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by Christopher Phillip Reicher

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In this paper, I estimate a series of long run reallocative shocks to sectoral employment using a stochastic volatility model of sectoral employment growth for the United States from 1960 through 2011. Reallocative shocks (which primarily measure construction and technology busts) have little effect on the natural rate of unemployment or on long run productivity, but there is mild evidence that they are recessionary. A broad class of theoretical models suggests that the contractionary effect of a reallocative shock should come from the direct aggregate effect of the underlying shock and not from human capital mismatch. Looking at the period of the Great Recession, reallocation has had no detectable effect on the natural rate of unemployment and can count for a 0.5% rise in cyclical unemployment from 2007 through the end of 2009 and 0.3% through the beginning of 2011.

Keywords: Mismatch, sectoral shifts, reallocation, natural rate, unemployment, Great Recession, stochastic volatility.

JEL classification: E24, E32, E66, J24, J62.

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The aggregate effects of long run sectoral reallocation

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In this paper, I estimate a series of long run reallocative shocks to sectoral employment using a stochastic volatility model of sectoral employment growth for the United States from 1960 through 2011. Reallocative shocks (which primarily measure construction and technology busts) have little effect on the natural rate of unemployment or on long run productivity, but there is mild evidence that they are recessionary. A broad class of theoretical models suggests that the contractionary effect of a reallocative shock should come from the direct aggregate effect of the underlying shock and not from human capital mismatch. Looking at the period of the Great Recession, reallocation has had no detectable effect on the natural rate of unemployment and can count for a 0.5% rise in cyclical unemployment from 2007 through the end of 2009 and 0.3% through the beginning of 2011.

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The aggregate effects of long run sectoral reallocation

1. Introduction

In this paper I estimate the aggregate effects of shocks which affect the long run cross-sectional dispersion of sectoral employment for the United States. I find that reallocative shocks do not have an obvious effect on either the natural rate of unemployment or on long run productivity, but there is moderate evidence that reallocative shocks are contractionary at business cycle frequencies. However, the statistical contribution of reallocative shocks to the variance of the U.S. business cycle appears to be limited, on the order of 12% of the total variance of the cycle. The estimated series of reallocative shocks for the most part measures busts in the construction sector, with the addition of the 2001 technology bust. Based on a theoretical set of models of human capital mismatch, I argue that the contractionary effect of a reallocative shock should come from the direct aggregate effect of the shock and not from the increased degree of human capital mismatch across sectors, which follows the shock.[1]

I also discuss the importance of reallocation in understanding the observed dynamics of the U.S. labor market after the Great Recession. Based on the estimates from this paper, reallocative shocks explain only a tiny portion of the rise in unemployment after the Great Recession. The natural rate does not appear to have moved substantially—the effects of reallocation appear to come entirely through the cycle, and the various sectors of the economy have behaved in a manner consistent with an unusually deep and persistent slump rather than a rise in trend unemployment. This is consistent with history. There were several large waves of reallocation before 2008 which were driven mostly by construction busts, and these waves of reallocation did not coincide with substantial changes in the natural rate. Based on the estimates from the statistical model, reallocative shocks should have contributed about 0.5 percentage points to the total rise in unemployment from early 2007 to late 2009, and about 0.3 percentage points to the total rise through early 2011. There is mild evidence that reallocative shocks are contractionary, but they appear to have a small aggregate effect.

The estimates are based upon a stochastic volatility model of long run sectoral employment growth estimated by Bayesian methods. Formulating the problem in this manner makes it

[1] This paper does not discuss the Beveridge Curve. Kocherlakota (2010), for instance, has claimed that mismatch accounts for a 2.5 percent rise in unemployment during the Great Recession. For a summary of the Beveridge Curve debate and a cautionary note, see Tasci and Lindner (2010).

possible to separate the short run movements in the data from long run trends and to explore any linkages between the two. It also makes it possible to quantify, within simulation error, the exact degree of confidence in the estimates. By concentrating on long run sectoral reallocation and its effects, and by ignoring short run movements in relative employment shares, this approach avoids the critique by Abraham and Katz (1986) of the original Lilien (1982) dispersion measure.

This paper appears to be the first paper which estimates sectoral reallocation in a stochastic volatility setting. Early approaches involved estimating sectoral reallocation based on other forward-looking observables. Loungani, Rush, and Tave (1990) identify sectoral shifts using stock prices, and Rissman (1993) identifies sectoral shifts using the Phillips curve. Since the 1990s, computing power has made it possible to estimate richer models of shocks and dynamics with a more direct focus on labor market data. Mills, Pelloni, and Zervoyianni (1995) and Campbell and Kuttner (1996) apply VAR techniques on a wider range of industries, and they find support for the sectoral shifts hypothesis. Pelloni and Polasek (2003) directly model sectoral shifts as relating to time-varying volatility. They extend Campbell and Kuttner's work using a vector autoregression on data for the United Kingdom, Germany, and the United States with GARCH errors; they claim that sectoral shifts have large aggregate effects. Pelloni and Polasek do not allow for different industries to exhibit different responses to the cycle, so their estimates remain subject to the Abraham-Katz critique.

Rissman (2009) has recently looked at the issue of reallocation using employment data, using the state space approach implemented by Rissman (1997) and Aaronson, Rissman, and Sullivan (2004). Rissman models sectoral employment dynamics as coming from an aggregate shock with different factor loadings across sectors, in addition to idiosyncratic employment growth shocks. Such an approach elegantly addresses the Abraham-Katz critique. Her focus is more on the performance of specific sectors in recent cycles and not on the time-varying nature of the reallocative process, though she does touch briefly upon the issue of unemployment. Her results regarding the relationship between reallocation and unemployment are ambiguous. She regresses the change in the unemployment rate on the dispersion of estimated mean idiosyncratic sectoral employment growth and also on demographic variables which may themselves be involved in the reallocative process. By contrast, my model directly integrates a stochastic reallocative process along with unemployment dynamics into the estimation procedure and thus avoids the issue of attenuation bias which comes from using a generated regressor. After doing this, I find that there is in fact moderate evidence that reallocative shocks are contractionary in the short to medium run, though the magnitude of their contractionary effect is most probably small.

The rest of this paper follows the traditional format. Section 2 outlines the theoretical effects of human capital mismatch in a simple market-clearing model as well as in a search and matching model. Both models predict that human capital mismatch should have no first order effect on aggregate employment or output, but it is reasonable to expect that some reallocative shocks ("dirty" reallocative shocks) may have direct aggregate effects if a negative shock to one sector is not exactly offset by a positive shock to another sector. Section 3 outlines the state space model used to estimate the effects of reallocation. Section 4 discusses the data and prior distributions used in the estimation exercise. Section 5 discusses the estimation results—reallocative shocks appear to be mildly contractionary at a cyclical frequency but have no obvious effect on aggregates in the long run, and their role during the Great Recession seems to be limited. Section 6 concludes.

2. A simple model of human capital mismatch in a general equilibrium context

2.1 The underlying structure of the model

This section lays out a simple model of human capital mismatch in a dynamic context. Workers belong to an industry and their human capital is sticky; they have to retrain in order to move between sectors. They will do so if economic opportunities in other sectors are better than those within their current sector, and those opportunities are subject to a series of reallocative shocks. Since it is costly to retrain, the population of workers at any time will not exactly reflect current economic reality; there will be mismatch between long-run labor demand and current labor supply. One should think of construction workers slowly retraining to become nurses after a reallocative shock which causes the demand for construction workers to fall and the demand for nurses to rise.

Sticky human capital is certainly relevant at the microeconomic level. Kambourov and Manovskii (2008) document occupational and industrial mobility rates in the United States in the low double digits per year—most unemployed people end up reemployed in their old occupation and industry, which suggests that there is a substantial cost to moving between industries and occupations. This model concentrates on the macroeconomic relevance of this fact. This paper instead models human capital mismatch at the worker level using the metaphor of islands popularized by Lucas and Prescott (1974). People live on islands, and it is costly to move between them. Islands specialize in different types of labor, the relative demand for which moves over time, and each island has its own labor market. Since economic opportunities for islanders evolve stochastically over time but islanders only move slowly, there will be a degree of mismatch between the population of a given island and long-run economic reality. This mismatch gives rise to different rates of unemployment across islands, but it turns out that mismatch has no first order aggregate effects. Mismatch does generate strong patterns of employment adjustment at the island level and may have interesting welfare effects, but human capital mismatch should have no first order effect in the aggregate. This result is relatively robust; it survives the inclusion of quadratic hiring frictions or search costs at the island level.

2.2 The islanders' problem

The economy consists of an archipelago of J islands which altogether have a unit mass of population. Each island j has a population $L_{j,t}$. To move between the islands requires a costly ferry trip and the islanders become seasick easily, so at any given point in time, the distribution of population across islands will not accurately reflect current economic conditions. Some islands will have too many inhabitants relative to the long run demand for their labor, while others have too few inhabitants. Within each island, labor supply is flexible and each worker earns his outside option; labor markets clear. Individual islanders have a convex disutility of work $v(h_{j,t})$ and they earn a real wage $W_{j,t}$ which they use to purchase a homogeneous consumption good C_t.

There is a central island where a complete set of state-contingent securities and each island's output are traded. Aggregators on that island combine each island's output into an aggregate consumption good, using a constant returns production technology whose optimal product mix varies over time. Both aggregators and the islanders take prices as given. As a result, islanders can perfectly insure their consumption against idiosyncratic shocks.[2] Preferences are separable in consumption and labor input, and they are compatible with the fact that unemployment does not grow or shrink on average over time. There is a disutility to

[2] This assumption is not crucial, as shown in Appendix A. Appendix B shows that the results of the model are robust to the inclusion of convex adjustment costs (such as search, hiring, and firing costs) at the firm level.

changing the composition of the labor force across islands, which without loss of generality is given by a well-behaved seasickness function γ.

The household sector's objective takes the reduced form:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\ln(C_{t+i}) - \sum_{j=1}^{J} L_{j,t+i} v(h_{j,t+i}) - \gamma(\{\{L_{j,t+i-k}\}_{j=1}^{J}\}_{k=0}^{-\infty}) \right]$$
$$- E_t \sum_{i=0}^{\infty} \beta^i \left[\lambda_{t+i} \left(C_{t+i} - \sum_{j=1}^{J} L_{j,t+i} W_{j,t+i} h_{j,t+i} \right) + \mu_{t+i} \left(\sum_{j=1}^{J} L_{j,t+i} - 1 \right) \right].$$

The first order conditions in labor supply for each island take the following form:

$$v'(h_{j,t}) = \lambda_t W_{j,t}, \tag{1}$$

where

$$\lambda_t = 1/C_t. \tag{2}$$

Based on the solution to the household's problem, it is possible to show that mismatch has no first order effects in the aggregate, though mismatch does have a large effect at the island level, and its welfare consequences depend on the thickness of insurance markets.

2.3 The constancy of aggregate labor input

Summing the equation (1) up across islands, weighting by each island's population and employment rate, yields the following expression:

$$\sum_{j=1}^{J} L_{j,t} h_{j,t} v'(h_{j,t}) = \sum_{j=1}^{J} \lambda_t L_{j,t} h_{j,t} W_{j,t}. \tag{3}$$

Since aggregate production exhibits constant returns to scale and factor markets are competitive, total wage payments must equal total output. Total consumption must also equal total output, so the right hand side of (3) must equal one.

Hatted variables denote first order percent deviations from a steady state without mismatch. In such a steady state, the per-capita labor input of each island is equal to the same long run value *h*; wages are the same across islands at *W*; so there is no incentive for people to move between islands. Taking first order deviations of the objects on the left hand side of (3) from a perfectly matched steady state yields the expression:

$$\sum_{j=1}^{J} L_j \left(\hat{L}_{j,t} + \phi \hat{h}_{j,t}\right) = 0, \tag{4}$$

for a constant ϕ which reflects the preferences of the households over labor supply.

Since the total population of the entire archipelago is one, its total deviation must be zero:

$$\sum_{j=1}^{J} L_j \hat{L}_{j,t} = 0, \tag{5}$$

so substituting (5) into (4) implies that:

$$\sum_{j=1}^{J} L_j \hat{h}_{j,t} = 0. \tag{6}$$

Archipelagowide labor input is given by the sum of its island-level labor input:

$$N_t = \sum_{j=1}^{J} L_{j,t} h_{j,t}, \tag{7}$$

which implies that in first order deviations:

$$\hat{N}_t = \sum_{j=1}^{J} L_j (\hat{L}_{j,t} + \hat{h}_{j,t}) = 0, \tag{8}$$

because the components of (8) are just given by the left hand portions of (5) and (6) which both equal zero. Mismatch therefore has no first order effects on the level of aggregate labor input. There is a bit of simple intuition behind this result. Mismatch drives a convex wedge between the disutility of work and the total wage. The population distribution across islands in the long run minimizes this wedge, which equals zero in steady state. The envelope theorem therefore implies that a small deviation in the island population distribution from its long run optimum results in at most a second-order effect on the wedge and on labor input.

2.4 The behavior of output after a clean reallocative shock

In the case of a CES aggregator, not only does mismatch by itself have no first order effect on labor input; it has no first order effect on productivity either, which precludes an RBC-style productivity effect of mismatch in a model with capital. The representative aggregator on the main island of the archipelago produces according to the production function:

$$Y_t = \left(\sum_{j=1}^{J} (a_{j,t})^{\frac{1}{\rho}} \left(L_{j,t} h_{j,t}\right)^{\left(\frac{\rho-1}{\rho}\right)} \right)^{\left(\frac{\rho}{\rho-1}\right)}. \tag{9}$$

A "clean" reallocative shock in the context of a CES aggregator is a shift in the relative factor weights $a_{j,t}$ so that their sum remains constant. Without loss of generalization, that sum can be set to one. In that case, a reallocative shock does not affect the long run level of productivity but it affects the relative productivity of each sector. In the construction worker-nurse example, a clean reallocative shock would lower the absolute productivity of construction workers and raise the absolute productivity of nurses to an exactly offsetting degree, so that in the long run, only their relative employment and output shares vary.

Price-taking behavior yields the demand curve for each type of labor:

$$L_{j,t} h_{j,t} = \left(W_{j,t}\right)^{-\rho} a_{j,t} Y_t . \tag{10}$$

Solving for wages gives labor demand on a sectoral basis:

$$W_{j,t} = \left(\frac{a_{j,t} Y_t}{L_{j,t} h_{j,t}} \right)^{\frac{1}{\rho}} . \tag{11}$$

Summing up the total wage bill gives:

$$\sum_{j=1}^{J} W_{j,t} L_{j,t} h_{j,t} = \sum_{j=1}^{J} \left(L_{j,t} h_{j,t}\right)^{\left(\frac{\rho-1}{\rho}\right)} \left(a_{j,t} Y_t\right)^{\left(\frac{1}{\rho}\right)}. \tag{12}$$

Total wage payments equal total income, simplifying (12):

$$\left(Y_t\right)^{\left(\frac{\rho-1}{\rho}\right)} = \sum_{j=1}^{J} \left(L_{j,t} h_{j,t}\right)^{\left(\frac{\rho-1}{\rho}\right)} \left(a_{j,t}\right)^{\left(\frac{1}{\rho}\right)}. \tag{13}$$

Expressed in first order deviations, (13) becomes:

$$(\rho-1) Y^{\left(\frac{\rho-1}{\rho}\right)} \hat{Y}_t = \sum_{j=1}^{J} \left(L_j h\right)^{\left(\frac{\rho-1}{\rho}\right)} \left(a_j\right)^{\left(\frac{1}{\rho}\right)} \left((\rho-1)(\hat{L}_{j,t} + \hat{h}_{j,t}) + \hat{a}_{j,t}\right). \tag{14}$$

In a matched steady state, L_j equals a_j, so since the first order deviations of aggregate labor input are zero, Equation (14) simplifies considerably:

$$(\rho-1) Y^{\left(\frac{\rho-1}{\rho}\right)} \hat{Y}_t = \sum_{j=1}^{J} a_j h^{\left(\frac{\rho-1}{\rho}\right)} \hat{a}_{j,t}. \tag{15}$$

Since the sum of CES weights is constant in response to a clean reallocation, the weighted sum of linearized CES weights equals zero:

$$\sum_{j=1}^{J} a_j \hat{a}_{j,t} = 0. \tag{16}$$

Therefore, the first order deviation of output must equal zero since (16) implies that the right hand side of (15) equals zero. The same intuition regarding this result holds as for the household's problem. Mismatch (i.e. a distribution of island populations that does not exactly reflect long-run labor demand) has no first order effect on output or employment, and it has no first order effect on productivity either. First order deviations in the distribution of island population and labor input from their optimum should have at most second-order effects.

2.5 Sectoral employment dynamics after a clean reallocative shock

Despite the lack of meaningful aggregate dynamics, sectoral employment dynamics are rather interesting. To fully investigate sectoral employment dynamics, it is necessary to parameterize the cost of moving workers from island to island and the disutility of work at the island level. I assume quadratic adjustment costs in island population and isoelastic labor supply. Algebraically, these functional forms would imply an objective of the sort:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\ln(C_{t+i}) - \sum_{j=1}^{J} L_{j,t+i} \frac{\varphi h_{j,t+i}^{1+\chi}}{1+\chi} - \sum_{j=1}^{J} \frac{\gamma}{2L_{j,t+i}} (L_{j,t+i} - L_{j,t+i-1})^2 \right]$$
$$- E_t \sum_{i=0}^{\infty} \beta^i \left[\lambda_{t+i} \left(C_{t+i} - \sum_{j=1}^{J} L_{j,t+i} W_{j,t+i} h_{j,t+i} \right) + \mu_{t+i} \left(\sum_{j=1}^{J} L_{j,t+i} - 1 \right) \right].$$

The quadratic adjustment cost term has $L_{j,t}$ in the denominator so that the total cost borne by the economy is proportional to the number of workers shifted between each island; larger islands incur larger absolute costs when transferring workers. Appendix C contains the detailed solution to the model under this specification. The first order conditions in labor supply for each island take the form previously discussed, and firm behavior is the same. Island-level population dynamics follow the forward-looking difference equation:

$$\frac{\varphi h_{j,t}^{1+\chi}}{1+\chi} + \mu_t + \frac{\gamma(L_{j,t} - L_{j,t-1})}{L_{j,t}} - \frac{\gamma(L_{j,t} - L_{j,t-1})^2}{2L_{j,t}^2} = \frac{W_{j,t} h_{j,t}}{Y_t} + \beta E_t \frac{\gamma(L_{j,t+1} - L_{j,t})}{L_{j,t+1}}. \quad (17)$$

It is possible to show that island-level employment rates are a simple function of mismatch by combining the island-level labor supply and demand curves:

$$\hat{h}_{j,t} = \frac{1}{1+\rho\chi} (\hat{a}_{j,t} - \hat{L}_{j,t}). \quad (18)$$

So long as island-level long run labor demand exceeds the island's population (i.e. there is mismatch), islanders will respond by working more until island population readjusts through migration. Conversely, unemployment in a particular industry will remain elevated so long as the population of workers in that industry exceeds long run labor demand in that industry. In particular, immediately after a construction bust combined with a nursing boom, construction workers should face a lower rate of employment while nurses should face a higher rate of employment.

Substituting (18) into the linearized version of (17) gives the law of motion for each island's population. The solution to that difference equation depends on the law of motion of sectoral labor demand $\hat{a}_{j,t}$. When $\hat{a}_{j,t}$ follows a random walk, the law of motion has a simple solution in error correction form. It is given by the reduced form solution:

$$\hat{L}_{j,t} = \rho_L \hat{L}_{j,t-1} + (1-\rho_L)\hat{a}_{j,t}, \tag{19}$$

where the error correction coefficient can be derived from the structural coefficients. The error correction coefficient is given by:

$$\rho_L = \frac{\left(1+\dfrac{\chi/\gamma}{1+\rho\chi}+\beta\right)-\sqrt{\left(1+\dfrac{\chi/\gamma}{1+\rho\chi}+\beta\right)^2-4\beta}}{2\beta}. \tag{20}$$

Unsurprisingly, persistence is increasing in γ. A high cost to migration between islands will result in more persistent mismatch and in more persistence in the deviation of the island-level employment rate from its trend. Less obviously, a high value of χ promotes mobility. When χ is low, the model economy behaves like a standard indivisible-labor economy. Well-insured islanders are indifferent as to which island they live on, so they see no reason ever to move. A high value of ρ has a similar effect but from the firm side. Firms become more indifferent as to where workers are physically located, so the dispersion of population across islands matters less.

The law of motion for total island-level employment $N_{j,t}$ is given by the laws of motion of $h_{j,t}$ and $L_{j,t}$. Combining (18) with (19) gives the law of motion for sectoral employment rates:

$$\hat{h}_{j,t} = \rho_L \hat{h}_{j,t-1} + \frac{\rho_L}{(1+\rho\chi)}(\hat{a}_{j,t} - \hat{a}_{j,t-1}). \tag{21}$$

Total employment at an island level is given by combining the law of motion of the island's population (19) with the law of motion of the island's employment rate (21):

$$\hat{N}_{j,t} = \rho_L \hat{N}_{j,t-1} + (1-\rho_L)\hat{a}_{j,t} + \left(\frac{\rho_L}{(1+\rho\chi)}\right)(\hat{a}_{j,t} - \hat{a}_{j,t-1}) . \qquad (22)$$

It is possible to easily simulate island-level dynamics after a clean reallocative shock using (19) through (22).

The top panel of Figure 1 shows the dynamics of island-level employment after a negative ten percent permanent reallocative shock to sector 1, with a quarterly persistence coefficient ρ_L of 0.75, a CES substitution parameter ρ of 2, and an inverse labor supply elasticity χ of 2. Sector 1 starts out at 10% of aggregate employment. The island-level employment rate in island 1 falls instantly since employment can adjust more quickly than population. Island-level population adjusts much more slowly because it is more costly to move between islands than to move between employment and nonemployment within an island. The evolution of total island employment is the sum of these two effects, with total employment adjusting more quickly than population. If mismatch and adjustment costs are important features of sectoral employment growth, they should show up in the data as slow adjustment of sectoral employment toward its long run trend.

2.6 The constancy of labor input in a search and matching model

Clean reallocative shocks have no aggregate effect on employment in a search and matching model as well. Phelan and Trejos (2000) deliver a somewhat similar result with regard to employment dynamics, though their model does not feature a segmented unemployment pool. No formal proof of the lack of an effect of mismatch is available, but the constancy of aggregate labor input can be demonstrated computationally using the gensys.m program of Sims (2002). This section presents a model which combines search frictions with mismatch. Households have the same preferences as they do in the original model. Appendix D presents the complete solution to the model with quadratic human capital adjustment costs and isoelastic labor supply. To simplify matters, workers have no bargaining power. Per-capita island labor input is now denoted by $n_{j,t}$, and the unemployment rate is $u_{j,t}$ on each island.

2.6.1 Households

Households wish to maximize the following objective (note the inclusion of a profit term Π in the households' income receipts):

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\ln(C_{t+i}) - \sum_{j=1}^{J} L_{j,t+i} \frac{\varphi n_{j,t+i}^{1+\chi}}{1+\chi} - \sum_{j=1}^{J} \frac{\gamma}{2L_{j,t+i}} (L_{j,t+i} - L_{j,t+i-1})^2 \right]$$

$$- E_t \sum_{i=0}^{\infty} \beta^i \left[\lambda_{t+i} \left(C_{t+i} - \sum_{j=1}^{J} L_{j,t+i} W_{j,t+i} n_{j,t+i} - \sum_{j=1}^{J} \Pi_{j,t+i} \right) + \mu_{t+i} \beta \left(\sum_{j=1}^{J} L_{j,t+i+1} - 1 \right) \right]$$

The first order condition in labor supply for each island (which gives the outside option) takes the following form:

$$\varphi n_{j,t}^{\chi} = \lambda_t W_{j,t}, \tag{23}$$

Employment and population are both effectively chosen one period in advance. The first order condition in $L_{j,t+1}$ is given by:

$$E_t \left[\frac{\varphi n_{j,t+1}^{1+\chi}}{1+\chi} + \frac{\gamma (L_{j,t+1} - L_{j,t})}{L_{j,t+1}} - \frac{\gamma (L_{j,t+1} - L_{j,t})^2}{2L_{j,t+1}^2} \right]$$

$$- E_t \left[\lambda_{t+1} W_{j,t+1} n_{j,t+1} + \beta \frac{\gamma (L_{j,t+2} - L_{j,t+1})}{L_{j,t+2}} \right] = -\mu_t . \tag{24}$$

The result that labor input is constant is not sensitive to the assumptions made in modeling island population flows, so long as steady state island populations reasonably reflect economic reality in the long run. I choose this particular setup because it mirrors the setup in the rest of the paper, and it implies that workers purposefully choose to retrain. Shimer (2007) follows a different route where construction workers wake up in the morning to find that they have become nurses; the results presented here are robust to any law of motion for island-level employment which allows long-run island populations to reflect economic reality. The first order condition with respect to consumption and the resource constraint are the same as in the flexible model.

2.6.2 Production firms and aggregators

Wholesale firms post vacancies using κ units of labor; the rest of their labor goes to producing their own output. They sell their output at a real price $P_{j,t}$, and they find workers from the island-level unemployment pool according to a Cobb-Douglas matching function with an elasticity on unemployment of ξ. Unemployment and vacancy pools are segmented by island; only workers trained in a certain occupation or industry search for jobs in that industry. The law of motion for sectoral employment is given by:

$$N_{j,t} = \phi(N_{j,t-1} + V_{j,t-1} m \theta_{j,t-1}^{-\xi}), \tag{25}$$

where ϕ is the hazard rate of being employed in the following period conditional on being employed in the current period; m is a matching function efficiency parameter; and $\theta_{j,t}$ is the tightness of the labor market on island j, which equals $V_{j,t} / U_{j,t}$.

Wholesale firms seek to maximize the present value of profits subject to the law of motion for employment, which implies maximizing the following objective function over current period vacancies and employment:

$$\begin{aligned} & E_t \sum_{i=0}^{\infty} \beta^i \frac{\lambda_{t+i}}{\lambda_t} \left[P_{j,t+i}(N_{j,t+i} - \kappa V_{j,t+i}) - W_{j,t+i} N_{j,t+i} \right] \\ & \qquad - E_t \sum_{i=0}^{\infty} \beta^i \frac{\lambda_{t+i}}{\lambda_t} \left[\omega_{j,t+i}(N_{j,t+i} - \phi(N_{j,t+i-1} + V_{j,t+i-1} m \theta_{j,t+i-1}^{-\xi})) \right]. \end{aligned}$$

The first order condition for vacancy creation is given by:

$$P_{j,t}\kappa = E_t \beta \frac{\lambda_{t+i}}{\lambda_t} \phi \omega_{j,t+1} m \theta_{j,t}^{-\xi}. \tag{26}$$

The first order condition for labor demand is given by:

$$P_{j,t} - W_{j,t} - \omega_{j,t} + E_t \beta \frac{\lambda_{t+i}}{\lambda_t} \phi \omega_{j,t+1} = 0. \tag{27}$$

The costate variable ω_t has an interpretation as a bargaining surplus in the context of a standard search and matching model. Condition (26) constitutes the free entry condition; and (27) defines the surplus.

Island-level production is given by the proportion of labor input which adds value:

$$Y_{j,t} = N_{j,t} - \kappa V_{j,t} \,. \tag{28}$$

Aggregators work on the main island; they sell their output as before, and they buy island-level input competitively. Their production function is given by the CES aggregator:

$$Y_t = \left(\sum_{j=1}^{J} (a_{j,t})^{\frac{1}{\rho}} (Y_{j,t})^{\left(\frac{\rho-1}{\rho}\right)} \right)^{\left(\frac{\rho}{\rho-1}\right)} . \tag{29}$$

Their demand for island-level output takes the form:

$$Y_{j,t} = (P_{j,t})^{-\rho} a_{j,t} Y_t \,. \tag{30}$$

2.6.3 Calibration and numerical results

The calibration of the model does not matter, so long as the model has a well-defined determinate nonexplosive solution. I choose a monthly calibration with $\beta = 0.99^{1/3}$; $\rho = 2$; $\chi = 2$; $\phi = 0.97$; and $N = 0.94$. The worker finding rate for a vacancy is 70% per month, and vacancy posting costs equal 1% of output, with a cost parameter γ of 100. All prices equal one in steady state; all wages equal W; and $L_j = a_j = Y_j/Y = N_j/N = U_j/U = V_j/V$ for all j. The matching function parameter ξ equals 0.4. Sector 1 starts out at 10% of aggregate employment.

The top panel of Figure 2 shows what happens to aggregate employment and sectoral employment after a -10% shock to a_1. This is meant to capture the effects of a relatively severe (but clean) construction bust, assuming that for every one worker employed in construction there is one more worker employed in construction-related activity. Aggregate employment does not move at all, while employment in both sectors moves in opposite directions. These effects completely offset each other as in the flexible model, and in addition, output does not move. The intuition underlying the flexible model holds for search and matching models as well; a clean reallocative shock has no aggregate effects. Employment in both sectors takes some time to move to its long run value, and employment dynamics heavily reflect population dynamics. The bottom panel of Figure 2 shows what happens in sector 1. Sector 1 behaves in much the same way as it does in the flexible model. These results are robust to different parameterizations of the model and to different specifications of the dynamics underlying island population.

2.7 A dirty reallocative shock: An aggregate shock in disguise

There is no reason as a statistical matter to expect reallocative shocks in the data to be entirely clean. Not every negative shock to the absolute productivity of construction workers must increase the absolute productivity of nurses by an offsetting amount. It is much easier to think of a shock which affects construction workers without offering an offsetting benefit for nurses—a severe storm, the collapse of an investment bubble, or a credit-driven housing bust, for example. This section shows that it is possible to model a "dirty" reallocative shock as the convolution of a "clean" reallocative shock and an aggregate shock in the flexible model.

The coefficients from the CES aggregator (9) no longer need to sum to one in the case that reallocative shocks are dirty. Instead, they sum to A_t which varies over time. It is possible to rewrite (9) as:

$$Y_t = (A_t)^{\frac{1}{\rho-1}}\left(\sum_{j=1}^{J}\left(\widetilde{a}_{j,t}\right)^{\frac{1}{\rho}}\left(L_{j,t}h_{j,t}\right)^{\left(\frac{\rho-1}{\rho}\right)}\right)^{\left(\frac{\rho}{\rho-1}\right)}, \tag{31}$$

where the change in the distribution of $\widetilde{a}_{j,t}$ (which sum to one) gives the clean part of a dirty reallocative shock. The change in A_t gives the overall effect of the shock on productivity. A_t is equivalent to an aggregate shock which accompanies a reallocative shock, while $\widetilde{a}_{j,t}$ gives the reallocative shock which accompanies an aggregate shock. In the construction worker example, a negative shock may hit construction without a positive shock to nursing. A_t would shrink. Workers would then leave the construction industry in response to the construction bust, and output in general would fall because nursing has not become more lucrative.

The degree to which reallocative shocks are clean or dirty is an empirical matter. Construction busts need not in general correspond with nursing booms. Even in the case of a dirty reallocative shock, the model suggests that human capital mismatch is not to blame for the performance of the economy after the shock. The data can say how clean or dirty reallocative shocks are, but there is not much that the data can say about what causes reallocative shocks to be dirty. Theory suggests that sticky human capital is unlikely to be the channel through which construction busts have a contractionary effect.

3. The state space model

3.1 Overview of the main components of the model

The state space model generalizes the theoretical model and makes it possible to estimate the degree to which reallocative shocks are clean or dirty. The main object of interest is a time-varying reallocation process S_t. When S_t is high, the economy experiences a wave of long run sectoral reallocation. When S_t is low, the economy experiences less reallocation. S_t feeds into the economy in several ways. Most importantly, when S_t is high, the cross-sectional variance of long run employment growth at the sectoral level is high; this means that workers subsequently find themselves moving across sectors at a faster rate. In addition, S_t can directly affect the aggregate economy—it can have an effect on trend productivity, the business cycle, and the natural rate of unemployment. S_t is unobserved by the econometrician and must be estimated along with its effects. To the degree that S_t is related to aggregate output and unemployment, it can be said that reallocative shocks are dirty and that reallocative shocks and aggregate shocks are statistically related.

The statistical model has three component blocks which govern output, employment, and unemployment. Throughout the analysis, output and employment are expressed in natural logarithms relative to the working-age population, while unemployment is expressed as a percent of the civilian labor force. The first block of the model governs the evolution of output, which has three components. Output has a nonstationary long run trend z_t^y which is governed by the long run levels of productivity and employment. Output also has a persistent but stationary component w_t^y which indexes the state of the business cycle. The final component of output is the idiosyncratic noise term c_t. The observation equation for log output has these three components:

$$y_t = z_t^y + w_t^y + c_t. \quad (32)$$

The second block of the model governs unemployment dynamics; unemployment has two components. It has a nonstationary permanent component z_t^u and a stationary temporary component w_t^u which may be related to the state of the business cycle. The component z_t^u equals the natural rate of unemployment, or the rate of unemployment that the economy would tend toward on average if all shocks were temporary in nature. Unemployment has the following observation equation consisting of its two components:

$$u_t = z_t^u + w_t^u. \quad (33)$$

Employment at the sectoral level has three components. The first two components are the common long run component of employment x_t^n, and a full array of nonstationary idiosyncratic trends $z_{i,t}^n$. Shocks to these idiosyncratic trends have a common stochastic volatility component indexed by S_t, which has a mean log of zero. When S_t is large, the economy undergoes a burst of long run reallocative activity, and this shows up as higher levels of cross-sectional dispersion in long run sectoral growth. The third component of sectoral employment is its stationary component $w_{i,t}^n$, which is related to the state of the business cycle. Written as an observation equation, the aggregate trend, idiosyncratic trend, and cyclical component of employment respectively sum up to observed log employment for sector *i*:

$$n_{i,t} = x_t^n + z_{i,t}^n + w_{i,t}^n. \quad (34)$$

3.2 The state space model in detail: Laws of motion for output

Output consists of its long run, cyclical, and noise factors. Shocks are independent and identically distributed across time. The change in aggregate long run output is governed by a drift coefficient, the change in the aggregate employment factor, and reallocation, with the residual representing a pure productivity shock:

$$\Delta z_t^y = \mu^y + \Delta x_t^n + \delta_{S,zy}\log(S_t) + \varepsilon_t^{zy}, \qquad \text{where } E\left[\left(\varepsilon_t^{zy}\right)^2\right] = \sigma_{zy}^2. \tag{35}$$

Including reallocation on the right hand side of (35) gives one test of the relationship between reallocation and aggregate productivity A_t as shown in (31). To the extent that reallocation is dirty in the long run, the coefficient $\delta_{S,zy}$ will differ from zero, and reallocative shocks will be related to aggregate long run productivity.

The cyclical output factor depends on its own lags. It can also respond to the process governing sectoral reallocation and to the process governing shocks to productivity. The parameter $\delta_{S,wy}$ captures the degree to which sectoral reallocation is recessionary in its direct effects. The parameter $\delta_{zy,wy}$ captures the effect of a shock to productivity on the cycle. Cyclical output evolves according to the following equation:

$$w_t^y = \sum_{p=1}^{P}\rho_p^{wy} w_{t-p}^y + \delta_{S,wy}\log(S_t) + \delta_{zy,wy}\left(\delta_{S,zy}\log(S_t) + \varepsilon_t^{zy}\right) + \varepsilon_t^{wy},$$

$$\text{where } E\left[\left(\varepsilon_t^{wy}\right)^2\right] = \sigma_{wy}^2. \tag{36}$$

Including reallocation on the right hand side of (36) gives another test of the cleanliness or dirtiness of reallocative shocks. To the extent that reallocation is dirty in its direct short run effects, the coefficient $\delta_{S,wy}$ will differ from zero. Reallocation can also have an indirect effect if productivity is systematically related to the cycle. The total effect of reallocation on the cycle is given by the composite coefficient $\delta_{S,wy} + \delta_{zy,wy}\delta_{S,zy}$. A negative value of that composite coefficient would indicate that reallocation tends to be recessionary on average, or in other words, that reallocative shocks tend to be dirty in the short run.

The idiosyncratic output factor (which mainly contains measurement error but also temporary shocks such as weather shocks or small strikes) is given by a white noise term:

$$c_t = \varepsilon_t^c, \qquad \text{where } E\left[\left(\varepsilon_t^c\right)^2\right] = \sigma_c^2. \tag{37}$$

Observed log output equals the sum of these three components given in (32)—trend, cycle, and error:

$$y_t = z_t^y + w_t^y + c_t .$$

3.3 The state space model in detail: Laws of motion for unemployment

Unemployment consists once again of its long run and short run factors. The aggregate long run unemployment factor (the natural rate) is given by:

$$z_t^u = z_{t-1}^u + \delta_{S,zu} \log(S_t) + \varepsilon_t^{zu} , \qquad \text{where } E\left[\left(\varepsilon_t^{zu}\right)^2\right] = \sigma_{zu}^2 . \qquad (38)$$

The theoretical model predicts that the coefficient $\delta_{S,zu}$ should equal zero. If reallocation is dirty, it should affect long run productivity and possibly have a short run effect as well, but it should not affect the natural rate of unemployment, which in the long run is governed by preferences over leisure. By contrast, much of the recent commentary blames a possible rise in the natural rate of unemployment on sectoral reallocation; such a situation would coincide with a positive value for $\delta_{S,zu}$. The estimated distribution for that coefficient will show the degree to which sectoral reallocation coincides with changes in the natural rate.

The short run unemployment factor may vary according to the cycle and is given by:

$$w_t^u = \sum_{p=0}^{P} \alpha_p^u w_{t-p}^y + \varepsilon_t^{wu} , \qquad \text{where } E\left[\left(\varepsilon_t^{wu}\right)^2\right] = \sigma_{wu}^2 . \qquad (39)$$

The factor loadings α_p^u capture the contemporaneous and lagged effects of the state of the business cycle on unemployment. Once again, observed unemployment equals the sum of its long run and short run factors given in (33):

$$u_t = z_t^u + w_t^u .$$

Specifying unemployment dynamics this way makes it easy to talk about trend versus cyclical unemployment. Much of the discussion about a possible rise in trend unemployment due to reallocation is a discussion about the dynamics of the long run factor z_t^u. However, a dirty reallocative event can also affect cyclical unemployment w_t^u through its effect on cyclical output w_t^y, even if it leaves z_t^u unchanged.[3] In that case, a reallocative shock does not cause the "new normal" unemployment rate to rise; instead, it simply contributes to a recession.

3.4 The state space model in detail: Laws of motion for sectoral employment

Sectoral employment consists of an aggregate trend factor, an idiosyncratic trend factor, and a cyclical factor with variable factor loadings. All structural shocks are iid across sectors and across time unless otherwise noted. The aggregate long run employment factor reflects shocks to the long run level of employment which have a common effect across sectors; this factor will primarily consist of changes in labor force participation and in the coverage of the establishment survey.

The aggregate trend employment factor follows a random walk and is given by:

$$x_t^n = x_{t-1}^n + \delta_{u,n}(z_t^u - z_{t-1}^u) + \varepsilon_t^{xn}, \qquad \text{where } E\left[\left(\varepsilon_t^{xn}\right)^2\right] = \sigma_{xn}^2 . \tag{40}$$

Changes in trend employment rates are a random walk but one might naturally expect trend employment to be related to trend unemployment. The degree to which an innovation in trend unemployment affects trend employment is given by the coefficient $\delta_{u,n}$, and that coefficient should be relatively close to negative one.

Each idiosyncratic long run employment factor reflects changes to trend employment across industries which are not related to either the common trend or cycle. Sectors may have their own long run growth intercepts given by μ_i^n; these differing coefficients capture the fact that manufacturing in general has shrunken over time, while services have expanded, as a share of employment. The idiosyncratic employment trends follow the law of motion:

$$\Delta z_{i,t}^n = \mu_i^n + \rho_i^{zn} \Delta z_{i,t-1}^n + \varepsilon_{i,t}^{zn}, \qquad \text{where } E\left[\varepsilon_t^{zn} \varepsilon_t^{zn'} \mid S_t\right] = \Sigma_{zn} S_t . \tag{41}$$

The theoretical model has strong predictions about trend employment at the sectoral level. It predicts that shocks to trend employment should covary negatively by sector, which would

[3] Trend unemployment and structural unemployment are not necessarily the same thing; one refers to the time-series properties of unemployment and the other refers to the economic causes of unemployment.

show up as negative off-diagonal elements of Σ_{zn}.[4] It also predicts that employment growth should be positively autocorrelated as in Figure 1. Equation (22) from the theoretical model, taken in first differences, actually suggests that sectoral employment growth should follow an ARMA(1,1) process. The use of an AR(1) is an approximation which is reasonable when the structural parameters ρ and χ are large. Attempts to introduce the MA term into the estimation procedure can cause numerical problems; Kleibergen and Hoek (2000) discuss the issue of numerical instability and the global identification of ARMA models in depth.

Each idiosyncratic short run employment factor may respond to the business cycle with different factor loadings and a different lag structure. These short run employment factors are given by:

$$w_{i,t}^{n} = \sum_{p=0}^{P} \alpha_{p,i}^{n} w_{t-p}^{y} + \varepsilon_{i,t}^{wn}, \qquad \text{where } E\left[\left(\varepsilon_{i,t}^{wn}\right)^2\right] = \sigma_{wn,i}^2. \tag{42}$$

The coefficients $\alpha_{p,i}^{n}$ capture the fact that the business cycle affects different sectors of the economy with differing degrees of intensity and possibly different lags; for instance, durable goods manufacturing is known to be very responsive to the cycle while government employment is much less responsive.

Once again, the three employment factors add up to the observed log of employment in each sector as shown in equation (34):

$$n_{i,t} = x_t^n + z_{i,t}^n + w_{i,t}^n.$$

An exploratory analysis of the model does not reveal any posterior autocorrelation in sectoral dispersion, so it is reasonable to assume that the volatility process underlying sectoral growth is independent and identically distributed over time. S_t can be modeled as independently and identically distributed according to a lognormal distribution, with a constant variance:

$$E\left[\log(S_t)^2\right] = \sigma_S^2. \tag{43}$$

[4] Actually, the model requires that Σ_{zn} be singular, but the employment data do not cover the entire economy.

The mean of $\log(S_t)$ is not identified since it is possible to scale the matrix Σ_{zn} up or down inversely with S_t while leaving the likelihood unchanged. To normalize S_t, it is convenient to assume that $\log(S_t)$ has a mean of zero; this makes it easy to work with the other equations where $\log(S_t)$ appears. It is necessary to make one additional normalizing assumption. I normalize the initial value of the common long run level factor x_1^n to zero since there are fourteen unit roots in the employment block but only thirteen nonstationary observables. Doing this gives the series x_t^n an interpretation as the cumulative level of excess employment growth from the beginning of the sample.

4. Data and priors

Sectoral establishment employment data come from the BLS's Current Employment Statistics program, broken out by the NAICS. The data cover thirteen sectors: Construction, durable goods manufacturing, nondurable goods manufacturing, wholesale trade, retail trade, transportation and utilities, leisure and hospitality, information, financial activities, professional and business services, education and health services, other services, and government. I omit mining and logging from the thirteen-sector model because that sector is small, volatile, and strike-prone. I manually smooth out the effects of large strikes, weather events, and census workers from the individual employment series, and I begin in 1960 in order to avoid having to manually correct for the extremely large strikes of the 1950s.

Data for the unemployment rate come from the CPS and data for GDP come from the NIPA; these are both economywide measures. The employment and output series are divided by the civilian noninstitutional population 16 and over, smoothed for breaks. I also estimate the state space model for five large sectors. For the 5-sector model I collapse mining and logging, durable manufacturing, and nondurable manufacturing into the production sector; construction stands on its own; I collapse wholesale trade, retail trade, leisure and hospitality, and transportation and utilities into the trade, leisure, and transportation sector; I collapse information, financial activities, and professional and business services into the financial and business services sector; and I collapse education and health services, other services, and government into the public and private services sector. I choose these sectoral classifications to approximate the industrial classification system used by foreign countries, and this classification also closely approximates the SIC. I place information into the financial and business services sector because the more volatile components of the information sector are in that supersector on an SIC basis.

Table 1 shows the prior distributions used in the estimation. Where possible, I use natural conjugate priors which are as uninformative as possible. The variance terms all have an inverse gamma or inverse Wishart prior distribution. The prior distributions on the variance terms are rather loose and they are very rough guesses as to the order of magnitude of these objects, with the prior variance on the variance of log(S_t) set large enough (to 4) so that observed sectoral dispersion lines up reasonably well with estimated dispersion. I use a tighter prior on the variances of the short run idiosyncratic employment terms since weather events, seasonal adjustment errors, benchmark errors, and strikes result in some fluctuations in sectoral employment which I do not wish to attribute to changes in the trend. I use weakly informative priors on the other variances so that the estimated variances stay within the numerical precision of the machine and away from zero. The results are robust to different priors on the variances, though if the variances on short run sectoral employment become too small, the model slightly overfits any observed blips in sectoral employment and attributes them to changes in trend.

I use two lags for each of the equations governing the cyclical components of output, employment, and unemployment. The data clearly indicate that the one lag is insufficient at describing cyclical dynamics; the estimated coefficients using two lags consistently give a hump-shaped response of the cycle to a cyclical shock. Moving beyond two lags does not yield a substantially different picture of business cycle dynamics than staying with two lags. I therefore use two lags in all of the estimated laws of motion. The system is estimated using the Markov Chain Monte Carlo (MCMC) algorithm discussed in Appendix E. I take 200,001 draws and discard the first 10,000, using the remainder to calculate posterior statistics for the parameters and unobserved processes of interest.

5. Estimation results

Table 2 shows the posterior median values of the major coefficients of interest—the coefficients governing the contribution of long run reallocation to the natural rate of unemployment and the contribution of reallocation to the cycle, respectively. Reallocation has no obvious effect on the natural rate of unemployment; the median estimates for the effect of reallocation $\delta_{S,zu}$ are very close to zero in both cases. The effect of reallocation on productivity given by $\delta_{S,zy}$ is also ambiguous; in the 13-sector model it appears to be slightly negative while it appears to be very slightly positive in the 5-sector model. Neither model estimates $\delta_{S,zy}$ with a great degree of statistical confidence. There is moderate evidence in both cases that reallocation is contractionary in the short run (given by the composite coefficient $\delta_{S,wy} + \delta_{S,zy}\delta_{zy,wy}$). That coefficient is negative with 95.5% confidence in the 13-sector model and with 89.6% confidence in the 5-sector model. The theoretical model and the data agree that reallocation has little to no effect on the natural rate of unemployment, and the effect of reallocation on productivity is ambiguous. There does seem to be moderate evidence that reallocation is dirty in the short run; shocks which are reallocative in their effects appear to be somewhat recessionary.

5.1 Parameter estimates: 13-sector model

Tables 3a and 3b show posterior percentiles and means for the model parameters for the United States using the 13-sector breakdown. Looking at the top of Table 3a, one can see that the business cycle clearly shows hump-shaped dynamics. The cyclical component of output has a total quarterly persistence of 0.976, with a 95% confidence interval running from 0.952 to 0.995.[5] In other words, business cycles are extremely persistent. Looking at factor loadings, unemployment moves about -0.78 to 1 with output over the cycle. Construction and durable goods manufacturing are the most cyclically sensitive sectors in terms of employment, with cyclical factor loadings between three and four. Government is the least cyclically sensitive sector, with a factor loading around 0.10 which cannot be distinguished from zero with much statistical confidence. In the long run, establishment employment falls by 0.78 percent for each percentage point increase in the natural rate of unemployment based on the coefficient $\delta_{u,n}$. Productivity growth also leads to a cyclical expansion based on the coefficient $\delta_{zy,wy}$; a one percent permanent increase in productivity causes cyclical output to rise by about 0.185 percent. A positive shock to productivity is slightly expansionary in the short run as well as in the long run.

Trend employment growth (at the top of Table 3b) within each sector is highly persistent, with idiosyncratic construction growth showing a persistence of 0.84 and the other idiosyncratic persistence coefficients running from 0.57 to 0.93. The theoretical and

[5] Technically I am abusing language in referring to a Bayesian credible interval as a “confidence interval”.

empirical models agree on the fact that the adjustment of sectoral employment to its new long run level proceeds slowly and that adjustment costs may determine the speed of adjustment. There do seem to be major frictions in the economy with regard to reallocating workers across sectors in response to sectoral shocks. The theoretical model and the state space model are consistent with one another when it comes to the persistence of sectoral employment growth.

Table 4 shows the covariance matrix of the long run sectoral shocks expressed as posterior correlation coefficients. The posterior mean of that matrix does have several interesting off-diagonal elements. Growth in construction is negatively correlated with growth in durable goods manufacturing (-0.58) and in nondurable goods manufacturing (-0.44), and also with growth in education and health services (-0.31). The demand for construction workers and nurses (or teachers) does actually seem to be negatively correlated in the long run. Growth in construction is positively related to growth in financial activities (+0.54). Shocks to both durable and nondurable manufacturing are positively related to each other as well as to growth in wholesale trade. The main negative off-diagonal relationships involve the construction sector; the parameter estimates seem to indicate that construction is a particularly important sector with respect to sectoral reallocation.

5.2 The effect of reallocation in more detail: The 13-sector model

Looking at the middle of Table 3a, it appears that reallocation has little quantitative effect on the natural rate of unemployment based on the coefficient $\delta_{S,zu}$. The posterior distribution gives a median and mean effect of reallocation on the natural rate of unemployment which is very small, on the order of 0.0002, and not distinguishable from zero. Given a standard deviation for the reallocative process of 0.64, this means that a one standard deviation reallocative event raises the natural rate by close to 0.01 percentage points. In the median scenario, reallocation contributes just over two percent of the variance to innovations in the natural rate, with the posterior distribution covering the range from about zero to one fifth with 95% confidence. There is simply no statistical evidence based on the thirteen-sector model that sectoral reallocation affects the natural rate of unemployment in any meaningful way. The effect of reallocation on long run productivity is ambiguously negative. Based on a coefficient $\delta_{S,zy}$ of -0.0037, a one standard deviation reallocative shock lowers long run productivity by 0.24 percent. Reallocation accounts for 20% of the variance of productivity growth over the long run at the posterior median, with an extremely wide degree of confidence running from almost none to about two thirds of the variance.

When it comes to the cycle, there is moderate statistical evidence that reallocation is contractionary. The composite effect of reallocation on the cycle is given by $\delta_{S,wy} + \delta_{zy,wy}\delta_{S,zy}$, which equals -0.0012 in the median case. A one standard deviation reallocative shock results in a shock to cyclical output of -0.08%. Even though there is moderate statistical evidence that reallocation is contractionary, the actual contractionary effect of reallocation over the cycle appears to be small. Despite the degree of statistical confidence, reallocation accounts for only about 12% of the variance of the cycle in the median case. It is difficult to say that reallocative shocks have an economically large effect on aggregate output, employment, and unemployment at a cyclical frequency, though they do appear to have some effect. To the extent that reallocative shocks are dirty, they appear to be more like dishwater and less like nuclear waste.

5.3 The historical behavior of reallocation and the natural rate in the 13-sector model

Figure 3 shows the estimated rate of reallocation S_t from 1960 through 2011 along with two related dispersion measures. I also plot two other long run dispersion estimates for the long run employment components, scaled to have a geometric mean of one. The traditional Lilien (1982) measure equals the variance of unexplained sectoral long run employment growth, weighted by observed employment shares. I also plot my own measure which is the maximum likelihood estimate of dispersion conditional on dispersion having no effect on aggregates. It equals the variance of unexplained sectoral employment growth weighted by the inverse of the covariance matrix Σ_{zn}. Both measures would weight small volatile sectors with small weights, while the latter would also slightly downweight volatile sectors such as construction. I calculate the weighted measure because it should closely resemble my estimates of S_t; it serves as a good diagnostic regarding the identification of reallocative events. In addition, Figure 4 shows the estimated natural rate of unemployment along with the actual rate and 95% posterior error bands.

The major bursts of reallocation occur throughout the sample; they are not limited to the 2000s. The most notable waves of reallocation occurred in 1966, 1970, 1974, 1982, 1990-91, 2001, and 2008. The 1974, 1990, 2001, and 2008 reallocative events were particularly notable events. The 1966 episode appears to be a clean reallocative episode which did not accompany a recession. The other episodes have accompanied recessions. These episodes do not appear to have systematically coincided with an upward move in the natural rate of unemployment; the natural rate does not seem to show much volatility in general. The natural rate is measured with a fairly wide degree of error because it is difficult to econometrically distinguish a persistent cycle from a very persistent trend. Nonetheless, it appears that reallocative events are associated with recessions and not with movements in the natural rate.

Figure 5 shows the posterior mean trends for log employment across each of the 13 sectors. By far the most volatile sector is construction, which goes through several low-frequency boom and bust cycles independently from the rest of the economy. There were a large construction bust during the mid 1960s, a large boom and then bust during the early to mid 1970s, a smaller construction bust in the early 1980s and a larger one in the early 1990s, and the large construction boom and bust cycle of the 2000s. The 2000s do not contain the first large construction boom and bust in history. All but one of the reallocative episodes found in the data coincides with a construction boom or bust. The other notable reallocative event, in 2001, appears primarily driven by a bust in the information and the professional and business services sectors. In general, though, the measure of reallocation S_t appears to be an excellent measure of volatility in trend employment in the construction sector.

Taking the evidence together, some clear facts about reallocative shocks emerge. Shocks to sectoral reallocation primarily affect reallocation into or especially out of the construction sector, though the technology bust of the early 2000s also shows up in the measured series of shocks. These shocks appear to be mildly contractionary; they do not contribute to changes in the natural rate of unemployment, but they are recessionary. While reallocative shocks may be mildly dirty, their aggregate effects are not particularly large. Put another way, the data indicate that construction and technology busts (i.e. investment busts) are associated with recessions, though the data cannot distinguish whether an investment bust causes a recession or vice versa.

5.4 Comparing the results from the 13-sector and 5-sector models

The conclusions from the 13-sector model also hold for the 5-sector model in detail; they are not an artifact of the sectoral breakdown. Table 5 indicates that the point estimates and degree of confidence surrounding the effects of reallocation fall slightly in comparison with the larger model since reallocation is less well-identified with a smaller number of sectors. However, the quantitative estimates are very similar. Production industries and construction are still the most cyclically sensitive sectors, and the service sectors are the least sensitive. In the median scenario, reallocative shocks contribute just less than two percent of the variance to innovations in the natural rate of unemployment, and they contribute just over 10% of the variance to innovations in long run productivity. Reallocative shocks may be mildly contractionary at a cyclical frequency, but they contribute just over 13% of the variance to innovations in the business cycle.

As with the 13-sector model, the 5-sector model indicates that reallocative shocks are basically shocks which affect the distribution of workers between construction and manufacturing. Table 6 shows the posterior correlations between the various long run sectoral shocks. Shocks to construction and the other production industries are highly negatively correlated over the long run (-0.53), while most other industries are either orthogonal or positively correlated. Altogether, these results for the 5-sector model are almost the same as the results for the 13-sector model. From a statistical standpoint, it does not appear that there is much to lose by aggregating the economy into five sectors. Both the 13-sector model and the 5-sector model indicate that sectoral reallocation does not substantially contribute to unemployment or productivity dynamics in the long run, while both models give moderate evidence of a slight recessionary effect of reallocative shocks.

A look at Figures 6 and 7 shows why the two models behave so similarly. The higher level of aggregation still captures the movements in construction trends, which are the same as before. It also captures the information and business services bust of the early 2000s. All of the episodes which show up in the 13-sector model can be identified using the 5-sector model, albeit with differing degrees of confidence. The reallocative measures based on the 5-sector model in Figure 5 seem to emphasize the relatively clean reallocative event of 1966 to a greater degree than the reallocative measures based on the larger model, and they do not indicate that there was a reallocative event in 1970. The remaining events share the same dates as those estimated under the 5-sector model. The estimates of S_t from the different models have a contemporaneous correlation of +0.52 which rises to +0.63 when taken as a four-quarter moving average.

The results from the 5-sector model indicate that the results from the 13-sector model are robust to a different level of aggregation, which is useful when looking at historical or cross-country evidence. The choice of a level of aggregation does not matter much, since reallocative activity shows up the strongest in the construction sector. Reallocative shocks, for the most part, are really investment busts, and they appear to be mildly dirty at the cyclical frequency. Reallocative shocks do not have obvious permanent effects in the aggregate; their main aggregate effects come at cyclical frequencies.

5.5 The Great Recession in perspective

Based on the parameter estimates presented in this paper, it seems improbable that sectoral reallocation is to blame for the persistently high unemployment experienced by the United States after the Great Recession. The data provide no statistical evidence that reallocation drives the natural rate, and the late 2000s saw a large but not extraordinary degree of reallocation in comparison with other historical episodes. The model does indicate that a reallocative shock (or investment bust) might have played some role in the cyclical downturn of 2008-2009 but it does not explain the persistent slump which has followed.

Over the last four years of the sample running from 2007.II through 2011.I, the 13-sector model does not give a strong indication that the natural rate of unemployment has risen. The median estimate of the change in the natural rate is +0.2% which has a 68% chance of being above zero. Changes between -0.8% and +1.3% would fit within a posterior 95% confidence interval. The 5-sector model gives a very similar result—a posterior 95% confidence interval would run from -0.9% to +1.5% with a median of +0.3%. Statistically it is difficult to conclusively say that the natural rate has risen based on the estimates given by the state space model. The evidence does indicate against a large rise in the natural rate since the natural rate does not usually move by much, but small movements of the natural rate in either direction cannot be ruled out.

There is more intuition as to why the point estimate of the natural rate in the United States does not appear to have moved by much. Figure 8 shows the posterior cyclical employment factors for the United States from the 5-sector model; the results from the 13-sector model are similar. The individual sectors show remarkable consistency in their cyclical behavior—when output and employment are low, production industry employment and construction employment show the greatest negative deviation from trend, while employment in the various service sectors falls by much less. Outside of the construction sector, sectors have behaved during and after the Great Recession as they have in response to past recessions. Much of the fall in construction employment can be blamed on the recession itself and not on a change in trend construction employment (which has gone in posterior geometric mean from 5.4% of total trend payroll employment in 2006 and early 2007 to 4.5% at the beginning of 2011). The behavior of the other sectors is typical of a serious recession and a slow recovery rather than a broad, persistent, economywide decrease in employment. Persistently low employment in the more acyclical of the service sectors would be a sign of higher trend unemployment and lower trend employment, but as of the first quarter of 2011, that has not yet occurred.

It is possible to estimate the rough statistical effect of reallocative shocks during the period of the Great Recession. Figure 9 plots the fluctuations in unemployment observed in the United States since the beginning of 2007 which have originated in reallocative shocks, based on the posterior median estimates of the 13-sector model. The narrow line shows the contributions of S_t to the natural rate (i.e. trend) of unemployment, and the heavy dashed line shows the total contribution of S_t to both the natural rate and the cycle. Reallocation has had almost no effect on the trend rate of unemployment since the beginning of 2007; in other words, the construction bust should not have led to an appreciably higher "new normal" rate of unemployment. The construction bust may have shown up somewhat in the cycle, accounting for a maximum of 0.5 percentage points of the unemployment gap as of the end of 2009 and 0.3 percentage points as of the beginning of 2011. Reallocative shocks appear to account for only a small portion of the rise in cyclical unemployment since 2007; the vast majority of the high unemployment after the Great Recession appears related to other cyclical factors. While reallocative shocks might have had a contractionary effect over the course of the Great Recession, the magnitude of their effect appears to be relatively small.

6. Conclusion

When long run sectoral reallocation, the natural rate of unemployment, and the business cycle are estimated consistently using sectoral employment data, the results do not show a clear effect of long run reallocative shocks on either the natural rate of unemployment or on trend productivity. There is moderate evidence that reallocative shocks are related to recessions, but the quantitative contribution of reallocative shocks to the business cycle appears to be small. These findings are robust to the level of sectoral aggregation. The measured series of reallocative shocks basically measures construction busts, with the addition of the technology bust of 2001. These busts have accompanied recessions, though their statistical contribution to the overall cycle is modest. The lion's share of unemployment dynamics come from shocks unrelated to reallocation; this statement also holds true for the period surrounding the Great Recession.

Viewed from the perspective of theory, the estimates indicate that reallocative shocks are somewhat dirty. Mismatch should have at most a second-order direct effect on aggregate employment and unemployment in a broad class of labor market models, based on the intuition that small deviations of economic aggregates from their long run optimum should have a first order effect of zero. In the context of human capital mismatch, the contractionary effect of sectoral busts should come from the direct aggregate effects of the shock which caused the bust in the first place, rather than from the human capital mismatch which results when workers face a cost of moving between sectors. A shock which causes an investment bust may be viewed as an aggregate shock with reallocative consequences.

Appendix A: The theoretical model with no insurance

It is not difficult to show that in an economy without functioning credit markets, the result regarding the neutrality of mismatch holds. The individual objective for a resident of island *j* now takes the reduced form:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\ln(C_{j,t+i}) - v(h_{j,t+i}) - \gamma_j^{(portion)}(\{\{L_{j,t+i-k}\}_{j=1}^{J}\}_{k=0}^{-\infty}) \right]$$
$$- E_t \sum_{i=0}^{\infty} \beta^i \left[\lambda_{j,t+i} \left(C_{j,t+i} - W_{j,t+i} h_{j,t+i} \right) \right].$$

The first order conditions in labor supply for each island take the following form:

$$v'(h_{j,t}) = \lambda_{j,t} W_{j,t} \,, \tag{A1}$$

where

$$\lambda_{j,t} = 1/C_{j,t} \,. \tag{A2}$$

Summing up across j and weighting by the islands' population and labor input yields the following expression:

$$\sum_{j=1}^{J} L_{j,t} h_{j,t} v'(h_{j,t}) = \sum_{j=1}^{J} \lambda_{j,t} L_{j,t} h_{j,t} W_{j,t} \,. \tag{A3}$$

The resource constraint now implies that household consumption equals the household wage bill, so the right hand side simplifies considerably:

$$\sum_{j=1}^{J} L_{j,t} h_{j,t} v'(h_{j,t}) = \sum_{j=1}^{J} L_{j,t} \,. \tag{A4}$$

Taking first order deviations of actual labor variables from a perfectly matched state, one can obtain the expression:

$$\sum_{j=1}^{J} L_j \left(\hat{L}_{j,t} + \phi \hat{h}_{j,t} \right) = \sum_{j=1}^{J} L_j \hat{L}_{j,t} = 0 . \tag{A5}$$

where ϕ is the same constant as before. Eliminating the deviations of population ensures that the first order effect of labor market mismatch on total labor input is still zero. However, the effect of mismatch on welfare becomes radically different—there are untapped opportunities to expand welfare by finding ways to insure against unemployment.

Appendix B: Solution to the household block of the model with additional quadratic adjustment costs for employment at the firm level

This appendix shows that the results from the paper are robust to the inclusion of quadratic adjustment costs at the firm level, given by δ. These costs may include vacancy posting costs, firing costs, job creation costs, training costs, and the like. If adjustment costs are quadratic and labor supply is isoelastic, then the objective for the household sector takes the following form:

$$
\begin{aligned}
& E_t \sum_{i=0}^{\infty} \beta^i \left[\ln(C_{t+i}) - \sum_{j=1}^{J} L_{j,t+i} v(h_{j,t+i}) \right] \\
& \quad - E_t \sum_{i=0}^{\infty} \beta^i \left[\sum_{j=1}^{J} \frac{\gamma}{2L_{j,t}} (L_{j,t+i} - L_{j,t+i-1})^2 + \sum_{j=1}^{J} \frac{\delta}{2L_{j,t} h_{j,t}} (L_{j,t+i} h_{j,t+i} - L_{j,t+i-1} h_{j,t+i-1})^2 \right] \\
& \quad - E_t \sum_{i=0}^{\infty} \beta^i \left[\lambda_{t+i} \left(C_{t+i} - \sum_{j=1}^{J} L_{j,t+i} W_{j,t+i} h_{j,t+i} \right) + \mu_{t+i} \left(\sum_{j=1}^{J} L_{j,t+i} - 1 \right) \right].
\end{aligned}
$$

The costs given by γ denote the cost of moving people from island to island, and the costs given by δ denote the cost to adjusting employment at the firm level. $L_{j,t}$ is in the denominator of both costs in order to make total adjustment costs paid by the household sector proportional to the size of the adjustments made in each sector.

The first order condition in labor supply for each island now takes the following form:

$$
\begin{aligned}
\lambda_t W_{j,t} &= v'(h_{j,t}) + \frac{\delta}{L_{j,t} h_{j,t}} (L_{j,t} h_{j,t} - L_{j,t-1} h_{j,t-1}) - \frac{\delta}{2L_{j,t} h_{j,t}^2} (L_{j,t} h_{j,t} - L_{j,t-1} h_{j,t-1})^2 \\
& \quad - \beta E_t \frac{\delta}{L_{j,t+1} h_{j,t+1}} (L_{j,t+1} h_{j,t+1} - L_{j,t} h_{j,t}) . \qquad \text{(B1)}
\end{aligned}
$$

Summing up the equation (B1) up across islands and weighting by each island's population and employment rate yields the following expression:

$$\sum_{j=1}^{J}\lambda_t L_{j,t}h_{j,t}W_{j,t} = \sum_{j=1}^{J}L_{j,t}h_{j,t}\left[v'(h_{j,t}) + \delta - \frac{\delta L_{j,t-1}h_{j,t-1}}{L_{j,t}h_{j,t}} - \beta\delta + E_t\frac{\beta\delta L_{j,t}h_{j,t}}{L_{j,t+1}h_{j,t+1}}\right]$$
$$-\sum_{j=1}^{J}L_{j,t}h_{j,t}\left[\frac{\delta(L_{j,t}h_{j,t} - L_{j,t-1}h_{j,t-1})^2}{2L_{j,t}h_{j,t}^2}\right]. \qquad \text{(B2)}$$

The left-hand side of (B2) must equal one since output equals consumption as before.

Taking first order deviations of the objects on the right hand side of (B2) from a perfectly matched steady state yields the expression:

$$\sum_{j=1}^{J}L_j hv'(h)\left[\hat{L}_{j,t} + \phi\hat{h}_{j,t}\right]$$
$$+\sum_{j=1}^{J}L_{j,t}h\left[(\beta+1)\delta(\hat{L}_{j,t} + \hat{h}_{j,t}) - \delta(\hat{L}_{j,t-1} + \hat{h}_{j,t-1}) - \delta\beta E_t(\hat{L}_{j,t+1} + \hat{h}_{j,t+1})\right] = 0\,. \qquad \text{(B3)}$$

A constant archipelago population eliminates the population deviation terms, implying that:

$$\sum_{j=1}^{J}L_{j,t}h\delta\beta E_t\hat{h}_{j,t+1} = \sum_{j=1}^{J}L_j hv'(h)\phi\hat{h}_{j,t} + \sum_{j=1}^{J}L_j h\left[(\beta+1)\delta\hat{h}_{j,t} - \delta\hat{h}_{j,t-1}\right] = 0\,. \qquad \text{(B4)}$$

(B4) is an unstable difference equation in the total labor input deviation $\sum_{j=1}^{J}L_j h\hat{h}_{j,t}$ for any parameters $\delta, \beta > 0$. Stability therefore requires that the labor input deviation equal zero at all times.

The results from the main body of the paper are therefore robust to the inclusion of quadratic adjustment costs in labor input at the firm level. Such adjustment costs will naturally affect the path of adjustment of island-level labor input after an aggregate or sectoral shock, but the theoretical results regarding mismatch still hold. Sectoral dynamics described in Section 2.5 become more complicated and more persistent than in the model without hiring frictions.

Appendix C: The solution to the basic model with linear quadratic adjustment costs

This appendix presents the complete solution to the model with quadratic adjustment costs and isoelastic labor supply. The households maximize the following objective:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\ln(C_{t+i}) - \sum_{j=1}^{J} L_{j,t+i} \frac{\varphi h_{j,t+i}^{1+\chi}}{1+\chi} - \sum_{j=1}^{J} \frac{\gamma}{2L_{j,t}} (L_{j,t+i} - L_{j,t+i-1})^2 \right]$$

$$- E_t \sum_{i=0}^{\infty} \beta^i \left[\lambda_{t+i} \left(C_{t+i} - \sum_{j=1}^{J} L_{j,t+i} W_{j,t+i} h_{j,t+i} \right) + \mu_{t+i} \left(\sum_{j=1}^{J} L_{j,t+i} - 1 \right) \right].$$

The first order conditions in labor supply for each island take the following form, after imposing market clearing in output markets:

$$\varphi h_{j,t}^{\chi} = \frac{W_{j,t}}{Y_t}, \tag{C1}$$

so in steady state, summing over all industries ensures that:

$$\varphi h^{1+\chi} = 1. \tag{C2}$$

The first order condition with respect to $L_{j,t}$ is given by:

$$\frac{\varphi h_{j,t}^{1+\chi}}{1+\chi} + \mu_t + \frac{\gamma(L_{j,t} - L_{j,t-1})}{L_{j,t}} - \frac{\gamma(L_{j,t} - L_{j,t-1})^2}{2L_{j,t}^2} = \frac{W_{j,t} h_{j,t}}{Y_t} + \beta E_t \frac{\gamma(L_{j,t+1} - L_{j,t})}{L_{j,t+1}}. \tag{C3}$$

Linearized, these equations become:

$$\chi \hat{h}_{j,t} = \hat{W}_{j,t} - \hat{Y}_t = \hat{W}_{j,t}; \tag{C4}$$

and

$$\hat{\mu}_t + \gamma(\hat{L}_{j,t} - \hat{L}_{j,t-1}) = \hat{W}_{j,t} - \hat{Y}_t + \beta E_t \gamma(\hat{L}_{j,t+1} - \hat{L}_{j,t}). \tag{C5}$$

After substituting in the labor supply condition, this becomes:

$$\hat{\mu}_t + \gamma(\hat{L}_{j,t} - \hat{L}_{j,t-1}) = \chi\hat{h}_{j,t} + \beta E_t \gamma(\hat{L}_{j,t+1} - \hat{L}_{j,t}) . \tag{C6}$$

By summing across all of the islands, weighted by population, it is possible to show that the first order deviation of μ_t is 0 at all times, which leads to the simplification:

$$\gamma(\hat{L}_{j,t} - \hat{L}_{j,t-1}) = \chi\hat{h}_{j,t} + \beta E_t \gamma(\hat{L}_{j,t+1} - \hat{L}_{j,t}) . \tag{C7}$$

The linearized demand curve for each type of labor is given by:

$$\hat{L}_{j,t} + \hat{h}_{j,t} = -\rho\hat{W}_{j,t} + \hat{a}_{j,t} + \hat{Y}_t = -\rho\hat{W}_{j,t} + \hat{a}_{j,t} = -\rho\chi\hat{h}_{j,t} + \hat{a}_{j,t} , \tag{C8}$$

giving the island-level employment rate as a function of mismatch:

$$\hat{h}_{j,t} = \frac{1}{1+\rho\chi}(\hat{a}_{j,t} - \hat{L}_{j,t}) . \tag{C9}$$

Substituting the equation governing within-island employment into the law of motion for island population gives a difference equation for the law of motion of each island's population as a function of mismatch:

$$\gamma(\hat{L}_{j,t} - \hat{L}_{j,t-1}) = \frac{\chi}{1+\rho\chi}(\hat{a}_{j,t} - \hat{L}_{j,t}) + \beta E_t \gamma(\hat{L}_{j,t+1} - \hat{L}_{j,t}) . \tag{C10}$$

When $\hat{a}_{j,t}$ follows a random walk, this equation has a simple solution, which is given in error correction form. It is given by the reduced form solution $\hat{L}_{j,t} = \rho_L \hat{L}_{j,t-1} + (1-\rho_L)\hat{a}_{j,t}$, where the error correction coefficient can be derived from the structural coefficients as follows.

First, substituting in the proposed solution eliminates the forward-looking terms from the equation:

$$\gamma(\hat{L}_{j,t} - \hat{L}_{j,t-1}) = \frac{\chi}{1+\rho\chi}(\hat{a}_{j,t} - \hat{L}_{j,t}) + \beta\gamma(1-\rho_L)(\hat{a}_{j,t} - \hat{L}_{j,t}) . \tag{C11}$$

Dividing through by γ and consolidating terms yields:

$$(\hat{L}_{j,t} - \hat{L}_{j,t-1}) = \left(\frac{\chi/\gamma}{1+\rho\chi} + \beta(1-\rho_L)\right)(\hat{a}_{j,t} - \hat{L}_{j,t}) . \tag{C12}$$

Solving for current island population yields the error-correction form:

$$\hat{L}_{j,t} = \frac{\left(\frac{\chi/\gamma}{1+\rho\chi} + \beta(1-\rho_L)\right)}{\left(1+\frac{\chi/\gamma}{1+\rho\chi} + \beta(1-\rho_L)\right)}\hat{a}_{j,t} + \frac{1}{\left(1+\frac{\chi/\gamma}{1+\rho\chi} + \beta(1-\rho_L)\right)}\hat{L}_{j,t-1} , \tag{C13}$$

which is consistent with the functional form proposed as a solution. The error correction parameter must therefore satisfy the equation:

$$\rho_L = \frac{1}{\left(1+\frac{\chi/\gamma}{1+\rho\chi} + \beta(1-\rho_L)\right)} . \tag{C14}$$

Solving out for ρ_L requires solving the quadratic equation:

$$\beta\rho_L{}^2 - \rho_L\left(1+\frac{\chi/\gamma}{1+\rho\chi} + \beta\right) + 1 = 0 , \tag{C15}$$

giving the solutions:

$$\rho_L = \frac{\left(1+\frac{\chi/\gamma}{1+\rho\chi} + \beta\right) \pm \sqrt{\left(1+\frac{\chi/\gamma}{1+\rho\chi} + \beta\right)^2 - 4\beta}}{2\beta} . \tag{C16}$$

The negative solution has the following properties:

$$\rho_L^- = \frac{\left(1+\frac{\chi/\gamma}{1+\rho\chi}+\beta\right)-\sqrt{\left(1+\frac{\chi/\gamma}{1+\rho\chi}+\beta\right)^2-4\beta}}{2\beta}$$

$$< \frac{(1+\beta)-\sqrt{(1+\beta)^2-4\beta}}{2\beta}, \tag{C17}$$

since

$$\frac{\partial \rho_L^-}{\partial\left(\frac{\chi/\gamma}{1+\rho\chi}\right)} < 0, \tag{C18}$$

and

$$\frac{\chi/\gamma}{1+\rho\chi} > 0. \tag{C19}$$

Since

$$\frac{(1+\beta)-\sqrt{(1+\beta)^2-4\beta}}{2\beta} = \frac{(1+\beta)-\sqrt{1-2\beta+\beta^2}}{2\beta} = 1, \tag{C20}$$

it must be the case that the negative solution is less than one. It is also trivial to show that the negative solution is greater than zero, so island populations do not oscillate.

The positive solution has the following properties:

$$\rho_L^+ = \frac{\left(1+\frac{\chi/\gamma}{1+\rho\chi}+\beta\right)+\sqrt{\left(1+\frac{\chi/\gamma}{1+\rho\chi}+\beta\right)^2-4\beta}}{2\beta}.$$

$$> \frac{(1+\beta)+\sqrt{(1+\beta)^2-4\beta}}{2\beta} \tag{C21}$$

Since

$$\frac{(1+\beta)+\sqrt{(1+\beta)^2-4\beta}}{2\beta}=\frac{(1+\beta)+\sqrt{1-2\beta+\beta^2}}{2\beta}=\frac{1}{\beta}>1, \tag{C22}$$

the positive solution for ρ_L is explosive. So, the negative solution for ρ_L gives the unique nonexplosive solution of the model.

To solve for the movement of aggregate sectoral employment, it is useful invert (C9):

$$\hat{L}_{j,t}=\hat{a}_{j,t}-(1+\rho\chi)\hat{h}_{j,t}. \tag{C23}$$

Substituting h into the law of motion for L gives the law of motion for sectoral employment rates:

$$\hat{h}_{j,t}=\rho_L\hat{h}_{j,t-1}+\frac{\rho_L}{(1+\rho\chi)}(\hat{a}_{j,t}-\hat{a}_{j,t-1}). \tag{C24}$$

The deviation of employment at an island level is given by adding together the deviation of that island's population and its employment rate:

$$\hat{N}_{j,t}=\hat{L}_{j,t}+\hat{h}_{j,t}=\rho_L\hat{N}_{j,t-1}+(1-\rho_L)\hat{a}_{j,t}+\left(\frac{\rho_L}{(1+\rho\chi)}\right)(\hat{a}_{j,t}-\hat{a}_{j,t-1}). \tag{C25}$$

If one wanted instead to solve the model with firm-level adjustment costs, the laws of motion for sectoral population and employment rates would become more complicated, but the same basic intuition would hold.

Appendix D: The constancy of labor input in a search and matching model

This appendix solves the search and matching model in more detail. There is no closed-form proof of the constancy of labor input in the search and matching model; it is necessary to solve the model computationally.

D.1 Households

Households have the same preferences as they do in the original model. This appendix presents the complete solution to the household's problem with quadratic human capital adjustment costs and isoelastic labor supply. To simplify matters, workers have no bargaining power. Per-capita island labor input is now denoted by $n_{j,t}$, and the unemployment rate is $u_{j,t}$ on each island.

Households wish to maximize the following objective (note the inclusion of a profit term Π in the households' income receipts):

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\ln(C_{t+i}) - \sum_{j=1}^{J} L_{j,t+i} \frac{\varphi n_{j,t+i}^{1+\chi}}{1+\chi} - \sum_{j=1}^{J} \frac{\gamma}{2L_{j,t+i}} (L_{j,t+i} - L_{j,t+i-1})^2 \right]$$
$$- E_t \sum_{i=0}^{\infty} \beta^i \left[\lambda_{t+i} \left(C_{t+i} - \sum_{j=1}^{J} L_{j,t+i} W_{j,t+i} n_{j,t+i} - \sum_{j=1}^{J} \Pi_{j,t+i} \right) + \mu_{t+i} \beta \left(\sum_{j=1}^{J} L_{j,t+i+1} - 1 \right) \right]$$

The first order conditions in labor supply for each island take the following form, after imposing market clearing in output markets:

$$\varphi n_{j,t}^{\chi} = \lambda_t W_{j,t}, \qquad \text{(D1)}$$

Employment and population are both effectively chosen one period in advance, in contrast with the flexible model. The first order condition in $L_{j,t+1}$ is given by:

$$E_t \left[\frac{\varphi n_{j,t+1}^{1+\chi}}{1+\chi} + \frac{\gamma (L_{j,t+1} - L_{j,t})}{L_{j,t+1}} - \frac{\gamma (L_{j,t+1} - L_{j,t})^2}{2L_{j,t+1}^2} - \lambda_{t+1} W_{j,t+1} n_{j,t+1} - \beta \frac{\gamma (L_{j,t+2} - L_{j,t+1})}{L_{j,t+2}} \right]$$
$$= -\mu_t . \qquad \text{(D2)}$$

The results from this model are not sensitive to the assumptions made in modeling island population flows, so long as steady state island populations reasonably reflect economic reality in the long run. I chose this particular setup because it mirrors the setup in the main body of the paper, and it gives economically intuitive results.

The first order condition with respect to consumption, as before, is given by:

$$\lambda_t = 1/C_t, \tag{D3}$$

and the resource constraint is given by:

$$Y_t = C_t. \tag{D4}$$

As a matter of notation, the unemployment rate and employment rate on an island level basis are given by:

$$U_{j,t} = u_{j,t} L_{j,t}, \tag{D5}$$

and

$$N_{j,t} = n_{j,t} L_{j,t}. \tag{D6}$$

D.2 Wholesale firms and matching

Wholesale firms post vacancies using κ units of labor; the rest of their labor goes to producing their own output. They sell their output at a real price $P_{j,t}$, and they find workers from the island-level unemployment pool according to a Cobb-Douglas matching function with an unemployment elasticity ξ. The law of motion for sectoral employment is given by:

$$N_{j,t} = \phi(N_{j,t-1} + V_{j,t-1} m \theta_{j,t-1}^{-\xi}), \tag{D7}$$

where ϕ is the hazard rate of being employed in the following period conditional on being employed in the current period; m is a matching function efficiency parameter and $\theta_{j,t}$ is the tightness of the labor market on island j, which equals $V_{j,t} / U_{j,t}$.

Wholesale firms seek to maximize the present value of profits subject to the law of motion for employment, which implies maximizing the following objective function over current period vacancies and employment:

$$E_t \sum_{i=0}^{\infty} \beta^i \frac{\lambda_{t+i}}{\lambda_t} \left[P_{j,t+i}(N_{j,t+i} - \kappa V_{j,t+i}) - W_{j,t+i} N_{j,t+i} \right]$$

$$- E_t \sum_{i=0}^{\infty} \beta^i \frac{\lambda_{t+i}}{\lambda_t} \left[\omega_{j,t+i}(N_{j,t+i} - \phi(N_{j,t+i-1} + V_{j,t+i-1} m \theta_{j,t+i-1}^{-\xi})) \right].$$

The first order condition for vacancy creation is given by:

$$P_{j,t}\kappa = E_t \beta \frac{\lambda_{t+i}}{\lambda_t} \phi \omega_{j,t+1} m \theta_{j,t}^{-\xi} . \tag{D8}$$

The first order condition for labor demand is given by:

$$P_{j,t} - W_{j,t} - \omega_{j,t} + E_t \beta \frac{\lambda_{t+i}}{\lambda_t} \phi \omega_{j,t+1} = 0 . \tag{D9}$$

Island-level production is given by the proportion of labor input which adds value:

$$Y_{j,t} = N_{j,t} - \kappa V_{j,t} . \tag{D10}$$

D.3 Aggregators

Aggregators work on the main island; they sell their output as before, and they buy island-level input competitively. Their production function is given by the CES aggregator:

$$Y_t = \left(\sum_{j=1}^{J} (a_{j,t})^{\frac{1}{\rho}} \left(Y_{j,t}\right)^{\left(\frac{\rho-1}{\rho}\right)} \right)^{\left(\frac{\rho}{\rho-1}\right)} . \quad \text{(D11)}$$

Their demand for island-level output takes the form:

$$Y_{j,t} = \left(P_{j,t}\right)^{-\rho} a_{j,t} Y_t . \quad \text{(D12)}$$

D.4 Calibration and steady state

Steady state output Y is given by $N - \kappa V$, and that in turn is equal to steady state consumption C. The consumption costate λ equals $1/C$. The number of new matches M is given by (D7) in steady state:

$$M = (1 - \phi) N . \quad \text{(D13)}$$

The costate variable ω is given by (D8):

$$\omega = \frac{\kappa V}{\beta M} . \quad \text{(D14)}$$

Wages are given by (D9):

$$W = 1 - \omega (1 - \phi \beta) . \quad \text{(D15)}$$

The costate variable μ from (D2) is given by:

$$\mu = \lambda W N - \frac{\varphi N^{1+\chi}}{1+\chi} . \quad \text{(D16)}$$

D.5 Linearization

The linearized model is given by the following sets of equations. First, the aggregate production function (D11) is linearized as follows:

$$(\rho-1)Y^{\left(\frac{\rho-1}{\rho}\right)}\hat{Y}_t=\sum_{j=1}^{J}\left(Y_j\right)^{\left(\frac{\rho-1}{\rho}\right)}\left(a_j\right)^{\left(\frac{1}{\rho}\right)}\left((\rho-1)(\hat{Y}_{j,t})+\hat{a}_{j,t}\right) \tag{D17}$$

Aggregate employment, unemployment, and vacancies are linearized as follows:

$$N\hat{N}_t=\sum_{j=1}^{J}N_j\hat{N}_{j,t}\ ; \tag{D18}$$

$$U\hat{U}_t=\sum_{j=1}^{J}U_j\hat{U}_{j,t}\ ; \tag{D19}$$

and

$$V\hat{V}_t=\sum_{j=1}^{J}V_j\hat{V}_{j,t}\ . \tag{D20}$$

Linearized demand for product j from (D12) is given by:

$$\hat{Y}_{j,t}=-\rho\hat{P}_{j,t}+\hat{a}_{j,t}+\hat{Y}_t \tag{D21}$$

Island-level production (D10) is linearized as:

$$Y_j\hat{Y}_{j,t}=N_j\hat{N}_{j,t}-\kappa V_j\hat{V}_{j,t} \tag{D22}$$

Island-level labor supply (D1) is given by:

$$\varphi\left(\frac{N_{j,t}}{L_{j,t}}\right)^{\chi}=\lambda_t W_{j,t}\ , \tag{D23}$$

which is linearized as:

$$\chi\left(\hat{N}_{j,t}-\hat{L}_{j,t}\right)=\hat{\lambda}_t+\hat{W}_{j,t}\ . \tag{D24}$$

The consumption costate variable is linearized as:

$$\hat{\lambda}_t = -\hat{C}_t \, . \tag{D25}$$

The vacancy posting condition (D8) is linearized as:

$$\hat{P}_{j,t} + \hat{\lambda}_t + \xi\hat{\theta}_{j,t} = E_t(\hat{\lambda}_{t+1} + \hat{\omega}_{j,t+1}) \, . \tag{D26}$$

The definition of tightness at the island level is linearized as:

$$\hat{\theta}_{j,t} = \hat{V}_{j,t} - \hat{U}_{j,t} \, . \tag{D27}$$

The labor demand equation (D9) is linearized as:

$$\hat{P}_{j,t} - W\hat{W}_{j,t} - \omega\hat{\omega}_{j,t} + \beta\phi\omega(E_t\hat{\lambda}_{t+1} + E_t\hat{\omega}_{j,t+1} - \hat{\lambda}_t) = 0 \, ,$$

so combining in (D26):

$$\hat{P}_{j,t} - W\hat{W}_{j,t} - \omega\hat{\omega}_{j,t} + \beta\phi\omega(\hat{P}_{j,t} + \xi\hat{\theta}_{j,t}) = 0 \, . \tag{D28}$$

Productivity for sector one follows a random walk:

$$\hat{a}_{1,t} = \hat{a}_{1,t-1} + \varepsilon_{1,t} \, . \tag{D29}$$

Total productivity remains constant, from (16):

$$\sum_{j=1}^{J} a_j \hat{a}_{j,t} = 0 \, . \tag{D30}$$

The law of motion for island population (D2) can be linearized as:

$$E_t\left[(\varphi N - \lambda WN)\hat{N}_{j,t+1} - \beta E_t \gamma \hat{L}_{j,t+2} - \lambda WN(\hat{\lambda}_{t+1} + \hat{W}_{j,t+1})\right]$$
$$= (\varphi N - \lambda WN - \beta\gamma - \gamma)\hat{L}_{j,t+1} + \gamma \hat{L}_{j,t} - \mu\hat{\mu}_t . \quad \text{(D31)}$$

Total population remains constant:

$$\sum_{j=1}^{J} L_j \hat{L}_{j,t+1} = 0 . \quad \text{(D32)}$$

Each island's population consists of its employed and unemployed:

$$L_j \hat{L}_{j,t} = N_j \hat{N}_{j,t} + U_j \hat{U}_{j,t} . \quad \text{(D33)}$$

Island level employment is given by the law of motion (D7):

$$N_j \hat{N}_{j,t} = \phi N_j \hat{N}_{j,t-1} + \phi V_j m \theta^{-\xi} (\hat{V}_{j,t-1} - \xi \hat{\theta}_{j,t-1}) . \quad \text{(D34)}$$

The resource constraint (D4) is linearized as:

$$Y\hat{Y}_t = C\hat{C}_t . \quad \text{(D35)}$$

The model has to be solved numerically, but it behaves very much like the other models when simulated.

Appendix E: Notes on the estimation procedure

E.1 Algorithm for the MCMC

The model outlined in the text is easily estimated by a Markov Chain Monte Carlo technique which I run over 200,001 draws, discarding the first 10,000 draws. It works in the following manner:

1. First draw a series of unobservables $\xi_t = \{W_t, X_t, Z_t, \Delta Z_t, c_t\}$ conditional on the observables Y_t, the parameters $\{\mu\}$, $\{\rho\}$, $\{\delta\}$, and $\{\sigma\}$, and the unobservable data $\{S_t\}$. This can be done exactly through the Kalman Filter; ξ_t is conditionally normal. The formula for the Kalman Filter is discussed by Hamilton (1994, Chapter 13).

2. Then draw a series containing the updated values of S_t given the observables Y_t, the parameters $\{\mu\}$, $\{\rho\}$, $\{\delta\}$, and $\{\sigma\}$, and the unobservables ξ_t. This requires a Metropolis-Hastings step since S_t appears in the Gaussian likelihood function in both the mean and variance terms. The Metropolis-Hastings step proceeds as follows:

   a. Draw a candidate series $\{S'_t\}$ from a known proposal distribution q, which in this case is a loglinear random walk.

   b. Calculate the prior density p of $\{S_t\}$ and $\{S'_t\}$ for each t, which is lognormal.

   c. For each t, calculate the (log) likelihood of observing the series obtained in step (1) for both $\{S_t\}$ and $\{S'_t\}$. These objects show up in four places—in the law of motion for long run unemployment z^u, the law of motion for long run output z^y, the law of motion for the business cycle w^y, and in the observed weighted variance of the shocks to sectoral employment trends z^n.

   d. For each t, either carry S_t forward into the next draw or accept S'_t with a probability given by the Metropolis-Hastings acceptance formula:

$$pr(S_t = S'_t) = \frac{q(S_t \mid S'_t)p(S'_t)L(\xi_t \mid S'_t)}{q(S'_t \mid S_t)p(S_t)L(\xi_t \mid S_t)}. \tag{E1}$$

3. Then draw a series of parameters $\{\mu_{zn}\}$, $\{\rho_{zn}\}$, and Σ_{zn} conditional on the observables Y_t and the unobservables $\{S_t, \xi_t\}$. This is done as a Bayesian SUR. First, draw Σ_{zn} conditional on the unobservables; its posterior distribution is Inverse Wishart. Then draw $\{\mu_{zn}\}$, $\{\rho_{zn}\}$ using a formula identical to that for a restricted VAR given by Hamilton (1994, Section 11.3).

4. Draw a series of parameters $\{\mu_{zy}\}$, $\{\rho_{wy}\}$, $\{\delta\}$, and $\{\sigma\}$ conditional on the observables Y_t and the unobservables $\{S_t, \xi_t\}$. For invalid explosive values of $\{\rho_{wy}\}$ in the law of motion for cyclical output, assign the draw a likelihood of zero; reject; and re-draw. The coefficient parameters can easily be calculated using the formulas for a Bayesian regression; they are conditionally normal since the previous draw of $\{\sigma\}$ is known. The parameters $\{\sigma\}$ have an inverse gamma distribution given the exact residuals calculated from the regression draw.

Some of the initial states come from a nonstationary process so the prior conditional variance of the first observation is technically infinite. As a numerical approximation, I approximate "infinity" with a large number. Basically any number above one will work under this circumstance, with the prior mean reset to the initial value of each employment level minus the prior mean of cyclical employment. This approximation gives very accurate results and is numerically stable.

After iterating 200,001 times through the MCMC and tossing the first 10,000 values, one has a sample from the posterior distribution of all of these objects. The estimation in general is well-behaved so long as the estimated variances remain away from zero. Within the first 100 draws the system substantially converges toward its posteriors and it remains within that region. All estimates come from that posterior distribution, which is calculated separately for the different sectoral breakdowns.

E.2 The calculation of dispersion measures

The Lilien (1982) dispersion measure is given by the formula:

$$Disp_{LIL,t}^2 = \sum_{i=1}^{N} w_{it} \left(\varepsilon_{i,t}^{zn}\right)^2 . \qquad \text{(E2)}$$

Lilien weights by the sectoral employment share of a given sector. This has the effect of overweighting volatile sectors and is not invariant to the level of sectoral aggregation. My measure instead weights squared residuals instead by the sectoral precision parameters Σ_{zn}^{-1}, to within a ratio. This is equivalent to the maximum likelihood estimate of S_t when S_t has no effect on aggregates:

$$Disp_{WEIGHTED,t}^{2} = \sum_{i=1}^{N} \varepsilon_t^{zn}{}' \Sigma_{zn}^{-1} \varepsilon_t^{zn} \,. \tag{E3}$$

The figures report the estimates of dispersion after they are recentered to have a geometric mean of zero.

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Table 1: Prior distributions for all parameters, state space model

Parameter	Prior distribution	Remarks
σ^2_{xn}	Inverse Gamma (Mean = (0.01^2), #Obs=1)	Keeps var. away from 0.
$\sigma^2_{wn,i}$	Inverse Gamma (Mean = (0.01^2), #Obs=T)	Filters out transitory noise.
Σ_{zn}	Inverse Wishart (Mean = (0.002^2)*I, #Obs=1)	Keeps cov. away from 0.
σ^2_{zu}	Inverse Gamma (Mean = (0.01^2), #Obs=1)	Keeps var. away from 0.
σ^2_{wu}	Inverse Gamma (Mean = (0.01^2), #Obs=1)	Keeps var. away from 0.
σ^2_{zy}	Inverse Gamma (Mean = (0.01^2), #Obs=1)	Keeps var. away from 0.
σ^2_{wy}	Inverse Gamma (Mean = (0.01^2), #Obs=1)	Keeps var. away from 0.
σ^2_{c}	Inverse Gamma (Mean = (0.01^2), #Obs=1)	Keeps var. away from 0.
σ^2_{S}	Inverse Gamma (Mean = 2^2, #Obs=1)	Gives good fit to dispersion.
$\mu_{zn,i}$	Normal (Mean = 0, Std. = ∞)	Diffuse prior.
μ_{zy}	Normal (Mean = 0, Std. = ∞)	Diffuse prior.
ρ^{zn}	Multivariate Normal (Mean = 0*I, Cov. = ∞*I)	Diffuse prior.
ρ^{wy}_{p}	Normal (Mean = 0, Std. = ∞)	Diffuse prior.
α^{u}_{p}	Normal (Mean = 0, Std. = ∞)	Diffuse prior.
$\alpha^{n}_{p,i}$	Normal (Mean = 0, Std. = ∞)	Diffuse prior.
$\delta_{u,n}$	Normal (Mean = 0, Std. = ∞)	Diffuse prior.
$\delta_{S,zu}$	Normal (Mean = 0, Std. = ∞)	Diffuse prior.
$\delta_{S,wy}$	Normal (Mean = 0, Std. = ∞)	Diffuse prior.
$\delta_{S,zy}$	Normal (Mean = 0, Std. = ∞)	Diffuse prior.

This table gives the prior distributions used for each of the model parameters.

Table 2: Posterior statistics on the effects of reallocation on the natural rate of unemployment, trend productivity, and the cycle

	$\delta_{S,zu}$		$\delta_{S,zy}$		$\delta_{S,wy} + \delta_{S,zy}\delta_{zy,wy}$	
Sectoral breakdown	**Median**	**pr > 0**	**Median**	**pr > 0**	**Median**	**pr > 0**
13 sectors	0.0002	0.705	-0.0037	0.086	-0.0012	0.045
5 sectors	0.0000	0.459	0.0012	0.722	-0.0017	0.104

Statistics are posterior percentiles and probabilities of being above zero for the model parameters which govern the long run and cyclical responses to reallocation, after 200,001 draws with a burnin of 10,000 draws.

Table 3a: Posterior percentiles for the thirteen-sector model

Group	Variable	2.5	5	10	50	90	95	97.5	Mean
	Cyclical AR parameter ρ_1^{wy}	1.613	1.636	1.662	1.746	1.821	1.841	1.857	1.743
	Cyclical AR parameter ρ_2^{wy}	-0.884	-0.867	-0.847	-0.771	-0.686	-0.659	-0.636	-0.768
	Sum of AR parameters ρ_p^{wy}	0.952	0.956	0.961	0.976	0.989	0.992	0.995	0.975
Sum of cyc. loadings α	Unemployment	-1.015	-0.974	-0.932	-0.781	-0.655	-0.624	-0.602	-0.789
Sum of cyc. loadings α	Construction	2.470	2.582	2.732	3.345	4.075	4.310	4.509	3.381
Sum of cyc. loadings α	Durable Mfg.	2.951	3.075	3.223	3.876	4.698	4.920	5.108	3.923
Sum of cyc. loadings α	Nondur. Mfg.	1.068	1.129	1.202	1.493	1.855	1.969	2.076	1.514
Sum of cyc. loadings α	Wholesale Trade	0.969	1.032	1.109	1.422	1.792	1.897	1.994	1.439
Sum of cyc. loadings α	Retail Trade	0.647	0.706	0.776	1.041	1.338	1.433	1.518	1.051
Sum of cyc. loadings α	Leisure/Hospitality	0.640	0.692	0.753	0.980	1.245	1.332	1.408	0.991
Sum of cyc. loadings α	Transp./Utilities	1.118	1.178	1.256	1.562	1.936	2.049	2.145	1.583
Sum of cyc. loadings α	Information	1.303	1.372	1.456	1.798	2.213	2.348	2.467	1.820
Sum of cyc. loadings α	Fin. Activities	0.198	0.260	0.331	0.587	0.868	0.954	1.034	0.594
Sum of cyc. loadings α	Prof./Bus. Svcs.	0.886	0.947	1.018	1.315	1.678	1.798	1.897	1.335
Sum of cyc. loadings α	Edu./Health Svcs.	0.238	0.284	0.338	0.535	0.763	0.837	0.907	0.545
Sum of cyc. loadings α	Other Svcs.	0.318	0.369	0.429	0.667	0.938	1.021	1.099	0.677
Sum of cyc. loadings α	Government	-0.258	-0.194	-0.126	0.098	0.313	0.375	0.432	0.095
	$\delta_{u,n}$	-1.200	-1.133	-1.054	-0.778	-0.505	-0.428	-0.361	-0.779
	$\delta_{zy,wy}$	0.039	0.068	0.098	0.185	0.269	0.297	0.323	0.184
	$\delta_{S,zu}$	-0.0007	-0.0005	-0.0003	0.0002	0.0008	0.0010	0.0011	0.0002
	Variance contrib. of S to z^u	0.0000	0.0002	0.0008	0.0217	0.1153	0.1572	0.1976	0.0443
	$\delta_{S,zy}$	-0.0085	-0.0077	-0.0069	-0.0037	-0.0003	0.0008	0.0019	-0.0036
	Variance contrib. of S to z^y	0.0008	0.0033	0.0128	0.1997	0.5226	0.6071	0.6713	0.2441
	$\delta_{S,wy}$	-0.0023	-0.0020	-0.0016	-0.0006	0.0003	0.0006	0.0009	-0.0006
	$\delta_{S,wy} + \delta_{S,zy}\delta_{zy,wy}$	-0.0027	-0.0024	-0.0021	-0.0012	-0.0003	0.0000	0.0002	-0.0012
	Variance contrib. of S to w^y	0.0009	0.0035	0.0124	0.1194	0.2994	0.3556	0.4065	0.1410
	Std. of log reallocation S	0.4661	0.4890	0.5182	0.6387	0.7954	0.8470	0.8941	0.6593
Intercept μ of n trends z^n	Construction	-0.0035	-0.0028	-0.0020	0.0003	0.0025	0.0032	0.0039	0.0003
Intercept μ of n trends z^n	Durable Mfg.	-0.0065	-0.0059	-0.0054	-0.0038	-0.0021	-0.0015	-0.0009	-0.0038
Intercept μ of n trends z^n	Nondur. Mfg.	-0.0074	-0.0069	-0.0065	-0.0051	-0.0037	-0.0032	-0.0028	-0.0051
Intercept μ of n trends z^n	Wholesale Trade	-0.0007	-0.0005	-0.0003	0.0004	0.0011	0.0013	0.0015	0.0004
Intercept μ of n trends z^n	Retail Trade	0.0004	0.0005	0.0008	0.0015	0.0022	0.0024	0.0026	0.0015
Intercept μ of n trends z^n	Leisure/Hospitality	0.0026	0.0027	0.0029	0.0034	0.0038	0.0040	0.0041	0.0034
Intercept μ of n trends z^n	Transp./Utilities	-0.0015	-0.0013	-0.0011	-0.0004	0.0003	0.0005	0.0008	-0.0004
Intercept μ of n trends z^n	Information	-0.0053	-0.0042	-0.0032	-0.0003	0.0028	0.0041	0.0055	-0.0002
Intercept μ of n trends z^n	Fin. Activities	0.0000	0.0004	0.0008	0.0021	0.0033	0.0037	0.0040	0.0021
Intercept μ of n trends z^n	Prof./Bus. Svcs.	0.0031	0.0033	0.0036	0.0045	0.0055	0.0057	0.0060	0.0045
Intercept μ of n trends z^n	Edu./Health Svcs.	0.0049	0.0051	0.0053	0.0060	0.0066	0.0068	0.0070	0.0060
Intercept μ of n trends z^n	Other Svcs.	0.0024	0.0027	0.0031	0.0042	0.0054	0.0057	0.0060	0.0042
Intercept μ of n trends z^n	Government	-0.0031	-0.0019	-0.0010	0.0012	0.0031	0.0039	0.0048	0.0011

Statistics are posterior percentiles and means for the parameters for the state space model, after 200,001 draws with a burnin of 10,000 draws. Posterior means for standard deviations are calculated as the square root of the posterior variance.

Table 3b: Posterior percentiles for the thirteen-sector model (continued)

Group	Variable	2.5	5	10	50	90	95	97.5	Mean
Persist. ρ of growth of n trends z^n	Construction	0.7331	0.7528	0.7737	0.8372	0.8876	0.9006	0.9114	0.8333
	Durable Mfg.	0.7223	0.7493	0.7771	0.8546	0.9104	0.9238	0.9349	0.8479
	Nondur. Mfg.	0.8095	0.8257	0.8430	0.8927	0.9314	0.9412	0.9494	0.8893
	Wholesale Trade	0.4885	0.5288	0.5711	0.6956	0.7949	0.8195	0.8398	0.6877
	Retail Trade	0.4938	0.5304	0.5707	0.6895	0.7819	0.8049	0.8240	0.6813
	Leisure/Hospitality	0.2392	0.3103	0.3777	0.5683	0.7060	0.7372	0.7623	0.5515
	Transp./Utilities	0.5707	0.6137	0.6565	0.7711	0.8509	0.8700	0.8858	0.7602
	Information	0.8663	0.8778	0.8900	0.9271	0.9590	0.9674	0.9744	0.9255
	Fin. Activities	0.7040	0.7256	0.7488	0.8201	0.8788	0.8935	0.9061	0.8162
	Prof./Bus. Svcs.	0.5875	0.6144	0.6435	0.7331	0.8070	0.8254	0.8412	0.7282
	Edu./Health Svcs.	0.5816	0.6190	0.6562	0.7627	0.8421	0.8611	0.8766	0.7540
	Other Svcs.	0.7462	0.7646	0.7843	0.8425	0.8899	0.9023	0.9127	0.8391
	Government	0.8348	0.8526	0.8710	0.9239	0.9639	0.9735	0.9811	0.9198
Std. of shock to n trends x^n, z^n	Common trend	0.0015	0.0016	0.0016	0.0019	0.0021	0.0022	0.0022	0.0019
	Construction	0.0028	0.0029	0.0031	0.0037	0.0045	0.0048	0.0050	0.0038
	Durable Mfg.	0.0016	0.0017	0.0018	0.0023	0.0030	0.0032	0.0034	0.0024
	Nondur. Mfg.	0.0010	0.0011	0.0012	0.0015	0.0019	0.0020	0.0021	0.0015
	Wholesale Trade	0.0014	0.0015	0.0017	0.0022	0.0028	0.0030	0.0032	0.0022
	Retail Trade	0.0014	0.0015	0.0017	0.0022	0.0029	0.0031	0.0033	0.0023
	Leisure/Hospitality	0.0013	0.0014	0.0015	0.0020	0.0027	0.0029	0.0031	0.0021
	Transp./Utilities	0.0011	0.0011	0.0012	0.0016	0.0021	0.0023	0.0024	0.0017
	Information	0.0016	0.0016	0.0017	0.0021	0.0025	0.0027	0.0028	0.0021
	Fin. Activities	0.0016	0.0017	0.0018	0.0022	0.0028	0.0029	0.0031	0.0023
	Prof./Bus. Svcs.	0.0018	0.0019	0.0020	0.0025	0.0031	0.0033	0.0035	0.0026
	Edu./Health Svcs.	0.0011	0.0011	0.0012	0.0016	0.0021	0.0022	0.0024	0.0017
	Other Svcs.	0.0013	0.0014	0.0015	0.0018	0.0023	0.0025	0.0026	0.0019
	Government	0.0010	0.0010	0.0011	0.0014	0.0018	0.0019	0.0021	0.0015
Std. of shock to n levels w^n	Construction	0.0077	0.0078	0.0079	0.0083	0.0087	0.0089	0.0090	0.0083
	Durable Mfg.	0.0074	0.0075	0.0076	0.0079	0.0083	0.0085	0.0086	0.0080
	Nondur. Mfg.	0.0072	0.0073	0.0074	0.0078	0.0081	0.0083	0.0084	0.0078
	Wholesale Trade	0.0071	0.0072	0.0073	0.0077	0.0080	0.0082	0.0083	0.0077
	Retail Trade	0.0071	0.0072	0.0073	0.0076	0.0080	0.0081	0.0082	0.0076
	Leisure/Hospitality	0.0071	0.0072	0.0073	0.0077	0.0080	0.0081	0.0082	0.0077
	Transp./Utilities	0.0072	0.0073	0.0074	0.0077	0.0081	0.0082	0.0083	0.0078
	Information	0.0072	0.0073	0.0073	0.0077	0.0081	0.0082	0.0083	0.0077
	Fin. Activities	0.0072	0.0072	0.0073	0.0077	0.0081	0.0082	0.0083	0.0077
	Prof./Bus. Svcs.	0.0072	0.0072	0.0073	0.0077	0.0081	0.0082	0.0083	0.0077
	Edu./Health Svcs.	0.0071	0.0072	0.0073	0.0076	0.0080	0.0081	0.0082	0.0076
	Other Svcs.	0.0071	0.0072	0.0073	0.0076	0.0080	0.0081	0.0082	0.0076
	Government	0.0071	0.0072	0.0073	0.0077	0.0081	0.0082	0.0083	0.0077
	Std. of shock to u trend z^u	0.0014	0.0014	0.0014	0.0016	0.0018	0.0018	0.0019	0.0016
	Std. of shock to u cycle w^u	0.0011	0.0011	0.0012	0.0013	0.0014	0.0014	0.0015	0.0013
	Std. of shock to Y trend z^y	0.0033	0.0035	0.0038	0.0048	0.0056	0.0058	0.0060	0.0048
	Std. of shock to Y cycle w^y	0.0015	0.0016	0.0017	0.0020	0.0024	0.0026	0.0027	0.0020
	Std. of shock to Y level c	0.0021	0.0022	0.0023	0.0027	0.0032	0.0033	0.0034	0.0027

Sources and notes: See table 3a.

Table 4: Posterior correlation matrix for z^n shocks based on Σ_{zn}, 13-sector model

Sector	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
(1)	1.00	-0.58	-0.47	-0.05	0.28	0.28	0.00	-0.14	0.54	0.17	-0.31	0.00
(2)	-0.58	1.00	0.57	0.46	0.00	-0.16	0.05	0.41	-0.27	0.10	0.14	0.10
(3)	-0.47	0.57	1.00	0.30	0.14	0.14	0.22	0.36	-0.06	0.13	0.42	0.26
(4)	-0.05	0.46	0.30	1.00	0.44	0.28	0.17	0.37	0.08	0.39	0.07	0.32
(5)	0.28	0.00	0.14	0.44	1.00	0.74	0.11	0.25	0.33	0.30	0.16	0.51
(6)	0.28	-0.16	0.14	0.28	0.74	1.00	0.28	0.15	0.25	0.25	0.27	0.37
(7)	0.00	0.05	0.22	0.17	0.11	0.28	1.00	0.49	-0.07	0.40	0.08	-0.06
(8)	-0.14	0.41	0.36	0.37	0.25	0.15	0.49	1.00	-0.15	0.56	-0.10	0.05
(9)	0.54	-0.27	-0.06	0.08	0.33	0.25	-0.07	-0.15	1.00	0.23	0.12	0.38
(10)	0.17	0.10	0.13	0.39	0.30	0.25	0.40	0.56	0.23	1.00	-0.06	0.25
(11)	-0.31	0.14	0.42	0.07	0.16	0.27	0.08	-0.10	0.12	-0.06	1.00	0.55
(12)	0.00	0.10	0.26	0.32	0.51	0.37	-0.06	0.05	0.38	0.25	0.55	1.00
(13)	-0.01	0.10	0.14	-0.04	0.25	0.15	-0.07	0.17	0.01	-0.06	0.04	0.19

This table shows the posterior correlations of the thirteen long run sectoral shocks. The sectors are numbered as follows: (1) Construction, (2) Durable Manufacturing, (3) Nondurable Manufacturing, (4) Wholesale Trade, (5) Retail Trade, (6) Leisure and Hospitality, (7) Transportation and Utilities, (8) Information, (9) Financial Activities, (10) Professional and Business Services, (11) Education and Health Services, (12) Other Services, (13) Government.

Table 5: Posterior percentiles for the five-sector model

	Variable	2.5	5	10	50	90	95	97.5	Mean
	Cyclical AR parameter ρ_1^{wy}	1.578	1.601	1.627	1.711	1.787	1.807	1.825	1.709
	Cyclical AR parameter ρ_2^{wy}	-0.870	-0.852	-0.832	-0.755	-0.671	-0.644	-0.621	-0.753
	Sum of AR parameters ρ_p^{wy}	0.925	0.930	0.936	0.956	0.975	0.980	0.984	0.956
Sum of cyc. loadings α	Unemployment	-0.670	-0.644	-0.617	-0.539	-0.470	-0.453	-0.438	-0.542
	Mining/Manufact.	1.501	1.565	1.638	1.910	2.228	2.333	2.430	1.924
	Construction	1.680	1.763	1.856	2.226	2.631	2.768	2.894	2.239
	Trade/Leisure/Transp.	0.551	0.586	0.626	0.771	0.932	0.981	1.023	0.776
	Fin. and Bus. Svcs.	0.560	0.598	0.644	0.809	0.996	1.052	1.100	0.816
	Pub. and Private Svcs.	-0.010	0.021	0.056	0.180	0.308	0.346	0.380	0.181
	$\delta_{u,n}$	-1.580	-1.489	-1.385	-1.024	-0.664	-0.560	-0.471	-1.025
	$\delta_{zy,wy}$	-0.499	-0.383	-0.260	0.107	0.387	0.469	0.547	0.083
	$\delta_{S,zu}$	-0.0008	-0.0007	-0.0005	0.0000	0.0005	0.0006	0.0008	0.0000
	Variance contrib. of S to z^u	0.0000	0.0002	0.0007	0.0195	0.1096	0.1514	0.1929	0.0418
	$\delta_{S,zy}$	-0.0031	-0.0024	-0.0015	0.0012	0.0034	0.0040	0.0046	0.0010
	Variance contrib. of S to z^y	0.0002	0.0010	0.0041	0.1028	0.4075	0.5005	0.5722	0.1662
	$\delta_{S,wy}$	-0.0052	-0.0045	-0.0039	-0.0018	0.0001	0.0008	0.0015	-0.0019
	$\delta_{S,wy} + \delta_{S,zy}\delta_{zy,wy}$	-0.0046	-0.0041	-0.0035	-0.0017	0.0000	0.0006	0.0011	-0.0017
	Variance contrib. of S to w^y	0.0005	0.0020	0.0076	0.1320	0.3941	0.4716	0.5384	0.1828
	Std. of log reallocation S	0.5354	0.5697	0.6152	0.8123	1.0649	1.1484	1.2277	0.8493
Intercept μ of n trends z^n, z^u	Mining/Manufact.	-0.0067	-0.0062	-0.0058	-0.0044	-0.0029	-0.0024	-0.0018	-0.0043
	Construction	-0.0040	-0.0032	-0.0023	0.0001	0.0025	0.0033	0.0040	0.0001
	Trade/Leisure/Transp.	0.0005	0.0007	0.0009	0.0015	0.0022	0.0024	0.0025	0.0015
	Fin. and Bus. Svcs.	0.0013	0.0016	0.0019	0.0030	0.0040	0.0043	0.0046	0.0029
	Pub. and Private Svcs.	0.0014	0.0018	0.0021	0.0030	0.0039	0.0042	0.0045	0.0030
Persistence ρ of n trends z^n	Mining/Manufact.	0.7536	0.7793	0.8058	0.8784	0.9298	0.9420	0.9518	0.8718
	Construction	0.7422	0.7626	0.7849	0.8508	0.9035	0.9169	0.9281	0.8468
	Trade/Leisure/Transp.	0.6015	0.6420	0.6833	0.7919	0.8680	0.8858	0.9004	0.7814
	Fin. and Bus. Svcs.	0.7202	0.7438	0.7686	0.8413	0.8979	0.9120	0.9236	0.8363
	Pub. and Private Svcs.	0.6859	0.7212	0.7571	0.8541	0.9219	0.9378	0.9506	0.8448
Std. of shock to n trends x^n, z^n	Common trend	0.0018	0.0018	0.0019	0.0022	0.0025	0.0026	0.0027	0.0022
	Mining/Manufact.	0.0010	0.0011	0.0012	0.0016	0.0021	0.0022	0.0024	0.0016
	Construction	0.0024	0.0025	0.0027	0.0033	0.0040	0.0043	0.0045	0.0034
	Trade/Leisure/Transp.	0.0008	0.0008	0.0009	0.0012	0.0016	0.0018	0.0019	0.0013
	Fin. and Bus. Svcs.	0.0010	0.0011	0.0012	0.0015	0.0019	0.0020	0.0021	0.0015
	Pub. and Private Svcs.	0.0008	0.0008	0.0009	0.0011	0.0015	0.0017	0.0018	0.0012
Std. of shock to n levels w^n	Mining/Manufact.	0.0073	0.0074	0.0075	0.0079	0.0083	0.0084	0.0085	0.0079
	Construction	0.0077	0.0078	0.0079	0.0083	0.0087	0.0089	0.0090	0.0083
	Trade/Leisure/Transp.	0.0071	0.0071	0.0072	0.0076	0.0079	0.0080	0.0081	0.0076
	Fin. and Bus. Svcs.	0.0071	0.0072	0.0073	0.0076	0.0080	0.0081	0.0082	0.0076
	Pub. and Private Svcs.	0.0071	0.0072	0.0073	0.0076	0.0080	0.0081	0.0082	0.0076
	Std. of shocks to u trend z^u	0.0014	0.0014	0.0015	0.0016	0.0018	0.0019	0.0019	0.0017
	Std. of shocks to u cycle w^u	0.0011	0.0012	0.0012	0.0013	0.0014	0.0015	0.0015	0.0013
	Std. of shocks to Y trend z^y	0.0028	0.0030	0.0032	0.0039	0.0045	0.0047	0.0049	0.0039
	Std. of shocks to Y cycle w^y	0.0025	0.0027	0.0028	0.0035	0.0043	0.0046	0.0048	0.0036
	Std. of shocks to Y level c	0.0022	0.0023	0.0024	0.0028	0.0033	0.0034	0.0036	0.0029

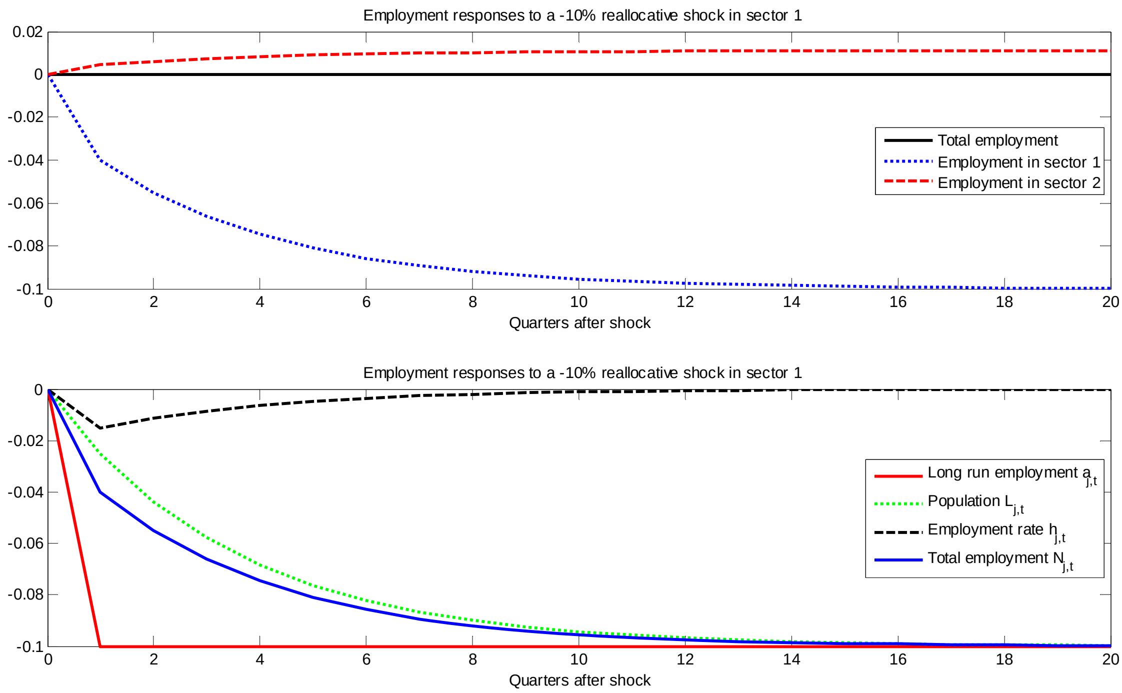
Sources and notes: See table 3a.

Table 6: Posterior correlation matrix for z^n shocks based on Σ_{zn}, 5-sector model

Sector	(1)	(2)	(3)	(4)	(5)
(1)	1.00	-0.53	0.09	0.10	0.21
(2)	-0.53	1.00	0.35	0.33	-0.08
(3)	0.09	0.35	1.00	0.49	0.22
(4)	0.10	0.33	0.49	1.00	0.02
(5)	0.21	-0.08	0.22	0.02	1.00

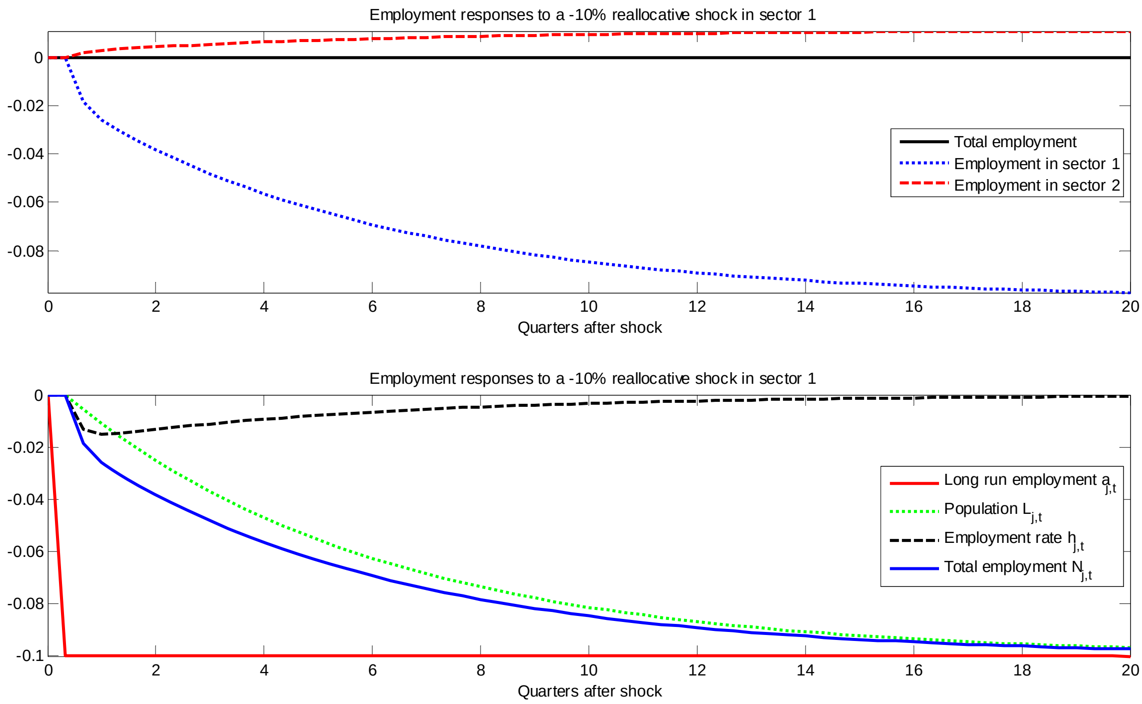
This table shows the posterior correlations of the five long run sectoral shocks. The sectors are numbered as follows: (1) Mining and Manufacturing (Production), (2) Construction, (3) Trade, Leisure, and Transportation, (4) Financial and Business Services, (5) Public and Private Services.

Figure 1: Simulated effect of a reallocative shock in the flexible model



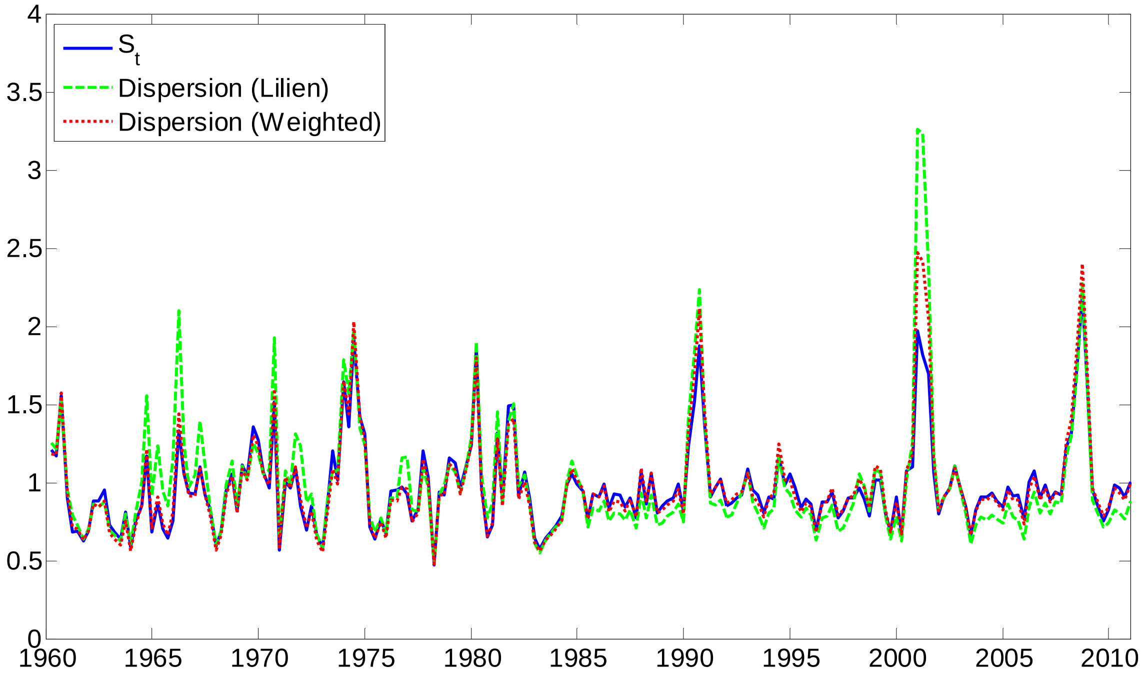
This figure shows the effect of a -10% clean reallocative shock to sector 1 in the flexible model with human capital mismatch. Time is in quarters.

Figure 2: Simulated effect of a reallocative shock in the two-sector search model



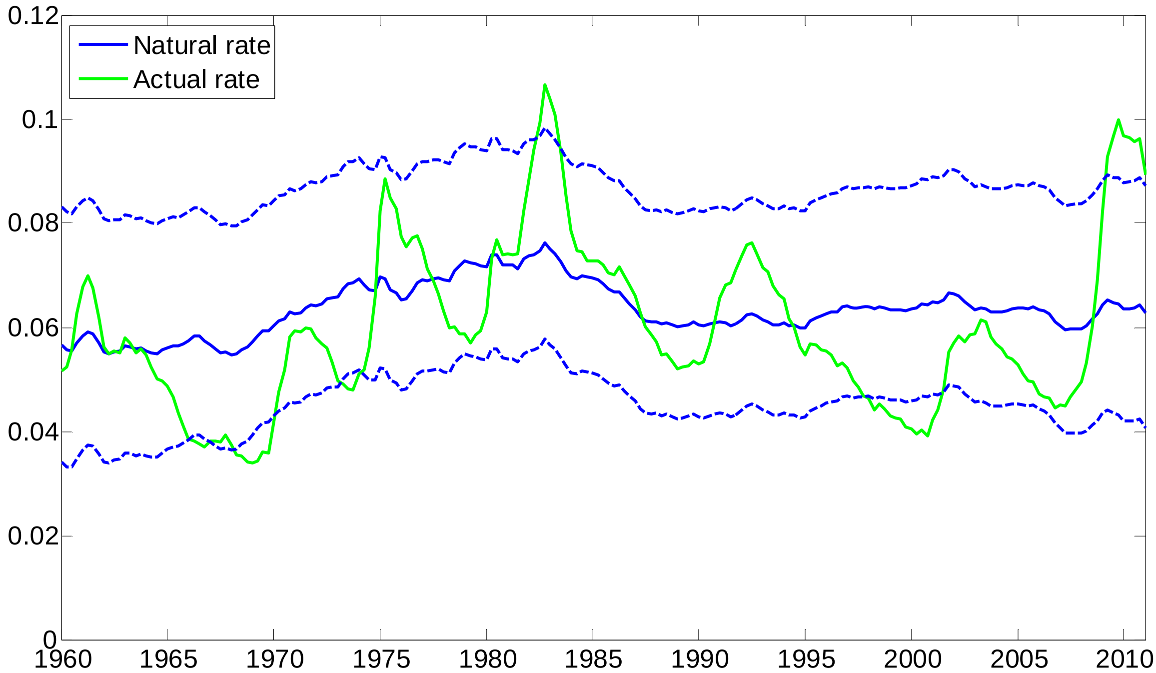
This figure shows the effect of a -10% clean reallocative shock to sector 1 in the two-sector search model with human capital mismatch. Time is in quarters.

Figure 3: Posterior long run dispersion measures (United States, 13 sectors)



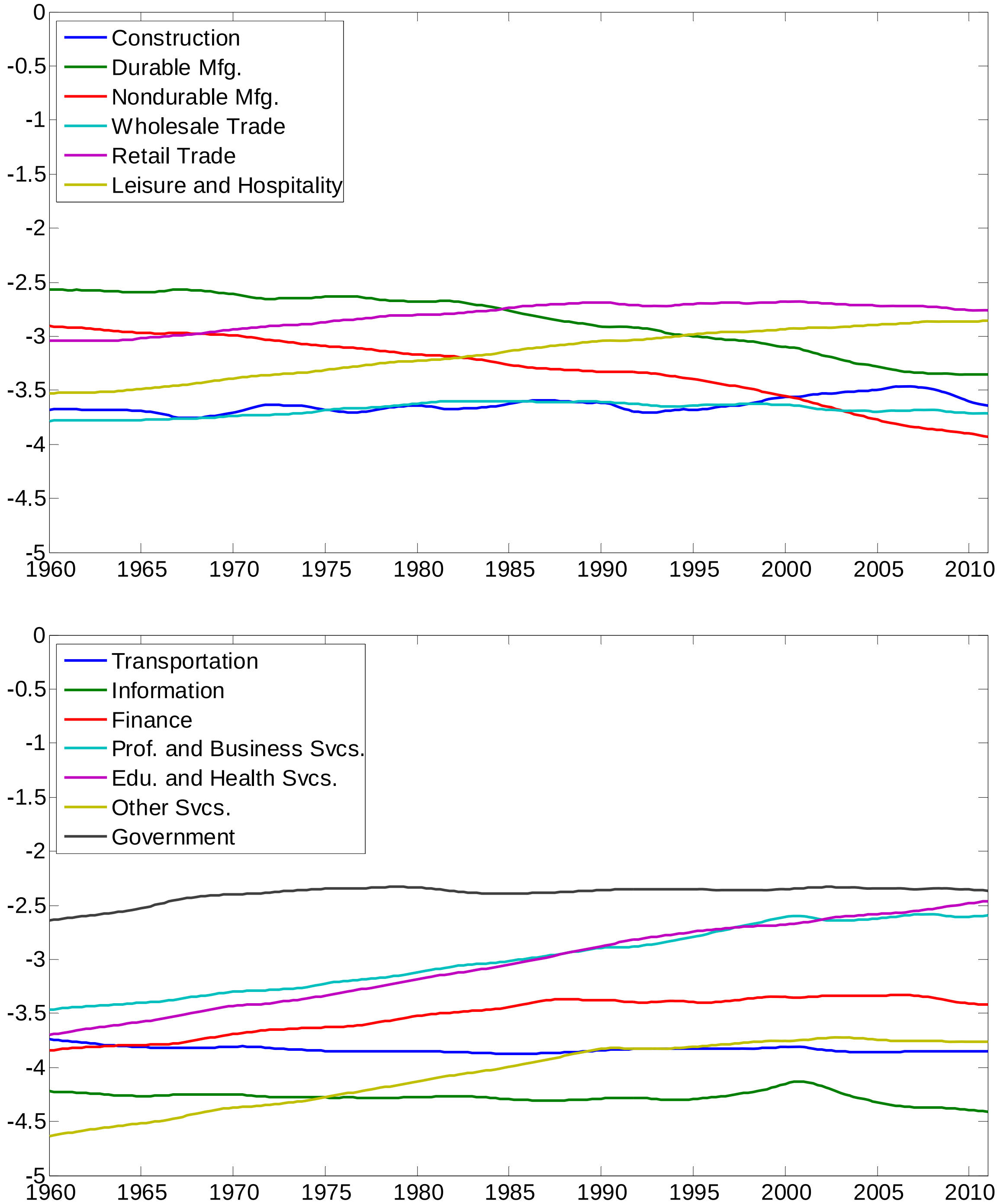
Numbers are scaled geometric means from the state space model.

Figure 4: Posterior mean natural rate of unemployment (United States, 13 sectors)



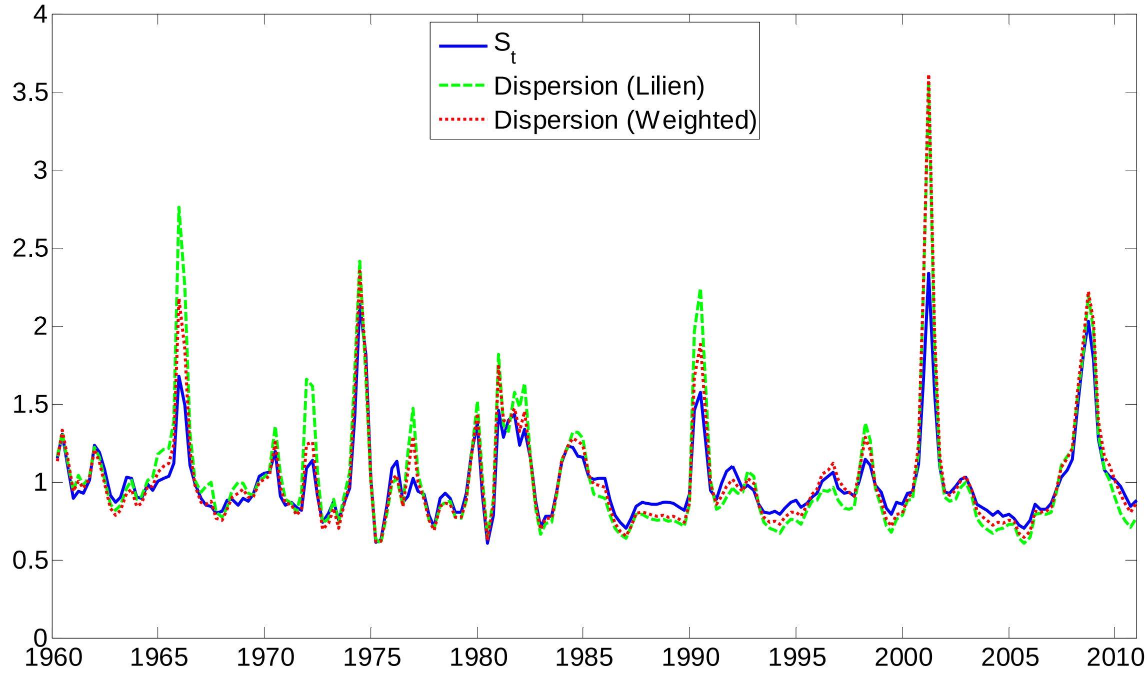
Error bands mark off the 2.5th percentile and the 97.5th percentile of the posterior distribution.

Figure 5: Posterior mean employment trends (United States, 13 sectors)



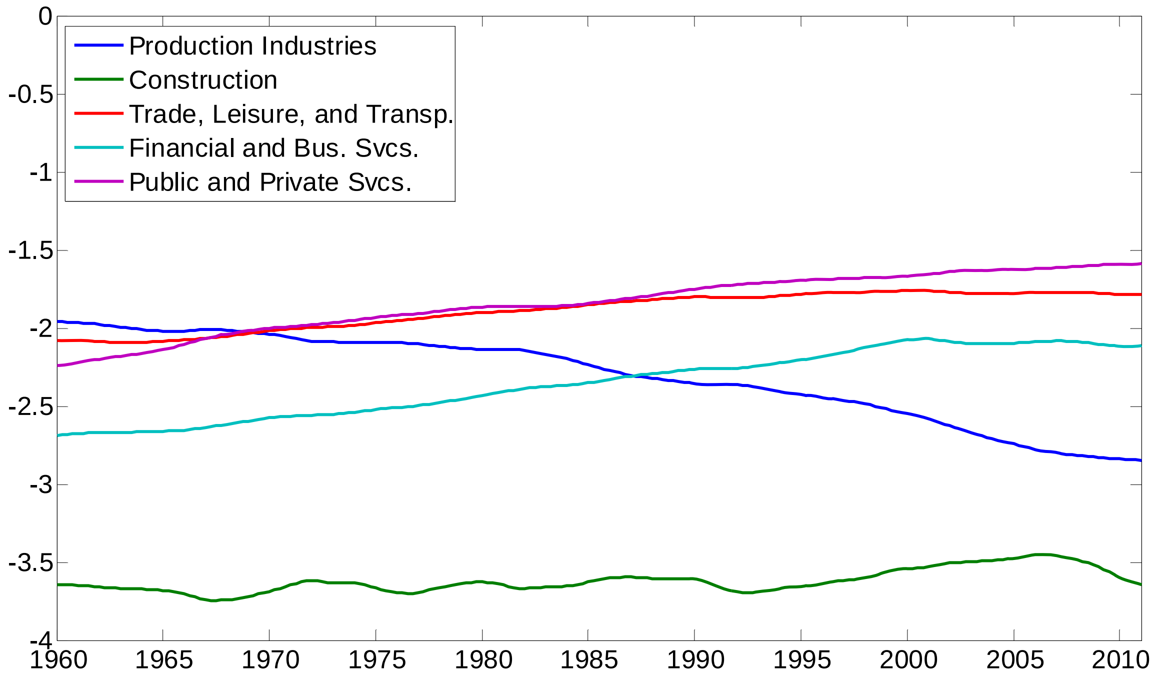
Numbers are posterior means for trend log employment from the state space model.

Figure 6: Posterior long run dispersion measures (United States, 5 sectors)



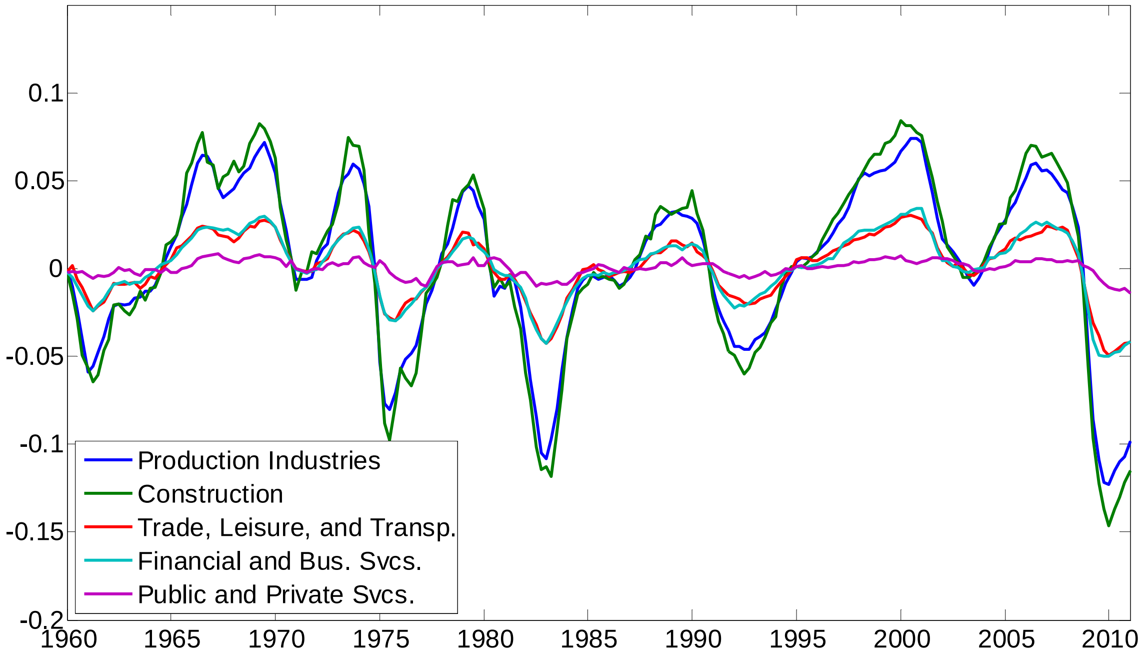
Numbers are scaled geometric means from the state space model.

Figure 7: Posterior mean employment trends (United States, 5 sectors)



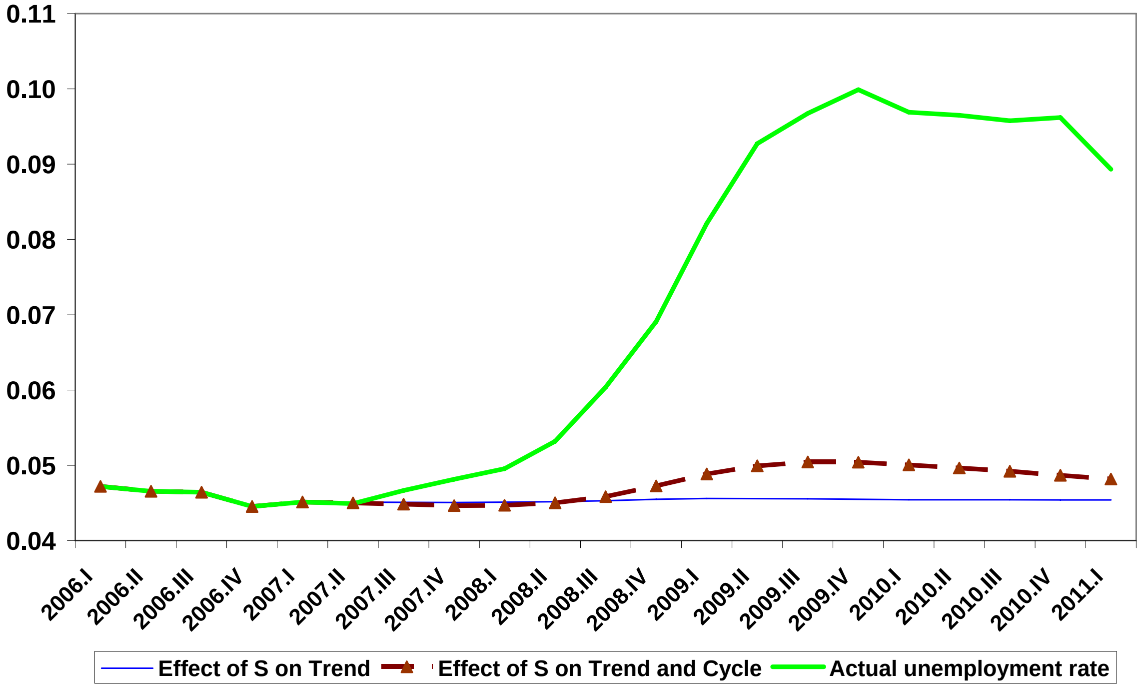
Numbers are posterior means for trend log employment from the state space model.

Figure 8: Posterior cyclical employment deviations (United States, 5 sectors)



Numbers are posterior means for the cyclical deviation of log employment.

Figure 9: Simulated effect of reallocative shocks on unemployment, 2007-2011.



This figure shows the effect of reallocative shocks on unemployment since 2007.II, using the median model coefficients and posterior geometric mean reallocation S_t.