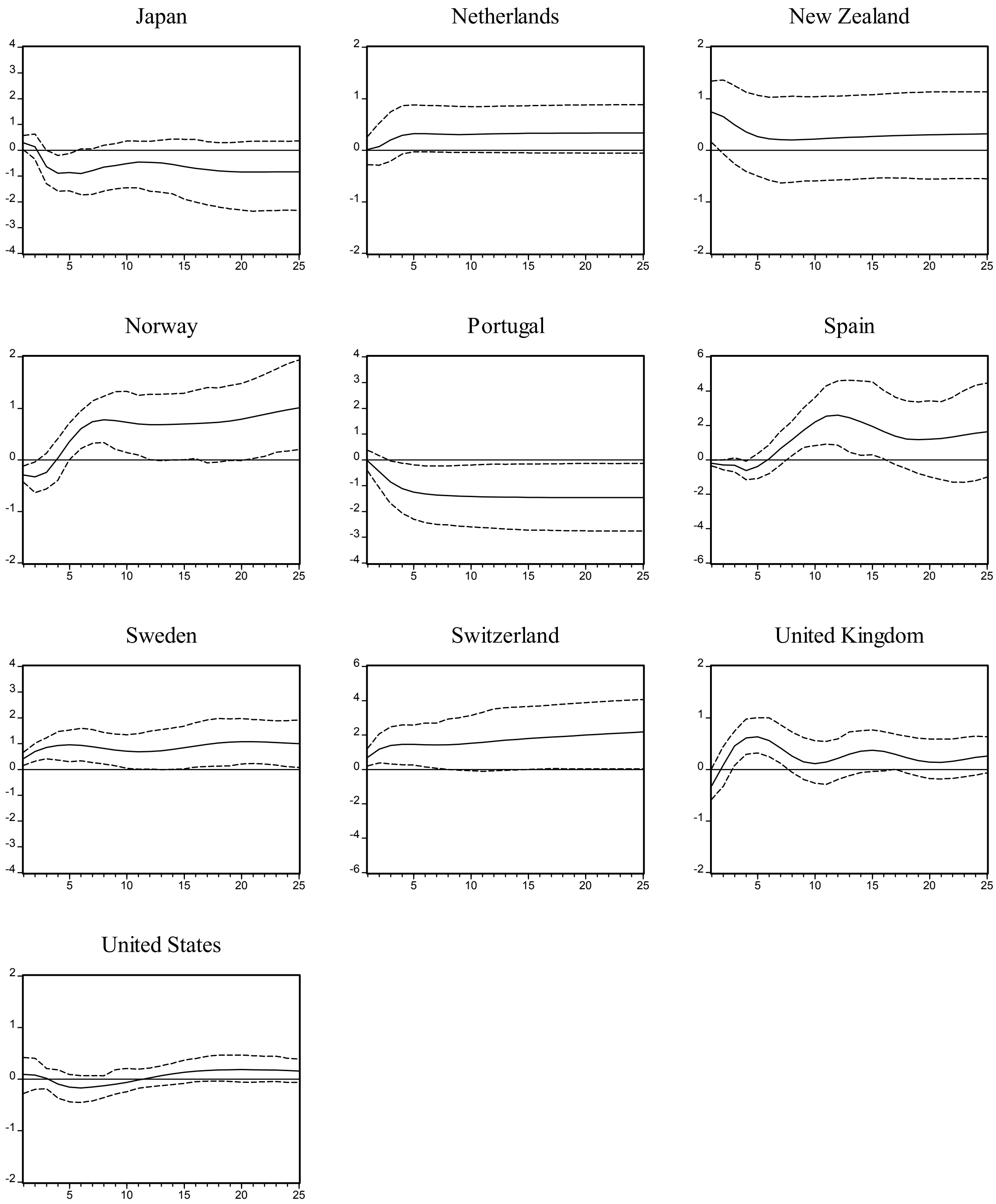
[a]The long-run elasticities give the long-run percentage change in private capital, employment and real GDP per one-percent long-run change in public capital. They are obtained by dividing the long-run response of private capital, employment and real GDP to a shock to public capital, respectively, by the long-run response of public capital to a shock to public capital. In the computations, we set the response horizon n=500 which ensures that for all countries the impulse responses have converged to their long-run levels.

Figure 1: Impulse Responses of GDP to a Shock to Public Capital



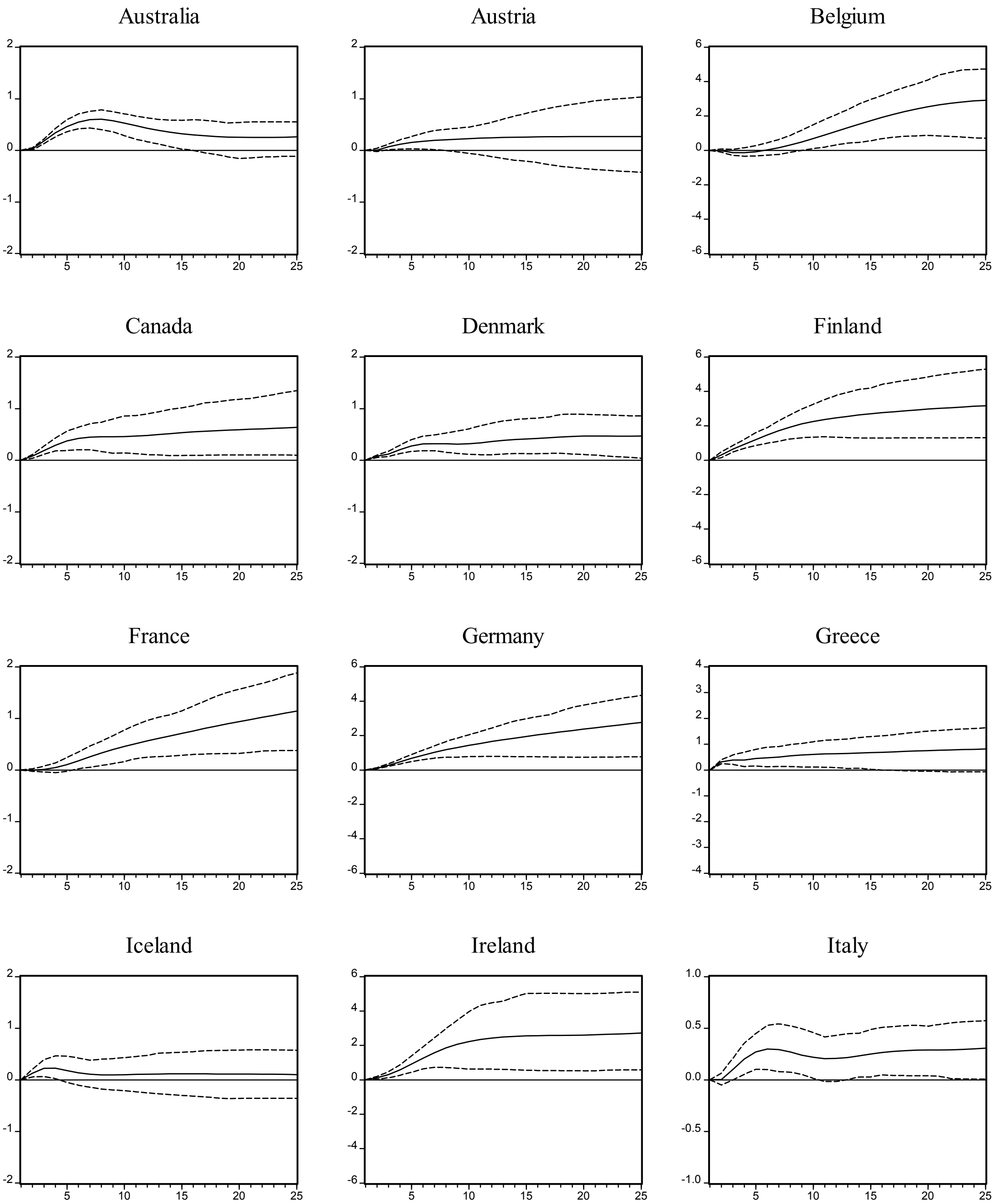
Notes: The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the error bands. They depict the percentage change in GDP in response to a one standard deviation shock to public capital for a horizon of 25 years. The dotted lines represent 68% bootstrap error bands. Identification of the model is achieved by a Choleski decomposition of the residual covariance matrix, employing the following ordering of variables: public capital, private capital, employment, GDP.

Figure 1 (continued): Impulse Responses of GDP to a Shock to Public Capital



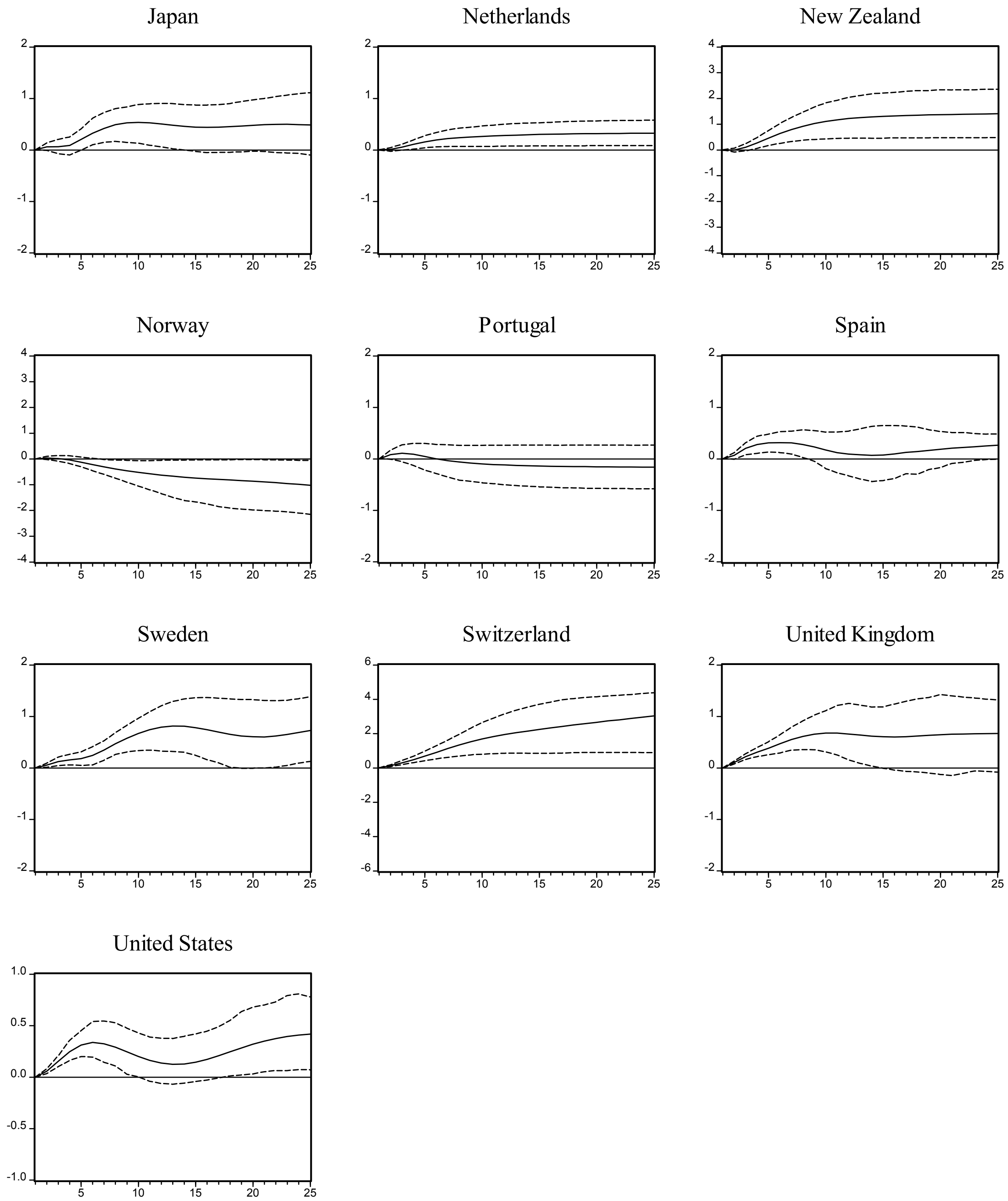
Notes: The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the error bands. They depict the percentage change in GDP in response to a one standard deviation shock to public capital for a horizon of 25 years. The dotted lines represent 68% bootstrap error bands. Identification of the model is achieved by a Choleski decomposition of the residual covariance matrix, employing the following ordering of variables: public capital, private capital, employment, GDP.

Figure 2: Impulse Responses of Public Capital to a Shock to GDP



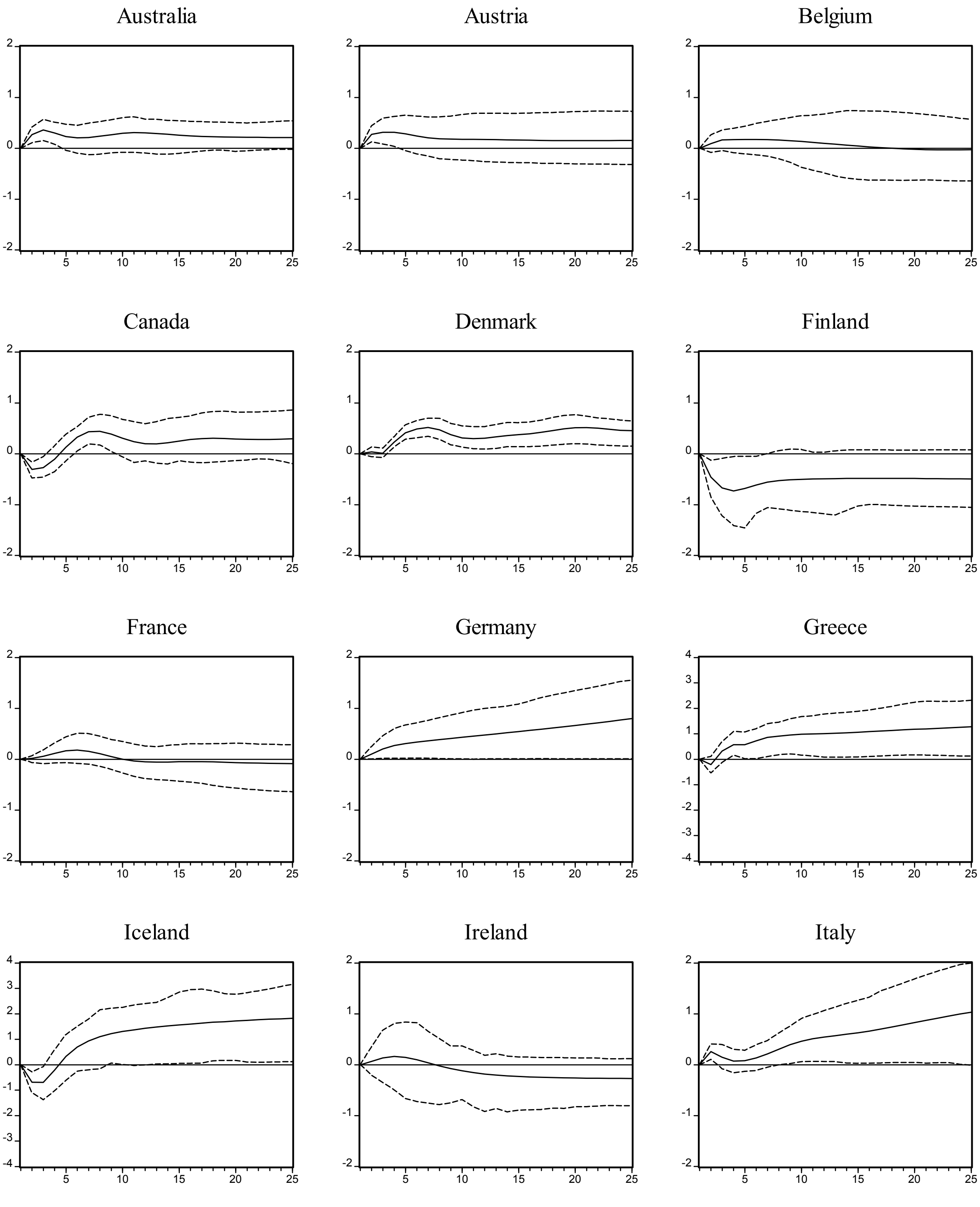
Notes: The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the error bands. They depict the percentage change in public capital in response to a one standard deviation shock to GDP for a horizon of 25 years. The dotted lines represent 68% bootstrap error bands. Identification of the model is achieved by a Choleski decomposition of the residual covariance matrix, employing the following ordering of variables: public capital, private capital, employment, GDP.

Figure 2 (continued): Impulse Responses of Public Capital to a Shock to GDP



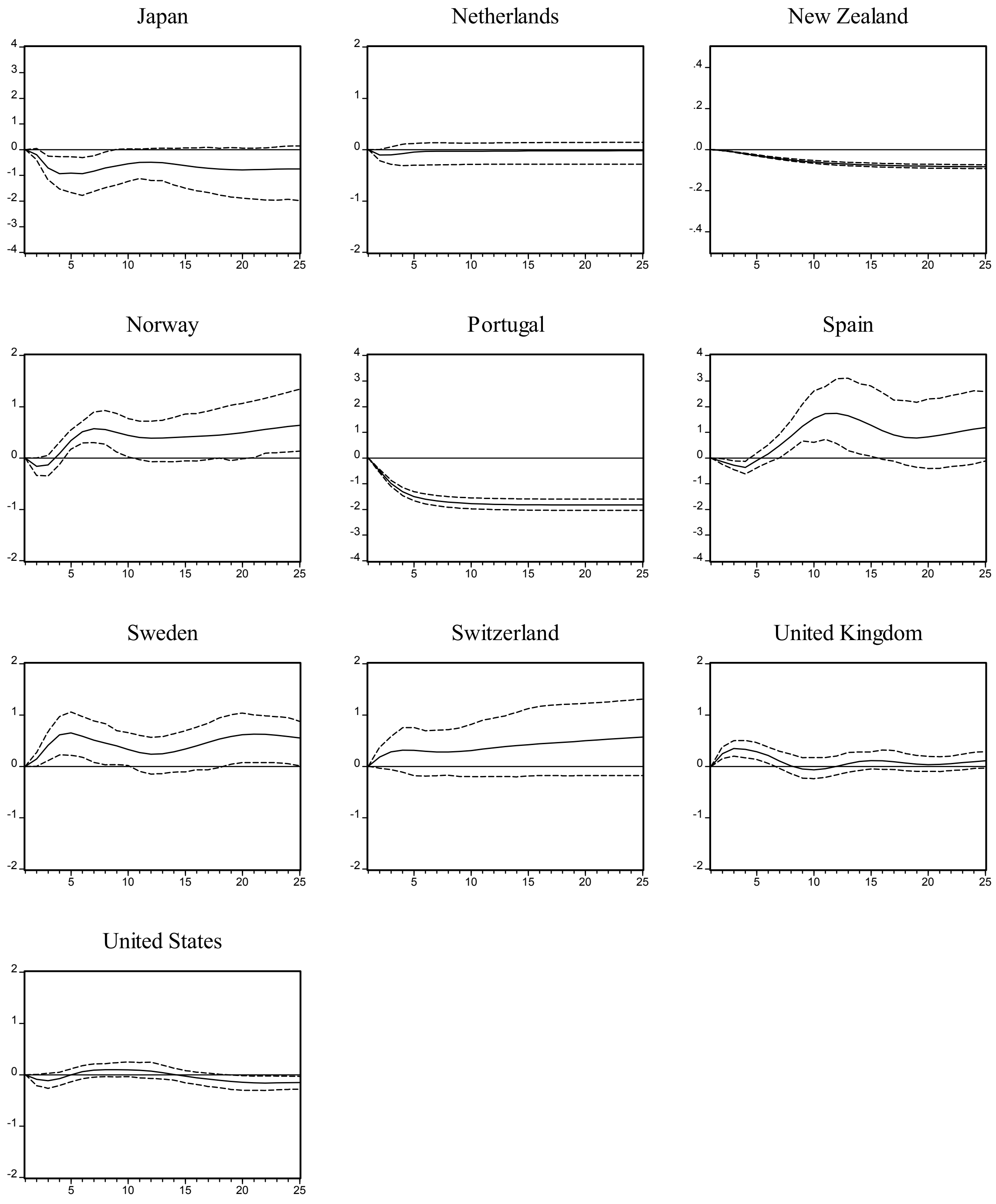
Notes: The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the error bands. They depict the percentage change in public capital in response to a one standard deviation shock to GDP for a horizon of 25 years. The dotted lines represent 68% bootstrap error bands. Identification of the model is achieved by a Choleski decomposition of the residual covariance matrix, employing the following ordering of variables: public capital, private capital, employment, GDP.

Figure 3: Impulse Responses of GDP to a Shock to Public Capital



Notes: The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the error bands. They depict the percentage change in GDP in response to a one standard deviation shock to public capital for a horizon of 25 years. The dotted lines represent 68% bootstrap error bands. Identification of the model is achieved by a Choleski decomposition of the residual covariance matrix, employing the following ordering of variables: private capital, employment, GDP, public capital.

Figure 3 (continued): Impulse Responses of GDP to a Shock to Public Capital



Notes: The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the error bands. They depict the percentage change in GDP in response to a one standard deviation shock to public capital for a horizon of 25 years. The dotted lines represent 68% bootstrap error bands. Identification of the model is achieved by a Choleski decomposition of the residual covariance matrix, employing the following ordering of variables: private capital, employment, GDP, public capital.