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by Christopher P. Reicher

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22 June 2010

1. Introduction and motivation

This paper shows that a standard frictionless DSGE macro model with heterogeneity can replicate the qualitative cyclical behavior of labor market search activity, job creation, and job destruction. The Diamond-Mortensen-Pissarides (DMP) model of labor market search and its close relatives have become the workhorse of the macro-labor literature because they ground their description of job creation and destruction as outcomes of optimizing behavior. In the DMP model, firms pay to post vacancies and search for workers, and in some variants of the model, workers and firms separate when it becomes profitable to do so. The search and matching model was originally developed to describe long-run levels of frictional unemployment, but much work has centered around DSGE versions of this model to describe what happens with unemployment, job and worker flows, and vacancies over the cycle. The main appeal of the model comes from its ability to describe frictional unemployment as the outcome of optimizing behavior.[1]

Actually getting the model to match basic business cycle facts has so far proven difficult, leaving the issue of the magnitude and mixture of shocks aside. In particular, the model faces several issues when it comes to simultaneously replicating the business cycle facts on both the hiring and firing margins. Hagedorn and Manovskii (2008) calibrate the model to match the behavior of separations and total employment over the cycle. In order to do so, a large mass of workers must continually reside near the firing margin, and workers and firms must therefore have small surpluses to bargain over. This also implies that search costs are fairly small as a percentage of output because of free entry in the vacancy creation sector. As Krause and Lubik (2007), Ramey (2008), and Reicher (2010) show, this strategy leads to problems with matching the data on the hiring margin. After a wave of job loss, the number of unemployed workers increases. This makes it less costly to hire a marginal worker since workers are easier to find, so vacancies and hiring should increase for most reasonable parameter values. This is exactly the opposite of what happens; during a downturn, vacancies

[1] Mortensen and Pissarides (1994) lay out the basic version of the model and discuss steady states, and den Haan, Ramey, and Watson (2000) lay out the stochastic version of the model with endogenous separations.

decrease sharply, and the reliable negative cyclical correlation between unemployment and vacancies is known as the Beveridge Curve.

Krause and Lubik as well as Reicher show that real and nominal wage rigidity can mitigate this problem somewhat, since too-high wages will dampen the hiring response to unemployment. This mechanism is not enough to generate a Beveridge curve; at most it generates a slightly negative correlation between unemployment and vacancies rather than the overwhelming one in the data. Ramey introduces on-the-job search, and this does somewhat better since it weakens the link between unemployment levels and hiring. He still finds problems with the performance of the search and matching model on the hiring margin. It seems that the search and matching model as currently formulated simply has a difficult time matching the cyclical behavior of hiring and vacancies, though it can match the behavior of job loss. Carlsson, Eriksson, and Gottfries (2008) examine Swedish firm-level data and find no evidence that the level of unemployment contributes to job creation at the firm level.

Perhaps surprisingly, extending the logic of Hagedorn and Manovskii to a zero-surplus model with heterogeneity (and no frictional unemployment) does fit the cyclical facts much better. While search costs and frictional unemployment are important features of the real world that the DMP model captures, it is interesting to abstract from these things to see what introducing heterogeneity alone can deliver. The indivisible-labor RBC model, when augmented with sector-level heterogeneity and costless labor market search, can match the qualitative cyclical behavior of job creation, job destruction, and vacancies. Since there are no surpluses in this model, job creation responds only to aggregate conditions and to the distribution of idiosyncratic movements in labor demand, and not directly to the unemployment rate. Unemployment rises, and the number of vacancies required to support the amount of hiring falls dramatically, giving rise to a Beveridge Curve relationship. The rest of this paper discusses the basic business cycle facts, lays out the model, and discusses this result.

2. Facts about job flows, vacancies, and firm heterogeneity

There are a number of US data sources which provide a picture of aggregate job and worker flows. The most comprehensive and longest-running are the annual job creation and data from the census's Business Dynamics Statistics (BDS) program which run from 1977 through 2005. This newly published series runs longer than the Business Employment Dynamics

(BDM) program which began in the early 1990s, and the BDS covers the entire economy unlike the Longitudinal Research Database. Looking at the BDS (Figure 1), expanding establishments contribute 33% to the variance of net job flows; opening establishments contribute 7%; contracting establishments contribute 50%; and closing establishments contribute 10%.[2] Interestingly, these data show rates of job creation and destruction that are much lower than those published by the Business Employment Dynamics (BDM, formerly BED) series, especially for continuing establishments. The BDS data are published at an annual frequency by taking March-March changes in establishment-level employment and therefore miss out on seasonal and temporary job flows. Both data sources show the same basic picture if one is careful to include the 1990 recession with its spike of job destruction in the analysis.

Figure 2 shows the Beveridge Curve from 1951 through 2009 for the United States. Vacancy rates come from regressing post-2000 JOLTS vacancy rates on the adjusted help wanted index constructed by Barnichon (2010) divided by employment. This is not a perfect measure of vacancies, but no perfect measure exists, and it displays qualitatively sensible behavior. One can clearly see the dynamics of vacancies during a recession. As unemployment rises, vacancies fall, and then vacancies begin to rise slightly as unemployment falls slowly. Unemployment and vacancies trace a long, thin counterclockwise loop. One could also see that the Beveridge Curve shifts at low frequencies. During certain periods (the 1950s and 2000s) the curve lay to the southwest, and in the late 1970s and early 1980s it lay to the northeast. While these shifts in the location of the curve are interesting and make it impossible to estimate the slope of this curve precisely, this paper concentrates on the major cyclical feature of the curve: Long thin counterclockwise loops with a slope of -0.552 (using monthly HP-filtered data with a smoothing parameter of one million).

At the establishment or firm level, there is a large literature on the topic of firm size, concentrating on a stylized fact known as Gibrat's Law. Under Gibrat's Law, the growth rate of a firm is independent of its size and age. Lotti, Santarelli, and Vivarelli (2003, 2009) and Contini and Revelli (1989) show that Gibrat's Law reasonably characterizes the behavior of larger and better-established firms in Italian data. Gibrat's Law is less accurate when it comes

[2] Regressing one component of job flows on net job flows will yield that component's contribution to the variance of net job flows, assuming a factor structure to the data. Appendix B contains the mathematics behind this. The data are HP-filtered with a smoothing parameter of one thousand in order to remove the downward trend in job creation and net job creation; this has the effect of attributing a few percent less of the variation in net job creation to gross job creation than would have otherwise been the case.

to describing new entrants and small firms. Contini and Revelli summarize the literature on US firm sizes as well, finding the same conditional adherence to Gibrat's Law. Rogers, Helmers, and Koch (2010) look at more recent British data and come to the same conclusion; Gibrat's law holds up fairly well for the third and higher deciles of firm size as measured by employment.

In a broad sense, it appears that a model would have to meet the following facts: That continuing firms contribute the overwhelming share (at least 82%) of net job flows, that for the most part these firms follow Gibrat's Law, and that job flows are almost equally divided between job creation and job destruction over the cycle. As it turns out, it is easy to set up an indivisible labor model with heterogeneity that delivers these facts. Furthermore, this model displays Beveridge Curve-type behavior over the cycle, whereby the search activity of employers and unemployment rates are negatively correlated and trace out long thin counterclockwise loops. This suggests that understanding firm-level heterogeneity may provide the key to understanding labor market search behavior over the cycle.

3. The basic indivisible labor model with heterogeneity

As one takes the limit of the DMP model in the absence of frictions, one ends up with an indivisible-labor RBC model. Workers in this model work either a set number of hours or not at all; this behavior can be generated by introducing a nonconvexity into the labor-leisure tradeoff with power utility. Since eighty percent of US labor market movements happen on the extensive margin, this seems to be a good starting point. The model has the property that workers are indifferent between working or not, but one could easily make unemployment involuntary in this model by introducing a more exotic bargaining environment with hold-ups or with complementarities between workers.

3.1. Households

Households have preferences which are balanced-growth compatible and separable. Lifetime utility takes the form:

$$E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[\ln(C_{t+i}) - AN_{t+i} \right].$$

The period by period budget constraint takes the form:

$$B_{t+1} + K_{t+1} + C_t + G_t = (1 + r_{t-1})B_t + (1 + \rho_t - \delta)K_t + W_t N_t + \Pi_t .$$

The first-order conditions for consumption, bonds (which are in zero net supply), capital, and labor inputs are the usual ones:

$$\lambda_t = 1/C_t ; \tag{1}$$

$$\lambda_t = (1 + r_t)\beta E_t \lambda_{t+1} ; \tag{2}$$

$$\lambda_t = \beta E_t \lambda_{t+1}(1 - \delta + \rho_{t+1}) ; \tag{3}$$

and

$$\lambda_t W_t = A, \tag{4}$$

with transversality conditions which rule out large bubbles in capital accumulation and Ponzi schemes between agents:

$$\lim_{T\to\infty} \beta^T E_t \frac{\lambda_{t+T}}{\lambda_t} K_{t+T+1} = 0 \text{, and}$$

$$\lim_{T\to\infty} \beta^T E_t \frac{\lambda_{t+T}}{\lambda_t} B_{t+T+1} = 0 .$$

3.2. Production and factor demand within and across sectors

Firms seek to maximize period by period economic profits, and they can adjust inputs immediately. They are price takers in output and factor markets. They produce according to the Cobb-Douglas production function with sector-specific productivity a_{it} which has an identical cross-sectional distribution across time, economywide productivity z_t, and factor inputs k_{it} and n_i:

$$y_{it} = a_{it}k_{it}^{\alpha}(z_t n_{it})^{1-\alpha} .$$

Profits depend on price, output, and factor costs:

$$\Pi_{it} = p_{it}a_{it}k_{it}^{\alpha}(z_t n_{it})^{1-\alpha} - W_t n_{it} - \rho_t k_{it} .$$

Firm-level output is aggregated according to the CES aggregator

$$Y_t = \left[\int_0^1 y_{jt}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}},$$

with the budget constraint

$$Y_t = \int_0^1 p_{jt} y_{jt} dj .$$

Appendix A goes through the math relating aggregate variables to sector-level variables. The economy ends up producing according to an economywide production function, for some constant l which reflects the aggregate of idiosyncratic productivity:

$$Y_t = l^{1-\theta}K_t^{\alpha}(z_t N_t)^{1-\alpha} . \tag{5}$$

Economywide factor demand is also standard, expressed as income shares:

$$W_t N_t = (1-\alpha)Y_t , \tag{6}$$

and

$$\rho_t K_t = \alpha Y_t . \tag{7}$$

Profits are zero in equilibrium. Market clearing implies that income equals expenditure and that net lending equals zero, yielding the economywide resource constraint:

$$K_{t+1} + C_t + G_t = (1-\delta)K_t + Y_t. \tag{8}$$

Technology follows a loglinear random walk with drift:

$$\ln(z_t) = \Gamma^z + \ln(z_{t-1}) + \varepsilon_t^z, \tag{9}$$

and autonomous demand G_t follows an AR(2) process which error-corrects toward the long-run level of productivity in order to ensure balanced growth:

$$\ln(G_t) = (1-\phi_1^G - \phi_2^G)(\ln(G) + \ln(z_t)) + \phi_1^G \ln(G_{t-1}) + \phi_2^G \ln(G_{t-2}) + \varepsilon_t^G. \tag{10}$$

An equilibrium in this economy is a series of allocations $\{K_{t+1}, C_t, G_t, Y_t, N_t, W_t, \rho_t, r_t, z_t, \lambda_t\}$ such that the first order conditions and budget constraints (1) through (10) are satisfied, along with the transversality conditions. Based on real allocations, one can back out the distribution of firm growth and shrinkage as well as unemployment and vacancies. Job growth at the individual firm level is given by:

$$\frac{n_{it}}{n_{it-1}} = \frac{a_{it}^{\theta-1}}{a_{it-1}^{\theta-1}}\frac{N_t}{N_{t-1}}.$$

Assumptions about firm growth may turn this problem into a tractable or an intractable one. One example of a tractable formulation which is consistent with the observed firm age-size profile would be to say that a fraction γ of firms closes exogenously in a given period and the same measure of firms opens up out of some initial well-behaved productivity distribution with employment $\mu_0 N_t$. For the rest of the firms, productivity grows according to an iid geometric random walk that obeys Gibrat's Law, so that:

$$\ln(g_{it}) \equiv \ln\left(\frac{a_{it}^{\theta-1}}{a_{it-1}^{\theta-1}}\frac{N_t}{N_{t-1}}\right) \sim N(\mu_g + \ln(N_t) - \ln(N_{t-1}), \sigma_g^2).$$

Since productivity growth in the model is independent of productivity levels at the micro level, it is possible to come up with the economywide job creation rate by accounting for exogenous job creation and destruction and then integrating over g_{it} to obtain the endogenous portion:

$$jc_t = \mu_0 \left(+ \Gamma^{LF} - 1 \right) \frac{N_t}{N_{t-1}} + (1-\gamma) \int_{g_{it}=1}^{\infty} (g_{it} - 1) df(g_{it}),$$

and

$$jd_t = \gamma + (1-\gamma) \int_{g_{it}=0}^{1} (1 - g_{it}) df(g_{it}).$$

Plugging in the formulas for a lognormal distribution yields the "iron law" relationships

$$jc_t = \mu_0 \left(+ \Gamma^{LF} - 1 \right) \frac{N_t}{N_{t-1}} + (1-\gamma) S_t, \tag{11}$$

and

$$jd_t = \gamma + (1-\gamma) \left[S_t + 1 - \frac{N_t}{N_{t-1}} m \right], \tag{12}$$

where

$$S_t = \frac{N_t}{N_{t-1}} e^{\mu_g + \frac{1}{2}\sigma_g^2} \Phi\left(\frac{\mu_g + \ln(N_t) - \ln(N_{t-1}) + \sigma_g^2}{\sigma_g} \right) - \Phi\left(\frac{\mu_g + \ln(N_t) - \ln(N_{t-1})}{\sigma_g} \right),$$

and

$$m = e^{\mu_g + \frac{1}{2}\sigma_g^2}.$$

In a frictionless model, vacancies are posted costlessly and bargaining surpluses between workers and firms are zero. Vacancies represent the search activity needed for employers to instantaneously fill new positions. The unemployment rate is given by the share of people who are not working at a given moment:

$$U_t = 1 - N_t, \tag{13}$$

and the number of jobs created as a function of unemployment and vacancies is given by the matching function:

$$jc_t N_{t-1} = \omega U_t^{\zeta} V_t^{1-\zeta}. \tag{14}$$

3.3. Calibration

Most of the parameters on the RBC side are chosen to fit a standard calibration based on US first moments. Capital's share in the production function is 0.3; depreciation is 2% per quarter; real interest rates are 1% per quarter; trend productivity growth is 0.23% per quarter; trend labor force growth is 0.56% per quarter (given by net BDS job creation). Government consumption is 16% of output with the rest of government spending lumped into investment, and the unemployment rate is 5%, while the vacancy rate is 3.8%. Unemployment's share in the matching function is 0.4 as estimated by Blanchard and Diamond (1989, 1991), and the coefficients on government spending (which give it a persistent hump shape) are 1.5 and -0.52. Nothing at all of what follows depends on the exact values of these coefficients; they are chosen for exposition purposes.

The BDS data provide a good conceptual fit with the model in that both are intended capture medium-run as opposed to short-run firm dynamics. From the BDS data, job destruction is 3.84% per quarter and job creation 4.40% per quarter. Establishment closings γ are 1.38% per quarter, and continuing establishments grow at the rate of 0.3% per quarter. New establishments are 50.7% the size of the average establishment, which gives the value of μ_0. The aggregate of idiosyncratic productivity l is normalized to 1.

This calibration strategy produces a quarterly dispersion parameter σ_g of 0.0797. There is external evidence on this issue. Davis, Faberman, Haltiwanger, Jarmin, and Miranda (2010) calculate that the cross-sectional dispersion (standard deviation) of establishment-level growth rates across all firms (including opening and closing firms) is 61% per year. They also present a measure of volatility that is cleansed of firm-specific effects, and this is measured at 46% per year. Both measures include establishment openings and closures, however, and this will overstate what happens to those firms which do not open or close. According to their formula which weights openings and closings at one half, the variance of measured idiosyncratic firm growth approximately equals:

$$\text{mean}^* = \frac{\mu_0\left(+ \Gamma^{LF} - 1\right) + (1-\gamma)\text{m} - \gamma}{.5\mu_0\left(+ \Gamma^{LF} - 1\right) + 1 + .5\gamma}$$

$$\text{disp}^{*2} = \frac{.5\mu_0\left(+ \Gamma^{LF} - 1\right)(2-\text{mean}^*)^2 + (1-\gamma)(\text{m}-1-\text{mean}^*)^2 + .5\gamma(2+\text{mean}^*)^2}{.5\mu_0\left(+ \Gamma^{LF} - 1\right) + 1 + .5\gamma}.$$

Using the calibrated parameters adjusted to annual rates, the Davis et al. measure of firm growth dispersion would equal approximately 45% per year, which is toward the bottom of the published measures.

There is more evidence on this issue, and this comes from the BDS aggregates. The coefficient on linearized job creation given by the linearized version of (11) after normalizing by labor force growth equals 0.56. This represents the statistical contribution of gross job creation to net job creation as implied by the model. Section II above discusses the contribution of gross job creation to the overall variance of net job creation, which equals approximately 60 percent as estimated by OLS. The model abstracts from establishment entry and exit, which do not matter much for short-run employment dynamics. It nonetheless captures aggregate job creation and destruction over the cycle quite well.

The standard deviation of annual total factor productivity growth, given by the BLS's multifactor productivity index, is 0.95% per year, or 0.47% per quarter, from 1987 onward. Since the standard deviation of unemployment during that same period is 0.92%, the demand shocks must have a quarterly standard deviation of 0.57% to match this. The exact mix of shocks does not matter since the shape of the Beveridge Curve conditional on both shocks is the same.

4. Simulation Results

Figure 3 shows what happens after a negative unit productivity shock is fed into the model. Aggregate variables move in their expected manner. Investment falls and labor input falls sharply, while the capital stock takes a while to fall. Job creation falls and job destruction spikes. During the recovery, job creation outpaces job destruction, but not by nearly the same pace. Since unemployment has now risen, it takes fewer vacancies to create the same number of jobs, so vacancies fall. Figure 4 shows what happens in unemployment-vacancy rate space. The initial impulse moves the economy far to the southeast in this space; unemployment rises and the initial crash in job creation requires fewer vacancies to support it. As the economy recovers, it moves back to the northwest in a counterclockwise fashion (as hiring slightly exceeds its long run average), forming a long thin loop. Figures 5 and 6 repeat the same exercise after a demand shock; the behavior of job creation, job destruction, and vacancies is almost the same as under a productivity shock. In this sense, the mix of shocks does not matter.

Figure 7 shows the Beveridge Curve after shocking the system for 100,000 quarters and keeping the last 240. Regressing the vacancy rate on the unemployment rate yields a coefficient of about -0.545, which is extremely close to the -0.552 found in the data. The Beveridge Curve does not exactly match the loops shown in Figure 1, mostly because unemployment rises instantaneously in this model while it rises more slowly in the data, but the general slope and shape are correct. The inexact match is an artifact of the lack of persistence in RBC models. A model which had more persistence built into it would possibly do a better job at matching the Beveridge Curve.

5. Conclusion

It appears that a frictionless model of unemployment captures the movements of job creation, job destruction, and vacancies over the cycle astoundingly well. When surviving-firm dynamics accord with Gibrat's Law, one can simultaneously match the volatility of firm growth at the micro level with the relative contributions of job creation and destruction to net employment growth at the macro level. Furthermore, by backing out the implicit search activity engaged in by firms, it is possible to generate a Beveridge Curve which looks strikingly like that observed in real life. The Beveridge Curve has the right slope and follows a long, thin counterclockwise loop, but the model itself does not quite get the persistence of recessions right, so the Beveridge Curve is not perfect.

The frictionless model offers insight into how the DMP model simultaneously succeeds and fails at capturing labor market dynamics. In the DMP model, vacancy posters can freely post vacancies in order to capture a bargaining surplus, and this would result in countercylical hiring activity. In the frictionless model, gross hiring responds to labor demand, so if labor demand falls, hiring falls. There is no channel for unemployment to positively affect hiring. Both models predict countercyclical job destruction and match the facts along that dimension, and they both do so because separating firms and workers have zero surplus. The results of this exercise suggest that modifying the DMP model to make hiring more responsive to overall labor demand might improve the performance of that model, and that one should focus on heterogeneity rather than on search costs as the prime driver of gross job flows over the cycle.

References

Barnichon, Regis, 2010. "Building a Composite Help Wanted Index," Working Paper, Federal Reserve Board.

Blanchard, Olivier Jean, and P. Diamond, 1989. "The Beveridge Curve," Brookings Papers on Economic Activity, pages 1-60.

Blanchard, Olivier Jean, and P. Diamond, 1991. "The Aggregate Matching Function," NBER Working Paper 3175.

Carlsson, Mikael, Stefan Eriksson, and Nils Gottfries, 2008. "How Important are Search Frictions for Employment Dynamics?" Working Paper, Uppsala University.

Contini, Bruno, and Ricardo Revelli, 1989. "The Relationship between Firm Growth and Labor Demand," Small Business Economics 1, pages 309-314.

Davis, Steven J., R. Jason Faberman, John Haltiwanger, Ron Jarmin, and Javier Miranda, 2010. "Business Volatility, Job Destruction, and Unemployment," American Economic Journal: Macroeconomics, vol. 2, pages 259-287.

den Haan, Wouter J., Garey Ramey, Joel Watson, 2000. "Job Destruction and Propagation of Shocks." American Economic Review 90(3), pages 482-98.

Hagedorn, Marcus, and Iourii Manovskii, 2008. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," American Economic Review 98(4), pages 1692-1706.

Lotti, Francesca, Enrico Santarelli, and Marco Vivarelli, 2003. "Does Gibrat's Law hold among young, small firms?" Journal of Evolutionary Economics 13, pages 213-235.

Lotti, Francesca, Enrico Santarelli, and Marco Vivarelli, 2009. "Defending Gibrat's Law as a Long-Run Regularity," Small Business Economics 32(1), pages 31-44.

Mortensen, Dale T., Christopher A. Pissarides, 1994. “Job Creation and Job Destruction in the Theory of Unemployment,” Review of Economic Studies 61(3), pages 397-415.

Ramey, Garey, 2008. “Exogenous versus Endogenous Separations,” Working Paper, UCSD.

Reicher, Christopher, 2010. “What Can a New Keynesian Labor Matching Model Match?” Kiel Working Paper 1496.

Rogers, Mark, Christian Helmers, and Christoffer Koch, 2010. “Firm growth and firm size.” Applied Economics Letters 2010, pages 1-4.

Appendix A: Factor demand and output across heterogeneous sectors

As a result of profit maximization, income shares are constant at the firm level:

$$W_t n_{it} = (1-\alpha)\, p_{it}\, y_{it},$$

and

$$\rho_t k_{it} = \alpha p_{it}\, y_{it},$$

or in aggregate terms this gives the income shares (6) and (7):

$$W_t N_t = (1-\alpha) Y_t,$$

and

$$\rho_t K_t = \alpha Y_t.$$

Free entry ensures that marginal costs and prices are equal across sectors, or that price times productivity is the same across sectors:

$$p_{it} a_{it} = p_{jt} a_{jt} \equiv l_t.$$

Price-taking behavior yields the demand curve

$$y_{it} = p_{it}^{-\theta} Y_t,$$

and the price index (normalized to 1 since output is its own numeraire) is given by

$$P_t = 1 = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}.$$

Substituting the free entry condition into the demand curve yields

$$y_{it} = l_t^{-\theta} a_{it}^{\theta} Y_t,$$

and substituting this into the output aggregator yields

$$l_t \left[\int_0^1 a_{it}^{\theta-1} dj \right]^{\frac{1}{1-\theta}} = 1.$$

Substituting the production function into the aggregator gives the ugly expression

$$Y_t = \left[\int_0^1 \left(p_{it}\, y_{it} a_{it} \left(\frac{\alpha}{\rho_t} \right)^{\alpha} \left(z_t \frac{1-\alpha}{W_t} \right)^{1-\alpha} \right)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}},$$

and substituting aggregate factor demand back in yields

$$Y_t = \left[\int_0^1 \left(p_{it}\, y_{it} a_{it} \left(\frac{K_t}{Y_t} \right)^{\alpha} \left(z_t \frac{N_t}{Y_t} \right)^{1-\alpha} \right)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}.$$

Substituting the aggregate versions of product demand yields

$$Y_t = \left[\int_0^1 \left(l_t^{1-\theta} a_{it}^{\theta} Y_t \left(\frac{K_t}{Y_t} \right)^{\alpha} \left(z_t \frac{N_t}{Y_t} \right)^{1-\alpha} \right)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}},$$

and after cleaning up one arrives at the economywide Cobb-Douglas production function

$$Y_t = l_t^{1-\theta} K_t^{\alpha} (z_t N_t)^{1-\alpha}.$$

With no loss of generalization (since the cross-sectional distribution of a_{it} does not change over time) we can set l_t to a constant, so that we arrive at (5):

$$Y_t = l^{1-\theta} K_t^{\alpha} (z_t N_t)^{1-\alpha}.$$

To get relative labor demand, one can divide firm-specific labor demand by economywide labor demand to get:

$$n_{it} = \frac{p_{it}\, y_{it}}{Y_t} N_t .$$

Substituting the firm's product demand yields the firm's share of employment as a function of firm-specific productivity:

$$n_{it} = 1^{1-\theta} a_{it}^{\theta-1} N_t ,$$

and taking growth rates yields:

$$\frac{n_{it}}{n_{it-1}} = \frac{a_{it}^{\theta-1}}{a_{it-1}^{\theta-1}} \frac{N_t}{N_{t-1}} .$$

Appendix B: Deriving a variance decomposition in a factor model

Imagine a vector of time series that share a common factor. The goal is to attribute movements in some aggregate of these time series to the systematic movements in the original series. One can represent the factor structure of the series as

$$y_t = af_t + \varepsilon_t ,$$

where y_t is a column vector of data; a is a column vector of factor loadings; and ε_t is a series of idiosyncratic components that is independent across rows and with the factor. Let b equal an accounting identity which links the rows of y to a scalar aggregate x. Then one could write, after restricting the weighted sum of idiosyncratic residuals to equal zero,

$$x_t = by_t = baf_t + b\varepsilon_t .$$

If one is willing to normalize the factor as being equal to x itself plus the nonsystematic part, then one could force ba=1 and write

$$x_t = f_t + b\varepsilon_t .$$

For each i, regressing y_i on x and then multiplying by b_i gives the coefficient:

$$c_i = \left(\frac{1}{T}\sum_{t=1}^{T} x_t x_t'\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T} x_t y_{it}'\right) b_i ,$$

which converges in probability to

$$c_i \underset{p}{\rightarrow} \left(E(x_t x_t')\right)^{-1} \left(E(x_t y_{it}')\right) b_i .$$

Writing out the variance terms, where subscripts under the Greek letter σ denote variances of individual objects,

$$c_i \underset{p}{\to} \left(\sum_{i=1}^{I} \left(b_i \sigma_f^2 + \sigma_{\varepsilon,i}^2 \right) b_i \right)^{-1} \left(b_i \sigma_f^2 + \sigma_{\varepsilon,i}^2 \right) b_i \, .$$

The right hand side is series i's contribution to the overall variance of the aggregate series, while the left hand side is the overall variance of the aggregate series. It is in this sense that the elements of c_i could be thought of as a variance decomposition. If one is willing to go further and assume that $b\varepsilon_t = 0$ at all times, then one can come up with the stronger statement:

$$c_i \underset{p}{\to} \left(\sum_{i=1}^{I} \left(b_i \sigma_f^2 b_i \right) \right)^{-1} \left(b_i \sigma_f^2 b_i \right).$$

Appendix C: Linearized indivisible-labor model with job flows.

Here are the linearized versions of the model, with the same equation numbers as in the main text. In some cases convenient substitutions are made:

$$\hat{\lambda}_t + \hat{C}_t = 0; \tag{1L}$$

$$\hat{\lambda}_t - \hat{r}_t = E_t \hat{\lambda}_{t+1}; \tag{2L}$$

$$\hat{r}_t = E_t \frac{\rho}{1-\delta+\rho} \hat{\rho}_{t+1}; \tag{3L}$$

$$\hat{\lambda}_t + \hat{W}_t = 0; \tag{4L}$$

$$\hat{Y}_t - \alpha\hat{K}_t - (1-\alpha)\hat{z}_t - (1-\alpha)\hat{N}_t = 0; \tag{5L}$$

$$\hat{W}_t + \hat{N}_t - \hat{Y}_t = 0; \tag{6L}$$

$$\hat{\rho}_t + \hat{K}_t - \hat{Y}_t = 0; \tag{7L}$$

$$(1-\delta)\frac{K}{Y}\hat{K}_t + \hat{Y}_t - \frac{C}{Y}\hat{C}_t - \frac{G}{Y}\hat{G}_t = \Gamma^z \Gamma^{LF} \frac{K}{Y}\hat{K}_{t+1}. \tag{8L}$$

Technology follows a loglinear random walk with drift:

$$\hat{z}_t = \hat{z}_{t-1} + \varepsilon_t^z, \tag{9L}$$

and autonomous demand G_t follows an AR(2) process:

$$\hat{G}_t = (1-\phi_1^G - \phi_2^G)\hat{z}_t + \phi_1^G \hat{G}_{t-1} + \phi_2^G \hat{G}_{t-2} + \varepsilon_t^G. \tag{10L}$$

Job creation and destruction are both linearized as a proportion of employment:

$$\mathrm{jc}_t = \left[\iota_0 \left(\kappa + \Gamma^{\mathrm{LF}} - 1\right) + (1-\gamma)\mathrm{a}\right]\left(\hat{\mathrm{N}}_t - \hat{\mathrm{N}}_{t-1}\right), \tag{11L}$$

and

$$\mathrm{jd}_t = (1-\gamma)\left[\mathrm{a} - \mathrm{e}^{\mu_g + \frac{1}{2}\sigma_g^2}\right]\left(\hat{\mathrm{N}}_t - \hat{\mathrm{N}}_{t-1}\right), \tag{12L}$$

where

$$\mathrm{a} = \mathrm{e}^{\mu_g + \frac{1}{2}\sigma_g^2}\Phi\left(\frac{\mu_g + \sigma_g^2}{\sigma_g}\right) + \frac{\mathrm{e}^{\mu_g + \frac{1}{2}\sigma_g^2}}{\sigma_g}\phi\left(\frac{\mu_g + \sigma_g^2}{\sigma_g}\right) - \frac{1}{\sigma_g}\phi\left(\frac{\mu_g}{\sigma_g}\right).$$

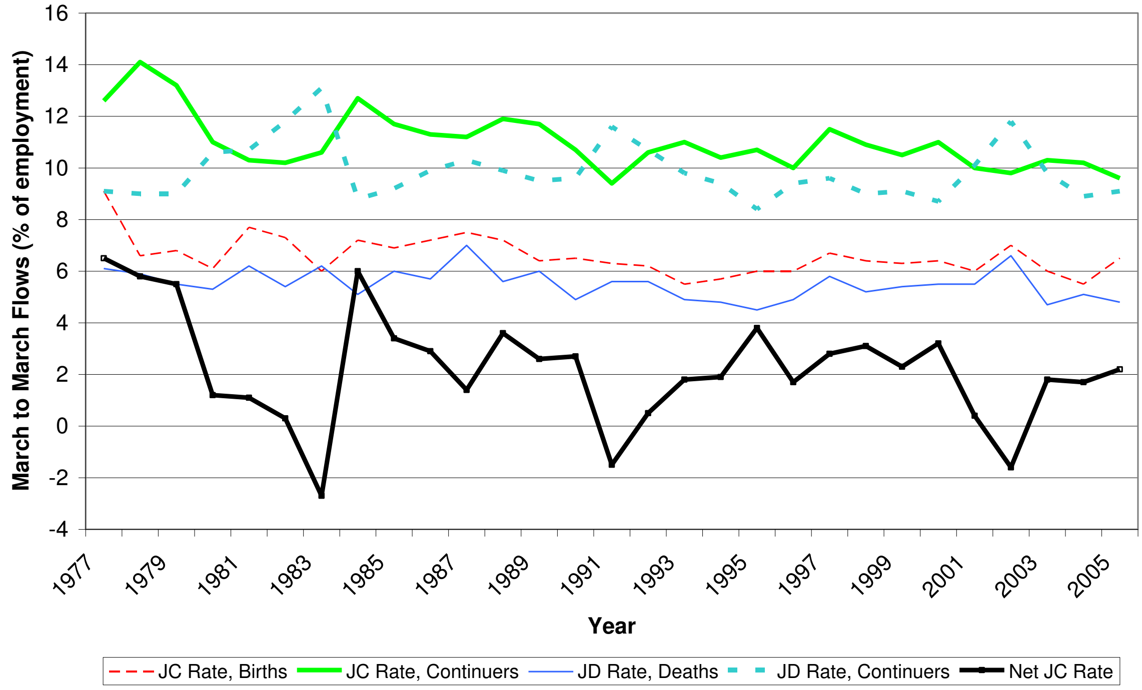
Unemployment is linearized as follows:

$$\hat{\mathrm{U}}_t = -\frac{\mathrm{N}}{1-\mathrm{N}}\mathrm{N}_t, \tag{13L}$$

and the matching function is given by:

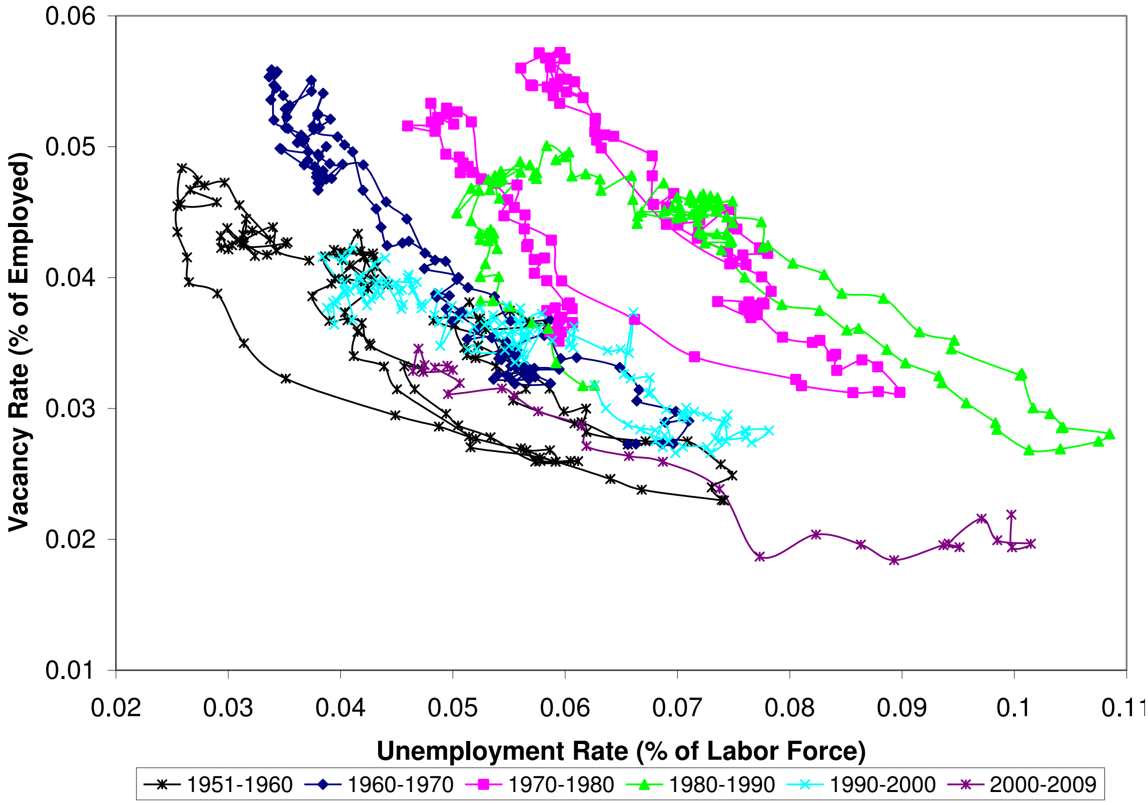
$$\hat{\mathrm{N}}_{t-1} = \zeta\hat{\mathrm{U}}_t + (1-\zeta)\hat{\mathrm{V}}_t - \frac{1}{\mathrm{jc}}\mathrm{jc}_t. \tag{14L}$$

Figure 1: Annual March-March BDS Job Flows, 1977-2005



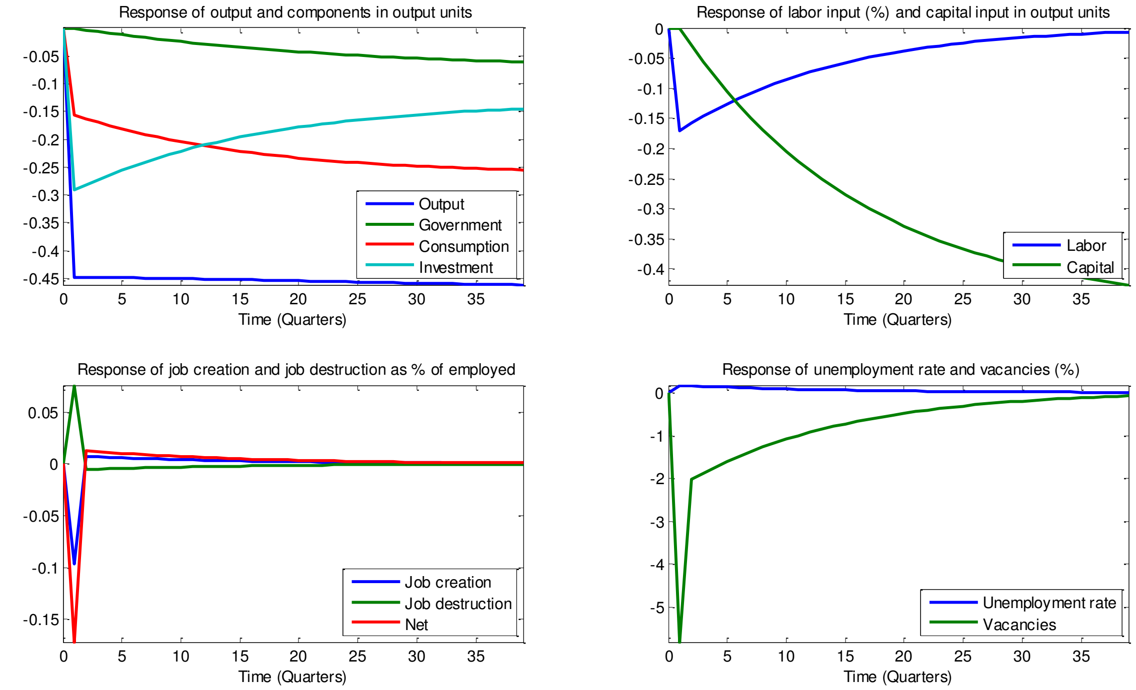
Source: Census Bureau Business Dynamics Statistics (BDS), and author's calculations.

Figure 2: Beveridge Curve by Decade



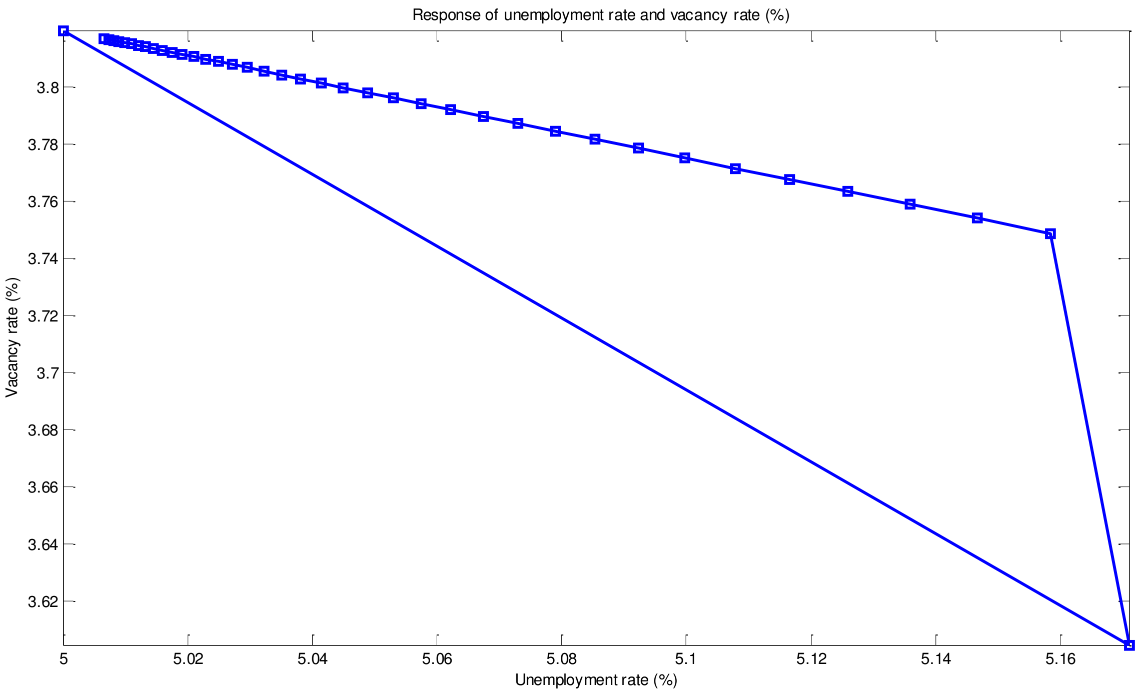
Source: Help wanted data from Barnichon (2010), nonfarm employment and vacancies from the BLS's CES and JOLTS programs, and author's calculations.

Figure 3: Impulse response to a -0.47% productivity shock



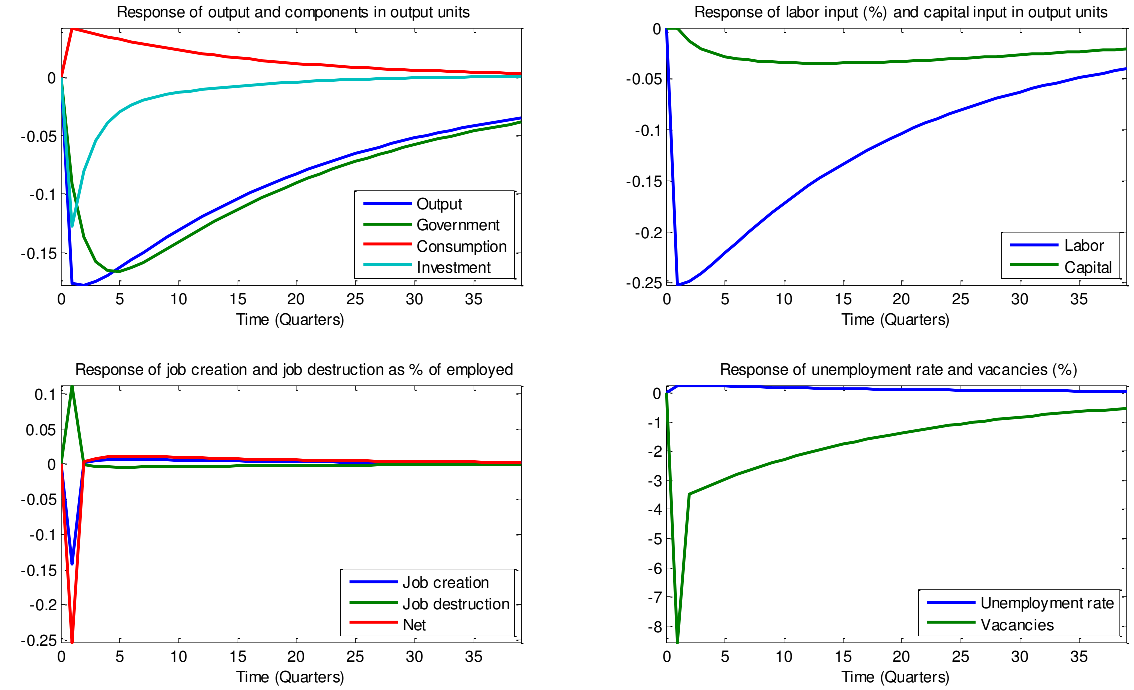
Source: Author's calculations using linearized model.

Figure 4: Response of vacancies and unemployment to a -0.47% productivity shock



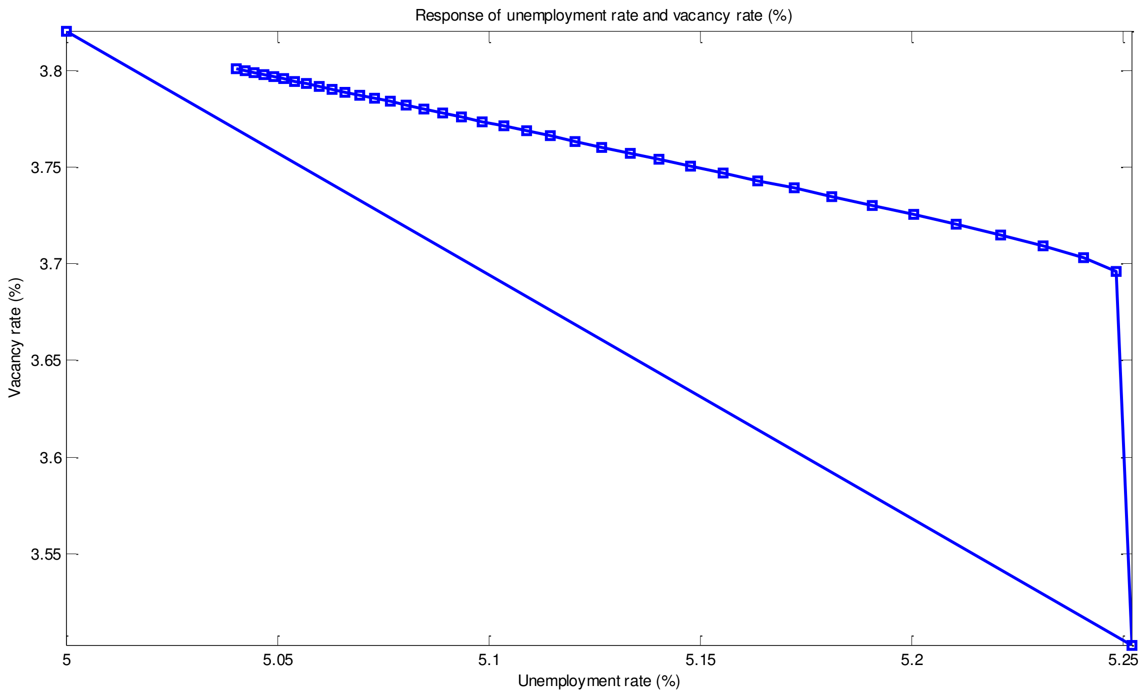
Source: Author's calculations using linearized model.

Figure 5: Impulse response to a -0.57% demand shock



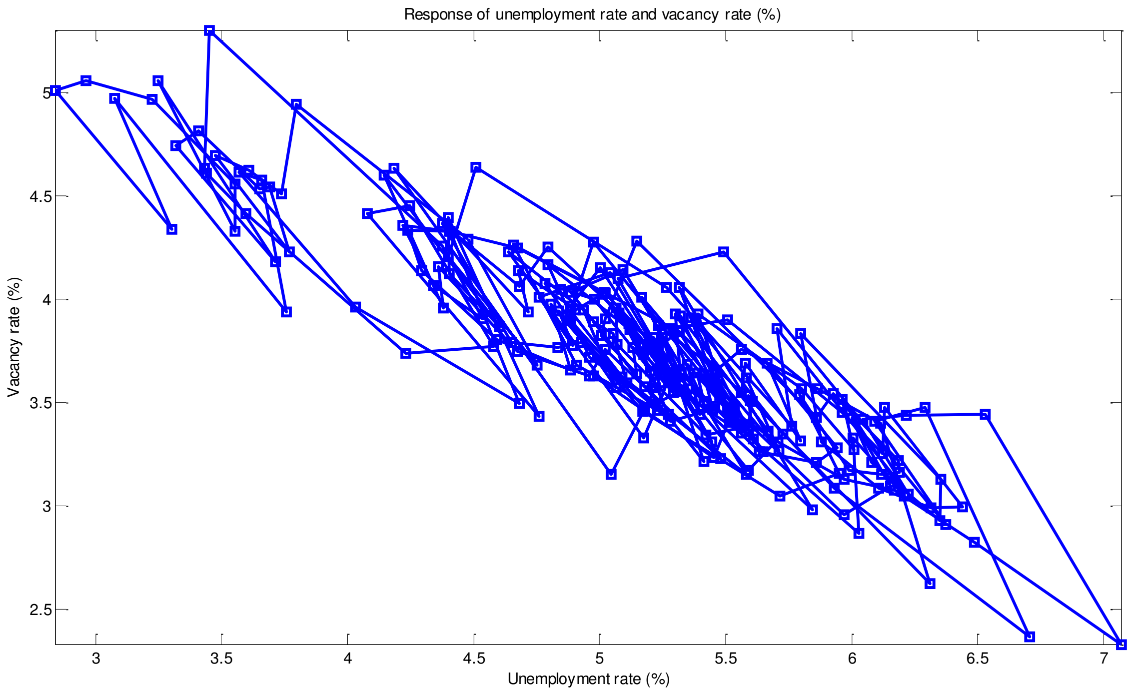
Source: Author's calculations using linearized model.

Figure 6: Response of vacancies and unemployment to a -0.57% demand shock



Source: Author's calculations using linearized model.

Figure 7: Simulated Beveridge Curve



Source: Author's calculations using linearized model. This represents a random draw of 240 quarters from the ergodic distribution of the Beveridge Curve.