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by **Mewael F. Tesfaselassie**

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**Keywords:** Trend growth, government spending shocks, fiscal multiplier.

**JEL classification:** E21, E62, H31

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# Trend Growth and the Dynamic Effects of Government Spending

Mewael F. Tesfaselassie\*

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## Abstract

The paper studies the macroeconomic effects of government spending shocks in an economy characterized by positive trend growth. It shows that the lower is the trend growth rate the less inflationary are government spending shocks and vice versa. Moreover, on impact output is higher but exhibits less persistence the lower is trend growth, an effect that also characterizes consumption and the fiscal multiplier given that consumption and labor are somewhat complementary.

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# 1 Introduction

In response to the recent global recession caused by the financial crisis of 2007, many governments resorted to fiscal policy once monetary policy was constrained by the zero lower bound of the short term interest rate. The intervention has prompted academic and policy debates on the effectiveness of fiscal policy (see, e.g., Bilbiie (2009) and Woodford (2010)). At the same time, concerns were raised about the long-term effect of the financial crisis, for example that the severity of the crisis could have lowered the potential growth of the economy. If such concerns are legitimate then discussions of the effectiveness of fiscal policy measures should take into account the role of lower trend growth.

This paper makes a contribution in that direction by incorporating trend growth into a workhorse New Keynesian model characterized by nominal price and wage rigidities. It then analyzes the dynamic effects of government spending shocks on key macroeconomic variables such as output, consumption, wage inflation and price inflation. The main finding is that the lower the trend growth rate the less inflationary are government spending shocks and vice versa. Moreover, on impact output is higher but less persistent the lower is trend growth, an effect that also characterizes consumption and the fiscal multiplier, given strong complementarity between consumption and labor in the utility function. When complementarity is weak consumption drops in response to a government spending shock but the effect is smaller (and thus fiscal multiplier increases) the lower is trend growth. As the model is stylized the focus is more on the qualitative effects of trend growth, in the face of government spending shocks, rather than on finding an exact quantitative value (in particular for the fiscal multiplier).

Since the model has both price and wage staggering, trend growth affects both wage and price setting. In this case, trend growth ultimately matter via its effects on

the shape of the Phillips curve. To help our intuition, we identify the key effects of trend growth. On the one hand, when setting prices, firms discount the future more heavily (lower stochastic discount factor) due to declining marginal utility from consumption on account of growth. On the other, the higher is trend growth, the higher is trend nominal wage inflation, given trend price inflation. This pushes the optimal reset wages up but this effect is mitigated through complementarity between consumption and employment.

The paper is organized as follows. Section 2 lays out a New Keynesian model that incorporates trend growth, where the effects of trend growth on short-run dynamics is shown to depend on the interaction of nominal price and wage rigidities. Then Section 3 presents the results in terms of the impulse response to government spending shocks. Moreover, the section discusses some limiting cases of the baseline model, pertaining to flexibility of prices and wages, as well as the degree of complementarity in consumption and labor. Results are also presented for a model with some degree of nominal indexation. Finally, Section 4 gives concluding remarks.

## **2 A New Keynesian model with trend growth**

We employ a New Keynesian model that features both nominal wage and price rigidities, where in any give period a fraction of households (firms) can not reset their wages (prices) optimally. Both sources of rigidities turn out to be important for trend growth to have meaningful effects on short-run dynamics.

In incorporating trend growth into a New Keynesian model, we use insights from the literature on balanced growth and business cycles (e.g., King, Plosser and Rebelo (1988a,b)) as well as the literature on long-run labor supply and consumption Euler equation (e.g., Basu and Kimball (2002), Kiley (2007)) to make sure that the business cycle model has implications that are consistent with balanced growth facts.

The introduction of trend growth is similar to Amano (2009) and Mattesini and Nistico (2010). The former examine optimal inflation under trend growth while the latter deal with optimal policy under trend growth. Unlike Amano (2009) our model allows for potential non-separable utility in household preferences, in line with some recent empirical evidence on the consumption Euler equation, in which parameter restrictions are imposed consistent with balanced growth facts (see e.g., Basu and Kimball (2002) and Kiley (2007)). The role of non-separable utility has also been emphasized in recent discussions of the government spending multiplier as a mechanism to generate positive response of consumption to fiscal spending shocks observed in the data (see, e.g., Monacelli and Perotti (2009) and Bilbiie (2009)).<sup>1</sup> Unlike Mattesini and Nistico (2010) we allow for staggered nominal wages. As will be shown later, the degree of stickiness in nominal wages and prices are important in the transmission mechanism of shocks under trend growth.

## 2.1 Trend growth

Following much of the New-Keynesian literature we assume labor to be the only input in the production function and let labor productivity,  $A_t$ , follow a deterministic trend, namely,  $A_{t+i} = \gamma A_{t+i-1} = \gamma^i A_t$ , where  $\gamma > 1$  is one plus the growth rate of productivity. In that case, the model has the property that, in a balanced growth path, aggregate output, consumption, government spending and real wages grow at the same rate as productivity, while aggregate employment is constant.

## 2.2 Government

The government sector is incorporated in a standard way (see, e.g., Monacelli and Perotti (2009) and Bilbiie (2009)). Aggregate government spending  $G_t$  is financed

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<sup>1</sup>Note, however, that our main focus is the effect of trend growth on short-run dynamics.

by lump-sum taxes  $T_t$  and in every period the government ensures a balance budget ( $G_t = T_t$ ). The process for  $G_t$  has two components—a trend component and a business cycle component—and is given by

$$G_t = A_t \epsilon_t \tag{1}$$

$$\epsilon_t = \epsilon_{t-1}^\rho e_t \tag{2}$$

where  $0 < \rho < 1$  captures the degree of persistence and  $e_t \sim iid(1, \sigma_e^2)$ .

Equation (1) can be rewritten in detrended form as

$$g_t = \epsilon_t \tag{3}$$

where  $g_t = G_t/A_t$  is detrended government spending.

## 2.3 Households

Household utility depends on consumption  $C_t$  and hours worked  $N_t$ . As has been shown by King, Plosser and Rebelo (1988a,b), for a business cycle model to be consistent with balanced steady state growth (and constancy of the great-ratios<sup>2</sup>), household utility must be of the form

$$U(C_t, N_t) = \frac{C_t^{1-\sigma} V(N_t)}{(1-\sigma)}; \sigma > 0 \tag{4}$$

The utility function may be potentially non-separable in consumption and leisure.<sup>3</sup> Consistent with available empirical estimates (see for e.g., Basu and Kimball (2002), Kiley (2007) and Guerron-Quintana (2008)) and recent theoretical analyses (e.g., Monacelli and Perotti Monacelli and Perotti (2009) and Bilbiie (2009)) we assume

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<sup>2</sup>See for e.g., Solow (1956).

<sup>3</sup>A limiting case of the utility function (4) is when  $\sigma = 1$ , so that  $u(C_t, N_t) = \log C_t - V(N_t)$ .

$U_{C,N} > 0$ .<sup>4</sup> In order to ensure concavity of  $V(N)$ ,  $v'(N) > 0$  is required. The specific functional form of  $V(N_t)$  is such that the elasticity of intertemporal substitution in consumption (holding  $N$  constant) should be independent of the level of consumption, consistent with balanced growth. As in Basu and Kimball (2002) and Kiley (2007) we assume  $V(N_t) = e^{-(1-\sigma)v(N_t)}$  so that the elasticity of intertemporal substitution in consumption is given by  $1/\sigma$ . Moreover, without loss of generality we let  $v(N_t) = N_t^{1+\eta}/(1+\eta)$ , where  $\eta > 0$ .

It is well known in the business cycle literature that models with non-separable utility and wage staggering turn out to be intractable, as consumption decisions of households depend on the entire history of wages, which complicates aggregation of consumption decisions. Most papers sidestep the aggregation problem by assuming separable utility (see e.g., Erceg, Henderson and Levin (2000)). By contrast, we assume a large representative household with a continuum of members, each supplying a differentiated labor service (this approach follows Schmidt-Grohe and Uribe (2005)). The household cares about per capita consumption  $C_t = \int_0^1 C_{j,t} dj$  and per capita hours worked  $N_t = \int_0^1 N_{j,t} dj$  (where  $j$  indexes household members) and sets wages for each of its members.

The household consumes a continuum of differentiated goods, indexed by  $k$ , which are transformed into a Dixit-Stiglitz composite good  $C_t$  as follows

$$C_t = \left( \int_0^1 C_{k,t}^{1/\mu_p} dk \right)^{\mu_p} \quad (5)$$

where  $\mu_p = \frac{\theta_p}{\theta_p - 1}$  and  $\theta_p$  is the elasticity of substitution between any two differentiated goods. We first solve for the household's consumption allocation across all goods for a given level of  $C_t$ . Minimizing total expenditure  $\int_0^1 P_{k,t} C_{k,t} dk$  subject to (5) gives

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<sup>4</sup>Note that  $U_{C,N} > 0$  does not necessarily imply that consumption and work are complementary (i.e., consumption and leisure are substitutes), in sense that the demand for consumption goods increases if real wages (the price of leisure) increases. For details discussion, see Bilbie (2009).



the consumption demand for each good  $k$ <sup>5</sup>

$$C_{k,t} = \left( \frac{P_{k,t}}{P_t} \right)^{-\theta_p} C_t \quad (6)$$

where  $P_t$  is the aggregate price index (or the price level), which is defined as

$$P_t = \left( \int_0^1 P_{k,t}^{1-\theta_p} dk \right)^{\frac{1}{1-\theta_p}} \quad (7)$$

Next we derive the optimal decisions regarding the paths of  $C_t$ ,  $N_t$  and  $W_t^j$ . The household maximizes

$$E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}, N_{t+i})$$

subject to the budget constraint

$$C_{t+i} + \frac{B_{t+i}}{P_{t+i}} = \int_0^1 \frac{W_{t+i}^j}{P_{t+i}} N_{t+i}^j dj - T_t + (1 + I_{t-1+i}) \frac{B_{t-1+i}}{P_{t+i}} + \frac{D_{t+i}}{P_{t+i}} \quad (8)$$

and the resource constraint

$$N_{t+i} = \int_0^1 N_{t+i}^j dj = N_{t+i}^d \int_0^1 \left( \frac{W_{t+i}^j}{W_{t+i}} \right)^{-\theta_w} dj \quad (9)$$

Here,  $\beta$  is the discount factor,  $I_t$  is the nominal interest rate on bond holdings  $B_t$ ,  $N_t^j$  is the number of hours worked of labor type  $j$ ,  $W_t^j$  is the nominal wage of labor type  $j$ ,  $W_t$  is the aggregate wage level,  $D_t$  is the nominal profit income from all firms and  $N_{t+i}^d$  is the aggregate labor demand, details of which are given below when solving each firms' labor demand.

Let  $\lambda_t^c$  and  $\lambda_t^n$  be, respectively, the lagrange multipliers associated with the budget constraint (8) and the resource constraint (9). Then, the maximization problem in

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<sup>5</sup>We follow the common assumption that the government's allocation of its spending across the differentiated goods is analogous to (6) so that  $G_{k,t} = (P_{k,t}/P_t)^{-\theta_p} G_t$ .

lagrangian form is

$$\begin{aligned}
\ell &= E_t \sum_{i=0}^{\infty} \beta^i \left\{ U(C_{t+i}, N_{t+i}) + \lambda_{t+i}^n \left[ N_{t+i} - N_{t+i}^d \int_0^1 \left( \frac{W_{t+i}^j}{W_{t+i}} \right)^{-\theta_w} dj \right] \right. \\
&+ \lambda_{t+i}^c \left[ C_{t+i} + \frac{B_{t+i}}{P_{t+i}} - N_{t+i}^d \int_0^1 \frac{W_{t+i}^j}{P_{t+i}} \left( \frac{W_{t+i}^j}{W_{t+i}} \right)^{-\theta_w} dj + T_t \right. \\
&\left. \left. - (1 + I_{t-1+i}) \frac{B_{t-1+i}}{P_{t+i}} - \frac{D_{t+i}}{P_{t+i}} \right] \right\}
\end{aligned}$$

and the first-order conditions with respect to  $C_{t+i}$  and  $N_{t+i}$  are, respectively,  $U_{C,t+i} = -\lambda_{t+i}^c$  and  $U_{N,t+i} = -\lambda_{t+i}^n$ ; moreover,  $W_{t+i}^j = W_{t+i}^*$  if set optimally and  $W_{t+i}^j = W_{t+i-1}^j$  otherwise.

The assumption of Calvo wage staggering implies that in any period only a fraction  $1 - \omega_w$  of labor markets are allowed to reset wages, so that the household's lagrangian that is relevant for resetting wages is

$$\ell_W = E_t \sum_{i=0}^{\infty} (\beta \omega_w)^i \left\{ \lambda_{t+i}^n \left[ -N_{t+i}^d \left( \frac{W_{t+i}^j}{W_{t+i}} \right)^{-\theta_w} \right] + \lambda_{t+i}^c \left[ -N_{t+i}^d \frac{W_{t+i}^j}{P_{t+i}} \left( \frac{W_{t+i}^j}{W_{t+i}} \right)^{-\theta_w} \right] \right\} \quad (10)$$

As all labor types whose wages are reset in period  $t$  face an identical optimization problem, all set an identical wage, which is denoted by  $W_t^*$ . Differentiating (10) with respect to  $W_t^*$  gives the first-order optimality condition

$$E_t \sum_{i=0}^{\infty} (\beta \omega_w)^i N_{t+i|t} \left( \frac{W_t^*}{P_{t+i}} U_{C,t+i} + \mu_w U_{N,t+i} \right) = 0 \quad (11)$$

where we made use of the first-order conditions for  $C_{t+i}$  and  $N_{t+i}$  to substitute out the lagrangian multipliers. The parameter  $\mu_w = \frac{\theta_w}{\theta_w - 1}$  represents the wage markup while

$$N_{t+i|t} = \left( \frac{W_t^*}{W_{t+i}} \right)^{-\theta_w} N_{t+i}^d \quad (12)$$

is the demand for labor in period  $t + i$  whose wages was last reset in period  $t$ .

The first order condition (11) shows that trend growth in consumption affects wage decisions via the marginal utility consumption and the marginal disutility of work. The functional form of the utility function implies that,  $U_{C,t+i} = C_{t+i}^{-\sigma} V(N_{t+i})$  and  $U_{N,t+i} = -C_{t+i}^{1-\sigma} N_{t+i}^\eta V(N_{t+i})$ . As long as  $\sigma > 1$  there are two countervailing effects of trend growth. On the one hand the marginal utility of consumption declines faster with trend consumption growth pushing newly set wages up. On the other hand, due to complementarity in the utility function consumption growth decreases the marginal disutility of work (i.e., work becomes less costly) and this moderates wage increases.

Solving (11) for  $W_t^*$ , making use of the demand for labor (12) and dividing through by  $W_t$  we get the optimal wage setting equation

$$\frac{W_t^*}{W_t} = \mu_W \frac{E_t \sum_{i=0}^{\infty} (\beta \omega_w)^i C_{t+i}^{1-\sigma} V(N_{t+i}) N_{t+i}^\eta N_{t+i}^d \left( \frac{W_{t+i}}{W_t} \right)^{\theta_w}}{\frac{W_t}{P_t} E_t \sum_{i=0}^{\infty} (\beta \omega_w)^i C_{t+i}^{-\sigma} V(N_{t+i}) N_{t+i}^d \left( \frac{W_{t+i}}{W_t} \right)^{\theta_w} \frac{P_t}{P_{t+i}}} \quad (13)$$

where  $W_t^*/W_t$  is the relative wage. Using  $C_{t+i} = \gamma A_t c_{t+i}$ , where  $c_{t+i} \equiv C_{t+i}/A_{t+i}$  is detrended consumption, the optimal wage setting (13) can be rewritten as

$$\frac{W_t^*}{W_t} = \mu_W \frac{E_t \sum_{i=0}^{\infty} (\beta \omega_w \gamma^{1-\sigma})^i c_{t+i}^{1-\sigma} V(N_{t+i}) N_{t+i}^\eta N_{t+i}^d \left( \frac{W_{t+i}}{W_t} \right)^{\theta_w}}{\frac{W_t}{A_t P_t} E_t \sum_{i=0}^{\infty} (\beta \omega_w \gamma^{-\sigma})^i c_{t+i}^{-\sigma} V(N_{t+i}) N_{t+i}^d \left( \frac{W_{t+i}}{W_t} \right)^{\theta_w} \frac{P_t}{P_{t+i}}} \quad (14)$$

To help get an intuition underlying the effect of trend growth, rewrite the term  $W_{t+i}/W_t$  in (14) as  $W_{t+i}/W_t = \Pi_w \tilde{w}_{t+i}/\tilde{w}_t$ , where  $\Pi_w$  is trend wage inflation,  $\tilde{w}_{t+i} \equiv W_{t+i}/\bar{W}_{t+i}$  is detrended nominal wage and  $\bar{W}_t$  is trend nominal wage. Likewise, rewrite  $P_{t+i}/P_t$  as  $P_{t+i}/P_t = \Pi_p \tilde{p}_{t+i}/\tilde{p}_t$  where  $\Pi_p$  is trend price inflation,  $\tilde{p}_{t+i} \equiv P_{t+i}/\bar{P}_{t+i}$  is detrended nominal price level and  $\bar{P}_t$  is trend nominal price. Then (14) can be rewritten as

$$\frac{W_t^*}{W_t} = \mu_W \frac{E_t \sum_{i=0}^{\infty} (\beta \omega_w \gamma^{1-\sigma+\theta_w} \Pi_p^{\theta_w})^i c_{t+i}^{1-\sigma} V(N_{t+i}) N_{t+i}^\eta N_{t+i}^d \left( \frac{\tilde{w}_{t+i}}{\tilde{w}_t} \right)^{\theta_w}}{\frac{W_t}{A_t P_t} E_t \sum_{i=0}^{\infty} (\beta \omega_w \gamma^{-\sigma+\theta_w} \Pi_p^{\theta_w-1})^i c_{t+i}^{-\sigma} V(N_{t+i}) N_{t+i}^d \left( \frac{\tilde{w}_{t+i}}{\tilde{w}_t} \right)^{\theta_w} \frac{\tilde{p}_t}{\tilde{p}_{t+i}}} \quad (15)$$

where under balanced growth  $\Pi_w = \gamma\Pi_p$ . The two terms  $\gamma^{1-\sigma+\theta_w}$  and  $\gamma^{-\sigma+\theta_w}$  in equation (15) capture the effects of trend growth. They affect the *effective* discount factors for discounting future payoffs. Thus trend growth can have significant effects on discounting future payoffs if there is sufficient degree of complementarity in consumption and labor ( $\sigma$  is sufficiently larger than 1). As was pointed out in the introductory section, wage stickiness is crucial in the transmission mechanism in the presence of trend growth.

Finally, the first order conditions for consumption and bond holdings imply the following consumption Euler equation

$$1 = \beta I_t E_t \left( \frac{C_{t+1}^{-\sigma} V(N_{t+1}) P_t}{C_t^{-\sigma} V(N_t) P_{t+1}} \right)$$

which can be rewritten in detrended form

$$1 = \beta \gamma^{-\sigma} I_t E_t \left( \frac{c_{t+1}^{-\sigma} V(N_{t+1}) P_t}{c_t^{-\sigma} V(N_t) P_{t+1}} \right) \quad (16)$$

## 2.4 Firms

The decision of firms regarding optimal prices for their goods is analogous to the wage setting decisions of households in the labor market. As is standard, assume there is a continuum of monopolistically competitive firms over the unit interval.

Let firm  $k$  has a production function of the form

$$Y_{k,t} = A_t N_{k,t} \quad (17)$$

where  $N_{k,t}$  is labor input in firm  $k$ , which is a composite made of a continuum of differentiated labor services

$$N_{k,t} = \left( \int_0^1 N_{k,t}^{j \frac{1}{\mu_w}} dj \right)^{\mu_w} \quad (18)$$

where  $\mu_w \equiv \frac{\theta_w}{\theta_w - 1}$  and  $\theta_w$  is the elasticity of substitution between any two different labor types. Minimizing firm  $k$ 's total wage bill  $\int_0^1 W_t^j N_{k,t}^j dj$  subject to (18) leads to the demand for labor of type  $j$

$$N_{k,t}^j = \left( \frac{W_t^j}{\bar{W}_t} \right)^{-\theta_w} N_{k,t} \quad (19)$$

(so that aggregate labor demand is then given by  $N_t^d = \int_0^1 N_{k,t} dk$ ) and the aggregate wage index

$$\bar{W}_t = \left( \int_0^1 W_t^j (1-\theta_w) dj \right)^{\frac{1}{1-\theta_w}} \quad (20)$$

While firms choose prices optimally, output is demand determined, which in turn pins down labor demand. Firm  $k$  chooses its price optimally in order to maximize the expected profit

$$E_t \sum_{i=0}^{\infty} \omega_p^i Q_{t,t+i} \left( \frac{P_{k,t}}{P_{t+i}} Y_{kt+i} - \phi_{t+i} Y_{kt+i} \right) \quad (21)$$

where  $Q_{t,t+i} = \beta^i \left[ \frac{C_{t+i}^{-\sigma} V(N_{t+i})}{C_t^{-\sigma} V(N_t)} \right]$  is the stochastic discount factor and  $\phi_t = \phi_{k,t} = \frac{W_t}{A_t P_t}$  is the real marginal cost. The stochastic discount factor depends on the ratio of future to current marginal utility of income, reflecting the fact that households own all firms in the economy. Note here that since households are risk averse the stochastic discount factor depends negatively on trend consumption growth.

The demand for good  $k$  is<sup>6</sup>

$$Y_{k,t+i} = \left( \frac{P_{k,t}}{P_{t+i}} \right)^{-\theta_p} (C_{t+i} + G_{t+i}) \quad (22)$$

Using the demand function for good  $k$  and the aggregate resource constraint  $Y_{t+i} = C_{t+i} + G_{t+i}$  in the profit function

$$E_t \sum_{i=0}^{\infty} \omega_p^i Q_{t,t+i} Y_{t+i} \left[ \left( \frac{P_{k,t}}{P_{t+i}} \right)^{1-\theta_p} - \phi_{t+i} \left( \frac{P_{k,t}}{P_{t+i}} \right)^{-\theta_p} \right] \quad (23)$$

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<sup>6</sup>Here we are assuming that the government's demand for each good is derived analogously to the household sector.

Let  $P_t^*/P_t$  denote the optimal relative price, identical for all firms optimizing in period  $t$ . Differentiating (23) with respect  $P_t^*$  leads to the first order condition, expressed in term of  $P_t^*/P_t$ ,

$$\frac{P_t^*}{P_t} = \mu_p \frac{E_t \sum_{i=0}^{\infty} (\beta \omega_p)^i Y_{t+i} C_{t+i}^{-\sigma} V(N_{t+i}) \phi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta_p}}{E_t \sum_{i=0}^{\infty} (\beta \omega_p)^i Y_{t+i} C_{t+i}^{-\sigma} V(N_{t+i}) \left( \frac{P_{t+i}}{P_t} \right)^{\theta_p - 1}} \quad (24)$$

Detrending output and consumption analogous to the households' wage setting problem, equation (24) can be rewritten as

$$\frac{P_t^*}{P_t} = \mu_p \frac{E_t \sum_{i=0}^{\infty} (\beta \omega_p \gamma^{1-\sigma})^i y_{t+i} c_{t+i}^{-\sigma} V(N_{t+i}) \phi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta_p}}{E_t \sum_{i=0}^{\infty} (\beta \omega_p \gamma^{1-\sigma})^i y_{t+i} c_{t+i}^{-\sigma} V(N_{t+i}) \left( \frac{P_{t+i}}{P_t} \right)^{\theta_p - 1}} \quad (25)$$

where  $y_t = Y_t/A_t$ .

Note that, similar to the wage setting problem, trend growth affects pricing decisions by reducing the effective discount factor,  $\beta \omega_p \gamma^{1-\sigma}$ , as long as  $\sigma > 1$ . However, unlike the wage setting equation, higher trend growth unequivocally reduces the stochastic discount factor, pushing newly set price down. Thus the overall effect of trend growth on short-run dynamics depends on the interplay of the individual effects.

## 2.5 Aggregation and market clearing

The aggregate wage index (20) can be rewritten as a weighted average of optimized and non-optimized wages

$$W_t = \left[ (1 - \omega_w) W_t^{*(1-\theta_w)} + \omega_w W_{t-1}^{(1-\theta_w)} \right]^{\frac{1}{1-\theta_w}} \quad (26)$$

Similarly, the aggregate price index (7) can be rewritten as a weighted average of optimized and non-optimized prices

$$P_t = \left[ (1 - \omega_p) (P_t^*)^{1-\theta_p} + \omega_p P_{t-1}^{1-\theta_p} \right]^{\frac{1}{1-\theta_p}} \quad (27)$$

Moreover, using the aggregate resource constraint on hours and imposing goods and labor market clearing, we get a relationship between aggregate hours worked  $N_t$ , aggregate labor demand  $N_t^d$  and aggregate detrended output  $y_t$ . First, use (19) when aggregating hours worked across labor types

$$\begin{aligned}
N_t &= \int_0^1 \int_0^1 N_{k,t}^j dj dk \\
&= \int_0^1 N_{k,t} \int_0^1 \left( \frac{W_t^j}{\bar{W}_t} \right)^{-\theta_w} dj dk \\
&= \Delta_{w,t} \int_0^1 N_{k,t} dk
\end{aligned} \tag{28}$$

where  $\Delta_{w,t} = \int_0^1 \left( \frac{W_t^j}{\bar{W}_t} \right)^{-\theta_w} dj$  measures wage dispersion. Moreover using (22) when aggregating labor demand across firms

$$\begin{aligned}
N_t^d &= \int_0^1 N_{k,t} dk \\
&= y_t \int_0^1 \left( \frac{P_{k,t}}{P_t} \right)^{-\theta_p} dk \\
&= \Delta_{p,t} y_t
\end{aligned} \tag{29}$$

where  $\Delta_{p,t} = \int_0^1 \left( \frac{P_{k,t}}{P_t} \right)^{-\theta_p} dk$  measures price dispersion.

Using backward recursion, the wage dispersion and price dispersion equations can be rewritten as

$$\Delta_{w,t} = (1 - \omega_w) \left( \frac{W_t^*}{\bar{W}_t} \right)^{-\theta_w} + \omega_w \left( \frac{W_{t-1}}{\bar{W}_t} \right)^{-\theta_w} \Delta_{w,t-1} \tag{30}$$

$$\Delta_{p,t} = (1 - \omega_p) \left( \frac{P_t^*}{P_t} \right)^{-\theta_p} + \omega_p \left( \frac{P_{t-1}}{P_t} \right)^{-\theta_p} \Delta_{p,t-1} \tag{31}$$

We close the model by specifying the setting of monetary policy according to a simple Taylor rule

$$\frac{I_t}{I} = \left( \frac{\Pi_{p,t}}{\Pi_p} \right)^{\varphi_p} \quad (32)$$

where  $I$  is steady state interest rate. We set  $\varphi_p > 1$  so that the model has a determinate equilibrium.<sup>7</sup>

### 3 The effects of government spending shocks

Having derived the key aggregate equations, we are now in a position to analyze the dynamic responses of output, consumption, interest rate, wage inflation and price inflation to a shock in government spending.

The complete system is given by equations (2), (3), (14), (16), (25), (26), (27), (28), (29), (30), (31), and (32), as well as the resource constraint  $y_t = c_t + g_t$ , and the definition of the real marginal cost  $\phi_t = W_t/(A_t P_t) = \phi_{t-1} \Pi_{w,t}/(\gamma \Pi_{p,t})$ . We linearize the model around a balanced growth path characterized by two alternative trend growth rates—positive trend growth ( $\gamma > 1$ ) and no trend growth ( $\gamma = 1$ ). We calibrate the model's other parameters using values that are standard in the business cycle literature (see, e.g., Schmidt-Grohe and Uribe (2005)). Our calibration of trend growth  $\gamma \in \{1, 1.005\}$  implies annual growth rates of zero percent and 2 percent, respectively.

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<sup>7</sup>One could also consider alternative specifications that allow for responses to output. However, it turns out that our main results remain intact. The reason is that the only shock in the system is a government spending shock, a type of demand shock. As is well known, a demand shock moves output and inflation in the same direction (there is no Phillips curve trade-off). The simple Taylor rule thus stabilizes both output and inflation in the face of the demand shock.



### Parameter configuration

Parameter		Calibrated values
$\beta$	discount factor	0.99
$\omega_p$	fraction of firms not resetting prices	0.75
$\omega_w$	fraction of workers not resetting wages	0.8
$\theta_p$	elasticity of substitution in goods	10
$\theta_w$	elasticity of substitution in labor services	5
$\rho$	degree of persistence in government spending	0.9
$\varphi_p$	response of interest rate to inflation	1.5
$\gamma$	productivity growth	{1, 1.005}
$\sigma$	degree of complementarity in utility/risk aversion	{1, 6.33}
$\eta$	labor supply elasticity	{1, .62}

The upper value of  $\sigma$  and the lower value of  $\eta$  are based on Guerron-Quintana (2008) who estimates a New Keynesian model with nonseparable utility as well as nominal price and wage staggering. We also show results for  $\sigma = 1$  (separable utility) and  $\eta = 1$ , values typically used in the literature. It is well known in the business cycle literature that the assumption of separable utility is made mainly for tractability reasons (for a detailed discussion, see, e.g., Basu and Kimball (2002)).

### 3.1 Impulse responses to a government spending shock

Our main result is shown in Figure 1, which plots the impulse responses to a positive government spending shock under the benchmark parameter values but for two alternative values of trend productivity growth  $\gamma$ . The impulse responses corresponding to zero percent trend growth ( $\gamma = 1$ ) are shown by the solid lines while those corresponding to 2 percent annual trend growth ( $\gamma = 1.005$ ) are shown by the dashed lines. As can be seen from the figure, a positive fiscal spending shock is

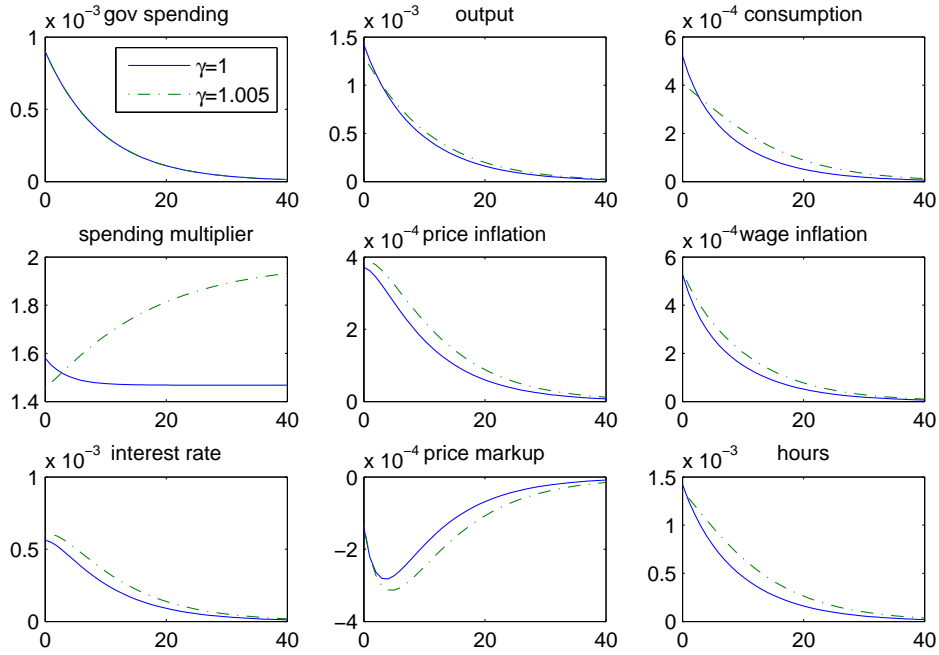


Figure 1: Impulse responses to a government spending shock. Baseline calibration.

more inflationary at trend growth rate of 2 percent than at zero percent, an effect also reflected in the dynamic response of interest rates, owing to the Taylor rule. Moreover, the shock is less expansionary in output and consumption on impact at trend growth rate of 2 percent than at zero percent, although both variables exhibit more persistence.<sup>8</sup> Consequently, the fiscal multiplier is lower on impact but higher after some periods than it would have been under no growth. Note also that the average price markup decreases by more at trend growth rate of 2 percent than at zero percent.

<sup>8</sup>Note here that consumption responds positively to a government spending shock because a strong enough complementarity between consumption and labor in the utility function. See for example, Bilbiie (2009) and Monacelli and Perotti (2009).

## 3.2 Two limiting cases

The two channels whereby trend growth affects short-run dynamics are related to the behavioral equations associated with price and wage settings when both are subject to nominal frictions. In order to get intuition on the relevance of the two channels, in this section we look at two limiting cases of the baseline model: (1) perfectly flexible prices and (2) perfectly flexible wages.

The impulse responses in the limiting case with perfectly flexible prices but sticky wages are shown in Figure 2. We see that price markup declines only marginally but exhibits more persistence at trend growth rate of 2 percent than at zero percent. Since prices are perfectly flexible, price inflation moves one for one with wage inflation. But unlike the baseline model, on impact the shock is not more inflationary under trend growth rate of 2 percent than at zero percent. Consumption follows a hump shape and becomes more persistent at trend growth rate of 2 percent than at zero percent, while the fiscal multiplier remains lower than it would be under no trend growth for somewhat longer duration. This is partly due to the fiscal multiplier under no trend growth being flat in all periods.

Figure 3 shows the impulse responses in the limiting case with perfectly flexible wages but sticky prices. In contrast to the flexible price model (Figure 2), in response to the spending shock price inflation responds less than wage inflation and the price markup declines, even on impact, at trend growth rate of 2 percent than at zero percent. Moreover, unlike the flexible price model, consumption increases by more also on impact so that the fiscal multiplier is larger for all periods at trend growth rate of 2 percent than at zero percent. This result is due to the fact that trend growth lowers the effective discounting factor for optimally set prices while the offsetting effects from wage setting under sticky wages are absent (wage setting is independent of trend growth). Note also the role of wage flexibility for the dynamic behavior of

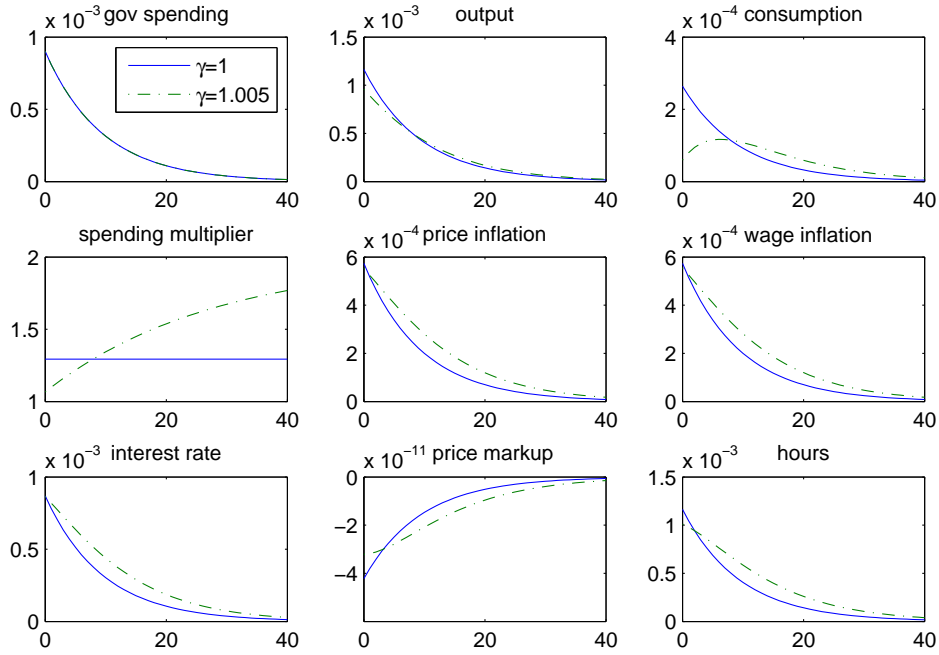


Figure 2: Impulse responses to a government spending shock. Flexible price model.

output and consumption. As is shown in Figure 2, sticky wages dampen the impact effect of the shocks by more at trend growth rate of 2 percent than at zero percent, a feature that is absent in Figure 3.

### 3.3 Impulse responses under separable utility

In order to see how trend growth in conjunction with nonseparable utility matters for our results, we also show impulse responses for the case  $\sigma = 1$  (separable utility) and  $\eta = 1$  (see Figure 4). The first thing to note from the figure is that consumption drops following a positive government spending shock, which is a well known result in models with separable utility. The fiscal multiplier is somewhat lower, implying the effect of the shocks on output is smaller, at trend growth rate of 2 percent than at zero percent. However, the effect of trend growth on most variables of interest is not visually detectable. This shows that, trend growth in conjunction with nonseparable

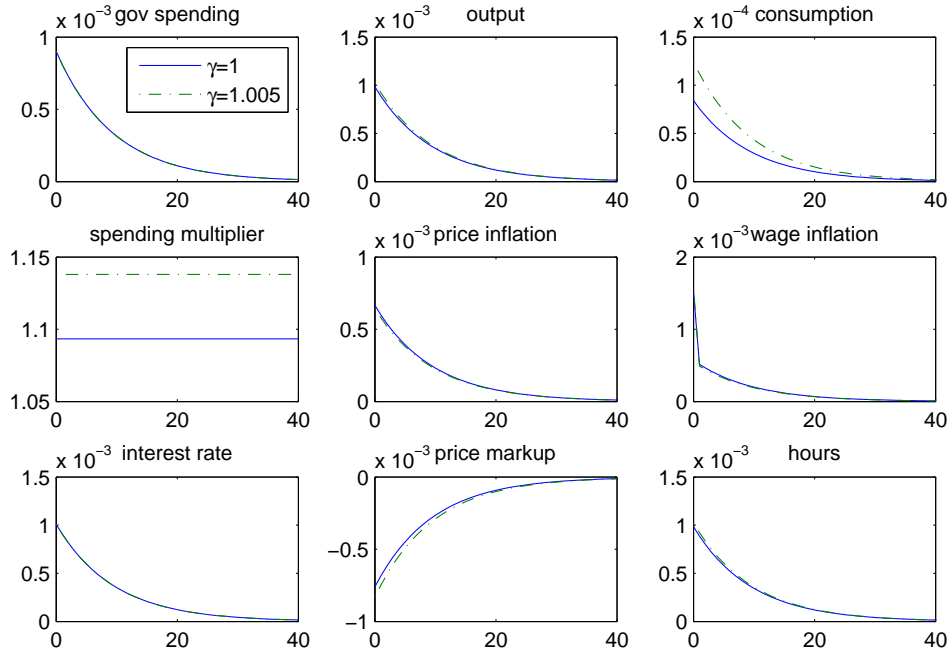


Figure 3: Impulse responses to a government spending shock. Flexible wage model.

utility matters for the propagation of fiscal spending shocks to the economy.

### 3.4 Impulse responses under partial indexation

We also analyze the sensitivity of our baseline results to the introduction of partial wage indexation (where the indexation parameter is  $0 \leq \varphi_w \leq 1$ ). Lacking firm empirical estimates, we set  $\varphi_w = 0.5$ . The results are shown in Figure 5. It turns out that consumption increases by more (and the fiscal multiplier is larger) in all periods under positive trend growth. The effects on wage and price inflation are similar to, although not as strong as, the baseline case.

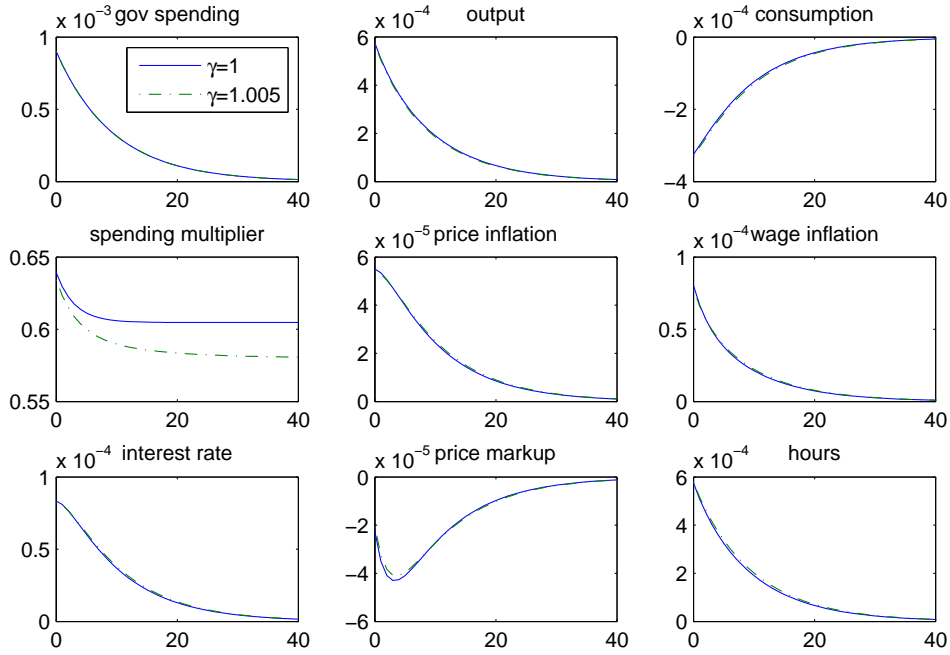


Figure 4: Impulse responses to a government spending shock. Separable utility.

## 4 Concluding remarks

This paper studies the macroeconomic effects of government spending shocks under different trend growth rates. It does this by incorporating trend growth into a New Keynesian model with both price and wage rigidity. It then shows trend growth affects the dynamic response of key variables such as inflation, output and consumption. In particular, the higher is the trend growth rate the more inflationary are government spending shocks and vice versa. Moreover, on impact both output and consumption are lower but are more persistent the higher is trend growth. These effects are mirrored in the dynamics of the fiscal multiplier.

As the model is stylized the focus of the paper is more on the qualitative effects of trend growth, in the face of government spending shocks, rather than on determining an exact quantitative value (in particular for the fiscal multiplier). Certainly the

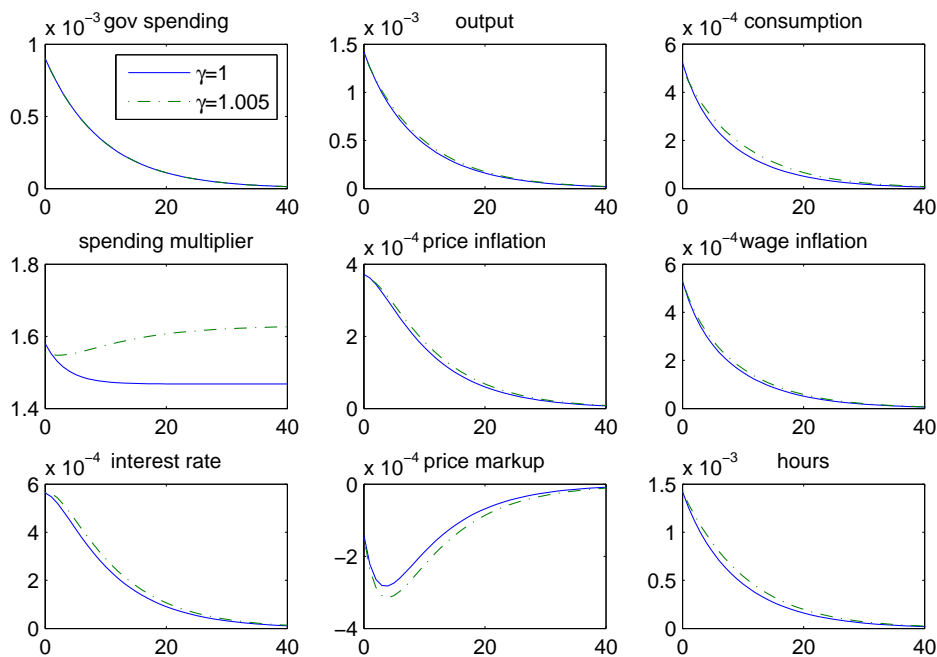


Figure 5: Impulse responses to a government spending shock with partial indexation.

model could be extended, for example by having investment and capital formation decisions, and allowing for additional sources of real frictions. We leave this for future research.

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