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Efficiency and Labor Market Dynamics in a Model of Labor Selection

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Sanjay Chugh and Christian Merkl

Abstract:

We characterize efficient allocations and business cycle fluctuations in a labor selection model. Due to forward-looking hiring and labor supply decisions, efficiency entails both static and intertemporal margins. We develop welfare-relevant measures of marginal rates of transformation and efficiency along each margin that nest their counterparts in frictionless labor markets. In a calibrated version of the model, efficient fluctuations feature highly volatile unemployment and job-finding rates, in line with empirical evidence. We show analytically in a simplified version of the model that volatility arises from selection effects, rather than general equilibrium effects. We also develop sufficient conditions on wages, which are independent of the wage-determination process, that decentralize efficient allocations. Unlike the Hosios condition for matching models, there is no simple restriction on Nash bargaining that guarantees that Nash wages can support efficient allocations. Cyclical fluctuations in the Nash-bargaining economy display even larger amplification of productivity shocks into labor market outcomes than in the efficient economy, without extreme assumptions about bargaining shares, inflexibility of wages, or the size of surpluses that govern labor demand. The results establish normative and positive foundations for DSGE labor selection models.

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1 Introduction

We characterize efficiency and business cycle fluctuations for a model of labor markets in which two primitives are prominent: because job applicants are heterogeneous in their characteristics, only a subset is selected for employment; and, conditional on hiring, integrating a new worker into a job is costly. The main positive result is that selection effects at the hiring margin lead to high time-series volatility of unemployment and job-finding rates in response to aggregate productivity shocks, aspects of the data that have received much attention in the macro-labor literature. Highly-volatile labor markets emerge in the efficient allocations of the model; possible inefficiencies in decentralization are thus not central for the model's transmission mechanism. Nonetheless, we do study decentralization, with a focus on wages and their normative implications. We develop a fairly weak set of sufficient conditions on the cross section of individuals' wages that supports efficiency. Based on these sufficient conditions, we show that (unlike the Hosios (1990) condition for matching models) there is no simple restriction on Nash-bargained wages that guarantees that these sufficient conditions are met. Taken together, the positive and normative results portray a view of labor markets complementary to the widely-used search and matching framework.

The central idea of the model is purposeful selection of a subset of heterogenous applicants. This aspect of labor markets is realistic and highlights frictions different from those usually emphasized in the standard search and matching analysis of labor markets. Selection as an important margin of adjustment in firms' hiring decisions is a long-standing idea. For example, Barron, Bishop, and Dunkelberg (1985) adopt and find strong evidence for the view that "...most employment is the outcome of an employer selecting from a pool of job applicants..." More recently, Davis, Faberman, and Haltiwanger (2010) add micro-level evidence to the view that the vacancy margin central to matching analysis is not always the most active margin for hiring decisions, and that selection issues can play an important role. Although our model spotlights selection as the driving force of long-run and short-run labor market outcomes, and abstracts from matching and other frictions, it does not deny that other frictions also play an important role. Focusing the model in this way allows us to obtain a number of sharp analytical and quantitative results, and thus provides benchmarks for a recently emerging class of quantitative macroeconomic models featuring selection effects in hiring.

On the positive dimension, the main result is that a calibrated version of the model reproduces quite well several facts regarding labor market dynamics, conditional on productivity shocks, that baseline matching models have difficulty replicating. In particular, the volatility and correlation structure of unemployment, labor force participation, and workers' job-finding rate fit the data remarkably well, despite the parsimony of the model. Empirically-relevant amplification effects arise in the efficient allocations of the model, and are a bit stronger in the decentralized economy with Nash-bargained wages. In neither case is the amplification due to mechanisms that have been used in the matching literature to generate large effects of productivity shocks, such as wage rigidity or extreme assumptions regarding surplus sharing. The large amplification of productivity shocks arises in both the general equilibrium model that is the focus of the analysis, as well as in a partial equilibrium setting that is analogous to the study of dynamics in matching models such as Shimer (2005), Hall (2005), and Hagedorn and Manovskii (2008).

On the normative dimension, there are four interrelated contributions, two of which are directly about efficient allocations and two of which are about their decentralization. First, we develop model-consistent measures of the marginal rates of transformation (MRT) between consumption and labor within a given time period and between consumption across different time periods. These static and intertemporal MRTs take into account the selection and hiring processes, which are primitives of the environment. Second, based on these model-consistent, and hence welfare-relevant, MRTs, efficiency is easily characterized by conditions that equate them to corresponding marginal rates of substitution (MRS). These conditions emerge from a simple social planning problem, but are also derived from the primitive transformation frontier of the economy, independent of any optimization.

A question of natural interest is whether and how efficient allocations may be decentralized. Our third main normative result is a set of sufficient conditions on wage determination that guarantee efficiency. These sufficient conditions are not predicated on any specific wage-setting process. In matching models, wages are typically assumed to be set in Nash bargaining. To connect with the matching literature and its efficiency results, we thus also consider Nash-bargained wages. This leads to our fourth main normative result, which is that there is no simple restriction on Nash bargaining that implements efficiency, unlike the Hosios (1990) condition for matching models.

On the theory front, Lechthaler, Merkl, and Snower (2010) incorporate a selection model into a DSGE New Keynesian framework absent labor matching frictions, and Guerreri (2007) studies business cycle dynamics when both matching frictions and selection issues are operative. Guerreri (2007) finds no amplification of productivity shocks to labor market fluctuations due to endogenous selection, while one of the main findings of Lechthaler, Merkl, and Snower (2010) — hereafter, LMS — is that fluctuations driven by monetary shocks and technology shocks can be quite large. However, LMS base their analysis on institutions such as European-style union wage bargaining, and do not derive normative results.[1] Because we adopt an efficiency view, when studying decentralization, we require a more flexible, individual-specific, type of bargaining. As Arseneau and Chugh (2010) show for DSGE matching models, once efficiency is properly defined, normative and positive insights can be easily described in terms of elementary economic theory. We view the results here

[1] A related model in Faia, Lechthaler, and Merkl (2009) studies optimal Ramsey monetary policy and optimal simple interest rules in a selection model with sticky prices. However, most of their analysis is numerical.

as similarly foundational for selection models.

The rest of the paper is organized as follows. Section 2 describes the economic environment. Section 3 characterizes efficient allocations analytically. Section 4 embeds the environment in a decentralized setting. Section 5 provides sufficient conditions on wage outcomes, for any arbitrary wage-setting process, that decentralize efficient allocations. Section 5 also considers whether and how efficient allocations can be supported as an equilibrium with Nash-bargained wages. Section 6 studies the cyclical properties of calibrated versions of both the efficient and Nash allocations. Section 7 uses a partial equilibrium version of the model to analytically and quantitatively inspect several important aspects of the model's mechanism; it highlights that the model's amplification is directly due to selection effects, rather than due to general equilibrium effects or reliance on leading explanations of amplification in matching models. Section 8 concludes. A detailed set of Appendices proves the main results and provides many technical details of the model that should be useful to future researchers using the selection framework.

2 The Environment

The model uses the "instantaneous hiring" view of transitions between unemployment and employment, in which new employees begin producing right away, rather than with a one-period delay. This is the timing assumption in the labor selection model of LMS, and it has also become standard in DSGE models with labor matching frictions.

2.1 Timing of Events

Suppose that n_{t-1} individuals produced output in period $t-1$. At the beginning of period t, a fraction ρ of these individuals separate from their production opportunities. Some of these newly-separated individuals may immediately enter the period-t labor force, as may some individuals who were non-participants in period $t-1$; these two groups taken together constitute the measure s_t of individuals available to begin work in period t. It will sometimes be useful to refer to these s_t individuals available for work as "unemployed." However, unlike models based on the Pissarides (1985) framework, there is no matching function that brings (with probability less than one) individuals available for work into contact with production opportunities. Rather, each individual available for work makes contact with ("matches" with) a production opportunity with probability one.[2]

Unemployed individual i has idiosyncratic characteristics, denoted by ε^i, which is a draw from a cumulative distribution function $F(\varepsilon)$, with associated density $f(\varepsilon)$. It is only unemployed individuals that are heterogenous; individuals who have been employed for more than one period

[2] Furthermore, each individual makes only one contact per period.

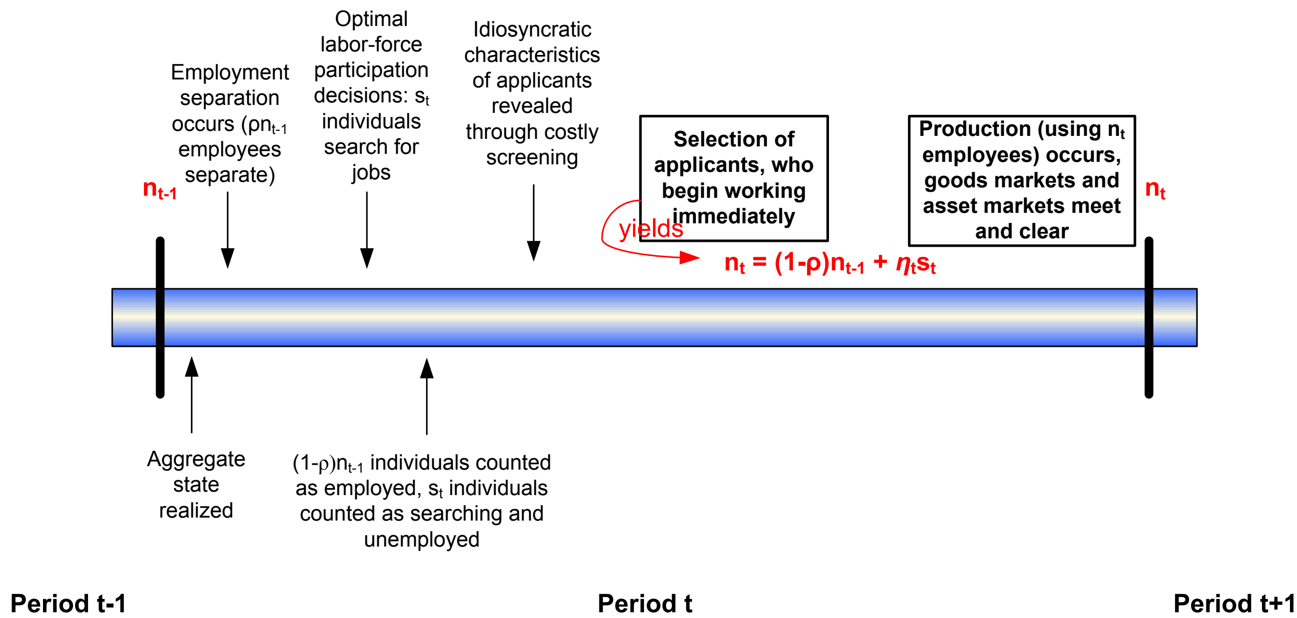

Figure 1: Timing of events.

are identical in their characteristics. For concreteness, we model and refer to an individual's ε as the "operating cost," measured in units of output, that is incurred in the *first* period of a new employment relationship that he begins. However, ε may be positive or negative.

Of the s_t unemployed individuals, $(1-\eta_t)s_t$ individuals turn out to be unsuccessful in becoming employed, where η_t is the probability that an individual available for work is selected and begins producing. This probability is taken as given by individuals, but, as described below, it is endogenous to the environment. The measure

$$n_t = (1-\rho)n_{t-1} + s_t\eta_t \tag{1}$$

of individuals are thus employed and produce in period t. Each of the $(1-\eta_t)s_t$ individuals who does not find a job receives nothing. With these definitions and timing of events, the measured labor force in period t is $lfp_t = n_t + (1-\eta_t)s_t$. By substituting (1), participation can alternatively be measured as $lfp_t = (1-\rho)n_{t-1} + s_t$. Figure 1 summarizes the timing of the model.

2.2 Selection of Unemployed Individuals

The central idea of the model is purposeful selection of a subset of individuals available for work. There are two steps in this process. First, each individual available for work is *screened* when he makes contact with a production opportunity (which occurs with probability one). Screening an individual entails a cost γ^s, and the screening process fully reveals the individual's ε.[3]

Second, because integrating a worker into production is also costly, it is efficient to move only those individuals with sufficiently attractive characteristics into the production process. Post-screening, integrating an individual into the production process entails a distinct cost γ^h, which is interpreted as a *hiring cost* that reflects training and other startup activities for a new worker.[4] There is thus a threshold $\tilde{\varepsilon}_t$, which is a function of the state of the economy, for selection of unemployed individuals once screening has revealed their types. Because individuals' idiosyncratic characteristics are defined as a *cost*, only those individuals with $\varepsilon_t^i \leq \tilde{\varepsilon}_t$ are brought into the production process. The probability that an unemployed individual is hired is thus $\eta(\tilde{\varepsilon}_t)$ $(= F(\tilde{\varepsilon}_t))$, and the aggregate number of individuals selected in period t is $\eta(\tilde{\varepsilon}_t)s_t$.[5]

2.3 Production

All workers, whether incumbent or newly selected, produce stochastic output z_t in period t. There is no intensive margin of labor adjustment. The operating cost imposed by newly-selected worker i is modeled as subtracting from the individual's production. Thus, the net production in period t of newly-selected worker i is $z_t - \varepsilon_t^i$. As noted above, ε_t^i may be positive or negative (given the distributional assumption we will make when considering quantitative properties of the model in Section 3).

The aggregate goods resource constraint of the economy is thus

$$\begin{aligned} c_t + \gamma^h \eta(\tilde{\varepsilon}_t)s_t + \gamma^s s_t &= z_t n_t - s_t \eta(\tilde{\varepsilon}_t)\frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \\ &= z_t n_t - s_t H(\tilde{\varepsilon}_t), \end{aligned} \tag{2}$$

in which $H(\tilde{\varepsilon}_t)/\eta(\tilde{\varepsilon}_t)$ denotes the average operating cost for each newly-selected worker, with $H(\tilde{\varepsilon}_t) \equiv \int_{-\infty}^{\tilde{\varepsilon}_t} \varepsilon f(\varepsilon) d\varepsilon$. Consumption is denoted c_t, and, because hiring costs and screening costs are denominated in goods, each type of cost is also a source of final absorption.

2.4 Preferences

There is a measure one of individuals in the economy. Each individual, whether employed, unemployed, or outside the labor force, has full consumption insurance, which is modeled by assuming

[3] There is no revealed heterogeneity of unemployed individuals *before* they make contact with a production opportunity, thus every individual gets screened. If there were pre-contact revealed heterogeneity amongst the unemployed, then which individuals to screen could also be endogenized.

[4] Note that the screening cost γ^s and the hiring cost γ^h are each distinct from the "operating cost" that is each new worker's idiosyncratic characteristics.

[5] Depending on context, we sometimes emphasize the dependence of the hiring probability on $\tilde{\varepsilon}_t$ and sometimes simply write η_t to conserve on notation.

that all individuals belong to a representative household that pools income and shares consumption. This "large household" assumption is a tractable way of modeling perfect consumption-risk insurance, and has been standard in the matching literature since Andolfatto (1996) and Merz (1995); LMS adopt this approach in their DSGE selection model.

The representative household has lifetime expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v\left(n_t + (1-\eta_t)s_t\right)\right]. \tag{3}$$

The subjective discount factor is β, the function $u(.)$ is a standard strictly-increasing and strictly-concave subutility function over consumption, and the function $v(.)$ is strictly increasing and strictly convex in the size of the labor force.[6] The measure of individuals in the labor force is endogenous, which is not the case in LMS but has become fairly common in the recent DSGE matching literature.[7] This allows for a greater degree of generality of our efficiency results than if participation were fixed; the main results do hold, however, if participation is fixed, as in Section 7. For intuition and because it facilitates analogy with both the RBC model and the basic matching model, it will be helpful to interpret the measure $1-lfp_t$ of individuals outside the labor force as enjoying leisure. We thus use the terms leisure and non-participation interchangeably.

3 Efficient Allocations

The main focus is on the nature of efficient allocations in this environment. Efficient allocations $\{c_t, s_t, \tilde{\varepsilon}_t, n_t\}_{t=0}^{\infty}$ are characterized by four (sequences of) conditions:

$$\frac{v'(lfp_t)}{u'(c_t)} = \tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) - \gamma^s, \tag{4}$$

$$\gamma^h = z_t - \tilde{\varepsilon}_t + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[\gamma^h + \tilde{\varepsilon}_{t+1} + \gamma^s + H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1})\right]\right\}, \tag{5}$$

$$c_t + \gamma^h \eta(\tilde{\varepsilon}_t)s_t + \gamma^s s_t + s_t H(\tilde{\varepsilon}_t) = z_t n_t, \tag{6}$$

and

$$n_t = (1-\rho)n_{t-1} + s_t \eta(\tilde{\varepsilon}_t). \tag{7}$$

The efficiency conditions (4) and (5) are obtained by maximizing household welfare (3) subject to the technological frontier defined by the sequence of goods resource constraints (6) and laws of motion for employment (7). The formal analysis of this problem appears in Appendix A.

[6] Given the definitions presented above, we sometimes will write $v(lfp_t)$.

[7] In a variety of applications, Veracierto (2008), den Haan and Kaltenbrunner (2009), Krusell, Mukoyama, Rogerson, and Sahin (2009), and Ebell (2010), among others, have introduced participation margins into matching models.

Condition (4) is a static dimension of efficiency and is analogous to static consumption-leisure efficiency in the RBC model. Condition (5) is an intertemporal dimension of efficiency, and it corresponds to the matching model's vacancy-creation, or job-creation, condition; it also corresponds to the RBC model's Euler equation for efficient capital accumulation. Even though the model does not have "physical capital" in the strict RBC sense, the creation of an employment match is an investment activity that yields a long-lasting asset. Because of frictions, employment thus inherently has both static and intertemporal dimensions in a selection framework, just as it does in a matching framework.[8] Together, conditions (4) and (5) define the two "zero-wedge" conditions for the model, both of which are statements about labor markets.

To highlight this "zero-wedges" view, it is useful to restate efficiency in terms of marginal rates of substitution (MRS) and corresponding marginal rates of transformation (MRT). For the intertemporal condition, this restatement is most straightforward for the non-stochastic case, which allows an informative disentangling of the preference and technology terms inside the $E_t(.)$ operator in (5).

Proposition 1. ***Efficient Allocations.*** *The MRS and MRT for the pairs* (c_t, lfp_t) *and* (c_t, c_{t+1}) *are defined by*

$$MRS_{c_t,lfp_t} \equiv \frac{v'(lfp_t)}{u'(c_t)} \qquad MRT_{c_t,lfp_t} \equiv \tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) + b - \gamma^s$$

$$IMRS_{c_t,c_{t+1}} \equiv \frac{u'(c_t)}{\beta u'(c_{t+1})} \qquad IMRT_{c_t,c_{t+1}} \equiv \frac{(1-\rho)\left(\gamma^h + \tilde{\varepsilon}_{t+1} + \gamma^s + H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1})\right)}{\gamma^h + \tilde{\varepsilon}_t - (z_t - b)}.$$

Static efficiency (4) is characterized by $MRS_{c_t,lfp_t} = MRT_{c_t,lfp_t}$*, and (for the non-stochastic case) intertemporal efficiency (5) is characterized by* $IMRS_{c_t,c_{t+1}} = IMRT_{c_t,c_{t+1}}$.

Proof. See Appendix A. □

Each MRS in Proposition 1 has the standard interpretation as a ratio of relevant marginal utilities. By analogy, each MRT has the interpretation as a ratio of the marginal products of an appropriately-defined transformation frontier.[9] As per elementary economic theory, then, efficient allocations are characterized by an MRS = MRT condition along each optimization margin, implying zero distortion on each margin. These efficiency conditions are the welfare-relevant ones in this environment and hence must be the basis for any normative analysis. However, rather than take the efficiency conditions as prima facie justification that the expressions in Proposition 1

[8] See also Faia et al. (2009), who point out that there are both intra-temporal and intertemporal wedges in selection models.

[9] We have in mind a very general notion of transformation frontier as in Mas-Colell, Whinston, and Green (1995, p. 129), in which every object in the economy can be viewed as either an input to or an output of the technology to which it is associated. Appendix A provides formal details.

are properly to be understood as MRTs, each can be derived from primitives, independent of the characterization of efficiency. Formal details of the following mostly intuitive discussion appear in Appendix A.

3.1 Static MRT

To understand the static MRT, MRT_{c_t,lfp_t}, in Proposition 1, consider how the economy can transform a unit of *non*-participation (leisure) in period t into a unit of output, and hence consumption, in period t. By construction, this within-period transformation holds fixed all allocations beyond period t. The transformation is described in terms of leisure because leisure is a good (and hence gives positive utility), while participation is a bad (and gives disutility); we proceed by describing transformation as occurring between goods.

A one-unit reduction in leisure allows a one-unit increase in s_t, which leads to a sequence of further transformations. First, because every individual available for work is screened, γ^s resources are used. Second, with probability $\eta(\tilde{\varepsilon}_t)$, the individual's revealed operating cost is below the cutoff $\tilde{\varepsilon}_t$ and he is selected to join a production opportunity; integrating the individual into production entails cost γ^h. Because a newly-selected individual has idiosyncratic characteristics $\varepsilon \leq \tilde{\varepsilon}_t$, the expected *savings* of operating costs conditional on being selected is $\int_{-\infty}^{\tilde{\varepsilon}_t} [\tilde{\varepsilon}_t - \varepsilon] f(\varepsilon)d\varepsilon = \tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)$, where the equality follows from the definitions of $\eta(\tilde{\varepsilon}_t)$ and $H(\tilde{\varepsilon}_t)$. These *savings* on period-t operating costs allows an *increase* in period-t consumption. The overall marginal transformation between leisure and consumption described thus far is $\eta(\tilde{\varepsilon}_t)\left[\frac{\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t)-H(\tilde{\varepsilon}_t)-\gamma^h}{\eta(\tilde{\varepsilon}_t)}\right] - \gamma^s = \tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) - \gamma^h - \gamma^s$.

However, this is not *ceteris paribus* because the larger stock of employment in period t has intertemporal consequences; measuring only within-period effects requires controlling for this intertemporal effect. We show using the implicit function theorem in Appendix A that the appropriate adjustment adds γ^h, which measures the lifetime social asset value of a match. Hence, the overall within-period MRT between leisure and consumption is $\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) - \gamma^s$, as shown in Proposition 1.

3.2 Intertemporal MRT

Now consider the intertemporal MRT (IMRT) in Proposition 1. The IMRT measures how many additional units of c_{t+1} the economy can achieve if one unit of c_t is foregone. By construction, this transformation across periods t and $t+1$ holds fixed all allocations *beyond* period $t+1$.

A one-unit reduction in c_t frees up resources that can be devoted to selection of individuals. As (6) shows, resources devoted to "investment" in selection of individuals can be increased by $\frac{1}{[\gamma^h\eta'(\tilde{\varepsilon}_t)+H'(\tilde{\varepsilon}_t)]s_t}$ units, by selecting some marginally worse (in terms of higher idiosyncratic operating costs) individuals who otherwise would not have been selected.[10] Relaxing the selection criteria increases period-t aggregate employment by $\frac{\eta'(\tilde{\varepsilon}_t)s_t}{\left[\gamma^h\eta'(\tilde{\varepsilon}_t)+H'(\tilde{\varepsilon}_t)\right]s_t} = \frac{1}{\gamma^h+\tilde{\varepsilon}_t}$ units.[11]

The addition of $\frac{1}{\gamma^h+\tilde{\varepsilon}_t}$ individuals to period-t employment has two effects. Because workers become productive in the period in which they are selected, period-t output, and hence period-t consumption, *rises* by $\frac{z_t}{\gamma^h+\tilde{\varepsilon}_t}$ units. This *rise* in period-t consumption must be netted from the one-unit *reduction* in period-t consumption that started the thought experiment. Thus, we can now view all effects on period-$t+1$ consumption as arising from a (net) reduction of c_t by $1-\frac{z_t}{\gamma^h+\tilde{\varepsilon}_t} = \frac{\gamma^h+\tilde{\varepsilon}_t-z_t}{\gamma^h+\tilde{\varepsilon}_t}$ (< 1) units.

The second effect of the additional $\frac{1}{\gamma^h+\tilde{\varepsilon}_t}$ units of period-t employment is that, in period $t+1$, there are $\frac{1-\rho}{\gamma^h+\tilde{\varepsilon}_t}$ additional units of employment — that is, n_{t+1} rises by $\frac{1-\rho}{\gamma^h+\tilde{\varepsilon}_t}$ units. Each of these additional units of employment produces z_{t+1} units of output, and hence consumption. The overall addition to period $t+1$ consumption (starting, recall from immediately above, from a reduction of period-t consumption by $\frac{\gamma^h+\tilde{\varepsilon}_t-z_t}{\gamma^h+\tilde{\varepsilon}_t}$ units) described thus far is $\frac{z_{t+1}(1-\rho)}{\gamma^h+\tilde{\varepsilon}_t}$ units.

However, this transformation is not *ceteris paribus* because the larger stock of employment in period $t+1$ has intertemporal consequences *beyond* period $t+1$; measuring the marginal transformation across only period t and period $t+1$ requires controlling for this additional intertemporal effect. We show using the implicit function theorem in Appendix A that the appropriate adjustment factor is $\frac{H(\tilde{\varepsilon}_{t+1})-\tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1})+\gamma^h+\gamma^s+\tilde{\varepsilon}_{t+1}}{z_{t+1}}$, which measures the one-period-ahead social asset value of a match in period $t+1$ (that is, valued from the perspective of period t). The z_{t+1} term in this asset value serves only to convert units of labor into units of consumption goods, so focus on the numerator. The social cost of screening and selecting a worker in period $t+1$ to replace a worker selected in period t is $\gamma^h+\gamma^s+\tilde{\varepsilon}_{t+1}$. Furthermore, due to uncertainty about individuals' idiosyncratic characteristics, a replacement worker selected in period $t+1$ entails an expected operating cost $H(\tilde{\varepsilon}_{t+1})-\tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) = \int_{-\infty}^{\tilde{\varepsilon}_{t+1}}\left[\varepsilon-\tilde{\varepsilon}_{t+1}\right]f(\varepsilon)d\varepsilon$. These total *costs* in period $t+1$ of selecting a replacement individual thus define the *value* of an individual selected in period t.

Putting together this logic leads to the IMRT shown in Proposition 1. The fully stochastic intertemporal efficiency condition can thus be represented as

$$1 = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[\frac{(1-\rho)\Big(H(\tilde{\varepsilon}_{t+1})-\tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1})+\gamma^h+\gamma^s+\tilde{\varepsilon}_{t+1}\Big)}{\gamma^h+\tilde{\varepsilon}_t-z_t}\right]\right\} = E_t\left\{\frac{IMRT_{c_t,c_{t+1}}}{IMRS_{c_t,c_{t+1}}}\right\}. \tag{8}$$

[10] Note that the screening cost γ^s is not included here because, as described above, it is assumed that all individuals are screened, which makes γ^s irrelevant on the margin.

[11] The simplification follows from using the Fundamental Theorem of Calculus to compute the derivatives of the functions $\eta(\tilde{\varepsilon}_t)$ and $H(\tilde{\varepsilon}_t)$.

3.3 Efficiency with One-Period Employment

These selection-based static and intertemporal MRTs apply basic economic theory to a general equilibrium labor selection model. They compactly describe the *two* technologies — the selection technology embodied by the costly screening and hiring processes, and the production technology $z_t n_t$ — that must operate for the within-period transformation of leisure into consumption and the transformation of consumption across time. Due to the participation decision and the investment nature of costly labor selection, employment inherently features both static and intertemporal dimensions.

To see how the efficiency concepts developed here nest the standard Walrasian notion of consumption-leisure efficiency, suppose first that $\rho = 1$, which makes employment a one-period, though not a frictionless, phenomenon. With one-period employment outcomes, the intertemporal condition (5) simplifies to

$$\gamma^h + \tilde{\varepsilon}_t = z_t. \tag{9}$$

This can be combined with the efficiency condition (4), so that overall efficiency in the case of one-period employment spells is characterized by the single within-period condition,

$$\begin{aligned} \frac{v'(lfp_t)}{u'(c_t)} &= \left[z_t - \gamma^h\right] \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) - \gamma^s \\ &= \int_{-\infty}^{\tilde{\varepsilon}_t} \left[z_t - \gamma^h\right] f(\varepsilon) d\varepsilon - \int_{-\infty}^{\tilde{\varepsilon}_t} \varepsilon f(\varepsilon) d\varepsilon - \gamma^s \\ &= \int_{-\infty}^{\tilde{\varepsilon}_t} \left[z_t - \gamma^h - \varepsilon\right] f(\varepsilon) d\varepsilon - \gamma^s. \end{aligned} \tag{10}$$

Viewed as a primitive, the "frictions" captured by the screening and hiring costs are formally part of the MRT of the economy, even though a neoclassical "labor wedge accounting" exercise as in Shimer (2009), Chari, Kehoe, and McGrattan (2007), or Ohanian, Raffo, and Rogerson (2008) would regard them as wedges between the MRS and the marginal product z_t of the production technology.

Moving all the way to the RBC model also requires discarding costly screening and hiring. The RBC model can be trivially viewed as featuring $\gamma^h = \gamma^s = 0$ and $\eta(\tilde{\varepsilon}_t) = 1 \ \forall t$ (in addition to $\rho = 1$). The one-period efficiency condition (10) then reduces to the familiar $\frac{v'(lfp_t)}{u'(c_t)} = z_t$, with "participation" now interchangeably interpretable as "employment" because there is no friction between the two.

4 Decentralized Economy

Now consider a decentralized economy in which a representative "large firm" hires and makes wage payments to many workers. Figure 2 summarizes the events in the decentralized economy. In addi-

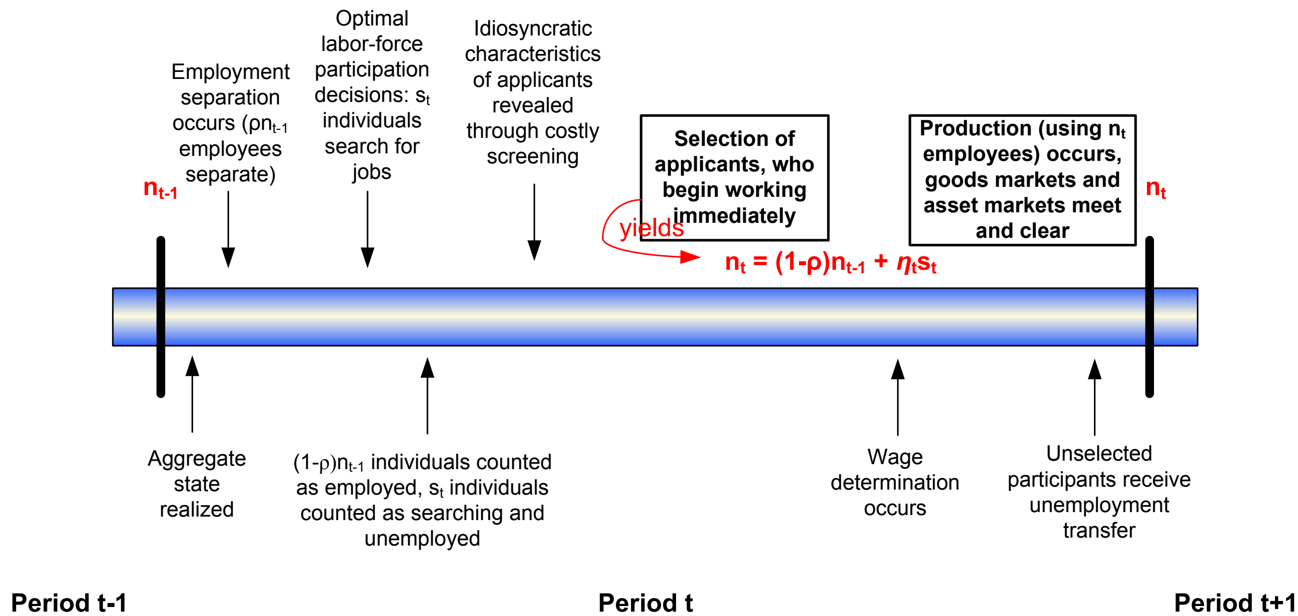

Figure 2: Timing of events in decentralized economy.

tion to the events described in Section 2, wages are set after period-t screening and selection have occurred, and each unselected individual receives an unemployment benefit from the government. To establish some notation, define w_t^I as the period-t wage earned by any incumbent worker. An incumbent worker is one that has completed at least one full period of employment and, because they are identical, all incumbent workers earn the same wage. Also define

$$\omega_e(\tilde{\varepsilon}_t) \equiv \int_{\infty}^{\tilde{\varepsilon}_t} w(\varepsilon) f(\varepsilon) d\varepsilon \tag{11}$$

as the *average* wage paid to a new hire in period t. The notation $w(\varepsilon)$ makes clear that in the period in which he is hired, a worker's wage may be conditioned on his idiosyncratic operating cost.

4.1 Firms

In period zero, the representative firm chooses state-contingent decision rules for its desired employment stock and the threshold operating cost $\tilde{\varepsilon}_t$ below which it is willing to hire in order to maximize discounted profits

$$E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left[z_t n_t - \gamma^s s_t - \gamma^h \eta(\tilde{\varepsilon}_t) s_t - \frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \eta(\tilde{\varepsilon}_t) s_t - (1-\rho) n_{t-1} w_t^I - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \eta(\tilde{\varepsilon}_t) s_t \right]. \tag{12}$$

In (12), $\Xi_{t|0}$ is the period-0 value to the representative household of period-t goods, which the firm uses to discount profit flows because households are the ultimate owners of firms. The five terms in square brackets, which represent the components of period-t profits are, respectively:

- Total firm output;
- Total screening costs;
- Total costs of hiring new workers;
- Total wages paid to newly-hired workers, which is measured as the average wage paid to a new hire conditional on being hired, $\omega_e(\tilde{\varepsilon}_t)/\eta(\tilde{\varepsilon}_t)$, times the measure of new hires $\eta(\tilde{\varepsilon}_t)s_t$;
- Total wages paid to incumbent workers, of which there is a measure $(1-\rho)n_{t-1}$;
- Total operating costs of the newly-hired workers, which is measured as the average operating cost for a newly-selected worker conditional on being hired, $H(\tilde{\varepsilon}_t)/\eta(\tilde{\varepsilon}_t)$, times the measure of new hires $\eta(\tilde{\varepsilon}_t)s_t$.

Without any confusion between firm-level variables and aggregate variables, the hiring rate η_t is understood in this section to be a consequence of the firm's decisions, while the firm takes as given the number of job-seekers s_t as well as, as is standard in search and matching models, the wage-setting process. Because output is sold in a perfectly-competitive market, the firm's problem is to choose $\tilde{\varepsilon}_t$ and n_t, $\forall t$, to maximize (12) subject to a sequence of perceived laws of motion for its employment level,

$$n_t = (1-\rho)n_{t-1} + s_t\eta(\tilde{\varepsilon}_t). \tag{13}$$

The formal analysis of the firm's problem appears in Appendix B; here we simply intuitively describe the outcome. The firm's *hiring (selection) condition* is

$$\gamma^h + \tilde{\varepsilon}_t = z_t - w(\tilde{\varepsilon}_t) + (1-\rho)E_t\left\{\Xi_{t+1|t}\left(\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w^I_{t+1}\right)\right\}, \tag{14}$$

which is the decentralized economy's counterpart to the intertemporal efficiency condition (5). At the optimum, the firm selects workers from the distribution of applicants until the cost of bringing an individual into its production activities, $\gamma^h + \tilde{\varepsilon}_t$, is equated to the payoff of hiring, which is the net marginal revenue product $z_t - w(\tilde{\varepsilon}_t)$ plus, conditional on the individual working beyond the first period of his employment relationship, a continuation value. The continuation value is composed of the worker's (future) replacement cost, $\gamma^h + \tilde{\varepsilon}_{t+1}$, and the differential between his future wage as an incumbent and a marginal (future) replacement hire. The hiring condition is thus a free-entry condition on the part of firms into the labor market, and it can be interpreted as the private economy's labor demand function.

4.2 Households

The representative household maximizes expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v\left(n_t + (1-\eta_t)s_t\right) \right] \tag{15}$$

subject to the sequence of budget constraints

$$c_t + T_t = (1-\rho)n_{t-1}w_t^I + \eta_t \frac{\omega_{et}}{\eta_t} s_t + (1-\eta_t)s_t b + \Pi_t, \tag{16}$$

and perceived laws of motion for its employment level

$$n_t = (1-\rho)n_{t-1} + \eta_t s_t. \tag{17}$$

In (16), Π_t is aggregate operating profits of the representative firm, which are distributed in lump-sum manner to households, b is an unemployment benefit paid to all unselected participants, and T_t is a lump-sum tax paid to the government (which is used by the government to provide unemployment benefits).[12] Thus, the terms on the right-hand side of (16) are, respectively, the total income of incumbent workers, the total income of newly-hired workers (conditional on hiring), total receipts of unemployment transfers, and receipts of profit distributions. Without any confusion between household-level variables and aggregate variables, the measure of participants s_t is understood in this section to be a consequence of the household's decisions, while the household takes as given the selection threshold $\tilde{\varepsilon}_t$ and thus any functions of it (in particular the hiring rate $\eta(.)$ and the average wage of a new hire $\omega_e(.)$).

The formal analysis of the household's problem appears in Appendix C; here we simply intuitively describe the outcome. The household's *labor-force participation (LFP) condition* is

$$\frac{v'(lfp_t)}{u'(c_t)} = \eta_t \left[\frac{\omega_{et}}{\eta_t} + (1-\rho)E_t \left\{ \Xi_{t+1|t} \left[w_{t+1}^I + \frac{\mu_{ht+1}}{u'(c_{t+1})} \right] \right\} \right] + (1-\eta_t)b, \tag{18}$$

in which μ_{t+1}^h is the shadow value at time $t+1$ of the household's beginning-of-period $t+1$ employment stock n_t. The LFP condition has straightforward interpretation: at the optimum, the household makes available for hiring a fraction of individuals such that the MRS between participation and consumption is equated to the expected payoff of participation. The payoff is either an unemployment benefit b in the event a given individual is not selected (which happens with probability $1-\eta_t$) or, if a given individual is selected, an immediate (expected) wage (where the expectation is with respect to the possible realizations of worker characteristics) plus an expected discounted continuation value. The LFP condition is thus simply a free-entry condition on the part of households into the labor market, and it can be interpreted as the private economy's labor supply function.

[12] Compared to the primitives of the environment described in Section 2, each unselected individual in the decentralized economy receives a government-provided unemployment payment; b is thus *not* a primitive of the environment.

4.3 Wage Determination

The economy's wage-determination process is left unspecified for now, which allows for maximum generality of the analysis in Section 5. Whatever the wage-setting process, however, three particular wages must be determined: the wage w_t^I paid to an incumbent worker, the wage $w(\tilde{\varepsilon}_t)$ paid to the *marginal* worker selected, and the *average* wage $\omega_e(\tilde{\varepsilon}_t)$ paid to a newly-selected worker.

4.4 Equilibrium

A symmetric private-sector equilibrium is made up of endogenous state-contingent processes $\{c_t, n_t, s_t, \tilde{\varepsilon}_t, w_t^I, w(\tilde{\varepsilon}_t), \omega_e(\tilde{\varepsilon}_t)\}_{t=0}^{\infty}$ that satisfy seven sequences of conditions: the goods resource constraint (6), the law of motion for employment (7), the representative firm's selection condition (14), the representative household's LFP condition (18), and the (three, unspecified) conditions that determine wages.

5 Decentralizing Efficient Allocations

For an arbitrary wage-setting process, we now derive conditions that decentralize efficient allocations.

5.1 Decentralization with One-Period Employment

To build intuition, consider first the case of one-period employment ($\rho = 1$). With $\rho = 1$, the selection condition (14) simplifies to

$$\gamma^h + \tilde{\varepsilon}_t = z_t - w(\tilde{\varepsilon}_t). \tag{19}$$

and the household LFP condition (18) simplifies to

$$\frac{v'(lfp_t)}{u'(c_t)} = \omega_e(\tilde{\varepsilon}_t) + (1 - \eta(\tilde{\varepsilon}_t))b. \tag{20}$$

The one-period LFP condition (20) shows that optimal participation equates the MRS between participation and consumption to the expected within-period return to participation, which is a weighted sum of unemployment benefits and new-hire wages (recall that $\omega_e(.)$ is itself defined as an expectation).

Unlike the one-period efficiency conditions (9) and (10), the decentralized economy counterparts cannot be expressed in a single condition. However, two sufficient conditions on wages that decentralize the efficient allocation are apparent: $w(\tilde{\varepsilon}_t) = 0$ makes the hiring condition (19) identical to the efficient selection condition (9), and $\omega_e(\tilde{\varepsilon}_t) = \tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) - \gamma^s - (1-\eta(\tilde{\varepsilon}_t))\, b$ makes the participation condition (20) identical to the static efficiency condition (10).[13]

5.2 Sufficient Conditions on Wage Determination for Efficiency

Drawing on these lessons from the one-period case, now return to the full model with $\rho < 1$. We have the following results.

Proposition 2. ***Efficient Selection.*** *Sufficient conditions on equilibrium wages for the decentralized economy to achieve intertemporal efficiency are*

$$w(\tilde{\varepsilon}_t) = 0, \forall t \tag{21}$$

and

$$w_t^I = \tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) - \gamma^s, \forall t. \tag{22}$$

Proof. Compare the decentralized economy's selection condition (14) with the intertemporal efficiency condition (5). □

Proposition 3. ***Efficient Participation.*** *If conditions (21) and (22) hold, so that the selection margin is efficient, a sufficient condition for the decentralized economy to achieve static efficiency is*

$$\omega_e(\tilde{\varepsilon}_t) = w_t^I - (1-\eta(\tilde{\varepsilon}_t))\, b. \tag{23}$$

Proof. Impose conditions (21) and (22) from Proposition 2 in the decentralized economy's LFP condition (18), which gives

$$\begin{aligned}\frac{v'(lfp_t)}{u'(c_t)} &= \omega_e(\tilde{\varepsilon}_t) + (1-\eta(\tilde{\varepsilon}_t))\, b \\ &+ \quad \eta(\tilde{\varepsilon}_t)(1-\rho)E_t\left\{\Xi_{t+1|t}\left[\tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) - H(\tilde{\varepsilon}_{t+1}) - \gamma^s + \frac{(1-\eta(\tilde{\varepsilon}_{t+1}))v'(lfp_{t+1})}{\eta(\tilde{\varepsilon}_{t+1})u'(c_{t+1})} - \frac{\omega_e(\tilde{\varepsilon}_{t+1}) + (1-\eta(\tilde{\varepsilon}_{t+1}))\, b}{\eta(\tilde{\varepsilon}_{t+1})}\right]\right\}\end{aligned}$$

in which the last two terms in the expectations operator measure the period-$(t+1)$ value to the household of a pre-existing employment relationship (further details appear in Appendix C). Imposing (23) in this expression gives

$$\begin{aligned}\frac{v'(lfp_t)}{u'(c_t)} &= \tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) - \gamma^s \\ &+ \quad \eta(\tilde{\varepsilon}_t)(1-\rho)E_t\left\{\Xi_{t+1|t}\left(\frac{1-\eta(\tilde{\varepsilon}_{t+1})}{\eta(\tilde{\varepsilon}_{t+1})}\right)\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - (\tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) - H(\tilde{\varepsilon}_{t+1}) - \gamma^s)\right]\right\},\end{aligned}$$

[13] This conclusion is more straightforward to see from the static efficiency condition (4).

which is a difference equation in the term $\frac{v'(lfp)}{u'(c)} - (\tilde{\varepsilon}\eta(\tilde{\varepsilon}) - H(\tilde{\varepsilon}) - \gamma^s)$. A solution to this difference equation is

$$\frac{v'(lfp_t)}{u'(c_t)} = \tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) - \gamma^s, \tag{24}$$

$\forall t$, which recovers the static efficiency condition (4). □

5.3 Discussion

Propositions 2 and 3 provide sufficient, though not necessary, conditions on wages that decentralize efficient allocations. They are independent of the economy's wage-determination process. We discuss the intuition of each of the three sufficient conditions.

Condition (21) in Proposition 2 states that the marginal new hire should be paid a zero wage, which is efficient because unselected individuals have zero social value. Comparing the economy's primitives (depicted in Figure 1) with the events that occur in the decentralized economy (Figure 2) shows that unemployment benefits paid to unselected individuals are a feature only of the decentralized economy; they are not a primitive of the economy because they reflect neither preferences nor technology. Efficiency thus requires that the wage of the marginal new hire should equal his social value if unselected, which is zero. Note also that this condition is the same as one of the two conditions that achieves efficiency in the case of one-period employment described above.

Condition (22) in Proposition 2 states that the wage of an incumbent worker should be exactly equal to the economy's within-period MRT between consumption and leisure. As described in Section 3, the static MRT measures the transformation required to screen and select an individual for production. Loosely speaking, and making an analogy with the RBC model, the within-period MRT can be thought of as an individual's contemporaneous social marginal product. An incumbent worker thus should be paid his marginal product, which is identical for all incumbent workers because it is only new workers that are heterogenous in their characteristics.

Condition (23) in Proposition 3 states that the average wage paid to a new hire should be equal to that of an incumbent worker net of any unemployment benefit he expects to receive if unselected. This is efficient because wages of incumbent workers reflect their "marginal product," as just described, but a downward adjustment is required because the expected payment $(1 - \eta(\tilde{\varepsilon}_t))\, b > 0$ is, from the point of view of the primitives of the environment, inefficient.

5.4 Nash-Bargained Wages

Labor-market models with frictions often assume generalized Nash bargaining over wages. It is thus of natural interest to explore some implications of Nash-bargained wages for the labor-selection model. Each worker is assumed to bargain individually with the firm, and vice-versa. That is, each bilateral worker-firm negotiation takes outcomes in all other worker-firm negotiations as given;

there are thus no strategic considerations in wage determination across employees. We assume that each worker's wage can be conditioned on his idiosyncratic characteristics ε_t^i and that all wages are re-bargained every period.[14] Finally, the bargaining power of each newly-selected worker is $\alpha^E \in [0,1]$, and the bargaining power of each incumbent worker is $\alpha^I \in [0,1]$. We prove the following in Appendix D.

Proposition 4. ***Individually-Bargained Nash Wages.*** *Suppose each worker-firm pair Nash bargains over the real wage independently of every other worker-firm pair. If the Nash bargaining power of every newly-hired worker is α^E and if the Nash bargaining power of every incumbent worker is α^I, then the real wage earned by the marginal new hire is*

$$w(\tilde{\varepsilon}_t) = b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\right\}, \tag{25}$$

the real wage earned by a new hire with idiosyncratic characteristics ε_t^i is

$$w(\varepsilon_t^i) = b + \alpha^E\left(\tilde{\varepsilon}_t - \varepsilon_t^i\right) + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\right\}, \tag{26}$$

and the real wage earned by every incumbent worker is

$$w_t^I = b + \alpha^I\left(\gamma^h + \tilde{\varepsilon}_t\right) + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\right\}. \tag{27}$$

Proof. See Appendix D. □

Several aspects of the wage functions (25), (26), and (27) are useful to highlight. First, the unemployment benefit b is the lower bound of all wages because it is the payoff an individual receives for sure if wage negotiations break down. Second, the continuation-value component of each wage function (the last term on the right-hand side of each wage expression) is identical because no matter a worker's type in period t, he will be a (homogenous) incumbent worker in period $t+1$ if he remains employed.[15] Third, both new hires with $\varepsilon_t^i < \tilde{\varepsilon}_t$ and incumbent workers receive a premium over a marginal new hire. These premia depend on their respective bargaining powers and the values they bring to the firm over and above that of a marginal new hire.

Indeed, the wage functions (25), (26), and (27) imply wage differentials that are intuitive to understand. A new hire with $\varepsilon_t^i < \tilde{\varepsilon}_t$ earns a premium over the marginal new hire

$$w(\varepsilon_t^i) - w(\tilde{\varepsilon}_t) = \alpha^E\left(\tilde{\varepsilon}_t - \varepsilon_t^i\right), \tag{28}$$

which is the share of the operating cost *savings* he provides the firm that he is able to extract through his bargaining power. An incumbent worker earns a premium over the marginal new hire

$$w_t^I - w(\tilde{\varepsilon}_t) = \alpha^I\left(\gamma^h + \tilde{\varepsilon}_t\right), \tag{29}$$

[14] These assumptions are also standard in DSGE matching models.

[15] The wage of the marginal new hire, (25), is simply the wage of an arbitrary new hire (26) evaluated at $\varepsilon_t^i = \tilde{\varepsilon}_t$.

which is the share of the replacement cost *savings* (relative to a marginal new hire) he provides the firm that he is able to extract through his bargaining power. The replacement cost savings includes both the cost γ^h of hiring a new worker and his operating cost $\tilde{\varepsilon}_t$.

Integrating (26) over $\varepsilon_t^i \leq \tilde{\varepsilon}_t$ gives the average wage paid to a new hire,

$$\omega_e(\tilde{\varepsilon}_t) = \eta(\tilde{\varepsilon}_t)b + \alpha^E\left[\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)\right] + \eta(\tilde{\varepsilon}_t)(1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\right\} \tag{30}$$

(details of the calculation appear in Appendix D). Finally, in terms of defining equilibrium, these Nash conditions for $w(\tilde{\varepsilon}_t)$, w_t^I, and $\omega_e(\tilde{\varepsilon}_t)$ close the definition presented in Section 4.4.

As noted above, the sufficient conditions (21), (22), and (23) on wages in Propositions 2 and 3 are independent of any particular wage-determination process. A natural question is thus what parameter restrictions — in particular, parameter restrictions on the bargaining problem itself, not on other primitives of the environment — would be required for Nash-bargained wages to satisfy our sufficient conditions for efficiency. If such a parameter restriction exists, it would be analogous to the benchmark Hosios (1990) efficiency condition in matching models, which amounts to a simple restriction on Nash bargaining powers that guarantees wages that are sufficient (as well as necessary) for supporting the planning outcome. Unfortunately, no such parameter restriction on Nash bargaining exists for the labor-selection model.

Proposition 5. ***Impossibility of Nash Wages Guaranteeing Efficiency.*** *Nash-bargained wages, characterized in Proposition 4, cannot satisfy the sufficient conditions on wages in Propositions 2 and 3. Thus, it is impossible to guarantee that Nash-bargained wages can support efficient allocations.*

Proof. See Appendix E. □

The idea of the proof is to compare (25), (27), and (30) with their respective counterparts (21), (22), and (23) to try to obtain a set of parameter restrictions such that the former coincide with the latter. However, this is impossible, as stated in Proposition 5.

6 Quantitative Analysis

Having established a number of analytical results, we now explore the model's business-cycle properties. The focus is on amplification of aggregate productivity shocks into labor-market fluctuations. Because many DSGE matching models have aimed to replicate the high empirical volatility of labor-market outcomes, this seems a natural point of quantitative comparison. Our aim is not to exactly replicate many dimensions of the data, which would be difficult given the rather parsimonious framework, but rather to illustrate that a reasonably-calibrated selection model generates greater amplification than does a baseline matching model. Amplification occurs in both the efficient and decentralized (Nash-bargaining) allocations.

Nearly all of the parameters are assumed identical in both the efficient and decentralized allocations. We conduct the baseline calibration of so that the steady state of the efficient allocation matches key dimensions of the data. We use the efficient, rather than the decentralized, outcome as the reference allocation because doing so makes clear that the quantitative properties of the model are driven by the selection mechanism, rather than the inefficiencies inherent in the Nash-bargaining economy. This point is important because, as has been known since Shimer (2005), *efficient* labor-market fluctuations in a baseline matching model are small. Several of the most prominent "solutions" to the so-called "Shimer puzzle" invoke inefficiencies in the wage-setting process in order to increase amplification, such as Hall (2005), Gertler and Trigari (2009), and Hagedorn and Manovskii (2008).[16] In contrast, in the selection model, efficient fluctuations of labor-market outcomes are sizable; the inefficiencies in the Nash economy make fluctuations even larger.

6.1 Parameterization

The model frequency is quarterly, so we set a subjective discount factor $\beta = 0.99$, which implies a steady-state real interest rate of about four percent. For utility, standard functional forms are used, $u(c_t) = \ln c_t$ and $v(lfp_t) = \frac{\kappa}{1+1/\phi} lfp_t^{1+1/\phi}$. The parameter ϕ is the elasticity of labor-force participation with respect to the real wage, which is set to $\phi = 0.18$ following Arseneau and Chugh's (2010) calibration of a matching model with endogenous participation fit to U.S. data. The scale parameter is set to $\kappa = 7$ to deliver a steady-state participation rate of 74 percent, the long-run U.S. empirical measure reported by Veracierto (2008). As noted above, these parameters are calibrated in the long-run efficient allocation and held fixed when considering the the decentralized economy.

The other parameters directly govern labor-market transitions. The separation rate is set to $\rho = 0.10$, consistent with the average quarterly job-destruction rate in the U.S. Without specific evidence on "screening costs," we set γ^s so that aggregate screening costs, $\gamma^s s$, absorb 0.25 percent of GDP in the efficient steady state, which we think is conservative. Both ρ and γ^s are held

[16] By "inefficiencies," we mean any allocation that does not coincide with the solution of the planning problem of Section 3. Arseneau and Chugh (2010) show that in achieving its descriptive success, a Hagedorn-and-Manovskii-style calibration of a matching model generates incredibly inefficient, in the precise sense just described, fluctuations. That is, *efficient* labor-market fluctuations in a standard matching model are small. For example, Figure 2 in Arseneau and Chugh (2010) shows that fluctuations in a matching model generated with this calibration strategy are orders of magnitude larger than the efficient fluctuations in the underlying model. Briefly, the essence of a Hagedorn-and-Manovskii style calibration is that, in the decentralized economy, workers receive a very small share of employment surpluses, and workers' payoffs outside employment are fairly close to those inside employment.

fixed when studying the decentralized economy. In the efficient allocation, neither unemployment benefits nor bargaining powers are defined. In the decentralized economy, we set the bargaining power of both incumbents and newly-selected workers to $\alpha^E = \alpha^I = 0.5$, a standard value in matching models; and the unemployment benefit b is chosen so that it constitutes 75 percent of incumbent workers' wages; the resulting value is $b = 0.65$. This calibration of unemployment benefits is an intermediate setting between Shimer's (2005) relatively low value and Hagedorn and Manovskii's (2008) very large value. It is thus not the case that the average newly-selected worker is virtually indifferent between working and not working in our decentralized economy; that is, a "small surplus" calibration strategy is not at the heart of the model's amplification mechanism, a point discussed further in Section 7.[17]

The remaining parameters are the distribution of workers' idiosyncratic characteristics and the hiring cost. The distribution from which workers' idiosyncratic characteristics are drawn is assumed to be Gaussian (in levels) with mean zero and standard deviation σ_ε. To maintain tractability, idiosyncratic shocks are assumed to have zero persistence. There is no direct empirical evidence on the cross-sectional variance of individuals' idiosyncratic traits, which, for the quantitative purposes here, are most usefully interpreted as idiosyncratic productivity. In matching models featuring endogenous separation due to idiosyncratic productivity shocks, such as Mortensen and Pissarides (1994), den Haan, Ramey, and Watson (2000), Walsh (2005), and Krause and Lubik (2007), the cross-sectional variance is assumed to be small. At a technical level, we cannot directly adopt their values because we assume a Gaussian distribution, whereas these previous studies assume log-normality. At a conceptual level, the cross-sectional heterogeneity in these studies is in the characteristics of *individuals who already are employed*, which is in contrast to our model, in which the cross-sectional heterogeneity is in the characteristics of *individuals who are being considered for employment*. It seems realistic that heterogeneity in the latter group would be larger than heterogeneity in the former group.

As an illustrative value that nonetheless tries to maintain some comparability, we set a cross-sectional standard deviation $\sigma_\varepsilon = 0.25$. Given that we normalize mean aggregate productivity of all individuals to unity in the steady state, this parameter setting implies that the individual with idiosyncratic realization $\varepsilon^i = -0.25$ (recall that idiosyncratic traits are defined as *costs*) *adds* 25 percent to output in his first period of employment, which seems plausible.[18] The parameter $\sigma_\varepsilon = 0.25$ is held fixed across the efficient and decentralized allocations.[19]

[17]Sensitivity analysis using lower values of b show that the model dynamics do not change much, conditional on appropriately recalibrating σ_ε and γ^h, described next.

[18]Symmetrically, the individual with idiosyncratic realization $\varepsilon^i = 0.25$ *subtracts* 25 percent from output, which also seems plausible.

[19]Idiosyncratic productivity should be reflected in the residual wage distribution in the decentralized economy. We can compare our illustrative parametrization with the residual wage distribution in the U.S. Heathcote, Perri,

The hiring cost parameter γ^h is set so that in each of the efficient and decentralized allocations, the steady-state job-finding rate is $\eta(\tilde{\varepsilon}) = 0.70$, consistent with U.S data. This requires setting a fairly large $\gamma^h = 7.7$ in the efficient allocation, and a much lower $\gamma^h = 2.26$ in the decentralized economy.[20] There is not much evidence on hiring/training costs, though Dolfin (2008) shows, using U.S. micro data, that training costs are large. Keeping in mind that she focuses only on training costs, Dolfin's (2008) evidence is of the same order of magnitude as implied by our parameter value for γ^h in the decentralized economy (if direct and indirect training costs are taken into account).[21]

Finally, the only source of aggregate uncertainty is aggregate productivity shocks, which follow the AR(1) process

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t^z, \tag{31}$$

with innovations ϵ_t^z distributed $N(0, \sigma^2_{\epsilon^z})$, and standard parameter values $\rho_z = 0.95$ and $\sigma_{\epsilon^z} = 0.01$.

6.2 Efficient Fluctuations

Table 1 presents results for both the efficient allocation and the decentralized economy.[22] In the efficient allocation, the volatility of unemployment, at 4.5 times that of GDP, is a close match

and Violante. (2010) find a variance of the residual wage distribution for males of about 0.25 in 1970 and about 0.4 in 2005. Conditional on the degree of cross-sectional dispersion in wages being similar in value to that of the cross-sectional dispersion of idiosyncratic characteristics, our parametrization is in line with the Heathcote, Perri, and Violante (2010) findings. A robustness check in Section 7 shows that the amplification of productivity shocks becomes smaller with larger σ_ε. However, even with substantially larger distribution, there are still strong amplification effects.

[20] The share of hiring costs in total output turns out to be large in the efficient allocation. Natural modifications that would dampen the share would be to incorporate government spending and physical capital investment. Introducing other types of fixed and/or variable costs in the employment formation process would also dampen the share of hiring costs in output. For example, integrating the selection framework with a matching framework would lower the share, as would introducing some type of "home production" by individuals not selected for employment; in New Keynesian applications of matching models, Ravenna and Walsh (2008, 2011) use this type of mechanism. In the interest of focusing on the selection mechanism, we chose to leave out other frictions. The quantitative analysis focuses on percentage deviations from long-run levels, not on long-run levels and shares themselves, so the results below still highlight the cyclical sensitivity generated by the model of labor markets to productivity shocks.

[21] It is known that hiring costs can be used to boost amplification effects in search and matching models (see Silva and Toledo (2009)). By contrast, for labor selection models, it can be shown analytically that the standard deviation of the idiosyncratic shocks is most important for the amplification effects, while hiring costs are more important for pinning down the appropriate steady states. In principle, there are various ways of reducing the hiring costs in our calibration. For example, one could assume that there is less than one contact per period per worker, or by assuming that the separation rate is larger than 0.10. Echoing the point above, we left such frictions out of the model to maintain analytical and conceptual clarity, as well as for comparability with the related literature.

[22] Deterministic steady states are computed using a standard nonlinear numerical solver. For dynamics, we use our own first-order implementation of the algorithm described in Schmitt-Grohe and Uribe (2004). Second moments are computed from HP-filtered simulated data (using HP smoothing parameter 1,600), with 500 simulations, each of length 200 periods.

to the empirical value of 5 reported by Gertler and Trigari (2009, Table 2) for U.S. quarterly data.[23] The first-order serial correlation of unemployment and its contemporaneous correlation with GDP are also extremely close to the empirical values (0.91 and -0.86, respectively) measured by Gertler and Trigari (2009, Table 2). The tight fit of the model with the data along several dimensions of umemployment dynamics is remarkable, considering that fluctuations are driven only by productivity shocks.

Efficient fluctuations also capture the dynamics of the participation rate fairly well. The relative volatility of participation, at 0.14, is close to the observed 0.20 for the U.S., as is the correlation with GDP, 0.26 in the efficient fluctuations vs. 0.38 in U.S. data (both empirical measures are from Veracierto (2008, Table 2)). The calibration strategy of Arseneau and Chugh (2010) for the labor subutility function thus seems to carry over well from a matching model to a selection model.

Efficient fluctuations are a bit less successful in matching the relative volatility of employment, however, which is about half the empirical value reported by Gertler and Trigari (2009, Table 2). The job-finding rate, $\eta(\tilde{\varepsilon})$, while slightly more volatile than both GDP and productivity, is not as volatile as in the data. However, in *efficient* fluctuations of a matching model, the volatility of the job-finding rate is miniscule.[24] Our overall conclusion from these results is that efficient fluctuations in the selection model do qualitatively well in jointly explaining several cyclical labor-market facts that have been a challenge for matching models, and thus entail substantially more amplification than efficient fluctuations in a matching model.

6.3 Decentralized Fluctuations

The lower panel of Table 1 presents results for the calibrated Nash-bargaining economy. Compared to the efficient fluctuations, the two most notable differences are that the relative volatility of unemployment and the job-finding rate are larger. There is thus more amplification of productivity shocks into labor-market outcomes in the (inefficient-)Nash-bargaining economy than in the efficient allocation, just as is the case with matching models.

However, unlike matching models, these results are driven neither by a small-surplus calibration (as in Hagedorn and Manovskii (2008)) nor wage rigidities (as in Hall (2005) and Gertler and Trigari (2009)). The last column of Table 1 shows that the aggregate wage (which we compute in each period as the weighted average of new hires' wages and incumbents' wages, $w_t^{AGG} = (1-\rho)n_{t-1}w_t^I + \eta_t s_t \omega_t$) is *more* volatile than GDP. While this excess volatility of wages is counterfactual, it clearly shows that the model mechanism in the decentralized economy has nothing to do with wage rigidity.

[23]We compute ue as the measure of individuals available for work that do not get selected for work — that is, $ue \equiv (1-\eta(\tilde{\varepsilon}))s$.

[24]See Shimer (2005) and Hagedorn and Manovskii (2008) for both the empirics and theory, or Arseneau and Chugh (2010) for a general-equilibrium analysis.

	gdp	n	ue	lfp	$\eta(\tilde{\varepsilon})$	z	w^{AGG}
			Efficient allocation				
Mean	0.72	0.72	0.03	0.74	0.70	1	—
Volatility (SD%)	1.41	0.35	6.29	0.20	1.45	1.29	—
Relative volatility (/gdp)	1	0.25	4.46	0.14	1.03	0.91	—
Autocorrelation	0.74	0.95	0.87	0.95	0.85	0.70	—
Correlation with gdp	1	0.28	-0.79	0.26	0.90	0.97	—
			Decentralized allocation				
Mean	0.72	0.72	0.03	0.74	0.70	1	0.55
Volatility (SD%)	1.65	0.40	9.18	0.05	2.89	1.28	1.91
Relative volatility (/gdp)	1	0.24	5.58	0.03	1.75	0.78	1.16
Autocorrelation	0.75	0.88	0.85	0.74	0.78	0.70	0.85
Correlation with gdp	1	0.66	-0.95	0.35	0.99	0.99	0.97

Table 1: Fluctuations driven by productivity shocks. Second moments computed from cyclical components of HP-filtered simulated data. Aggregate wage w^{AGG} is computed as weighted average of new hires' wages and incumbents' wages.

The quantitative results demonstrate that aggregate shocks are transmitted into labor markets in a fundamentally different way in selection models than in matching models. We explore the mechanism in more detail in Section 7, but a brief description is first nonetheless useful. In selection models, it is the endogenous determination of hiring thresholds, which picks the most productive end of the distribution of heterogenous applicants, that is critical for dynamics. If the mass of (heterogenous) individuals near the hiring thresholds at different points along the business cycle is sufficiently large, the measure of individuals that flow between employment and unemployment can be large, with associated (large) swings in job-finding probabilities. In contrast, in matching models, it is the free-entry condition that governs firms' incentives to post vacancies that is critical for aggregate dynamics.[25] It is largely a quantitative question, then, which framework generates more intrinsic propagation when appropriately calibrated. In matching models, it is often some aspect of decentralized wage-setting that requires an extreme assumption in order to generate meaningful volatility. In the selection model, one does not have to take a stand on the nature of wage determination whatsoever to generate meaningful amplification. We see this feature as a virtue of the selection model, because prices (wages) themselves are indeterminate in equilibrium in the presence of surplus-sharing, the key insight (in the matching class of models) of Hall (2005) and a point re-emphasized by Rogerson and Shimer (2010).

7 Inspecting the Mechanism

To shed further light on the model's mechanism, we now focus on its main components. Examining the partial equilibrium core of the model isolates several important aspects of the results. As we inspect the mechanism, we draw comparisons with matching models to highlight that amplification of productivity shocks arises from a fundamentally different source in the selection framework than the sources underlying prominent explanations of amplification in matching models. These comparisons are drawn not in order to argue that the selection framework is superior to the matching framework in this regard, but rather only to emphasize that, because employment formation in reality entails many margins — a point that also emerges from the firm-level evidence of Davis, Faberman, and Haltiwanger (2010) — amplification can arise from many sources. Our model emphasizes the selection margin.

The focus throughout has been on efficient allocations, so we establish our points using the efficient hiring and employment dynamics of a single firm that employs multiple workers and operates in partial equilibrium. The gross real interest rate and participation are thus treated as constant throughout this section. Maintaining the calibration of the full general equilibrium model, $1 + r$ is

[25] For more details, see Brown, Merkl, and Snower. (2009).

	n	ue	$\eta(\tilde{\varepsilon})$	z
Efficient partial equilibrium allocation				
Mean	0.71	0.03	0.70	1
Volatility (SD%)	0.38	9.71	3.09	1.29
Autocorrelation	0.81	0.81	0.70	0.70

Table 2: Partial-equilibrium fluctuations driven by productivity shocks. Second moments computed from cyclical components of HP-filtered simulated data.

fixed at $\beta^{-1} = 0.99^{-1}$, and participation is fixed at $\bar{l} = 0.74$. The latter implies that the number of individuals searching in period t is $s_t = \bar{l} - (1-\rho)n_{t-1}$, which is predetermined at the start of period t. The fixed-participation assumption also establishes that endogeneity of labor supply, while useful for the efficiency analysis above, is not central to the model's quantitative predictions.

7.1 The Importance of Selection Effects

The most important aspect of the results to highlight is that the volatility of labor markets arises directly from selection effects, rather than through general equilibrium effects such as fluctuations of labor supply (participation) or of the real interest rate. The selection condition is the heart of the model, making it the appropriate starting point for the analysis. In the partial equilibrium version of the model, efficient allocations $\{\tilde{\varepsilon}_t, n_t\}_{t=0}^{\infty}$ are characterized by the selection condition

$$\gamma^h + \tilde{\varepsilon}_t = z_t + \frac{1-\rho}{1+r} E_t \left\{ H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) + \gamma^s + \gamma^h + \tilde{\varepsilon}_{t+1} \right\}, \tag{32}$$

which is derived from a version of the social planner problem that takes r and $\bar{l}$ as given, in conjunction with the law of motion

$$\begin{aligned} n_t &= (1-\rho)n_{t-1} + \eta(\tilde{\varepsilon}_t)\left(\bar{l} - (1-\rho)n_{t-1}\right) \\ &= (1-\eta(\tilde{\varepsilon}_t))(1-\rho)n_{t-1} + \eta(\tilde{\varepsilon}_t)\bar{l}. \end{aligned} \tag{33}$$

Table 2 presents simulation-based dynamics from a first-order approximation of the decision rules of this partial equilibrium model. Fluctuations are driven by the same sequences of productivity shocks that generated the general-equilibrium fluctuations in Table 1, and all parameter values are held fixed from Section 6. As comparison of Table 2 with Table 1 shows, productivity shocks induce even sharper fluctuations in the selection rate $\eta(\tilde{\varepsilon})$ and, as a consequence, in unemployment than in the general-equilibrium model. Figure 3 also illustrates this result with an

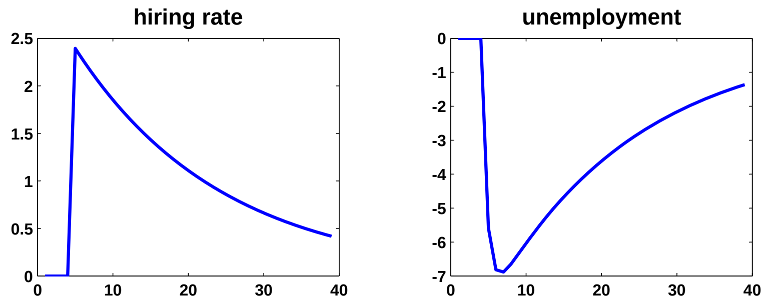

Figure 3: Impulse response to one-time, one-standard-deviation positive shock to productivity. Horizontal axes plot quarters, vertical axes plot percentage deviation from steady state.

impulse response to a one-time, one-standard-deviation positive shock to productivity. The result that the efficient allocation displays such sharp fluctuations stands in contrast to the very small fluctuations of efficient allocations in a standard matching model.[26]

7.2 The Outside Option

A second important aspect of the results to emphasize is that a "small surplus" calibration is not part of the model's amplification mechanism.

7.2.1 Efficient Allocation

To demonstrate this point in the planning allocation, introduce "production" by unselected individuals, and consider it part of the technology respected by a social planner. In particular, suppose that each unselected individual produces $\upsilon < z_t$ units of the homogenous final good. Because this production is outside the long-lasting employment relationships created by the selection process, it defines the outside option of employment.[27] The only modification in the efficient selection condition (32) is that z_t is replaced by $z_t - \upsilon$, the flow social surplus of a selected individual.[28] A small social surplus of employment is part of the Hagedorn and Manovskii (2008) calibration strategy that enables matching models to generate large amplification of productivity shocks, so it is important

[26]See, for example, Shimer (2005) or Arseneau and Chugh (2010).

[27]This production defines, in terms of primitives, an outside option for an individual who has been selected for employment and is on the verge of beginning to produce. It can be thought of as a type of "home production" or "informal sector production," although we are not modeling either in a serious way. Also not that this extended model is not a two-sector model.

[28]Formally, introduce the term $(1-\eta(\tilde{\varepsilon}_t))s_t\upsilon$ on the right hand side of the resource constraint (6) and then conduct the planning optimization. The term $z_t - \upsilon$ replaces z_t in the efficient selection condition (5) of the general equilibrium model. Assuming participation and the real interest rate are constant then implies replacing z_t with $z_t - \upsilon$ in the partial equilibrium version (32). The flow social surplus in the baseline model is thus simply z_t.

to understand that the selection model's mechanism does not rely on a similar channel.

In our setup, choosing a value υ to hit any arbitrary surplus, including a small surplus, in the deterministic steady state requires simultaneously re-setting the hiring cost parameter γ^h to hold fixed the long-run hiring rate at its calibration target $\eta(\tilde{\varepsilon}) = 0.70$. Because the hiring rate is a function of only $\tilde{\varepsilon}$ (recall $\eta(\tilde{\varepsilon})$ is the cumulative distribution), this immediately implies that the long-run selection threshold $\tilde{\varepsilon}$ is invariant to any such re-parameterization. Furthermore, the steady-state version of (33) shows that long-run employment n is also unchanged, given that $\bar{l}$ is fixed. The steady state is thus identical for all (υ, γ^h) pairs that hold fixed the long-run selection rate, including ones in which the surplus $z - \upsilon$ is small.

Productivity-induced fluctuations of the model, up to a first-order approximation, are also identical. This can be established quantitatively by simulating conditions (32) and (33) for alternative pairs (υ, γ^h); we find dynamics identical to those in Table 2 (hence the results are not shown).

A suggestive analytical argument also indicates this first-order invariance result. To gauge the sensitivity of the selection threshold $\eta(\tilde{\varepsilon})$ to changes in z, consider the steady state version of the selection condition (32). As shown in Appendix A, the steady state elasticity is

$$\frac{\partial \ln \tilde{\varepsilon}}{\partial \ln z} = \left(\frac{z}{\tilde{\varepsilon}}\right)\left(\frac{1}{1-\beta(1-\rho)(1-\eta(\tilde{\varepsilon}))}\right). \tag{34}$$

The responsiveness of the efficient selection threshold $\tilde{\varepsilon}$ to changes in productivity does *not* depend *directly* on υ and hence does not depend directly on the social surplus $z - \upsilon$. This suggests that local dynamics around the steady state are, up to first-order effects, are also insensitive to the size of the social surplus.

Given $\upsilon = 0$, the calibrated parameters, and the endogenous steady state threshold $\tilde{\varepsilon}$, this elasticity is about 15 in magnitude, which is large. Intuitively, this suggests that productivity-induced fluctuations around the steady state may be large. This intuition is confirmed by the numerical results noted above for the case of $\upsilon > 0$.[29] This result is important because it establishes that the heart of the model is the selection decision and the fluctuations induced in it by productivity shocks.[30]

[29] At higher-order approximations, the sensitivity of $\tilde{\varepsilon}$ to productivity shocks in general will depend on υ because $\tilde{\varepsilon}$ is fundamentally a function of υ (albeit a nonlinear one) through the selection condition. But these effects are indirect (i.e., of second- and higher order), unlike the effects of small surpluses in matching models, which have first-order effects.

[30] The logic of the preceding argument is similar to that of Hagedorn and Manovskii (2008, p. 1695), who compute the sensitivity of the hiring rate (technically, of labor-market tightness, which has unit correlation with the hiring rate in their and most matching models) with respect to productivity to show that, in matching models, the elasticity *does* depend *directly* on the (inverse of the) size of the social surplus. That is, Hagedorn and Manovskii (2008) establish that productivity-induced fluctuations in a matching model depend in a *first-order* way on the size of the social surplus. The fact that they compute this elasticity in a decentralized model with Nash wage bargaining is a

7.2.2 Decentralized Allocation

The decentralized model featured government-provided unemployment benefits, an outside option to employment, so there is no need to introduce production by unselected individuals as in the preceding analysis. Nonetheless, the surplus created by production over unemployment transfers ($z - b$ in the steady state of the decentralized economy) is not of first-order importance for the magnitude of fluctuations in the decentralized economy. To see this, insert the steady-state Nash wage for the marginal new hire (25) and the wage differential between the marginal new hire and incumbent workers (29) into the steady-state version of the decentralized selection condition (14). This gives a steady-state partial equilibrium form of the selection condition,

$$\gamma^h + \tilde{\varepsilon} = \frac{z}{1-\beta(1-\rho)} - b - \frac{\beta(1-\rho)}{1-\beta(1-\rho)} \frac{v'\left(\bar{l}\right)}{u'\left(\bar{c}\right)}, \tag{35}$$

in which $\bar{c}$ is viewed as a constant. Constructing the same arguments as for the efficient case above, it is easy to show that, for uniformly-distributed idiosyncratic operating costs, the steady-state elasticity of the selection threshold $\eta(\tilde{\varepsilon})$ to productivity z is independent of the magnitude $z - b$. Thus, the size of the surplus $z - b$ does not have first-order effects on the magnitude of fluctuations; quantitatively, this result also holds for the case of normally-distributed operating costs.

7.3 Cross-Sectional Dispersion of Idiosyncratic Operating Costs

As discussed in Section 6, we set the cross-sectional standard deviation of idiosyncratic operating costs to $\sigma_\varepsilon = 0.25$ as a compromise between an illustrative parameter value and one that maintains some comparability with DSGE matching models featuring cross-sectional dispersion in worker productivity such as den Haan, Ramey, and Watson (2000), Walsh (2005), and Krause and Lubik (2007).[31] To illustrate robustness of the model's results with respect to changes in σ_ε, Table 3 presents dynamics for cross-sectional standard deviations that are three times smaller and three times larger than the benchmark $\sigma_\varepsilon = 0.25$.[32] Fluctuations are much larger (smaller) in magnitude the less (more) diffuse are individuals' characteristics in the cross section. This result is intuitive. For a given size change in the selection threshold (induced by a shock to productivity), if the cross-sectional variance is small (large), the measure of individuals that move across the selection

detail, because the elasticity (Hagedorn and Manovskii (2008, condition (1)) can just as easily be evaluated at the efficient allocation, which requires imposing the Hosios (1990) condition, and for any size of the social surplus. Even in this case, the elasticity remains a function directly of the social surplus.

[31] As discussed in Section 6, there are both conceptual reasons and technical reasons for not directly adopting their values for σ_ε.

[32] In each of the small- and large-dispersion scenarios, the hiring cost parameter γ^h is re-set to hold constant the long-run hiring rate at its calibration target $\eta(\tilde{\varepsilon}) = 0.70$: the small-dispersion case requires $\gamma^h = 8.66$, and the large-dispersion case requires $\gamma = 4.4$.

	n	ue	$\eta(\tilde{\varepsilon})$	z
Small cross-sectional dispersion ($\sigma_\varepsilon = 0.08$)				
Mean	0.71	0.03	0.70	1
Volatility (SD%)	1.35	89.3	12.1	1.29
Autocorrelation	0.81	0.81	0.70	0.70
Large cross-sectional dispersion ($\sigma_\varepsilon = 0.75$)				
Mean	0.71	0.03	0.70	1
Volatility (SD%)	0.14	3.39	1.15	1.29
Autocorrelation	0.80	0.80	0.70	0.70

Table 3: Partial-equilibrium fluctuations driven by productivity shocks for small and large cross-sectional dispersion of idiosyncratic operating costs. Second moments computed from cyclical components of HP-filtered simulated data.

threshold is large (small) because there is a relatively large (small) mass of individuals very close to the threshold.

8 Conclusion

Labor-selection models depict realistic micro-frictions that affect employment, frictions that are distinct from those highlighted in search and matching models. Based on our view that selection models may develop into another class of DSGE models in which a variety of positive and normative macro-labor issues can be studied, a precise characterization of efficiency and an understanding of the dynamics generated by a baseline version of the environment are important for further application of selection models.

On the positive dimension, the calibrated versions of the model generate high volatilities of unemployment and job-finding rates, which are aspects of empirically-relevant labor-market fluctuations that have received much modeling attention in the matching literature. This amplification occurs in both the efficient allocations of the model as well as allocations decentralized through Nash-bargained wages. In neither case is the amplification due to prominent explanations of amplification in the matching literature.

On the normative dimension, our efficiency concepts and results rely on the basic principles of transformation frontiers and "wedges," appropriately defined for the environment. In terms of decentralization, we derived a set of sufficient conditions on wages that implement efficiency. These conditions put restrictions on the shape of the distribution of wages across workers, and are weak conditions in that they are independent of the wage-determination process. However, Nash bargaining, which is commonly used in the matching literature as the wage-determination process, generically does not satisfy these sufficient conditions, nor does it generically implement efficiency. There thus seems to be no simple Hosios-like condition for labor selection models.

Many extensions of our work suggest themselves. In terms of capturing cyclical fluctuations even more realistically, introducing physical capital accumulation and other real frictions common in quantitative macro models is straightforward. To arrive at an even richer model of labor market dynamics, the selection framework can be integrated with the matching framework. Such an integrated framework could allow, among many other questions, exploration of how and why different margins of adjustment seem to be more important than others, at least in aggregate, in different business cycles and even different points along a given business cycle; such questions are broadly motivated by the empirical evidence of Davis, Faberman, and Haltiwanger (2010). Perhaps most important for the continued quantitative development of selection models are calibration strategies based on direct micro evidence of the worker heterogeneity that lies at the core of the model.

A Efficient Allocations

A social planner in this economy optimally allocates the measure one of individuals in the representative household to leisure, unemployment, and employment. There are several representations of the planning problem available: suppose that c_t, lfp_t, n_t, and $\tilde{\varepsilon}_t$ are the formal objects of choice. Given the accounting identities of the model, the measure of individuals available for work can thus be expressed $s_t = lfp_t - (1-\rho)n_{t-1}$.

The social planner problem is to maximize lifetime expected utility of the representative household

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v\left(lfp_t\right) \right] \tag{36}$$

subject to the sequence of goods resource constraints

$$c_t + \gamma^h \eta(\tilde{\varepsilon}_t)[lfp_t - (1-\rho)n_{t-1}] + \gamma^s [lfp_t - (1-\rho)n_{t-1}] = z_t n_t - [lfp_t - (1-\rho)n_{t-1}] H(\tilde{\varepsilon}_t), \tag{37}$$

and laws of motion for the employment stock

$$n_t = (1-\rho)n_{t-1} + [lfp_t - (1-\rho)n_{t-1}]\eta(\tilde{\varepsilon}_t). \tag{38}$$

The social planner takes into account the dependence of the hiring rate and the average operating cost of a newly-selected worker on the threshold $\tilde{\varepsilon}_t$, which is made explicit in the notation here. Recalling that $\eta(\tilde{\varepsilon}_t) \equiv \int_{-\infty}^{\tilde{\varepsilon}_t} f(\varepsilon) d\varepsilon$ and $H(\tilde{\varepsilon}_t) \equiv \int_{-\infty}^{\tilde{\varepsilon}_t} \varepsilon f(\varepsilon) d\varepsilon$, we have $\eta'(\tilde{\varepsilon}_t) = f(\tilde{\varepsilon}_t)$ and $H'(\tilde{\varepsilon}_t) = \tilde{\varepsilon}_t f(\tilde{\varepsilon}_t)$, by the Fundamental Theorem of Calculus.

Let $\beta^t \lambda_t$ be the Lagrange multiplier on the period-t goods resource constraint, and $\beta^t \mu_t$ be the Lagrange multiplier on the period-t law of motion for employment. The first-order conditions of the social planner problem with respect to c_t, lfp_t, n_t, and $\tilde{\varepsilon}_t$ are, respectively,

$$u'(c_t) - \lambda_t = 0, \tag{39}$$

$$-v'(lfp_t) - \lambda_t \left[\gamma^h \eta(\tilde{\varepsilon}_t) + \gamma^s + H(\tilde{\varepsilon}_t) \right] + \mu_t \eta(\tilde{\varepsilon}_t) = 0, \tag{40}$$

$$\lambda_t z_t - \mu_t + (1-\rho)\beta E_t \left\{ \mu_{t+1} \left[1 - \eta(\tilde{\varepsilon}_{t+1}) \right] + \lambda_{t+1} \left[\gamma^h \eta(\tilde{\varepsilon}_{t+1}) + \gamma^s + H(\tilde{\varepsilon}_{t+1}) \right] \right\} = 0, \tag{41}$$

and

$$-\lambda_t s_t \left[\gamma^h \eta'(\tilde{\varepsilon}_t) + H'(\tilde{\varepsilon}_t) \right] + \mu_t s_t \eta'(\tilde{\varepsilon}_t) = 0. \tag{42}$$

A.1 Static Efficiency (Participation)

Isolating the multiplier μ_t from (42),

$$\begin{aligned} \mu_t &= \frac{u'(c_t) \left[\gamma^h \eta'(\tilde{\varepsilon}_t) + H'(\tilde{\varepsilon}_t) \right]}{\eta'(\tilde{\varepsilon}_t)} \\ &= u'(c_t) \left[\gamma^h + \tilde{\varepsilon}_t \right], \end{aligned} \tag{43}$$

in which we have substituted (39). Substituting this expression for μ_t in (40) gives

$$\begin{aligned}\frac{v'(lfp_t)}{u'(c_t)} &= \tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) - \gamma^s \\ &= \int_{-\infty}^{\tilde{\varepsilon}_t} [\tilde{\varepsilon}_t - \varepsilon] f(\varepsilon)d\varepsilon - \gamma^s, \end{aligned} \tag{44}$$

in which the second line substitutes the definitions of $H(\tilde{\varepsilon}_t)$ and $\eta(\tilde{\varepsilon}_t)$. The term in square brackets in the integral is unambiguously positive. Expression (44) is the static efficiency condition that appears as condition (4) in the main text.

A.2 Intertemporal Efficiency (Hiring)

Next, substituting expression (43) for μ_t (and its time $t+1$ counterpart) in (41), we have

$$\begin{aligned}u'(c_t)\left[\gamma^h + \tilde{\varepsilon}_t\right] &= u'(c_t)z_t \\ &\quad +(1-\rho)\beta E_t\left\{u'(c_{t+1})\left[\gamma^h\eta(\tilde{\varepsilon}_{t+1}) + \gamma^s + H(\tilde{\varepsilon}_{t+1})\right]\right\} \\ &\quad +(1-\rho)\beta E_t\left\{u'(c_{t+1})\left[\gamma^h + \tilde{\varepsilon}_{t+1}\right][1-\eta(\tilde{\varepsilon}_{t+1})]\right\} \\ &= u'(c_t)z_t + (1-\rho)\beta E_t\left\{u'(c_{t+1})\left[\gamma^h\eta(\tilde{\varepsilon}_{t+1}) + \gamma^s + H(\tilde{\varepsilon}_{t+1}) + (\gamma^h + \tilde{\varepsilon}_{t+1})(1-\eta(\tilde{\varepsilon}_{t+1}))\right]\right\} \\ &= u'(c_t)z_t + (1-\rho)\beta E_t\left\{u'(c_{t+1})\left[H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) + \gamma^s + \gamma^h + \tilde{\varepsilon}_{t+1}\right]\right\}. \end{aligned} \tag{45}$$

Dividing by $u'(c_t)$,

$$\gamma^h = z_t - \tilde{\varepsilon}_t + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) + \gamma^s + \gamma^h + \tilde{\varepsilon}_{t+1}\right]\right\}, \tag{46}$$

which is the representation of efficiency along the intertemporal margin that appears as condition (5) in the main text. In the deterministic steady state, intertemporal efficiency is characterized by

$$\gamma^h + \tilde{\varepsilon} = \frac{z}{1-\beta(1-\rho)} + \frac{\beta(1-\rho)}{1-\beta(1-\rho)}\left(H(\tilde{\varepsilon}) - \tilde{\varepsilon}\eta(\tilde{\varepsilon})\right) + \frac{\beta(1-\rho)}{1-\beta(1-\rho)}\gamma^s. \tag{47}$$

A.2.1 Steady State Elasticity of Selection Threshold to Productivity

As per the partial equilibrium analysis in Section 7, suppose each unselected individual produces output $\upsilon > 0$. For this subsection only, suppose z_t is replaced in the derivations above by $z_t - \upsilon$, so that the efficient selection condition becomes

$$\gamma^h = z_t - \upsilon - \tilde{\varepsilon}_t + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) + \gamma^s + \gamma^h + \tilde{\varepsilon}_{t+1}\right]\right\}, \tag{48}$$

which in deterministic steady state is

$$\gamma^h + \tilde{\varepsilon} = \frac{z-\upsilon}{1-\beta(1-\rho)} + \frac{\beta(1-\rho)}{1-\beta(1-\rho)}\left(H(\tilde{\varepsilon}) - \tilde{\varepsilon}\eta(\tilde{\varepsilon})\right) + \frac{\beta(1-\rho)}{1-\beta(1-\rho)}\gamma^s. \tag{49}$$

Define

$$G(\tilde{\varepsilon}, z; \cdot) \equiv \gamma^h + \tilde{\varepsilon} - \frac{z-\upsilon}{1-\beta(1-\rho)} - \frac{\beta(1-\rho)}{1-\beta(1-\rho)}\left(H(\tilde{\varepsilon}) - \tilde{\varepsilon}\eta(\tilde{\varepsilon})\right) - \frac{\beta(1-\rho)}{1-\beta(1-\rho)}\gamma^s. \tag{50}$$

The elasticity of $\tilde{\varepsilon}$ with respect to z is $\frac{\partial \tilde{\varepsilon}}{\partial z}\frac{z}{\tilde{\varepsilon}}$, which by the implicit function theorem can be computed as $-\frac{G_z}{G_{\tilde{\varepsilon}}}\frac{z}{\tilde{\varepsilon}}$. We have $-G_z = \frac{1}{1-\beta(1-\rho)}$ and $G_{\tilde{\varepsilon}} = 1 - \frac{\beta(1-\rho)}{1-\beta(1-\rho)}\left[H'(\tilde{\varepsilon}) - \tilde{\varepsilon}\eta'(\tilde{\varepsilon}) - \eta(\tilde{\varepsilon})\right]$. The elasticity is thus

$$\begin{aligned}
\frac{\partial \ln \tilde{\varepsilon}}{\partial \ln z} &= -\frac{G_z}{G_{\tilde{\varepsilon}}}\frac{z}{\tilde{\varepsilon}} \\
&= \frac{z}{\tilde{\varepsilon}}\frac{1}{1-\beta(1-\rho)-\beta(1-\rho)\left[H'(\tilde{\varepsilon}) - \tilde{\varepsilon}\eta'(\tilde{\varepsilon}) - \eta(\tilde{\varepsilon})\right]} \\
&= \frac{z}{\tilde{\varepsilon}}\frac{1}{1-\beta(1-\rho)+\beta(1-\rho)\eta(\tilde{\varepsilon})} \\
&= \frac{z}{\tilde{\varepsilon}}\frac{1}{1-\beta(1-\rho)(1-\eta(\tilde{\varepsilon}))},
\end{aligned} \tag{51}$$

in which the third line follows from application of the Fundamental Theorem of Calculus, by which $\eta'(\tilde{\varepsilon}) = f(\tilde{\varepsilon})$ and $H'(\tilde{\varepsilon}) = \tilde{\varepsilon} f(\tilde{\varepsilon})$. This elasticity appears in the main text as condition (34).

The rest of the derivations proceed by again fixing $\upsilon = 0$.

A.3 MRS-MRT Representation of Efficiency

The efficiency conditions (44) and (46) can be described in terms of appropriately-defined concepts of marginal rates of substitution (MRS) and corresponding marginal rates of transformation (MRT). Defining MRS and MRT in a model-appropriate way allows us to describe efficiency in terms of the basic principle that efficient allocations are characterized by MRS = MRT conditions along all optimization margins.

Consider the static efficiency condition (44). The left-hand side is clearly the within-period MRS between consumption and participation in any period t. We claim that the right-hand side is the corresponding MRT between consumption and participation. Rather than take the efficiency condition (44) as prima facie evidence that the right-hand side *must be* the static MRT, however, this MRT can be derived from the primitives of the environment (i.e., independent of the context of any optimization).

First, though, define MRS and MRT relevant for intertemporal efficiency. To do so, first restrict attention to the non-stochastic case because it makes especially clear the separation of components of preferences from components of technology (due to endogenous covariance terms inherent in the $E_t(.)$ operator). The non-stochastic intertemporal efficiency condition can be expressed as

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{(1-\rho)\left(H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) + \gamma^h + \gamma^s + \tilde{\varepsilon}_{t+1}\right)}{\gamma^h + \tilde{\varepsilon}_t - z_t}. \tag{52}$$

The left-hand side of (52) is clearly the intertemporal MRS (hereafter abbreviated IMRS) between c_t and c_{t+1}. We claim that the right-hand side is the corresponding intertemporal MRT (hereafter abbreviated IMRT). Applying this definition to the fully stochastic condition (46), we can thus express intertemporal efficiency as

$$1 = E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[\frac{(1-\rho)\left(H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) + \gamma^h + \gamma^s + \tilde{\varepsilon}_{t+1} \right)}{\gamma^h + \tilde{\varepsilon}_t - z_t} \right] \right\} = E_t \left\{ \frac{IMRT_{c_t,c_{t+1}}}{IMRS_{c_t,c_{t+1}}} \right\}. \tag{53}$$

Rather than take the efficiency condition (52) as prima facie evidence that the right-hand side *must be* the IMRT, however, the IMRT can be derived from the primitives of the environment (i.e., independent of the context of any optimization), which is shown next.

A.4 Proof of Proposition 1: Transformation Frontier and Derivation of MRTs

Based only on the primitives of the environment — that is, independent of the context of any optimization — we now prove that the right-hand sides of (44) and (52) are, respectively, the model-appropriate concepts of the static MRT and deterministic IMRT. Doing so thus proves Proposition 1 in the main text. This requires defining the transformation frontier of the economy, a joint description of the goods resource constraint and the law of motion for employment.

In order to define the within-period MRT between c_t and lfp_t, the within-period transformation frontier needs to be viewed in the space (c_t, lfp_t). In principle, this requires eliminating the variable $\tilde{\varepsilon}_t$ between the period-t goods resource constraint (37) and the period-t law of motion (38) to express them as a single condition. However, this cannot be done explicitly. The within-period transformation is thus implicitly defined by the pair of functions

$$\Psi^{RC}(c_t, lfp_t, \tilde{\varepsilon}_t; .) \equiv z_t n_t - c_t - \left[\gamma^h \eta(\tilde{\varepsilon}_t) + \gamma^s + H(\tilde{\varepsilon}_t) \right] [lfp_t - (1-\rho)n_{t-1}] = 0, \tag{54}$$

which is condition (37), and

$$\Psi^{LOM}(lfp_t, \tilde{\varepsilon}_t; .) \equiv n_t - (1-\rho)n_{t-1} - [lfp_t - (1-\rho)n_{t-1}]\, \eta(\tilde{\varepsilon}_t) = 0, \tag{55}$$

which is condition (38). The within-period transformation frontier is implicitly defined by the pair of functions (54) and (55).

Computing the MRT between c_t and lfp_t requires total differentiation of (54) and (55), due to the fact that $\tilde{\varepsilon}_t$ is the variable that we would like to, but cannot, eliminate between the two expressions. Total differentiation gives:

$$MRT_{c_t, lfp_t} \quad = \quad -\frac{\Psi^{RC}_{lfp_t}}{\Psi^{RC}_{c_t}} + \frac{\Psi^{LOM}_{lfp_t}}{\Psi^{LOM}_{\tilde{\varepsilon}_t}} \frac{\Psi^{RC}_{\tilde{\varepsilon}_t}}{\Psi^{RC}_{c_t}}$$

$$
\begin{aligned}
&= -\frac{\Psi^{RC}_{lfp_t}}{\Psi^{RC}_{\tilde{\varepsilon}_t}}\frac{\Psi^{RC}_{\tilde{\varepsilon}_t}}{\Psi^{RC}_{c_t}} + \frac{\Psi^{LOM}_{lfp_t}}{\Psi^{LOM}_{\tilde{\varepsilon}_t}}\frac{\Psi^{RC}_{\tilde{\varepsilon}_t}}{\Psi^{RC}_{c_t}} \\
&= -\frac{\Psi^{RC}_{\tilde{\varepsilon}_t}}{\Psi^{RC}_{c_t}}\left[\frac{\Psi^{RC}_{lfp_t}}{\Psi^{RC}_{\tilde{\varepsilon}_t}} - \frac{\Psi^{LOM}_{lfp_t}}{\Psi^{LOM}_{\tilde{\varepsilon}_t}}\right]. \qquad (56)
\end{aligned}
$$

The first term on the right-hand side of the first line is, by the implicit function theorem, the slope $\frac{\partial c_t}{\partial lfp_t}$ embodied *directly* in the period-t goods resource constraint. The second term on the right-hand side of the first line is, by the implicit function theorem, the slope $\frac{\partial c_t}{\partial lfp_t}$ computed *through the marginal effect of a change in lfp_t on $\tilde{\varepsilon}_t$ embodied in the period-t law of motion* — hence the need for total differentiation.

Based on this computation and using the functions (54) and (55), the MRT is

$$
\begin{aligned}
MRT_{c_t,lfp_t} &= -\left[\frac{-\left(\gamma^h\eta'(\tilde{\varepsilon}_t)+H'(\tilde{\varepsilon}_t)\right)s_t}{-1}\right]\left[\frac{-\left(\gamma^h\eta(\tilde{\varepsilon}_t)+\gamma^s+H(\tilde{\varepsilon}_t)\right)}{-\left(\gamma^h\eta'(\tilde{\varepsilon}_t)+H'(\tilde{\varepsilon}_t)\right)s_t} - \frac{-\eta(\tilde{\varepsilon}_t)}{-\eta'(\tilde{\varepsilon}_t)s_t}\right] \\
&= -\left(\gamma^h\eta'(\tilde{\varepsilon}_t)+H'(\tilde{\varepsilon}_t)\right)\left[\frac{\gamma^h\eta(\tilde{\varepsilon}_t)+\gamma^s+H(\tilde{\varepsilon}_t)}{\gamma^h\eta'(\tilde{\varepsilon}_t)+H'(\tilde{\varepsilon}_t)} - \frac{\eta(\tilde{\varepsilon}_t)}{\eta'(\tilde{\varepsilon}_t)}\right] \\
&= -\left(\gamma^h\eta(\tilde{\varepsilon}_t)+\gamma^s+H(\tilde{\varepsilon}_t)\right) + \frac{\eta(\tilde{\varepsilon}_t)\left(\gamma^h\eta'(\tilde{\varepsilon}_t)+H'(\tilde{\varepsilon}_t)\right)}{\eta'(\tilde{\varepsilon}_t)} \\
&= -\left(\gamma^h\eta(\tilde{\varepsilon}_t)+\gamma^s+H(\tilde{\varepsilon}_t)\right) + \frac{\eta(\tilde{\varepsilon}_t)\left(\gamma^h f(\tilde{\varepsilon}_t)+\tilde{\varepsilon}_t f(\tilde{\varepsilon}_t)\right)}{f(\tilde{\varepsilon}_t)} \\
&= -\gamma^h\eta(\tilde{\varepsilon}_t)-\gamma^s-H(\tilde{\varepsilon}_t)+\gamma^h\eta(\tilde{\varepsilon}_t)+\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}) \\
&= \tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t)-H(\tilde{\varepsilon}_t)-\gamma^s \qquad (57) \\
&= \int_{-\infty}^{\tilde{\varepsilon}_t}\left[\tilde{\varepsilon}_t-\varepsilon\right]f(\varepsilon)d\varepsilon-\gamma^s, \qquad (58)
\end{aligned}
$$

which formalizes, independent of the solution to the social planning problem, the notion of the static MRT on the right-hand side of the efficiency condition (44).

Before computing the IMRT, note that the implicit function theorem allows us to also compute

$$
\begin{aligned}
\frac{\partial c_t}{\partial n_t} &= -\frac{\Psi^{RC}_{n_t}}{\Psi^{RC}_{c_t}} + \frac{\Psi^{RC}_{\tilde{\varepsilon}_t}}{\Psi^{RC}_{c_t}}\frac{\Psi^{LOM}_{n_t}}{\Psi^{LOM}_{\tilde{\varepsilon}_t}} \\
&= -\frac{z_t}{-1} + \frac{-s_t\left[\gamma^h\eta'(\tilde{\varepsilon}_t)+H'(\tilde{\varepsilon}_t)\right]}{-1}\frac{1}{-s_t\eta(\tilde{\varepsilon}_t)} \\
&= z_t - \frac{\gamma^h\eta'(\tilde{\varepsilon}_t)+H'(\tilde{\varepsilon}_t)}{\eta'(\tilde{\varepsilon}_t)} \\
&= z_t - \frac{\gamma^h f(\tilde{\varepsilon}_t)+\tilde{\varepsilon}_t f(\tilde{\varepsilon}_t)}{f(\tilde{\varepsilon}_t)} \\
&= z_t - \gamma^h - \tilde{\varepsilon}_t, \qquad (59)
\end{aligned}
$$

which measures the marginal effect on period-t consumption of a change in period-t employment. This effect has intertemporal consequences because n_t is the stock of employment entering period

$t+1$. In constructing this, note that the first term on the right-hand side of the first line is, by the implicit function theorem, the slope $\frac{\partial c_t}{\partial n_t}$ embodied *directly* in the period-t goods resource constraint; and the second term on the right-hand side of the first line is, by the implicit function theorem, the slope $\frac{\partial c_t}{\partial n_t}$ computed *through the marginal effect of a change in n_t on $\tilde{\varepsilon}_t$ embodied in the period-t law of motion* — hence, as above, the need for total differentiation. The second line computes the necessary partials of the functions (54) and (55), and the fourth line uses the Fundamental Theorem of Calculus to compute the derivatives of $\eta(\tilde{\varepsilon}_t)$ and $H(\tilde{\varepsilon}_t)$.

Next, define the period $t+1$ analogs of the functions (54) and (55):

$$G^{RC}(c_{t+1}, lfp_{t+1}, \tilde{\varepsilon}_{t+1}, c_t; .) \equiv z_{t+1}n_{t+1} - c_{t+1} - \left[\gamma^h \eta(\tilde{\varepsilon}_{t+1}) + \gamma^s + H(\tilde{\varepsilon}_{t+1})\right]\left[lfp_{t+1} - (1-\rho)n_t\right] = 0 \tag{60}$$

and

$$G^{LOM}(lfp_{t+1}, \tilde{\varepsilon}_{t+1}, c_t; .) \equiv n_{t+1} - (1-\rho)n_t - \left[lfp_{t+1} - (1-\rho)n_t\right]\eta(\tilde{\varepsilon}_{t+1}). \tag{61}$$

The functions $G^{RC}(.)$ and $G^{LOM}(.)$ clearly have the same form as (54) and (55), but, for the purpose of computing the IMRT, it is useful to view them as generalizations in that $G^{RC}(.)$ and $G^{LOM}(.)$ are viewed as functions of *both* period-t *and* period $t+1$ allocations. This generalization is emphasized by using the notation $G(.)$, rather than $\Psi(.)$, and by highlighting both c_{t+1} and c_t as arguments. The two-period (across period t and $t+1$) transformation frontier is implicitly defined by the pair of functions (60) and (61).

Computing the IMRT between c_t and c_{t+1} thus requires computing the total derivative

$$\begin{aligned}
\underbrace{\frac{\partial c_{t+1}}{\partial c_t}} + \underbrace{\frac{\partial c_{t+1}}{\partial c_t}} &= \frac{\partial c_{t+1}}{\partial n_t \frac{\partial c_t}{\partial n_t}} + \frac{\partial c_{t+1}}{\partial \tilde{\varepsilon}_{t+1}} \frac{\partial \tilde{\varepsilon}_{t+1}}{\partial n_t \frac{\partial c_t}{\partial n_t}} \\
&= \left\{ \frac{\partial c_{t+1}}{\partial n_t} + \frac{\partial c_{t+1}}{\partial \tilde{\varepsilon}_{t+1}} \frac{\partial \tilde{\varepsilon}_{t+1}}{\partial n_t} \right\} \frac{1}{\partial c_t / \partial n_t} \\
&= \left\{ -\frac{G^{RC}_{n_t}}{G^{RC}_{c_{t+1}}} + \frac{G^{RC}_{\tilde{\varepsilon}_{t+1}}}{G^{RC}_{c_{t+1}}} \frac{G^{LOM}_{n_t}}{G^{LOM}_{\tilde{\varepsilon}_{t+1}}} \right\} \frac{1}{\partial c_t / \partial n_t}.
\end{aligned}$$

The right-hand side of the first line highlights that the effect of c_t on c_{t+1} occurs through its effect on n_t (which is why we computed $\frac{\partial c_t}{\partial n_t}$), and the third line uses the implicit function theorem. Using the functions (60) and (61), the IMRT is

$$\begin{aligned}
&IMRT \\
&= -\left[-\frac{(1-\rho)\left(\gamma^h \eta(\tilde{\varepsilon}_{t+1}) + \gamma^s + H(\tilde{\varepsilon}_{t+1})\right)}{-1} + \frac{-s_{t+1}\left(\gamma^h \eta'(\tilde{\varepsilon}_{t+1}) + H'(\tilde{\varepsilon}_{t+1})\right)}{-1} \frac{-(1-\rho)(1-\eta(\tilde{\varepsilon}_{t+1}))}{-s_{t+1}\eta'(\tilde{\varepsilon}_{t+1})} \right] \frac{1}{\partial c_t / \partial n_t} \\
&= -(1-\rho)\left[\gamma^h \eta(\tilde{\varepsilon}_{t+1}) + \gamma^s + H(\tilde{\varepsilon}_{t+1}) + (1-\eta(\tilde{\varepsilon}_{t+1}))\left(\frac{\gamma^h \eta'(\tilde{\varepsilon}_{t+1}) + H'(\tilde{\varepsilon}_{t+1})}{\eta'(\tilde{\varepsilon}_{t+1})} \right) \right] \frac{1}{\partial c_t / \partial n_t}
\end{aligned}$$

$$
\begin{aligned}
&= -(1-\rho)\left[\gamma^h \eta(\tilde{\varepsilon}_{t+1}) + \gamma^s + H(\tilde{\varepsilon}_{t+1}) + (1-\eta(\tilde{\varepsilon}_{t+1}))\left(\frac{\gamma^h f(\tilde{\varepsilon}_{t+1}) + \tilde{\varepsilon}_{t+1} f(\tilde{\varepsilon}_{t+1})}{f(\tilde{\varepsilon}_{t+1})}\right)\right]\frac{1}{\partial c_t/\partial n_t} \\
&= -(1-\rho)\left[\gamma^h \eta(\tilde{\varepsilon}_{t+1}) + \gamma^s + H(\tilde{\varepsilon}_{t+1}) + (1-\eta(\tilde{\varepsilon}_{t+1}))\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\frac{1}{\partial c_t/\partial n_t} \\
&= -(1-\rho)\left[\gamma^h \eta(\tilde{\varepsilon}_{t+1}) + \gamma^s + H(\tilde{\varepsilon}_{t+1}) + \gamma^h + \tilde{\varepsilon}_{t+1} - \gamma^h \eta(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1})\right]\frac{1}{\partial c_t/\partial n_t} \\
&= -(1-\rho)\left[H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) + \gamma^h + \gamma^s + \tilde{\varepsilon}_{t+1}\right]\frac{1}{\partial c_t/\partial n_t} \\
&= \frac{(1-\rho)\left(H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) + \gamma^h + \gamma^s + \tilde{\varepsilon}_{t+1}\right)}{\gamma^h + \tilde{\varepsilon}_t - z_t}, \qquad (62)
\end{aligned}
$$

in which the third line makes use of the Fundamental Theorem of Calculus to compute the derivatives of $\eta(\tilde{\varepsilon}_t)$ and $H(\tilde{\varepsilon}_t)$, and the last line makes use of the slope (59). The sign convention is that the IMRT is the negative of the slope of the two-period transformation frontier. This derivation formalizes, independent of the social planning problem, the notion of the IMRT on the right-hand side of the (deterministic) efficiency condition (46).

B Firm Optimization

The representative firm chooses state-contingent processes $\{\tilde{\varepsilon}_t, n_t\}_{t=0}^{\infty}$ to maximize the present value of discounted profits

$$E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left[z_t n_t - \gamma^s s_t - \gamma^h \eta(\tilde{\varepsilon}_t) s_t - \frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \eta(\tilde{\varepsilon}_t) s_t - (1-\rho) n_{t-1} w_t^I - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \eta(\tilde{\varepsilon}_t) s_t \right] \tag{63}$$

subject to the sequence of perceived laws of motion for its employment stock

$$n_t = (1-\rho) n_{t-1} + s_t \eta(\tilde{\varepsilon}_t). \tag{64}$$

Letting $\beta^t \mu_{ft}$ denote the Lagrange multiplier on the period-t law of motion (64), the first-order conditions with respect to $\tilde{\varepsilon}_t$ and n_t are

$$\mu_{ft} \eta'(\tilde{\varepsilon}_t) s_t - \omega_e'(\tilde{\varepsilon}_t) s_t - \gamma^h \eta'(\tilde{\varepsilon}_t) s_t - H'(\tilde{\varepsilon}_t) s_t = 0 \tag{65}$$

and

$$z_t - \mu_{ft} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\mu_{ft+1} - w_{t+1}^I \right) \right\} = 0, \tag{66}$$

in which $\Xi_{t+1|t} \equiv \Xi_{t+1|0} / \Xi_{t|0}$ is the one-period stochastic discount factor. From (65), the value to the firm of an employee can be measured as

$$\begin{aligned} \mu_{ft} &= \frac{\omega_e'(\tilde{\varepsilon}_t) + \gamma^h \eta'(\tilde{\varepsilon}_t) + H'(\tilde{\varepsilon}_t)}{\eta'(\tilde{\varepsilon}_t)} \\ &= \frac{w(\tilde{\varepsilon}_t) f(\tilde{\varepsilon}_t) + \gamma^h f(\tilde{\varepsilon}_t) + \tilde{\varepsilon}_t f(\tilde{\varepsilon}_t)}{f(\tilde{\varepsilon}_t)} \\ &= w(\tilde{\varepsilon}_t) + \gamma^h + \tilde{\varepsilon}_t, \end{aligned} \tag{67}$$

where the second line follows from the Fundamental Theorem of Calculus.

Substituting (67) into (66),

$$\gamma^h + \tilde{\varepsilon}_t = z_t - w(\tilde{\varepsilon}_t) + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left[\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right] \right\}, \tag{68}$$

which is the firm's hiring (selection) condition that appears as expression (14) in the main text.

Define the value function associated with the firm problem as $\mathbf{F}(n_{t-1})$. The envelope condition is thus

$$\begin{aligned} \mathbf{F}'(n_{t-1}) &= (1-\rho) \left[\mu_{ft} - w_t^I \right] \\ &= (1-\rho) \left[\gamma^h + \tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t) - w_t^I \right], \end{aligned} \tag{69}$$

where the second line makes use of (67). For use in the analysis of the Nash bargaining problems in Appendix D, the period $t+1$ envelope condition can be expressed in discounted terms as

$$\begin{aligned} \frac{\beta u'(c_{t+1})}{u'(c_t)} \mathbf{F}'(n_t) &= (1-\rho) \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right] \\ &= (1-\rho) \Xi_{t+1|t} \left[\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right]. \end{aligned} \tag{70}$$

C Household Optimization

The representative household maximizes expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v\left((1-\eta_t)s_t + n_t\right)\right] \tag{71}$$

subject to the sequence of budget constraints

$$c_t + T_t = (1-\rho)n_{t-1}w_t^I + \eta_t \frac{\omega_{et}}{\eta_t} s_t + (1-\eta_t)s_t b + \Pi_t, \tag{72}$$

and perceived laws of motion for its employment level

$$n_t = (1-\rho)n_{t-1} + \eta_t s_t. \tag{73}$$

Let $\beta^t \phi_t$ denote the Lagrange multiplier on the period-t budget constraint, and $\beta^t \mu_{ht}$ denote the Lagrange multiplier on the household's period-t perceived law of motion. The first-order conditions with respect to c_t, s_t, and n_t are

$$u'(c_t) - \phi_t = 0, \tag{74}$$

$$-(1-\eta_t)v'((1-\eta_t)s_t + n_t) + \phi_t\left(\omega_{et} + (1-\eta_t)b\right) + \mu_{ht}\eta_t = 0, \tag{75}$$

and

$$-\mu_{ht} - v'((1-\eta_t)s_t + n_t) + (1-\rho)\beta E_t\left\{\phi_{t+1}w_{t+1}^I + \mu_{ht+1}\right\} = 0. \tag{76}$$

With first-order conditions now computed, switch to the notation $lfp_t = (1-\eta_t)s_t + n_t$, which follows from the accounting identities of the model.

From (75), we can isolate

$$\mu_{ht} = \frac{(1-\eta_t)v'(lfp_t) - u'(c_t)\left(\omega_{et} + (1-\eta_t)b\right)}{\eta_t}. \tag{77}$$

Substituting this into (76),

$$\begin{aligned} \frac{(1-\eta_t)v'(lfp_t) - u'(c_t)\left(\omega_{et} + (1-\eta_t)b\right)}{\eta_t} &= -v'(lfp_t) \\ + \quad (1-\rho)\beta E_t&\left\{u'(c_{t+1})w_{t+1}^I + \left(\frac{(1-\eta_{t+1})v'(lfp_{t+1}) - u'(c_{t+1})\left(\omega_{et+1} + (1-\eta_{t+1})b\right)}{\eta_{t+1}}\right)\right\}. \end{aligned} \tag{78}$$

Dividing by $u'(c_t)$ and using the notation $\Xi_{t+1|t} \equiv \beta u'(c_{t+1})/u'(c_t)$,

$$\begin{aligned} \frac{(1-\eta_t)v'(lfp_t) - u'(c_t)\left(\omega_{et} + (1-\eta_t)b\right)}{\eta_t u'(c_t)} &= -\frac{v'(lfp_t)}{u'(c_t)} \\ + \quad (1-\rho)E_t&\left\{\Xi_{t+1|t}\left[w_{t+1}^I + \left(\frac{(1-\eta_{t+1})v'(lfp_{t+1}) - u'(c_{t+1})\left(\omega_{et+1} + (1-\eta_{t+1})b\right)}{\eta_{t+1}u'(c_{t+1})}\right)\right]\right\}, \end{aligned} \tag{79}$$

which is a representation of the LFP condition that is useful for the Nash bargaining problem in Appendix D because it is recursive in the term $\frac{(1-\eta_t)v'(lfp_t) - u'(c_t)(\omega_{et} + (1-\eta_t)b)}{\eta_t u'(c_t)}$.

To obtain the representation that appears in the main text, first recognize that the second additive term under the expectation operator in the previous expression can be written compactly as $\frac{\mu_{ht+1}}{u'(c_{t+1})}$, so that

$$\frac{(1-\eta_t)v'(lfp_t) - u'(c_t)\left(\omega_{et} + (1-\eta_t)b\right)}{\eta_t u'(c_t)} = -\frac{v'(lfp_t)}{u'(c_t)} + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[w^I_{t+1} + \frac{\mu_{ht+1}}{u'(c_{t+1})}\right]\right\}. \tag{80}$$

Rearranging,

$$\frac{(1-\eta_t)v'(lfp_t)}{\eta_t u'(c_t)} = \frac{\omega_{et} + (1-\eta_t)b}{\eta_t} - \frac{v'(lfp_t)}{u'(c_t)} + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[w^I_{t+1} + \frac{\mu_{ht+1}}{u'(c_{t+1})}\right]\right\}. \tag{81}$$

Expanding the terms on the left-hand side,

$$\frac{v'(lfp_t)}{\eta_t u'(c_t)} - \frac{v'(lfp_t)}{u'(c_t)} = \frac{\omega_{et} + (1-\eta_t)b}{\eta_t} - \frac{v'(lfp_t)}{u'(c_t)} + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[w^I_{t+1} + \frac{\mu_{ht+1}}{u'(c_{t+1})}\right]\right\}, \tag{82}$$

which allows canceling a couple of terms, to give

$$\frac{v'(lfp_t)}{\eta_t u'(c_t)} = \frac{\omega_{et} + (1-\eta_t)b}{\eta_t} + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[w^I_{t+1} + \frac{\mu_{ht+1}}{u'(c_{t+1})}\right]\right\}. \tag{83}$$

Multiplying by η_t gives

$$\frac{v'(lfp_t)}{u'(c_t)} = \eta_t\left[\frac{\omega_{et}}{\eta_t} + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[w^I_{t+1} + \frac{\mu_{ht+1}}{u'(c_{t+1})}\right]\right\}\right] + (1-\eta_t)b, \tag{84}$$

which is the representation of the LFP condition that appears as condition (18) in the main text. It is also useful to express this as

$$\frac{v'(lfp_t)}{u'(c_t)} - b = \eta_t\left[\frac{\omega_{et}}{\eta_t} - b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[w^I_{t+1} + \frac{\mu_{ht+1}}{u'(c_{t+1})}\right]\right\}\right] \tag{85}$$

because the term on the left-hand side of this latter representation turns out to appear in the continuation values of all of the Nash wage equations derived in Appendix D. Furthermore, this form of the LFP condition also allows for expression in terms of the value equations derived in Appendix D, as shown below.

For the Nash bargaining problem in Appendix D, define the value function associated with the household problem as $\mathbf{V}(n_{t-1})$. The associated envelope condition is thus

$$\begin{aligned}\mathbf{V}'(n_{t-1}) &= (1-\rho)\left[\phi_t w^I_t + \mu_{ht}\right] \\ &= (1-\rho)\left[u'(c_t)w^I_t + \frac{(1-\eta_t)v'(lfp_t) - u'(c_t)\left(\omega_{et} + (1-\eta_t)b\right)}{\eta_t}\right],\end{aligned} \tag{86}$$

where the second line follows from (77). Finally, for use in Appendix D, the period $t+1$ envelope condition can be expressed in discounted terms as

$$\begin{aligned}\frac{\beta u'(c_{t+1})}{u'(c_t)}\frac{\mathbf{V}'(n_t)}{u'(c_{t+1})} &= (1-\rho)\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[w^I_{t+1} + \frac{\mu_{ht+1}}{u'(c_{t+1})}\right] \\ &= (1-\rho)\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[w^I_{t+1} + \frac{(1-\eta_{t+1})v'(lfp_{t+1}) - u'(c_{t+1})\left(\omega_{et+1} + (1-\eta_{t+1})b\right)}{\eta_{t+1}u'(c_{t+1})}\right]\end{aligned} \tag{87}$$

D Nash-Bargained Wages

This section presents the details of the derivation of the Nash wage equations given in Proposition 4. This requires first defining the values to both the household and the firm of a newly-hired worker with idiosyncratic characteristics ε_t^i and any given incumbent worker *at the time bargaining occurs*. Because of the timing of events in the model (see Figure 2), these values are properly defined in the "second subperiod" of period t, immediately after worker selection has taken place and thus each individual's measured labor market status for period t is known. In contrast, household-level decisions (in particular, the participation decision of how many individuals to send to look for jobs) occurs in the "first subperiod" of period t, *before* selection has taken place. The temporal separation of events in the model requires that we construct the bargaining-relevant value equations by simply accounting for the payoffs (viewed from the perspectives of the household and the firm).

A further observation is in order, one that applies to firms and households. Regardless of whether a given individual is a newly-hired or an incumbent worker in period t, he will be an incumbent worker in period $t+1$ if he remains employed. From the perspective of the household (or the firm), the continuation value of any worker is thus identical to that of any other worker (because it is only in the first period of employment that workers are heterogeneous) and is measured by the envelope condition of the household (or firm) problem. It is thus already apparent that the envelope conditions derived above measure the values of an incumbent worker, although this is verified below.

D.1 Value Equations for Household

A labor-market participant who either was not selected in period t or was selected (or was an incumbent) but fails to successfully complete wage negotiations is classified as "unemployed" and receives a transfer from the government, and thus has value (measured in goods) to the household

$$\mathbf{U}_t = b. \tag{88}$$

There is zero continuation payoff to the household of an unemployed individual because the household re-optimizes participation at the start of period $t+1$, and unemployment is *not* a state variable for the household at the start of period $t+1$. Note that, because the solution to the Nash bargaining problem will yield an interior solution, in equilibrium it is only individuals that were looking for work but were not selected that receive the unemployment transfer (which justifies including only unemployment transfers for this group of individuals in the household budget constraint (72)).

D.1.1 Incumbent Workers

An incumbent worker in period t has value (measured in period-t goods) to the household

$$\begin{aligned}\mathbf{W}_{It} &= w_t^I + E_t\left\{\Xi_{t+1|t}\frac{\mathbf{V}'(n_t)}{u'(c_{t+1})}\right\} \\ &= w_t^I + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[w_{t+1}^I + \frac{(1-\eta_{t+1})v'(lfp_{t+1}) - u'(c_{t+1})\left(\omega_{et+1} + (1-\eta_{t+1})b\right)}{\eta_{t+1}u'(c_{t+1})}\right]\right\}\end{aligned} \tag{89}$$

in which the first line follows from the discussion above, and the second line makes use of the expression for the household's envelope condition (87).

The surplus earned by the household from having an incumbent worker successfully complete wage negotiations is thus

$$\mathbf{W}_{It}-\mathbf{U}_t = w_t^I - b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[w_{t+1}^I + \frac{(1-\eta_{t+1})v'(lfp_{t+1}) - u'(c_{t+1})\left(\omega_{et+1} + (1-\eta_{t+1})b\right)}{\eta_{t+1}u'(c_{t+1})}\right]\right\}. \tag{90}$$

The goal of the next several steps is to rewrite this expression in a form convenient for the Nash bargaining problem in Appendix D.

Comparing expression (90) with the LFP condition (79) allows for expressing the surplus $\mathbf{W}_{It} - \mathbf{U}_t$ as

$$\mathbf{W}_{It} - \mathbf{U}_t = w_t^I - b + \frac{(1-\eta_t)v'(lfp_t) - u'(c_t)\left(\omega_{et} + (1-\eta_t)b\right)}{\eta_t u'(c_t)} + \frac{v'(lfp_t)}{u'(c_t)}. \tag{91}$$

Rearranging,

$$w_t^I + \frac{(1-\eta_t)v'(lfp_t) - u'(c_t)\left(\omega_{et} + (1-\eta_t)b\right)}{\eta_t u'(c_t)} = \mathbf{W}_{It} - \mathbf{U}_t + b - \frac{v'(lfp_t)}{u'(c_t)}, \tag{92}$$

which is the period-t counterpart of the term inside expectations in expression (90). Making this substitution,

$$\begin{aligned}\mathbf{W}_{It} - \mathbf{U}_t &= w_t^I - b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1} + b - \frac{v'(lfp_{t+1})}{u'(c_{t+1})}\right]\right\} \\ &= w_t^I - b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1}\right]\right\} + (1-\rho)bE_t\Xi_{t+1|t} \\ &\quad -(1-\rho)E_t\left\{\Xi_{t+1|t}\frac{v'(lfp_{t+1})}{u'(c_{t+1})}\right\},\end{aligned} \tag{93}$$

D.1.2 Newly-Hired Workers

A newly-hired worker with idiosyncratic characteristics ε_t^i in period t has value (measured in period-t goods) to the household

$$\mathbf{W}_E(\varepsilon_t^i) = w(\varepsilon_t^i) + E_t\left\{\Xi_{t+1|t}\frac{\mathbf{V}'(n_t)}{u'(c_{t+1})}\right\}, \tag{94}$$

in which the first line again follows from the discussion above, and the second line makes use of the expression for the household's envelope condition (87). Note that the wage payment to a newly-hired worker $w(\varepsilon_t^i)$ can be conditioned on his idiosyncratic characteristics.

The surplus earned by the household from having a newly-selected individual successfully complete wage negotiations is thus

$$\begin{aligned}
\mathbf{W}_E(\varepsilon_t^i)-\mathbf{U}_t &= w(\varepsilon_t^i)-b+E_t\left\{\Xi_{t+1|t}\frac{\mathbf{V}'(n_t)}{u'(c_{t+1})}\right\}\\
&= w(\varepsilon_t^i)-b+(1-\rho)E_t\left\{\Xi_{t+1|t}\left[w_{t+1}^I+\frac{(1-\eta_{t+1})v'(lfp_{t+1})-u'(c_{t+1})\left(\omega_{et+1}+(1-\eta_{t+1})b\right)}{\eta_{t+1}u'(c_{t+1})}\right]\right\},\\
&= w(\varepsilon_t^i)-b+(1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1}-\mathbf{U}_{t+1}+b-\frac{v'(lfp_{t+1})}{u'(c_{t+1})}\right]\right\}, \qquad (95)
\end{aligned}$$

in which the second line makes use of (87), and the third line uses expression (92) from the derivation of the surplus expression $\mathbf{W}_{It}-\mathbf{U}_t$ above. Breaking apart the terms inside the expectation, we have

$$\mathbf{W}_E(\varepsilon_t^i)-\mathbf{U}_t = w(\varepsilon_t^i)-b+(1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1}-\mathbf{U}_{t+1}\right]\right\}+(1-\rho)bE_t\Xi_{t+1|t}-(1-\rho)E_t\left\{\Xi_{t+1|t}\frac{v'(lfp_{t+1})}{u'(c_{t+1})}\right\}, \qquad (96)$$

the form of the household's surplus from a new employment relationship used in the derivation of the Nash wage function below. Before proceeding, however, we note that integrating the surplus $\mathbf{W}_E(\varepsilon_t^i)-\mathbf{U}_t$ expressed as in the first line above gives

$$\begin{aligned}
\int_{-\infty}^{\tilde{\varepsilon}_t}\mathbf{W}_E(\varepsilon_t^i)f(\varepsilon_t^i)d\varepsilon_t^i-\mathbf{U}_t &= \int_{-\infty}^{\tilde{\varepsilon}_t}w(\varepsilon_t^i)f(\varepsilon_t^i)d\varepsilon_t^i-b\int_{-\infty}^{\tilde{\varepsilon}_t}f(\varepsilon_t^i)d\varepsilon_t^i+E_t\left\{\Xi_{t+1|t}\frac{\mathbf{V}'(n_t)}{u'(c_{t+1})}\right\}\int_{-\infty}^{\tilde{\varepsilon}_t}f(\varepsilon_t^i)d\varepsilon_t^i\\
&= \omega_e(\tilde{\varepsilon}_t)-b\eta(\tilde{\varepsilon}_t)+\eta(\tilde{\varepsilon}_t)E_t\left\{\Xi_{t+1|t}\frac{\mathbf{V}'(n_t)}{u'(c_{t+1})}\right\}\\
&= \eta(\tilde{\varepsilon}_t)\left[\frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}+E_t\left\{\Xi_{t+1|t}\frac{\mathbf{V}'(n_t)}{u'(c_{t+1})}\right\}\right]-\eta(\tilde{\varepsilon}_t)b. \qquad (97)
\end{aligned}$$

With this expression for the *expected* surplus to the household of having one of its unemployed members selected for work, the LFP condition (85) derived in Appendix C can be expressed as

$$\begin{aligned}
\frac{v'(lfp_t)}{u'(c_t)}-b &= \eta(\tilde{\varepsilon}_t)\left[\frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}-b+(1-\rho)E_t\left\{\Xi_{t+1|t}\left[w_{t+1}^I+\frac{\mu_{ht+1}}{u'(c_{t+1})}\right]\right\}\right]\\
&= \eta(\tilde{\varepsilon}_t)\left[\frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}+(1-\rho)E_t\left\{\Xi_{t+1|t}\left[w_{t+1}^I+\frac{\mu_{ht+1}}{u'(c_{t+1})}\right]\right\}\right]-\eta(\tilde{\varepsilon}_t)b\\
&= \eta(\tilde{\varepsilon}_t)\left[\frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}+E_t\left\{\Xi_{t+1|t}\frac{\mathbf{V}'(n_t)}{u'(c_{t+1})}\right\}\right]-\eta(\tilde{\varepsilon}_t)b\\
&= \int_{-\infty}^{\tilde{\varepsilon}_t}\mathbf{W}_E(\varepsilon_t^i)f(\varepsilon_t^i)d\varepsilon_t^i-\eta(\tilde{\varepsilon}_t)\mathbf{U}_t, \qquad (98)
\end{aligned}$$

in which the third line uses the household-level envelope condition derived above and the fourth line uses the definition of $\mathbf{U}_t$. This expression states that optimal participation equates the (net) marginal utility cost (denominated in goods) to the household of participation to the expected surplus from having an unemployed individual selected for work. The expectation is taken over both the probability of being selected as well as an individual's idiosyncratic characteristics, which are unknown at the time participation decisions are made.

D.2 Value Equations for Firm

D.2.1 Incumbent Workers

An incumbent worker in period t has value (measured in period-t goods) to the firm

$$\begin{aligned}\mathbf{J}_{It} &= z_t - w_t^I + E_t\left\{\Xi_{t+1|t}\mathbf{F}'(n_t)\right\}\\ &= z_t - w_t^I + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\},\end{aligned} \tag{99}$$

in which the first line follows from the discussion above, and the second line makes use of the expression for the firm's envelope condition (70).

Next, note from the hiring condition (68) that the last term on the right-hand side of (99) is $\gamma^h + \tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t) - z_t$; substituting this in (99) gives

$$\mathbf{J}_{It} = \gamma^h + \tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t) - w_t^I. \tag{100}$$

Comparing this expression with (99), we see that the value of an incumbent worker to the firm can be expressed recursively,

$$\mathbf{J}_{It} = z_t - w_t^I + (1-\rho)E_t\left\{\Xi_{t+1|t}\mathbf{J}_{It+1}\right\}, \tag{101}$$

and, furthermore, the relationship between the value to the firm of an incumbent worker and the firm's envelope condition is $\mathbf{J}_{It} = (1-\rho)\mathbf{F}'(n_{t-1})$.

D.2.2 Newly-Hired Workers

Similarly, a newly-hired individual with idiosyncratic characteristics ε_t^i in period t has value (measured in period-t goods) to the firm

$$\begin{aligned}\mathbf{J}_E(\varepsilon_t^i) &= z_t - \varepsilon_t^i - w(\varepsilon_t^i) - \gamma^h + E_t\left\{\Xi_{t+1|t}\mathbf{F}'(n_t)\right\}\\ &= z_t - \varepsilon_t^i - w(\varepsilon_t^i) - \gamma^h + (1-\rho)E_t\left\{\Xi_{t+1|t}\mathbf{J}_{It+1}\right\}\\ &= z_t - \varepsilon_t^i - w(\varepsilon_t^i) - \gamma^h + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[w(\tilde{\varepsilon}_{t+1}) + \gamma^h + \tilde{\varepsilon}_{t+1} - w_{t+1}^I\right]\right\}.\end{aligned} \tag{102}$$

The first line again follows from the discussion above, the second line uses the relationship proven above between the envelope condition and the value to the firm of an incumbent worker, and the third line substitutes the expression for the firm's envelope condition (70). Once again note that the wage payment $w(\varepsilon_t^i)$ made to a newly-hired worker can be conditioned on his idiosyncratic characteristics.

Again noting from the hiring condition (68) that the last term on the right-hand side of (102) is $\gamma^h + \tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t) - z_t$, the value of a newly-hired worker with idiosyncratic characteristics ε_t^i can be expressed as

$$\mathbf{J}_E(\varepsilon_t^i) = \tilde{\varepsilon}_t - \varepsilon_t^i + w(\tilde{\varepsilon}_t) - w(\varepsilon_t^i). \tag{103}$$

Clearly, the value of a new hire with the threshold idiosyncratic characteristics $\tilde{\varepsilon}_t$ has value

$$\mathbf{J}_E(\tilde{\varepsilon}_t) = 0, \tag{104}$$

that is, and as is intuitive, the firm earns zero value from a new worker who was at exactly the selection threshold.

D.3 Nash Bargaining

The firm bargains individually with each of its workers, whether an incumbent or a new hire with idiosyncratic characteristics $\varepsilon_t^i < \tilde{\varepsilon}_t$, in every period. For every worker, the firm and the worker choose the real wage that maximizes the generalized Nash product

$$(\mathbf{W}_t - \mathbf{U}_t)^{\alpha^K} \mathbf{J}_t^{1-\alpha^K}, \tag{105}$$

in which $\alpha^K \in [0,1]$, $K \in \{E, I\}$, measures the bargaining power of the worker (α^I is the bargaining power of an incumbent worker, α^E is the bargaining power of a newly-hired worker). For the different types of workers, $\mathbf{W}_t$ is replaced by either $\mathbf{W}_{It}$ or $\mathbf{W}_E(\varepsilon_t^i)$, $\mathbf{J}_t$ is replaced by either $\mathbf{J}_{It}$ or $\mathbf{J}_E(\varepsilon_t^i)$, and α^K is replaced by either α^I or α^E.

Using the generic notation $\mathbf{W}_t$, $\mathbf{J}_t$, and α^K, the first-order condition of (105) with respect to the period-t real wage (which in the various cases below is either w_t^I or $w(\varepsilon_t^i)$ — here, denote it simply w_t) is

$$\alpha^K (\mathbf{W}_t - \mathbf{U}_t)^{\alpha^K - 1} \mathbf{J}_t^{1-\alpha^K} \left(\frac{\partial \mathbf{W}_t}{\partial w_t} - \frac{\partial \mathbf{U}_t}{\partial w_t} \right) + (1-\alpha^K)(\mathbf{W}_t - \mathbf{U}_t)^{\alpha^K} \mathbf{J}_t^{-\alpha^K} \frac{\partial \mathbf{J}_t}{\partial w_t} = 0. \tag{106}$$

To simplify, multiply by $\mathbf{J}_t^{\alpha^K}$, and also multiply by $(\mathbf{W}_t - \mathbf{U}_t)^{1-\alpha^K}$, which gives

$$\alpha^K \mathbf{J}_t \left(\frac{\partial \mathbf{W}_t}{\partial w_t} - \frac{\partial \mathbf{U}_t}{\partial w_t} \right) + (1-\alpha^K)(\mathbf{W}_t - \mathbf{U}_t) \frac{\partial \mathbf{J}_t}{\partial w_t} = 0. \tag{107}$$

It is clear from the value equations above that, no matter the type of worker, the marginals are $\frac{\partial \mathbf{J}_t}{\partial w_t} = -1$, $\frac{\partial \mathbf{U}_t}{\partial w_t} = 0$, and $\frac{\partial \mathbf{W}_t}{\partial w_t} = 1$. Substituting these, the first-order condition simplifies to

$$\mathbf{W}_t - \mathbf{U}_t = \frac{\alpha^K}{1-\alpha^K} \mathbf{J}_t, \tag{108}$$

which is the usual Nash sharing rule, independent of worker type.

D.3.1 Incumbent Workers

To obtain an expression for the period-t bargained wage of an incumbent, begin with the sharing rule

$$\mathbf{W}_{It} - \mathbf{U}_t = \frac{\alpha^I}{1-\alpha^I} \mathbf{J}_{It}, \tag{109}$$

and substitute (93). This gives

$$w_t^I - b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1}\right]\right\} + (1-\rho)bE_t\Xi_{t+1|t} - (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{v'(lfp_{t+1})}{u'(c_{t+1})}\right\} = \frac{\alpha^I}{1-\alpha^I}\mathbf{J}_{It}. \tag{110}$$

Then, substitute the time-$(t+1)$ sharing rule (109) in the third term on the left-hand side,

$$w_t^I - b + (1-\rho)\left(\frac{\alpha^I}{1-\alpha^I}\right)E_t\left\{\Xi_{t+1|t}\mathbf{J}_{It+1}\right\} + (1-\rho)bE_t\Xi_{t+1|t} - (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{v'(lfp_{t+1})}{u'(c_{t+1})}\right\} = \frac{\alpha^I}{1-\alpha^I}\mathbf{J}_{It}. \tag{111}$$

Next, substitute using (100) for both $\mathbf{J}_{It}$ and $\mathbf{J}_{It+1}$, which gives

$$\begin{aligned} w_t^I - b + (1-\rho)\left(\frac{\alpha^I}{1-\alpha^I}\right)E_t\left\{\Xi_{t+1|t}\left[\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\} \\ + (1-\rho)bE_t\Xi_{t+1|t} - (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{v'(lfp_{t+1})}{u'(c_{t+1})}\right\} = \frac{\alpha^I}{1-\alpha^I}\left[\gamma^h + \tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t) - w_t^I\right]. \end{aligned} \tag{112}$$

Rearranging,

$$\begin{aligned} w_t^I\left[1 + \frac{\alpha^I}{1-\alpha^I}\right] &= b + \frac{\alpha^I}{1-\alpha^I}\left[\gamma^h + \tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t)\right] \\ &+ (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \left(\frac{\alpha^I}{1-\alpha^I}\right)\left(\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right]\right\}. \end{aligned} \tag{113}$$

Multiplying by $(1-\alpha^I)$,

$$\begin{aligned} w_t^I &= (1-\alpha^I)b + \alpha^I\left[\gamma^h + \tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t)\right] \\ &+ (1-\rho)E_t\left\{\Xi_{t+1|t}\left[(1-\alpha^I)\left(\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b\right) - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right]\right\} \end{aligned} \tag{114}$$

Finally, because it will be useful in further manipulations below, multiply and divide the last term on the right-hand side by $(1-\alpha^I)$, which gives

$$\begin{aligned} w_t^I &= (1-\alpha^I)b + \alpha^I\left[\gamma^h + \tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t)\right] \\ &+ (1-\rho)(1-\alpha^I)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \left(\frac{\alpha^I}{1-\alpha^I}\right)\left(\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right]\right\}. \end{aligned} \tag{115}$$

Note that this expression is not a closed-form expression for w_t^I (even taking the continuation value as given) because the endogenous wage $w(\tilde{\varepsilon}_t)$ also appears. A closed-form expression requires also solving for a new hire's wage, which is done next.

D.3.2 Newly-Hired Workers

To obtain an expression for the period-t bargained wage of a new hire, begin with the sharing rule

$$\mathbf{W}_E(\varepsilon_t^i) - \mathbf{U}_t = \frac{\alpha^E}{1-\alpha^E}\mathbf{J}_E(\varepsilon_t^i), \tag{116}$$

and substitute (96). This gives

$$w(\varepsilon_t^i) - b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1}\right]\right\} + (1-\rho)bE_t\Xi_{t+1|t} - (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{v'(lfp_{t+1})}{u'(c_{t+1})}\right\} = \frac{\alpha^E}{1-\alpha^E}\mathbf{J}_E(\varepsilon_t^i). \tag{117}$$

Then, substitute the time-$(t+1)$ sharing rule (109) *for incumbents* in the third term on the left-hand side,

$$w(\varepsilon_t^i) - b + (1-\rho)\left(\frac{\alpha^I}{1-\alpha^I}\right)E_t\left\{\Xi_{t+1|t}\mathbf{J}_{It+1}\right\} + (1-\rho)bE_t\Xi_{t+1|t} - (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{v'(lfp_{t+1})}{u'(c_{t+1})}\right\} = \frac{\alpha^E}{1-\alpha^E}\mathbf{J}_E(\varepsilon_t^i). \tag{118}$$

Next, substitute using (100) for $\mathbf{J}_{It+1}$ and using (103) for $\mathbf{J}_E(\varepsilon_t^i)$, which gives

$$\begin{aligned} &w(\varepsilon_t^i) - b + (1-\rho)\left(\frac{\alpha^I}{1-\alpha^I}\right)E_t\left\{\Xi_{t+1|t}\left[\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\} \\ &\quad + (1-\rho)bE_t\Xi_{t+1|t} - (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{v'(lfp_{t+1})}{u'(c_{t+1})}\right\} = \frac{\alpha^E}{1-\alpha^E}\left[\tilde{\varepsilon}_t - \varepsilon_t^i + w(\tilde{\varepsilon}_t) - w(\varepsilon_t^i)\right] \end{aligned} \tag{119}$$

Rearranging

$$\begin{aligned} w(\varepsilon_t^i)\left[1 + \frac{\alpha^E}{1-\alpha^E}\right] &= b + \frac{\alpha^E}{1-\alpha^E}\left[\tilde{\varepsilon}_t - \varepsilon_t^i + w(\tilde{\varepsilon}_t)\right] \\ &\quad + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \left(\frac{\alpha^I}{1-\alpha^I}\right)\left(\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right]\right\}. \end{aligned} \tag{120}$$

Multiplying by $(1-\alpha^E)$,

$$\begin{aligned} w(\varepsilon_t^i) &= (1-\alpha^E)b + \alpha^E\left[\tilde{\varepsilon}_t - \varepsilon_t^i + w(\tilde{\varepsilon}_t)\right] \\ &\quad + (1-\rho)(1-\alpha^E)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \left(\frac{\alpha^I}{1-\alpha^I}\right)\left(\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right]\right\}, \end{aligned} \tag{121}$$

which is the bargained wage for a newly-hired worker with idiosyncratic characteristics ε_t^i. This expression is not a closed-form solution for $w(\varepsilon_t^i)$ because the endogenous wage $w(\tilde{\varepsilon}_t)$ also appears. However, at this stage of the analysis, it is not difficult to obtain closed-form solutions.

D.3.3 Closed-Form Solutions for Bargained Wages

There are three more steps required to obtain closed-form expressions for bargained wages, which completes the proof of Proposition 4. First, construct expressions for each of the three period-t wages in which no other *contemporaneous* wage appears. Second, compute wage differentials. Third, substitute wage differentials into the *continuation value* components of each wage expression.

Begin by evaluating (121) at $\varepsilon_t^i = \tilde{\varepsilon}_t$, which gives the bargained wage of the threshold new hire,

$$w(\tilde{\varepsilon}_t) = b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \left(\frac{\alpha^I}{1-\alpha^I}\right)\left(\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right]\right\}. \tag{122}$$

Then, substitute (122) in (121), which gives the bargained wage for a new hire with idiosyncratic characteristics ε_t^i,

$$w(\varepsilon_t^i) = b+\alpha^E\left(\tilde{\varepsilon}_t - \varepsilon_t^i\right)+(1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \left(\frac{\alpha^I}{1-\alpha^I}\right)\left(\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right]\right\}. \tag{123}$$

Next, substitute (122) in (115), which gives the bargained wage for an incumbent worker,

$$w_t^I = b+\alpha^I\left(\gamma^h + \tilde{\varepsilon}_t\right)+(1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \left(\frac{\alpha^I}{1-\alpha^I}\right)\left(\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right]\right\}. \tag{124}$$

Note that in all three wages (122), (123), and (124), the continuation value is identical because no matter what type a worker is in period t, he will be a (homogenous) incumbent worker in period $t+1$ if he remains employed. Moreover, the period-$(t+1)$ wage differential $w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I$ appears in all three.

These expressions allow us to explicitly compute wage differentials between the different types of workers, which are intuitive to understand. A new hire with $\varepsilon_t^i < \tilde{\varepsilon}_t$ earns a premium over the threshold new hire

$$w(\varepsilon_t^i) - w(\tilde{\varepsilon}_t) = \alpha^E\left(\varepsilon_t^i - \tilde{\varepsilon}_t\right), \tag{125}$$

which is the share of the operating cost *savings* he provides the firm that he is able to extract through his bargaining power α^E. An incumbent worker earns a premium over the threshold new hire

$$w_t^I - w(\tilde{\varepsilon}_t) = \alpha^I\left(\gamma^h + \tilde{\varepsilon}_t\right), \tag{126}$$

which, similarly, is the share of the replacement cost *savings* (relative to a marginal new hire) he provides the firm that he is able to extract through his bargaining power α^I.

Substitute $w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I = -\alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)$ into each of (122), (123), and (124) to obtain

$$w(\tilde{\varepsilon}_t) = b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\right\}, \tag{127}$$

$$w(\varepsilon_t^i) = b + \alpha^E\left(\tilde{\varepsilon}_t - \varepsilon_t^i\right) + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\right\}, \tag{128}$$

and

$$w_t^I = b + \alpha^I\left(\gamma^h + \tilde{\varepsilon}_t\right) + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\right\}, \tag{129}$$

which are the forms of the wage solutions that appear in Proposition 4. Integrating (128) gives the average wage paid to a new hire

$$\omega_e(\tilde{\varepsilon}_t) \quad = \quad \int_{-\infty}^{\tilde{\varepsilon}_t} w(\varepsilon_t^i) f(\varepsilon_t^i) d\varepsilon_t^i$$

$$
\begin{aligned}
&= b\int_{-\infty}^{\tilde{\varepsilon}_t} f(\varepsilon_t^i)d\varepsilon_t^i + \alpha^E \int_{-\infty}^{\tilde{\varepsilon}_t}\left(\tilde{\varepsilon}_t - \varepsilon_t^i\right) f(\varepsilon_t^i)d\varepsilon_t^i \\
&\quad +(1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\right\}\int_{-\infty}^{\tilde{\varepsilon}_t} f(\varepsilon_t^i)d\varepsilon_t^i \\
&= b\eta(\tilde{\varepsilon}_t) + \alpha^E\left[\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)\right] \\
&\quad +\eta(\tilde{\varepsilon}_t)(1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\right\},
\end{aligned} \tag{130}
$$

which appears as expression (30) in the main text.

Finally, from the definitions of the value equations $\mathbf{W}_E(\varepsilon_t^i)$ and $\mathbf{U}_t$ above, the continuation value that appears on the right-hand side of all three wage functions (127), (128), and (129) can be expressed as

$$
\begin{aligned}
&(1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\right\} \\
&= (1-\rho)E_t\left\{\int_{-\infty}^{\tilde{\varepsilon}_{t+1}} \mathbf{W}_E(\varepsilon_{t+1}^i)f(\varepsilon_{t+1}^i)d\varepsilon_{t+1}^i - \eta\left(\tilde{\varepsilon}_{t+1}\right)\mathbf{U}_{t+1} - \left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right\}.
\end{aligned} \tag{131}
$$

E Impossibility of Nash Wages Guaranteeing Efficiency: Proof of Proposition 5

Here we prove Proposition 5. To determine parameter restrictions under which the Nash wages described in Proposition 4 can coincide with the sufficient conditions on wages presented in Propositions 2 and 3, compare the Nash-bargained wages

$$w(\tilde{\varepsilon}_t) = b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\right\}, \tag{132}$$

$$w_t^I = b + \alpha^I\left(\gamma^h + \tilde{\varepsilon}_t\right) + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\right\}, \tag{133}$$

and

$$\omega_e(\tilde{\varepsilon}_t) = b\eta(\tilde{\varepsilon}_t) + \alpha^E\left(\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)\right) + \eta(\tilde{\varepsilon}_t)(1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\right\} \tag{134}$$

(which are expressions (25), (26), and (27) in the main text) with the sufficient conditions on wages

$$w(\tilde{\varepsilon}_t) = 0, \tag{135}$$

$$w_t^I = \tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) - \gamma^s, \tag{136}$$

and

$$\omega_e(\tilde{\varepsilon}_t) = \tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) - \gamma^s - \left(1 - \eta(\tilde{\varepsilon}_t)\right)b \tag{137}$$

(which are expressions (21), (22), and (23) in the main text).

Setting these respectively equal to each other,

$$0 = b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\right\}, \tag{138}$$

$$\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) - \gamma^s = b + \alpha^I\left(\gamma^h + \tilde{\varepsilon}_t\right) + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\right\}, \tag{139}$$

and

$$\begin{aligned}&\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) - \gamma^s - \left(1 - \eta(\tilde{\varepsilon}_t)\right)b \\ = \;& \eta(\tilde{\varepsilon}_t)b + \alpha^E\left(\tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)\right) + \eta(\tilde{\varepsilon}_t)(1-\rho)E_t\left\{\Xi_{t+1|t}\left[\frac{v'(lfp_{t+1})}{u'(c_{t+1})} - b - \alpha^I\left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)\right]\right\}\end{aligned} \tag{140}$$

To conserve on notation, denote the continuation value in each of the previous three expressions (the last term on the right-hand side of each) as $(1-\rho)E_t\left\{\Xi_{t+1|t}\Omega_{t+1}\right\}$. To prove the impossibility of Nash-determined wages satisfying the sufficient conditions on wages, we proceed constructively in three steps.

Step 1. Given that $\Omega_{t+1} > 0$ in any interesting equilibrium and given the natural parameter restrictions $b \geq 0$ and $\rho \leq 1$, the only way that (138) can be satisfied is if $b = 0$ and $\rho = 1$. Thus, suppose hereafter that $b = 0$ and $\rho = 1$, in which case the Nash outcome (132) is $w(\tilde{\varepsilon}_t) = 0$, as required by (135). Nash wages thus satisfy one of the three sufficient conditions on wages.

Step 2. Conditional on $b = 0$ and $\rho = 1$, note that (136) and (137) require the wage differential $\omega_e(\tilde{\varepsilon}_t) - w_t^I = 0$. The Nash-bargained wages (133) and (134) imply the differential

$$\omega_e(\tilde{\varepsilon}_t) - w_t^I = \alpha^E \left(\tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t)\right) - \alpha^I \left(\gamma^h + \tilde{\varepsilon}_t\right). \tag{141}$$

If $\alpha^E = 0$ and $\alpha^I > 0$, then $\gamma^h + \tilde{\varepsilon}_t = 0$ is required for the differential to equal zero. However, while $\tilde{\varepsilon}_t < 0$ can occur in equilibrium, there is nothing that guarantees $\tilde{\varepsilon}_t = -\gamma^h$ in equilibrium. If instead $\alpha^I = 0$ and $\alpha^E > 0$, then $\tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) = 0$ is required for the differential to equal zero. By the definitions of $\eta(\tilde{\varepsilon}_t)$ and $H(\tilde{\varepsilon}_t)$, this would require that

$$\int_{-\infty}^{\tilde{\varepsilon}_t} \left[\tilde{\varepsilon}_t - \varepsilon\right] f(\varepsilon) d\varepsilon = 0, \tag{142}$$

which is impossible for a nondegenerate distribution of ε. Thus, the only way the Nash differential (141) can equal zero is if $\alpha^E = \alpha^I = 0$; suppose this condition holds hereafter.

Step 3. Conditional on $b = 0$, $\rho = 1$, and $\alpha^E = \alpha^I = 0$, the Nash wage for incumbents (133) is $w_t^I = 0$. For this to coincide with the sufficient condition on incumbents' wages (136), the condition $\tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t) = \gamma^s$ must hold in equilibrium; however, nothing guarantees this outcome.

Thus, even though parameter restrictions can be obtained that ensure Nash wages satisfy two of the three conditions on wages sufficient for implement efficiency, all three sufficient conditions cannot simultaneously be met. Hence, there are no restrictions on parameters that guarantees that Nash-bargained wages can support the efficient allocation.

F Nash-Bargained Wages with Fixed Participation

Nash-bargained wages for the case of fixed participation are derived analogously to those in Appendix D for the baseline model. The main difference is in the setup of the bargaining-relevant household value equations. Unaffected by the endogeneity or exogeneity of participation are the Nash sharing rules and the equilibrium expressions for the firm's value equations derived in Appendix D. These are repeated here for convenience:

$$\mathbf{W}_{It} - \mathbf{U}_t = \left(\frac{\alpha^I}{1-\alpha^I}\right)\mathbf{J}_{It} \tag{143}$$

is the Nash sharing rule for incumbent workers;

$$\mathbf{W}_E(\varepsilon_t^i) - \mathbf{U}_t = \left(\frac{\alpha^E}{1-\alpha^E}\right)\mathbf{J}_E(\varepsilon_t^i) \tag{144}$$

is the Nash sharing rule for a newly-hired individual with idiosyncratic characteristics ε_t^i;

$$\mathbf{J}_{It} = \gamma^h + \tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t) - w_t^I \tag{145}$$

is the equilibrium value to the firm of an incumbent worker; and

$$\mathbf{J}_E(\varepsilon_t^i) = \tilde{\varepsilon}_t - \varepsilon_t^i + w(\tilde{\varepsilon}_t) - w(\varepsilon_t^i) \tag{146}$$

is the equilibrium value to the firm of a newly-hired worker with idiosyncratic characteristics ε_t^i.

F.1 Value Equations for Household

F.1.1 Unemployed Individuals

Because of the absence of a household-level envelope condition with respect to participation (which is inherent with endogenous participation), specifying the bargaining-relevant value equations for the household proceeds in a different way than in Appendix D. More specifically, the value $\mathbf{U}_t$ of an unemployed individual to the household in the endogenous-participation model (condition (88) in Appendix D) did not explicitly feature a continuation term. However, continuation values appeared implicitly though the household envelope conditions. With fixed participation and thus no envelope conditions to exploit, we must instead directly take into account the continuation values of individuals in every labor-market state, not just employed individuals.

It is useful to refer to Figure 2 for the following discussion. Consider an individual who, after the selection phase of period t, will *not* work, either because he was not selected or because he was selected but his wage negotiations broke down. Because such an individual is identical to every other individual from the perspective of the household (differences in idiosyncratic characteristics between individuals are only relevant if they are employed), his value to the household at this point in time (specifically, at the time marked "wage determination occurs" in Figure 2) is identical to the value of every other unemployed individual. Denote this value, as in Appendix D, $\mathbf{U}_t$. Such an individual receives the flow payoff b in period t. In period $t+1$, he will again be available for work with probability one; in contrast, with endogenous participation, whether or not a *particular* individual will be available for work in period $t+1$ is a matter of household choice and randomization due to the "lottery" interpretation of the full-consumption insurance household setup (see Rogerson (1988), Hansen (1985), and Andolfatto (1996) for more details).

In period $t+1$, the individual will again *not* work with probability $(1-\eta(\tilde{\varepsilon}_{t+1}))$, in which case his value to the household at the time wage determination occurs in period $t+1$ is $\mathbf{U}_{t+1}$. With probability $\eta(\tilde{\varepsilon}_{t+1})$, he will be selected and successfully complete wage negotiations in period $t+1$. Because the given individual's characteristics ε^i_{t+1} are unknown and impossible to predict in period t (recall ε is *iid* over time) *and* because bargaining occurs at the individual level and thus *is* conditional on idiosyncratic characteristics, the expected value to the household conditional on selection in period $t+1$ is $\frac{\int_{-\infty}^{\tilde{\varepsilon}_{t+1}} \mathbf{W}_E(\varepsilon^i_{t+1}) f(\varepsilon^i_{t+1}) d\varepsilon^i_{t+1}}{\eta(\tilde{\varepsilon}_{t+1})}$, where $\mathbf{W}_E(\varepsilon^i_{t+1})$ is defined below.

Putting these events together, an unemployed individual thus has value (measured in period-t goods) to the household

$$\mathbf{U}_t = b + E_t\left\{\Xi_{t+1|t}\left[\eta(\tilde{\varepsilon}_{t+1})\left(\frac{\int_{-\infty}^{\tilde{\varepsilon}_{t+1}} \mathbf{W}_E(\varepsilon^i_{t+1}) f(\varepsilon^i_{t+1}) d\varepsilon^i_{t+1}}{\eta(\tilde{\varepsilon}_{t+1})}\right) + (1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right]\right\}. \tag{147}$$

To conserve on notation, denote hereafter $\bar{\mathbf{W}}_{Et+1} \equiv \int_{-\infty}^{\tilde{\varepsilon}_{t+1}} \mathbf{W}_E(\varepsilon^i_{t+1}) f(\varepsilon^i_{t+1}) d\varepsilon^i_{t+1}$.

F.1.2 Incumbent Workers

Following similar logic, an incumbent worker in period t has value (measured in period-t goods) to the household

$$\mathbf{W}_{It} = w^I_t + E_t\left\{\Xi_{t+1|t}\left[(1-\rho)\mathbf{W}_{It+1} + \rho\left(\eta(\tilde{\varepsilon}_{t+1})\bar{\mathbf{W}}_{Et+1} + (1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right)\right]\right\}. \tag{148}$$

A previously-employed worker who loses his job at the start of period t can be hired anew in the same period, hence the composite probabilities $\rho\eta(\tilde{\varepsilon}_{t+1})$ and $\rho(1-\eta(\tilde{\varepsilon}_{t+1}))$ on the right-hand side. The surplus to the household of an incumbent worker who successfully completes wage negotiations is thus

$$\begin{aligned}
&\mathbf{W}_{It} - \mathbf{U}_t \\
&= w^I_t - b + E_t\left\{\Xi_{t+1|t}\left[(1-\rho)\mathbf{W}_{It+1} - (1-\rho)\eta(\tilde{\varepsilon}_{t+1})\bar{\mathbf{W}}_{Et+1} - (1-\rho)(1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right]\right\} \\
&= w^I_t - b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \eta(\tilde{\varepsilon}_{t+1})\bar{\mathbf{W}}_{Et+1} - (1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right]\right\} \\
&= w^I_t - b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1}\right]\right\} - (1-\rho)E_t\left\{\Xi_{t+1|t}\eta(\tilde{\varepsilon}_{t+1})\left[\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right]\right\}.
\end{aligned} \tag{149}$$

F.1.3 Newly-Hired Workers

Again based on the same logic as above, a newly-hired worker with idiosyncratic characteristics ε_t^i in period t has value (measured in period-t goods) to the household

$$\mathbf{W}_E(\varepsilon_t^i) = w(\varepsilon_t^i) + E_t\left\{\Xi_{t+1|t}\left[(1-\rho)\mathbf{W}_{It+1} + \rho\left(\eta(\tilde{\varepsilon}_{t+1})\bar{\mathbf{W}}_{Et+1} + (1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right)\right]\right\}. \quad (150)$$

The surplus to the household of a newly-hired worker who successfully completes wage negotiations is thus

$$\begin{aligned}
&\mathbf{W}_E(\varepsilon_t^i) - \mathbf{U}_t \\
&= w(\varepsilon_t^i) - b + E_t\left\{\Xi_{t+1|t}\left[(1-\rho)\mathbf{W}_{It+1} - (1-\rho)\eta(\tilde{\varepsilon}_{t+1})\bar{\mathbf{W}}_{Et+1} - (1-\rho)(1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right]\right\} \\
&= w(\varepsilon_t^i) - b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \eta(\tilde{\varepsilon}_{t+1})\bar{\mathbf{W}}_{Et+1} - (1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right]\right\} \\
&= w(\varepsilon_t^i) - b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1}\right]\right\} - (1-\rho)E_t\left\{\Xi_{t+1|t}\eta(\tilde{\varepsilon}_{t+1})\left[\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right]\right\}.
\end{aligned} \quad (151)$$

F.2 Wages

With surpluses now defined, we now obtain expressions characterizing bargained wages. The derivations parallel those in Appendix D.

F.2.1 Incumbent Workers

To obtain an expression for the period-t bargained wage of an incumbent, first substitute (149) in the Nash sharing rule (143), which gives

$$w_t^I - b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1}\right]\right\} - (1-\rho)E_t\left\{\Xi_{t+1|t}\eta(\tilde{\varepsilon}_{t+1})\left[\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right]\right\} = \left(\frac{\alpha^I}{1-\alpha^I}\right)\mathbf{J}_{It}. \quad (152)$$

Then, substitute the period $t+1$ sharing rule (143) in the third term on the left-hand side,

$$w_t^I - b + (1-\rho)\left(\frac{\alpha^I}{1-\alpha^I}\right)E_t\left\{\Xi_{t+1|t}\mathbf{J}_{It+1}\right\} - (1-\rho)E_t\left\{\Xi_{t+1|t}\eta(\tilde{\varepsilon}_{t+1})\left[\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right]\right\} = \left(\frac{\alpha^I}{1-\alpha^I}\right)\mathbf{J}_{It}. \quad (153)$$

Next, substitute using (145) for both $\mathbf{J}_{It}$ and $\mathbf{J}_{It+1}$, which gives

$$\begin{aligned}
&w_t^I - b + (1-\rho)\left(\frac{\alpha^I}{1-\alpha^I}\right)E_t\left\{\Xi_{t+1|t}\left[\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\} \\
&\quad -(1-\rho)E_t\left\{\Xi_{t+1|t}\eta(\tilde{\varepsilon}_{t+1})\left[\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right]\right\} = \left(\frac{\alpha^I}{1-\alpha^I}\right)\left[\gamma^h + \tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t) - w_t^I\right].
\end{aligned}$$

Rearranging,

$$w_t^I \left[1 + \frac{\alpha^I}{1-\alpha^I}\right] = b + \left(\frac{\alpha^I}{1-\alpha^I}\right)\left[\gamma^h + \tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t)\right]$$
$$-(1-\rho)\left(\frac{\alpha^I}{1-\alpha^I}\right) E_t\left\{\Xi_{t+1|t}\left[\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\} + (1-\rho)E_t\left\{\Xi_{t+1|t}\eta(\tilde{\varepsilon}_{t+1})\left[\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right]\right\}.$$

Multiplying by $(1-\alpha^I)$,

$$w_t^I = (1-\alpha^I)b + \alpha^I\left[\gamma^h + \tilde{\varepsilon}_t + w(\tilde{\varepsilon}_t)\right]$$
$$-(1-\rho)\alpha^I E_t\left\{\Xi_{t+1|t}\left[\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\} + (1-\rho)(1-\alpha^I)E_t\left\{\Xi_{t+1|t}\eta(\tilde{\varepsilon}_{t+1})\left[\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right]\right\}. \tag{154}$$

This expression is not a closed-form expression for w_t^I (even taking the continuation value as given) because the endogenous wage $w(\tilde{\varepsilon}_t)$ also appears. A closed-form expression requires also solving for a new hire's wage, which is done next.

F.2.2 Newly-Hired Workers

To obtain an expression for the period-t bargained wage of a new hire, first substitute (151) in the Nash sharing rule (144),

$$w(\varepsilon_t^i) - b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1}\right]\right\}$$
$$-(1-\rho)E_t\left\{\Xi_{t+1|t}\eta(\tilde{\varepsilon}_{t+1})\left[\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right]\right\} = \left(\frac{\alpha^E}{1-\alpha^E}\right)\mathbf{J}_E(\varepsilon_t^i).$$

Then, substitute using (146) for $\mathbf{J}_E(\varepsilon_t^i)$, which gives

$$w(\varepsilon_t^i) - b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1}\right]\right\}$$
$$-(1-\rho)E_t\left\{\Xi_{t+1|t}\eta(\tilde{\varepsilon}_{t+1})\left[\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right]\right\} = \left(\frac{\alpha^E}{1-\alpha^E}\right)\left[\tilde{\varepsilon}_t - \varepsilon_t^i + w(\tilde{\varepsilon}_t) - w(\varepsilon_t^i)\right].$$

Rearranging,

$$w(\varepsilon_t^i)\left[1 + \frac{\alpha^E}{1-\alpha^E}\right] = b + \left(\frac{\alpha^E}{1-\alpha^E}\right)\left[\tilde{\varepsilon}_t - \varepsilon_t^i + w(\tilde{\varepsilon}_t)\right]$$
$$+(1-\rho)E_t\left\{\Xi_{t+1|t}\eta(\tilde{\varepsilon}_{t+1})\left[\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right]\right\} - (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1}\right]\right\}.$$

Multiplying by $(1-\alpha^E)$,

$$w(\varepsilon_t^i) = (1-\alpha^E)b + \alpha^E\left[\tilde{\varepsilon}_t - \varepsilon_t^i + w(\tilde{\varepsilon}_t)\right]$$
$$+(1-\rho)(1-\alpha^E)E_t\left\{\Xi_{t+1|t}\eta(\tilde{\varepsilon}_{t+1})\left[\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right]\right\} - (1-\rho)(1-\alpha^E)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1}\right]\right\}.$$

Now substitute the period $t+1$ sharing rule (143) for *incumbents*,

$$w(\varepsilon_t^i) = (1-\alpha^E)b + \alpha^E\left[\tilde{\varepsilon}_t - \varepsilon_t^i + w(\tilde{\varepsilon}_t)\right]$$
$$+(1-\rho)(1-\alpha^E)E_t\left\{\Xi_{t+1|t}\eta(\tilde{\varepsilon}_{t+1})\left[\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right]\right\} - (1-\rho)(1-\alpha^E)\left(\frac{\alpha^I}{1-\alpha^I}\right)E_t\left\{\Xi_{t+1|t}\mathbf{J}_{It+1}\right\}.$$

Then, substituting using (145) for $\mathbf{J}_{It+1}$, gives

$$\begin{aligned} w(\varepsilon_t^i) &= (1-\alpha^E)b + \alpha^E\left[\tilde{\varepsilon}_t - \varepsilon_t^i + w(\tilde{\varepsilon}_t)\right] \\ &+(1-\rho)(1-\alpha^E)E_t\left\{\Xi_{t+1|t}\eta(\tilde{\varepsilon}_{t+1})\left[\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right]\right\} \\ &-(1-\rho)(1-\alpha^E)\left(\frac{\alpha^I}{1-\alpha^I}\right)E_t\left\{\Xi_{t+1|t}\left[\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right]\right\}. \end{aligned} \tag{155}$$

This expression characterizes the bargained wage for a newly-hired worker with idiosyncratic characteristics ε_t^i, but is not a closed-form solution because the endogenous wage $w(\tilde{\varepsilon}_t)$ also appears. However, at this stage of the analysis, it is not difficult to obtain closed-form solutions.

F.2.3 Closed-Form Solutions for Bargained Wages

As in Appendix D, there are three more steps required to obtain closed-form expressions for bargained wages. First, construct expressions for each of the three period-t wages in which no other *contemporaneous* wage appears. Second, compute wage differentials. Third, substitute wage differentials into the *continuation value* components of each wage expression.

Begin by evaluating (155) at $\varepsilon_t^i = \tilde{\varepsilon}_t$, which gives the bargained wage of the threshold new hire,

$$w(\tilde{\varepsilon}_t) = b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\eta(\tilde{\varepsilon}_{t+1})\left(\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right) - \left(\frac{\alpha^I}{1-\alpha^I}\right)\left(\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right]\right\}. \tag{156}$$

Then, substitute (156) in (155), which gives the bargained wage for a new hire with idiosyncratic characteristics ε_t^i,

$$\begin{aligned} w(\varepsilon_t^i) &= b + \alpha^E\left(\tilde{\varepsilon}_t - \varepsilon_t^i\right) \\ &+(1-\rho)E_t\left\{\Xi_{t+1|t}\left[\eta(\tilde{\varepsilon}_{t+1})\left(\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right) - \left(\frac{\alpha^I}{1-\alpha^I}\right)\left(\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right]\right\}. \end{aligned} \tag{157}$$

Next, substitute (156) in (154), which gives the bargained wage for an incumbent worker,

$$\begin{aligned} w_t^I &= b + \alpha^I\left(\gamma^h + \tilde{\varepsilon}_t\right) \\ &+(1-\rho)E_t\left\{\Xi_{t+1|t}\left[\eta(\tilde{\varepsilon}_{t+1})\left(\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right) - \left(\frac{\alpha^I}{1-\alpha^I}\right)\left(\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I\right)\right]\right\}. \end{aligned} \tag{158}$$

As in Appendix D, the continuation value is identical in all three wage functions (156), (157), and (158) because, just as in the endogenous LFP case, no matter what type a worker is in period t, he will be a (homogenous) incumbent worker in period $t+1$ if he remains employed. Moreover, the period-$(t+1)$ wage differential $w(\tilde{\varepsilon}_{t+1}) - w^I_{t+1}$ appears in all three.

Expressions (156), (157), and (158) allow us to explicitly compute wage differentials between the different types of workers, which turn out to be identical to those in the endogenous LFP case: a new hire with $\varepsilon^i_t < \tilde{\varepsilon}_t$ earns a premium over the threshold new hire

$$w(\varepsilon^i_t) - w(\tilde{\varepsilon}_t) = \alpha^E \left(\varepsilon^i_t - \tilde{\varepsilon}_t\right), \tag{159}$$

and an incumbent worker earns a premium over the threshold new hire

$$w^I_t - w(\tilde{\varepsilon}_t) = \alpha^I \left(\gamma^h + \tilde{\varepsilon}_t\right). \tag{160}$$

Substitute $w(\tilde{\varepsilon}_{t+1}) - w^I_{t+1} = -\alpha^I \left(\gamma^h + \tilde{\varepsilon}_{t+1}\right)$ into each of (156), (157), and (158) to obtain

$$w(\tilde{\varepsilon}_t) = b + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left[\eta(\tilde{\varepsilon}_{t+1}) \left(\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1} \right) - \alpha^I \left(\gamma^h + \tilde{\varepsilon}_{t+1}\right) \right] \right\}, \tag{161}$$

$$w(\varepsilon^i_t) = b + \alpha^E \left(\tilde{\varepsilon}_t - \varepsilon^i_t\right) + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left[\eta(\tilde{\varepsilon}_{t+1}) \left(\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1} \right) - \alpha^I \left(\gamma^h + \tilde{\varepsilon}_{t+1}\right) \right] \right\}, \tag{162}$$

and

$$w^I_t = b + \alpha^I \left(\gamma^h + \tilde{\varepsilon}_t\right) + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left[\eta(\tilde{\varepsilon}_{t+1}) \left(\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1} \right) - \alpha^I \left(\gamma^h + \tilde{\varepsilon}_{t+1}\right) \right] \right\}. \tag{163}$$

These wage functions are identical to (127), (128), and (129) for the endogenous LFP model once we recognize that optimal participation is characterized by

$$\begin{aligned} \frac{v'(lfp_t)}{u'(c_t)} - b &= \eta(\tilde{\varepsilon}_t) \left[\bar{\mathbf{W}}_{Et} - \mathbf{U}_t \right] \\ &= \eta(\tilde{\varepsilon}_t) \left(\frac{\int_{-\infty}^{\tilde{\varepsilon}_t} \mathbf{W}_E(\varepsilon^i_t) f(\varepsilon^i_t) d\varepsilon^i_t}{\eta(\tilde{\varepsilon}_t)} \right) - \eta(\tilde{\varepsilon}_t) \mathbf{U}_t \\ &= \int_{-\infty}^{\tilde{\varepsilon}_t} \mathbf{W}_E(\varepsilon^i_t) f(\varepsilon^i_t) d\varepsilon^i_t - \eta(\tilde{\varepsilon}_t) \mathbf{U}_t, \end{aligned} \tag{164}$$

a result that was shown in Appendix C. With exogenous participation, however, this equality (which is simply the LFP condition) does not hold, so the left-hand side of (164) cannot be substituted for the right-hand side of (164). The value expressions that appear in the wage functions (161), (162), and (163) thus cannot be eliminated. Computationally, this means that the integrals inherent in these expressions must be computed in numerically working with the fixed-LFP version of the model.

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