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by Wilfried Rickels

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I investigate the optimal role of carbon sequestration for mitigation in the presence of a ceiling on atmospheric carbon concentration and consider aspects that have so far only been analyzed in the context of a damage function to measure the consequences of climate change for society. I assume extraction costs to be stock-dependent, replace the proportional decay description of the global carbon cycle by a two-box model, investigate the differences resulting from linear versus convex sequestration costs, and consider oceanic instead of geological carbon storage. Using a two-box model allows the non-renewable aspects of the global carbon cycle to be accounted for and implies that carbon emissions have to decline at the ceiling due to the ongoing saturation of the ocean with respect to anthropogenic carbon. Convex sequestration costs result in a continuous use of such a technology and allow the ceiling to be reached later than without sequestration, whereas linear sequestration costs result in a discontinuous use of such a technology and earlier reaching of the ceiling. Consequently, taking into the account the uncertainties in defining an appropriate ceiling, the policy recommendations with respect to carbon sequestration differ crucially according to the underlying assumptions of sequestration costs. Furthermore, the ocean might be a storage option for captured carbon, but even though its storage capacity is probably not scarce by itself, the ongoing saturation of the complete carbon cycle has to be taken into account.

Keywords: atmospheric ceiling, global carbon cycle, ocean sequestration

JEL classification: Q30, Q54

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The role of sequestration costs with a ceiling on atmospheric carbon concentration☆

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Abstract

I investigate the optimal role of carbon sequestration for mitigation in the presence of a ceiling on atmospheric carbon concentration and consider aspects that have so far only been analyzed in the context of a damage function to measure the consequences of climate change for society. I assume extraction costs to be stock-dependent, replace the proportional decay description of the global carbon cycle by a two-box model, investigate the differences resulting from linear versus convex sequestration costs, and consider oceanic instead of geological carbon storage. Using a two-box model allows the non-renewable aspects of the global carbon cycle to be accounted for and implies that carbon emissions have to decline at the ceiling due to the ongoing saturation of the ocean with respect to anthropogenic carbon. Convex sequestration costs result in a continuous use of such a technology and allow the ceiling to be reached later than without sequestration, whereas linear sequestration costs result in a discontinuous use of such a technology and earlier reaching of the ceiling. Consequently, taking into the account the uncertainties in defining an appropriate ceiling, the policy recommendations with respect to carbon sequestration differ crucially according to the underlying assumptions of sequestration costs. Furthermore, the ocean might be a storage option for captured carbon, but even though its storage capacity is probably not scarce by itself, the ongoing saturation of the complete carbon cycle has to be taken into account.

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1. Introduction

Unhindered climate change implies the risk of catastrophic damage to society. Quantifying of this risk is complicated or even impossible due to our still limited understanding of the earth's climate system and in particular to the presence of tipping points in the climate system. Exceeding thresholds corresponding to such tipping points might involve sharp and non-linear changes in climate dynamics, determined intrinsically by the climate system and the feedback mechanisms involved. The location of such thresholds with respect to atmospheric carbon concentration or temperature increase and the degree of irreversibility of crossing such thresholds are still very uncertain (e.g., Lenton et al., 2008; Hoffmann and Rahmsdorf, 2009; Zickfeld et al., 2010). With respect to these uncertainties, countries agreed to limit temperature increase to 2°C as it has again been confirmed at the recent global warming summit in Cancun (UNFCCC, 2010). This temperature increase limit can be converted to a ceiling on atmospheric carbon concentration or a cumulative budget for carbon emissions into the atmosphere (Meinshausen et al., 2009).

Actual greenhouse gas emission (GHG) trends and corresponding reduction announcements challenge the credibility of this target. Furthermore, postponement of the necessary emission reductions implies increasing mitigation costs.[1] However, this target or the corresponding corresponding ceilings can still be met if substantial emission cuts are made. To achieve these emissions cuts, in addition to improving energy efficiency and making increased use of renewable energies, increased carbon sequestration, i.e. carbon capture and storage, within energy consumption is expected to be necessary (IEA, 2010).

The optimal global role of carbon sequestration for mitigation in the presence of a ceiling on atmospheric carbon concentration has so far mainly been analyzed quantitatively by numerous integrated assessment models that consider various atmospheric stabilization levels (e.g., Gerlagh and van der Zwaan, 2006; Azar et al., 2006; van der Zwaan and Gerlagh, 2009; Lemoine et al., 2011). The theoretical literature related to carbon sequestration addresses the problem of increasing atmospheric carbon concentration primarily by a damage function that measures the related consequences of climate change for society. As pointed out above, it is obviously difficult to determine or agree on such a damage function. Theoretical implications of imposing a ceiling on atmospheric carbon concentration while carbon emissions can be sequestered have been studied first of all by Chakravorty et al. (2006) and Lafforgue et al. (2008). Chakravorty et al. investigate the implication of a ceiling while energy consumption is provided by fossil fuels and a clean backstop technology. Additionally, in their model it is possible to reduce fossil-fuel-related carbon emissions to the atmosphere by costly abatement. However, there is little potential to abate carbon emissions once fossil fuels have been combusted, implying that abatement implies sequestration but without any scarcity related to potential storage sites. This has been further investigated by Lafforgue et al.. They consider geological

[1] The rather moderate emission reductions in the Copenhagen Accord until 2020 are estimated to cost an additional 1 trillion USD of investment costs in the period from 2010 to 2035 compared to a more efficient mitigation path (IEA, 2010).

carbon sequestration into a single reservoir or multiple reservoirs where carbon storing capacity is limited. Chakravorty et al. and Lafforgue et al. find that sequestration only takes place in a discontinuous manner once the ceiling has been reached and the availability of sequestration determines endogenously the period when the ceiling is binding.[2] Both papers assume extraction and sequestration units costs to be constant, describe the global carbon cycle by proportional decay of atmospheric carbon, and consider geological carbon storage. In this paper, I take a closer look at these assumptions and clarify the implications of four relevant aspects related to the issue: the implications of stock-dependent extraction cost, (2) the implications of modeling the global carbon cycle with a two-box model instead of a proportional decay, (3) the implications of modeling sequestration costs convexly rather than linearly, and (4) the implications of oceanic instead of geologic carbon storage.

Extraction costs are expected to be determined not only by the extraction rate, but also by the stock of fossil resource left in the ground. Decreasing resource availability in existing deposits and exploitation of economically less favorable deposits might be reflected by increasing marginal costs for a given rate of extraction (Farzin, 1992). The recoverable amount of the resource might therefore not be determined by geological constraints but by economic costs and the opportunity costs of backstop technologies (e.g., Farzin, 1992; Epple and Londregan, 1993; Farzin, 1996).

Furthermore, the carbon fluxes in the global carbon cycle are only roughly approximated by the proportional decay of carbon in the atmosphere that implies that the atmospheric carbon storing capacity is a renewable resource. The dimension and length of the anthropogenic disturbances to atmospheric carbon concentration is, besides the carbon emission path, mainly influenced by oceanic carbon uptake (e.g., Najjar, 1992; Sabine et al., 2004). Oceanic carbon uptake is estimated to increase disproportionally with increasing atmospheric carbon concentration as the buffer capacity declines (e.g., Sarmiento et al., 1995).[3] Even though uptake is currently mainly limited by kinetic constraints implied by the slow mixing of surface waters with the deep ocean (e.g., Sarmiento and Gruber, 2006), it is important to note that a renewable description of the atmospheric carbon concentration overestimates the storing capacity of the global carbon cycle on timescales reasonable to mankind. Consequently, a proportional decay description oversimplifies the atmospheric carbon accumulation problem (Farzin and Tahvonen, 1996; Rickels and Lontzek, 2011).

The IPCC (2005) special report on carbon dioxide capture and storage provides cost ranges for carbon sequestration. The ranges indicate that sequestration costs vary by differences in the design of carbon capture systems and by differences in the operating and financing of the reference plant to which the capture technology is applied. Additionally, their estimates show that the costs increase as the transportation

[2] Chakravorty et al. (2006) distinguishes in his analysis between decreasing, constant, and increasing demand for energy. For the first two assumptions, abatement takes place at the beginning of the ceiling, for the latter assumption at the end of the ceiling.

[3] Note the buffer capacity should not be confused with the buffer or Revelle factor, as the latter measures the ratio of the change in atmospheric carbon concentration to the change in oceanic carbon concentration, $\frac{\partial \ln pCO_2}{\partial \ln DIC}$, and is therefore increasing with anthropogenic carbon perturbation (Sarmiento and Gruber, 2006).

distance on land and on sea increases. By assuming that increasing the amount of carbon capture implies that this technology has also to be applied to less efficient plants and plants that are located farther from storage sites or the shore, the sequestration costs are expected to increase in a convex manner. This is also usually assumed in quantitative analyses, like Gerlagh and van der Zwaan (2006), who assume carbon sequestration to be described by an effort variable which is assumed to be a second-order polynomial function.

The captured carbon can be stored in geological formations like depleted oil fields or saline formations (IPCC, 2005). However, it could also be injected into the deep ocean via pipelines or ships (Marchetti, 1977; IPCC, 2005). As pointed out above, the ocean uptake is not linear, but it is expected that on timescales of several hundred years it will take up about 80 percent of the anthropogenic carbon emitted to the atmosphere (e.g., Archer et al., 1997; Körtzinger and Wallace, 2002; Sarmiento and Gruber, 2006). Consequently, when this fraction of anthropogenic carbon enters the deep ocean, is just a matter of time. Even though currently implicitly prohibited by the London protocol,[4] deep ocean sequestration would be an option to accelerate this process by overcoming kinetic constraints (Keeling, 2009). With respect to the overall carbon storage capacity of the ocean, there are practically no physical limits to oceanic carbon sequestration. However, carbon injected into the deep ocean in excess of the atmosphere-ocean equilibrium amount corresponding to the atmospheric stabilization goal is expected to leak back into the atmosphere, because the ocean will become supersaturated in relation to the atmosphere (IPCC, 2005).

Even though there are further papers that analyze the implications of a ceiling on atmospheric carbon, none of these papers addresses the issue from the perspective used by Chakravorty et al. (2006) and Lafforgue et al. (2008). The papers of Chakravorty et al. (2008) and Smulders and van der Werf (2008) focus on the order of extraction of two fossil fuel resources when these differ with respect to their related carbon emissions. Henriet (2010) focuses on the role of the backstop price and the optimal R&D effort to develop a backstop technology. Dullieux et al. (2010) focus on the strategic interaction between consumers, who set the carbon tax to comply with the ceiling, and fossil fuel producers, who respond by adjusting the fossil fuel prices. However, none of these papers includes carbon sequestration. Hoel and Jensen (2010) focus on the strategic implications involved in carbon sequestration by analyzing its implications in a two-stage decision problem. Amigues et al. (2010) extend the model of Chakravorty et al. (2006) by including stock-dependent extraction costs and the possibility of air capture. They analyze the optimal solution from a decentralized perspective by assuming two different sectors that are distinguished by the availability of abatement options and costs related to these options. They show that abatement can take place before the ceiling has been reached in the sector with the lower abatement costs. However, their result originates from the decentralized perspective

[4]Paragraph 1.8 in Appendix 1 of the London Protocol allows dumping of "Carbon dioxide streams from carbon dioxide capture processes for sequestration". However, this is restricted by paragraph 4: "Carbon dioxide streams referred to in paragraph 1.8 may only be considered for dumping, if: (1) disposal is into a sub-seabed geological formation; and (2) they consist overwhelmingly of carbon dioxide. They may contain incidental associated substances derived from the source material and the capture and sequestration processes used; and (3) no wastes or other matter are added for the purpose of disposing of those wastes or other matter."

and not from the curvature of the related costs, as those are again assumed to be linear. All the papers assume proportional atmospheric carbon decay, except Dullieux et al. (2010) and Hoel and Jensen (2010), who assume no natural decay at all.[5]

The present paper is structured as follows. In Section 2, I explain the optimization problem involved in using fossil fuels and a clean backstop-technology, while extraction costs are stock-dependent. Furthermore, I explain the description of the global carbon cycle using a two-box model. In Section 3 I show the results. Section 3.1 provides the general conditions for the optimal solution. Section 3.2 analyzes first the simple extraction-backstop decision and shows then that the description of the global carbon cycle using a two-box model implies declining carbon emissions at the ceiling. Consequently, it is possible to observe the simultaneous use of fossil fuels and the clean backstop technology from some point at the ceiling onwards even if carbon sequestration is not available. Section 3.3 shows first the difference between convex and linear sequestration costs by using the proportional decay description for the global carbon cycle. Whereas linear sequestration costs imply that the ceiling is reached earlier than without sequestration, convex sequestration costs allow the ceiling to be reached later. Additionally, convex sequestration costs imply that such a technology is used in a continuous manner. Then I show that ocean sequestration allows the kinetic constraints of oceanic carbon uptake to be overcome and that, even though the storage capacity of the ocean is not scarce in and of itself, the ongoing saturation of the global carbon cycle determines the optimal amount of sequestration. Finally, I present a numerical example to demonstrate the dynamics of the simple extraction-backstop decision while atmospheric carbon concentration is limited with a ceiling and how the dynamics change if ocean sequestration is included. I doing so, I distinguish whether sequestration costs are linear or convex. Section 4 concludes.

2. Two-box model with oceanic carbon storage and ceiling

I investigate the dynamic global optimal sequestration decision in the presence of a ceiling on atmospheric carbon stock as a social planner's problem where the social rate of discount is assumed to be positive and constant. The optimal sequestration decision is embedded in the decision regarding the global optimal energy consumption. Energy consumption is composed of fossil-fuel-based energy, $x(t)$, and non-fossil-fuel-based energy like solar or wind power, $y(t)$, which I denote as backstop technology. The total amount of energy consumption, $x(t) + y(t)$, generates gross utility in the social welfare function at any instant in time. Gross utility is described by $U(x(t) + y(t))$ and is assumed to have the properties

Assumption 1. $U' > 0, \quad U'' < 0.$

[5] Smulders and van der Werf (2008) assume a ceiling for the flow of emissions and abstract therefore from atmospheric carbon accumulation. Note that a ceiling on the flow of emissions, e.g., $\bar{q}$, implies an atmospheric carbon stock with proportional decay, e.g., $-\beta S(t)$, where the emission flow ceiling is equal to natural decay, so that $\bar{q} = \beta S(t)$.

The extraction of fossil fuels generates extraction costs at any instant in time which are assumed to depend on the resource stock $R(t)$ and the extraction cost function $C(R)$ is assumed to have the property

Assumption 2. $C'(R) < 0$,

so that extraction costs increase with the rate of exploitation of the available resource resource stock:[6]

$$\dot{R} = -x(t) \quad \text{with} \quad R(t_0) = R_0. \tag{1}$$

Following Chakravorty et al. (2006) and Lafforgue et al. (2008), I assume that the backstop technology has constant unit costs c_y and can be provided at an extent such that $U'(y(t)) = c_y$ with $y(t) = \bar{y} > 0$ is feasible at each point in time. Due to the presence of a backstop technology with the constant unit costs c_y, there is no need to specify whether the utility function satisfies the Inada condition or entails a choke price, $U'(0) = b < \infty$, as long as $b > c_y$. With respect to the constant unit costs of the backstop technology, I assume

Assumption 3. $R_0 > C^{-1}(c_y)$,

because otherwise energy consumption would only be provided by the backstop technology. In Section 3.2, I investigate this basic extraction-backstop optimization problem and refer to it as Scenario *Ext*.

The proportional amount of carbon emissions related to fossil fuels consumption (the proportionality factor is assumed to be one) increases the amount of carbon in the atmosphere and therefore in the global carbon cycle. The global carbon cycle is represented by a two-box model, where the upper box $S(t)$ entails the sum of the carbon stock in the atmosphere and the upper ocean and the lower box $W(t)$ entails the carbon stock in the deep ocean:

$$\dot{S} = x(t) - \gamma(S(t) - \omega W(t)) \quad \text{with} \quad S(t_0) = S_0, \tag{2}$$

$$\dot{W} = \gamma(S(t) - \omega W(t)) \quad \text{with} \quad W(t_0) = W_0. \tag{3}$$

The atmospheric carbon stock is assumed to be a constant fraction of $S(t)$.[7] In the equations of motion for the upper and lower box, (2) and (3), the amounts $\gamma S(t)$ and $\gamma\omega W(t)$ represent the natural fluxes between the boxes, which amount to a net transfer if there is a difference between the relative stock sizes, e.g., $S(t) > \omega W(t)$. Consequently, an increase in the atmospheric carbon concentration and therefore an

[6] An alternative formulation would be to model extraction costs to be increasing in the cumulative amount of fossil fuels extracted: $C(X)$ with $C' > 0$, while $X(t) = \int_0^t q(\tau)d\tau$ (Farzin, 1992).

[7] There is a net transfer of carbon between the atmosphere and the upper mixed layer of the ocean if there is a difference in the partial pressure of carbon dioxide (pCO_2) between these two reservoirs. It takes around one year for the upper layer of the ocean to equilibrate with the atmosphere. Consequently, I assume that the atmosphere and the upper mixed layer are always in equilibrium and focus on the transport of anthropogenic carbon to the deeper parts of the ocean, which is the limiting factor of oceanic carbon uptake.

increase in the carbon stock in the upper box causes a net downward transfer of excess carbon into the deep ocean because up-welling water is still free of excess anthropogenic carbon. The deep ocean saturates with anthropogenic carbon only at the rate ω because anthropogenic carbon reacts with carbonate ions to bicarbonate ions. Consequently, the term inside the parenthesis of the net exchange can be interpreted as a simplified representation of the chemical reactions caused by the uptake of anthropogenic carbon by the ocean.[8] The parameter γ represents the kinetic constraint, as it measures the speed of the adjustment process. Taking into account the inertia of the carbon cycle with respect to the carbon exchange with the deep ocean, realistic initial values are restricted to satisfy $S_0 \geq \omega W_0$. For simplicity I impose

Assumption 4. $S_0 = \omega W_0$.

Even though the two-box model is a very simple representation of the global carbon cycle, it allows a more appropriate description of the inertia and the non-renewable aspects of the global carbon cycle than the proportional-decay description does. A more detailed description of this two-box model can be found in Rickels and Lontzek (2011), where it is also shown that the two-box model does not provide any advantages in itself compared to modeling the atmospheric carbon stock as a partially renewable resource, as is done in Farzin and Tahvonen (1996), but becomes indispensable if options like ocean sequestration are considered. An overview of such box models used to represent the dynamics of the global carbon cycle can be found in Sarmiento and Gruber (2006). Even though the carbon stock in the upper box entails atmospheric and oceanic carbon, I refer to it as the atmospheric carbon stock and to the carbon stock in the lower box as the oceanic carbon stock. If we abstract from the ongoing saturation of the ocean with anthropogenic carbon, the oceanic uptake would only be limited by the kinetic constraint and we would regain the proportional decay description as used by Chakravorty et al. (2006) and Lafforgue et al. (2008): $\dot{S} = x(t) - \gamma_d S(t)$, implying that all anthropogenic carbon will be taken up by the ocean in the long run, $\dot{W} = \gamma_d S(t)$. The subscript d indicates that the parameter value does not necessarily coincide in the two-box model and the proportional decay description because in the latter the initial value of S is normalized to represent the deviation from its preindustrial level.

Further, I assume that society has agreed on that atmospheric carbon concentration should not exceed a certain ceiling:

Assumption 5. $\bar{S} \geq S(t)$ *for* $t \in (0, \infty)$ *with* $\bar{S} > S_0$.

In Section 3.2, I investigate in the Scenario *Ceil* how the basic extraction-backstop optimization problem changes in the presence of a ceiling on atmospheric carbon concentration when the global carbon cycle is described by the two-box model.

[8] I abstract from chemical reactions with the sediments and chemical reactions due to enhanced weathering. These reactions operate on timescales of 1000 to 100 000 years and are assumed to be beyond the economic optimization horizon due to discounting.

Additional to releasing carbon emissions into the atmosphere, I assume that these emissions can also be captured and injected into the deep ocean for purposes of ocean sequestration. I subdivide the total amount of fossil fuels extracted into those with related emissions released into the atmosphere $q(t)$ and those with related emissions captured and injected into the deep ocean $a(t)$, so that $x(t) = q(t) + a(t)$ with $q(t), a(t) \geq 0$.[9]

Ocean sequestration generates additional costs in the social welfare function at any instant in time. These costs are assumed to be described by $A(a(t))$ and summarize the costs of carbon capture in energy generation, transportation of carbon to the shore, and injecting it via pipelines or ships into the deep ocean. I distinguish two cases for the properties of $A(a(t))$:

Assumption 6. *Case C: $A' = a_c$ and Case X: $A' > 0$, $A'' > 0$.*

The costs are measured in the same units as utility. In Section 3.3, I investigate the optimal sequestration decision for both cases when the global carbon cycle is described either by proportional decay or the two-box model. With the former description, this model version coincides with the one used by Chakravorty et al. (2006) apart from stock-dependent extraction costs. Even though Chakravorty et al. refer to the control variable $a(t)$ as abatement, it should be noted that there exist literally no abatement measures for fossil-fuel-related carbon emissions once the fossil fuels have been combusted.[10] Consequently, abatement in this context can be interpreted as sequestration without scarcity of the carbon storing facility. I refer to the two scenarios with the proportional decay description as AC and AX, and to the two scenarios with the two-box model description as $SeqC$ and $SeqX$.

3. Results

3.1. Optimal solution conditions

The social welfare function can be formalized as follows:

$$\max_{q(t),a(t),y(t)} \int_0^\infty (U(q(t) + a(t) + y(t)) - C(R(t))(q(t) + a(t)) - A(a(t)) - c_y y(t))e^{-\rho t} dt, \tag{4}$$

$$\text{with} \quad q(t), a(t), y(t) \geq 0, \tag{5}$$

[9] Note that I could also apply the control variables $x(t)$ and $a(t)$ instead of $q(t)$ and $a(t)$, which would imply that only the net emissions $x(t) - a(t)$ would be released into the upper box and the resource stock would decrease by $x(t)$. In doing so, I would retain the model description of Chakravorty et al. (2006) and Lafforgue et al. (2008), where it becomes necessary to include the additional control constraints $x(t) - a(t) \geq 0$ if you do not want to consider the possibility of air capture.

[10] With respect to abatement the variable $a(t)$ could also be interpreted as costly additional efficiency gains, so that the amount of energy provided by $q(t)$ increases to $q(t) + a(t)$. A more profound description of this interpretation would be $a(t)q(t)$, with $a(t)\epsilon(a_{BAU}, a_{max})$, where an increase in energy efficiency to above its business-as-usual (BAU) level would be associated with additional costs.

where ρ is the constant social rate of discount and the dynamic and state variable constraints are given by

$$\begin{aligned} \dot{S} &= q(t) - \gamma S(t) - \gamma\omega W(t) \quad \text{with} \quad S(t_0) = S_0, & (6)\\ \dot{W} &= a(t) + \gamma S(t) - \gamma\omega W(t) \quad \text{with} \quad W(t_0) = W_0, & (7)\\ \dot{R} &= -q(t) - a(t) \quad \text{with} \quad R(t_0) = R_0, & (8)\\ S(t) &\leq \bar{S}. & (9) \end{aligned}$$

If the carbon cycle is described using the proportional decay description, the term $\gamma\omega W(t)$ drops out in (6) and (7) and γ is replaced by γ_d. From now on, I drop the time variable whenever it is convenient and the optimization problem described in (4) to (9) leads to the corresponding current value Lagrangian

$$L_c = U(q+a+y) - A(a) - C(R)(q+a) - c_y y - \lambda_R \dot{R} - \lambda_S \dot{S} - \lambda_W \dot{W} - \theta_1(-q) - \theta_2(-a) - \theta_3(-y) - \theta_4(S(t) - \bar{S}), \quad (10)$$

where

$$\lim_{t\to\infty} R(t) \geq 0, \quad \lim_{t\to\infty} S(t) \geq 0, \quad \lim_{t\to\infty} W(t) \geq 0. \quad (11)$$

I have changed the signs of the costate variables, λ_S and λ_W, in order to facilitate their economic interpretation as taxes. According to Proposition 6.2 and Propostion 7.5 in Feichtinger and Hartl (1986), the admissible solution has to fulfill the necessary conditions

$$\begin{aligned} \frac{\partial L_c}{\partial q} &= 0 \quad \Rightarrow \quad U' - C(R) - \lambda_R - \lambda_S + \theta_1 = 0, & (12)\\ \frac{\partial L_c}{\partial a} &= 0 \quad \Rightarrow \quad U' - C(R) - A' - \lambda_R - \lambda_W + \theta_2 = 0, & (13)\\ \frac{\partial L_c}{\partial y} &= 0 \quad \Rightarrow \quad U' - c_y + \theta_3 = 0, & (14)\\ -\frac{\partial L_c}{\partial R} &= \dot{\lambda}_R - \rho\lambda_R \quad \Rightarrow \quad C'(R)(q+a) = \dot{\lambda}_R - \rho\lambda_R, & (15)\\ -\frac{\partial L_c}{\partial S} &= -\dot{\lambda}_S + \rho\lambda_S \quad \Rightarrow \quad \gamma\lambda_S - \gamma\lambda_W - \theta_4 = \dot{\lambda}_S - \rho\lambda_S, & (16)\\ -\frac{\partial L_c}{\partial W} &= -\dot{\lambda}_W + \rho\lambda_W \quad \Rightarrow \quad -\gamma\omega\lambda_S + \gamma\omega\lambda_W = \dot{\lambda}_W - \rho\lambda_W, & (17)\\ \frac{\partial L_c}{\partial \theta_1} &\geq 0 \quad \theta_1 \geq 0 \quad \theta_1(-q) = 0, & (18)\\ \frac{\partial L_c}{\partial \theta_2} &\geq 0 \quad \theta_2 \geq 0 \quad \theta_2(-a) = 0, & (19)\\ \frac{\partial L_c}{\partial \theta_3} &\geq 0 \quad \theta_3 \geq 0 \quad \theta_3(-y) = 0, & (20)\\ \frac{\partial L_c}{\partial \theta_4} &\geq 0 \quad \theta_4 \geq 0 \quad \theta_4(S - \bar{S}) = 0, & (21) \end{aligned}$$

and also the transversality conditions[11]

$$\lim_{t\to\infty} e^{-\rho t}\lambda_R R = 0, \quad \lim_{t\to\infty} e^{-\rho t}\lambda_S = 0, \quad \lim_{t\to\infty} e^{-\rho t}\lambda_W = 0, . \tag{22}$$

Given the necessary conditions (12) to (22) are fulfilled, the solution is optimal because the control constraints fulfill the constraint qualification (see Appendix A) and are quasi-concave, because the equations of motion are described by linear equations and because the maximized Hamiltonian is concave in the state variables (see Appendix A)(Feichtinger and Hartl, 1986, p.181, Proposition 7.5). Given the optimization problem did not entail the backstop technology y and sequestration costs had the properties according to Case X in Assumption 6, the Hamiltonian would be strictly concave in the control variables (see Appendix A), which implies that the control variables are continuous (Feichtinger and Hartl, 1986, p.167, Corollary 6.2). Without this restriction, jumps in the controls variables can be observed when using sequestration (Case C) and switching to the backstop technology. Even though there can be jumps in the control variables, the costate variables are continuous if the ceiling constraint for the atmospheric carbon stock is not just tangentially approached (Feichtinger and Hartl, 1986, p.168, Corollary 6.3).

Note the equations of motion (6) to (8) constitute a closed system, so that ,e.g., a decrease in the resource stock must be balanced by an equivalent increase in the atmospheric and oceanic carbon stock, $\dot{S} + \dot{W} + \dot{R} = 0$. Therefore, it is possible to reduce the system by replacing one of the state variables. I refer to such a reduced system as a $Redux^X$ system, where the superscript indicates the dropped state variable. If the oceanic carbon stock has been excluded, the equation of motions for $Redux^W$ would read as follows:

$$\dot{R}^W = -q - a, \tag{23}$$

$$\dot{S}^W = q - \gamma(S - \omega(K_0 - S - R)), \tag{24}$$

where K_0 is the sum of the initial values of the stock variables $K_0 = S_0 + W_0 + R_0$. In the $Redux^X$ system the corresponding costate variable λ_X vanishes and the remaining two costate variables then also measure the influence of the omitted state variable on the objective function.

3.2. Stock-dependent extraction costs and the two-box model

I consider first the implications of our model without the option of oceanic carbon storage to clarify the implications of stock-dependent extraction costs and the two-box model description of the global carbon cycle. However, independent of the carbon cycle representation, without a ceiling on the atmospheric carbon

[11] As any admissible path for the state and costate variables is non negative and as any admissible path for the state variables is bounded due to the description of the carbon cycle as a closed system, the fulfillment of the transversality conditions, (22), is sufficient for the fulfillment of the general transversality conditions in a infinity horizon problem (Feichtinger and Hartl, 1986, Remark 2.9 and Remark 7.5).

stock in Scenario *Ext*, the optimization problem reduces to the simple extraction-backstop decision. The optimal solution is determined by the stock-dependent extraction costs $C(R)$, the shadow resource scarcity rent λ_R, and the constant price of the backstop technology c_y. As atmospheric or oceanic carbon stocks do not affect the objective function, the corresponding costate variables λ_S and λ_W are zero.[12] The marginal costs for fossil fuel extraction are increasing whereas the marginal costs of using the backstop technology are constant. Consequently, fossil fuels and backstop technology are not used simultaneously (e.g., Dasgupta and Heal, 1979). From Assumption 3 follows that there will be first a period when energy consumption is only provided by fossil fuels. The dynamics are described by

$$\dot{q} = \frac{\rho(U' - C(R))}{U''}, \tag{25}$$

$$\dot{R} = -q, \tag{26}$$

indicating that both q and R are monotonically decreasing until $q = \bar{y}$. At this point in time fossil fuel extraction drops to zero and energy consumption switches to the backstop technology for $t \,\epsilon\, (t_b, \infty)$ where t_b denotes the switching point. As shown by Farzin (1996), the inclusion of stock-dependent extraction costs changes the behavior of the shadow scarcity rent and the total amount of fossil fuel extraction, which is summarized in the context of our model in the presence of a backstop technology in the following proposition.

Proposition 1. *If $C(0) \geq c_y$ and Assumption 3 holds, $R(t_b) = R_\infty \geq 0$, and $\lambda_R(t_b) = 0$, with $\dot{\lambda}_R < 0$ for $t \,\epsilon\, (0, t_b)$, and $\dot{\lambda}_R = 0$ for $t \,\epsilon\, (t_b, \infty)$. If $C(0) < c_y$ and Assumption 3 holds, $R(t_b) = 0$, $\lambda_R(t_b) = c_y - C(0)$, with $\dot{\lambda}_R < 0$ for $t \,\epsilon\, (0, t_b)$, and $\dot{\lambda}_R > 0$ for $t \,\epsilon\, (t_b, \infty)$.*

Proof. At the switching point t_b, the marginal costs for extraction and the backstop technology have to be equal. Any solution including $C(R(t_b) + \lambda_R(t_b) = c_y$ cannot be optimal because a lower $\lambda_R(t_b)$ would allow $q(t_b) > \bar{y}$, so that the objective could be raised by increasing t_b. Consequently, λ_R is decreasing until either $C(R(t_b)) = c_y$ with $\lambda_R(t_b) = 0$ or until $R(t_b) = 0$ with $\lambda_R(t_b) = c_y - C(0)$. The closed form solution for λ_R is

$$\begin{array}{lll} C(0) \geq c_y & \lambda_R(t) = & \int_{t_b}^{t} C'(R(\tau))q(\tau)e^{-\rho(\tau - t)}d\tau, \\ C(0) < c_y & \lambda_R(t) = & (c_y - C(0))e^{-\rho(t_b - t)} + \int_{t_b}^{t} C'(R(\tau))q(\tau)e^{-\rho(\tau - t)}d\tau, \end{array} \tag{27}$$

which shows that the transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda_R R = 0$ is fulfilled because either $\lambda_R(t_b) = \lambda_R(t)_{t\to\infty} = 0$ or $R_\infty = 0$. □

In contrast to Chakravorty et al. (2006) and Lafforgue et al. (2008), the shadow scarcity rent is declining

[12] This can also be seen from the closed-form solution for λ_S in, e.g., the *Redux*^{W} system: $\lambda_S^W(t) = \lambda_{S0}e^{(\gamma+\gamma\omega+\rho)t}$. If the initial level λ_{S0} is positive, the transversality condition would be violated because θ_4 is zero due to the non existent ceiling and $\lim_{t\to\infty} \lambda_S(t) = \infty$.

and the fossil fuel stock is not necessarily entirely extracted because extraction costs are modeled as stock-dependent. Moreover, the possibility that $C(0) < c_y$ is more a theoretical concept, as it implies that extraction costs are still below the cost of the backstop technology when fossil fuel stocks are completely depleted. The more reasonable concept seems to be $C(0) > c_y$ so that the amount of the economic recoverable resource is endogenously determined by $C^{-1}(c_y) = R(t_b) = R_b > 0$ (Farzin, 1996).

The carbon emissions related to the extraction in Scenario *Ext* affect the dynamics of the global carbon cycle. According to Assumption 4, atmospheric carbon concentration is initially increasing and thereby also the net transfer into the deep ocean $nt = \gamma S - \gamma\omega W$ because the downward flux increases by γ, whereas the upward flux only by $\gamma\omega$. Even if Assumption 4 is not fulfilled, oceanic carbon stock is monotonically increasing given the initial values fulfill $S_0 > \omega W_0$.[13] However, atmospheric carbon concentration increases at a declining rate ($\ddot{S} < 0$) or might even reverse its sign ($\dot{S} < 0$) because of the monotonically decreasing carbon emissions.

Proposition 2. *If Assumption 4 is fulfilled, atmospheric carbon concentration approaches a unique peak concentration S_p at t_p with $0 < t_p \leq t_b$ if $U''' > 0$.*

Proof. If atmospheric carbon concentration is monotonically increasing until t_b, the peak concentration will be approached at $t_p = t_b$ because from $\dot{S} = q(t_b) - \gamma S(t_b) - \gamma\omega W(t_b) > 0$, from the continuity in the state variables, and from $q(t_b) = \bar{y} > 0$ follows that $\dot{S} = \gamma S(t_b + \epsilon) - \gamma\omega W(t_b + \epsilon) < 0$ with $\epsilon \to 0$. If atmospheric carbon concentration is not monotonically increasing until t_b, the peak concentration will be approached at $t_p < t_b$ with $-\frac{\dot{q}}{q} > \gamma\omega$ which follows from $\ddot{S}^R = \dot{q} - \gamma\omega\dot{R}^R < 0$. The atmospheric carbon concentration can only once reverse its sign between $t \,\epsilon\, (0, t_b)$. Observing two extrema would require

$$\begin{aligned} S_{max}: &\quad -\dot{q}/q > \gamma\omega \\ S_{min}: &\quad -\dot{q}/q < \gamma\omega, \end{aligned} \tag{28}$$

requiring an inflection point for q between S_{max} and S_{min}. From

$$\ddot{q} = \frac{\rho(U''\dot{q} + C'(R)q)U'' - \rho(U' - C(R))U'''\dot{q}}{(U'')^2} \tag{29}$$

it can be seen that $\ddot{q} = 0$ is not feasible if $U''' > 0$ because $U' - C(R) < 0$ follows from the necessary optimality condition (12). □

The proposition would also be valid if Assumption 4 only required $q_0^{Ext} > \gamma(S_0 - \omega W_0)$, where q_0^{Ext} is

[13] This can be seen from the closed-form solution for $W(t)$ if emissions are zero: $W(t) = e^{-(\gamma+\gamma\omega)t}(\frac{\omega W_0 - S_0}{1+\omega}) + \frac{1}{1+\omega}W_0$, where the first term inside the parentheses is zero if the initial values constitute a carbon cycle equilibrium but is negative if initial atmospheric carbon concentration has already increased. As the parentheses are multiplied by a declining exponential term, the oceanic carbon stock is monotonically increasing. This property does not change if carbon emissions are included because these are only released into the atmosphere.

the initial amount of carbon emissions in Scenario Ext. From the proposition, it follows that if $q_0^{Ext} < \gamma(S_0 - \omega W_0)$, the atmospheric carbon concentration is either monotonically decreasing until $t \to \infty$ or is U-shaped until t_b with $S_p(t_b) \lesseqgtr S_0$. However, no matter when the atmospheric peak concentration is reached, the carbon cycle is in disequilibrium at t_b because $q(t_b) = \bar{y} > 0$. Consequently, the steady state values for atmospheric and oceanic carbon stocks are approached by the natural adjustment process, as $t \to \infty$, and are given by

$$S_\infty = \frac{\omega}{1+\omega}(S_0 + W_0 + R_0 - R_b), \quad W_\infty = \frac{1}{1+\omega}(S_0 + W_0 + R_0 - R_b). \tag{30}$$

I turn now to Scenario $Ceil$, where atmospheric carbon stock is limited by a ceiling $S(t) \leq \bar{S}$. If $S_\infty > \bar{S}$, the ceiling would be limiting for the total carbon accumulation in the carbon cycle and the total amount of fossil fuel extraction would decrease to

$$\int_0^{t_b^{Ceil}} q^{Ceil}(\tau)d\tau = \frac{1+\omega}{\omega}\bar{S} - S_0 - W_0 = R_0 - R_b^{Ceil} < R_0 - R_b^{Ext}. \tag{31}$$

If $S_p < \bar{S}$ holds, the ceiling would never be binding and the optimal solution would coincide with the solution in Scenario Ext. With respect to the total storing capacity of the global carbon cycle but also the inertia of oceanic carbon uptake, I focus on the situation where the ceiling only limits the atmospheric peak concentration $S_p^{Ext} > \bar{S} > S_\infty > S_0$. $S_p^{Ext} > \bar{S}$ implies that $t_{cr}^{Ceil} < t_{cl}^{Ceil}$, where t_{cr} and t_{cl} denote the points in time when the ceiling is reached and left, respectively. $\bar{S} > S_\infty$ implies that the total amount of fossil fuel extraction is not affected ($S_\infty^{Ext} = S_\infty^{Ceil}$) but that the extraction dynamics are. $\bar{S} > S_0$ implies a period in the beginning when atmospheric carbon stock is below the ceiling. The binding ceiling requires θ_4 to be positive between t_{cr}^{Ceil} and t_{cl}^{Ceil}, resulting in positive costate variables λ_S and λ_W for $t \,\epsilon\, (0, t_{cl}^{Ceil})$, where λ_S measures the shadow environmental scarcity rent of the atmospheric carbon storing capacity and λ_W measures the shadow environmental scarcity rent of the oceanic carbon storing capacity (Farzin, 1996). The overall carbon storing capacity is only scarce before and at the ceiling. Once the ceiling has been left, there is no scarcity and therefore $\lambda_S(t) = \lambda_W(t) = 0$ for $t \,\epsilon\, (t_{cl}, \infty)$. The dynamics of the costate variables can be seen from the closed form for λ_S in the $Redux^W$ system[14]:

$$\lambda_S^W(t) = \lambda_{S0}^W e^{(\rho+\gamma+\gamma\omega)t} - \int_0^t \theta_4(\tau)e^{-(\rho+\gamma+\gamma\omega)(\tau-t)}d\tau \tag{32}$$

The closed form shows that the costate variables in the $Redux^W$ system and therefore both costate variables in the complete system are increasing on the path towards the ceiling and decreasing at the ceiling because $\theta_4(t) = 0$ for $t \,\epsilon\, (0, t_{cr})$ and $\theta_4(t) \geq 0$ for $t \,\epsilon\, (t_{cr}, t_{cl})$ with $\lambda_S(t) \geq \lambda_W(t)$ for $t \,\epsilon\, (0, t_{cl})$ For the

[14]The costate variable associated to the atmospheric carbon stock in the $Redux^W$ system is equal to the tax difference in the complete system: $\dot{\lambda}_S - \dot{\lambda}_W = (\lambda_S - \lambda_W)(\rho + \gamma + \gamma\omega) + \theta_4$.

atmospheric carbon stock at the ceiling, the entire dynamics are exogenously determined by the two-box model description.

Proposition 3. *When atmospheric carbon concentration is at the ceiling, extraction is monotonically decreasing at a constant contraction rate* $-\gamma\omega$*.*

Proof. From $\dot{S}^W = 0 = q - (\gamma + \gamma\omega)\bar{S} + \gamma\omega(K_0 - R(t))$ follows that $\frac{\dot{q}}{q} = -\gamma\omega$. □

In contrast to Chakravorty et al. (2006) and Lafforgue et al. (2008) extraction is not constant but decreasing at the ceiling because of the increasing saturation of the carbon cycle with anthropogenic carbon. This is implicitly confirmed by Farzin and Tahvonen (1996) who capture the non-renewable aspect of the global carbon cycle by artificially dividing the atmospheric carbon stock into two stock, on with decay and the other one without. However, they consider a damage function to measure the social costs of increasing atmospheric carbon concentration instead of a ceiling. They show that for certain functional forms and initial levels the situation of a stationary policy arises, where atmospheric carbon concentration is constant and extraction is decreasing at a constant contraction rate. The differences between a two-box model description versus an artificially division of the atmospheric carbon stock are further discussed in Rickels and Lontzek (2011). They show that the two-box model does not provide an advantage in itself, but becomes indispensable if options like ocean sequestration are investigated.

With the two-box model the relationship between the resource stock and the amount of extraction at the ceiling is linear:

$$q_c^{Ceil}(R) = K_1 + \gamma\omega R \quad \text{with} \quad K_1 = (\gamma + \gamma\omega)\bar{S} - \gamma\omega K_0, \tag{33}$$

which follows from $\dot{S}^W = 0$. Using this relation, three cases can be distinguished for switching to the backstop technology.

Case 1: If $q_c^{Ceil}(R_b) > \bar{y}$ holds, the ceiling will be left before energy consumption switches to the backstop technology. The point in time when the ceiling is left t_{cl}^{Ceil} is determined by $q_c^{Ceil}(R) = q^{Ext}(R, \lambda_R)$ and I denote the corresponding resource stock by R_{cl}^{Ceil}. From t_{cl}^{Ceil} until t_b, the dynamics are determined by Scenario *Ext*, where the initial level for the resource stock is R_{cl}^{Ceil}. Energy consumption switches to the backstop technology at t_b^{Ceil} with $t_{cl}^{Ceil} < t_b^{Ceil}$.

Case 2: If $q_c^{Ceil}(R_b) < \bar{y}$ holds, the backstop technology will already be used at the ceiling and t_b^{Ceil} is determined by $q_c^{Ceil}(R) = \bar{y}$ with $R > R_b^{Ext}$. Instead of a complete switch to the backstop technology at t_b^{Ceil}, both energy sources will be used simultaneously in this case. From t_b^{Ceil} onwards, energy consumption is fixed at $\bar{y} = q_c^{Ceil}(R) + y(t)$, where $q_c^{Ceil}(R)$ is monotonically decreasing (according to Proposition 3) and in turn $y(t)$ is monotonically increasing. Simultaneous use requires the marginal costs for both energy sources to be equal $c_y = C(R) + \lambda_R^{Ceil} + \lambda_S^{Ceil}$, which holds true until $R(t) = R_b$, so that $c_y = C(R_b)$ and $\lambda_R^{Ceil} = \lambda_S^{Ceil} = 0$. At this point in time, the ceiling is left and the consumption of the backstop technology jumps from $\bar{y} - q_c^{Ceil}(R_b)$ up to $\bar{y}$, whereas consumption of fossil-fuel-based energy jumps to zero. Consequently, in this case $t_{cl}^{Ceil} > t_b^{Ceil}$ holds.

Case 3: If $q_c^{Ceil}(R_b) = \bar{y}$ holds, the ceiling will be left at the point in time when energy consumption switches to the backstop technology. Consequently, in this case $t_{cl}^{Ceil} = t_b^{Ceil}$ holds.

In Case 1 and Case 3, the marginal costs of extraction are monotonically decreasing, but in Case 2 marginal costs are constant during the simultaneous use of both energy sources, implying λ_S to be decreasing at a slower rate after t_b^{Ceil}. This case is investigated in more detail in Chakravorty et al. (2006) but with the carbon cycle being described by proportional decay so that extraction is constant at the ceiling, $\bar{q_c} = \gamma_d \bar{S}$. If $\gamma_d \bar{S} < \bar{y}$, backstop technology utilization will start at a level of $y(t) = \bar{y} - \gamma_d \bar{S}$ once the ceiling is approached. Observing starting points for backstop technology utilization later at the ceiling requires the inclusion of sequestration so that energy consumption is above $\bar{y}$ at the beginning of the binding ceiling. The two-box model description allows such a result to be observed without the inclusion of sequestration. Following Lafforgue et al. (2008), I focus on Case 1 where $q_c^{Ceil}(R_b) > \bar{y}$.

Obviously, the ceiling is reached later in Scenario *Ceil* than it is exceeded in Scenario *Ext*, $t_{cr}^{Ceil} > t_{cr}^{Ext}$ because $q^{Ext}(R_0, \lambda_{R0}) > q^{Ceil}(R_0, \lambda_{R0}, \lambda_{S0})$ and $q^{Ext}(t_{cr}^{Ext}) > q^{Ceil}(t_{cr}^{Ext})$. Accordingly, the resource stock $R(t_{cr}) = R_{cr}$ also changes.

Proposition 4. *The ceiling is approached with a lower fossil fuel resource stock in Scenario Ceil than in Scenario Ext:* $R_{cr}^{Ceil} < R_{cr}^{Ext}$.

Proof. This can be seen from the closed-form solution for $S(t)$ in the $Redux^W$ system:

$$S(t) = e^{-(\gamma+\gamma\omega)t} \left(\int_0^t ((q(\tau) - \gamma\omega R(\tau))e^{(\gamma+\gamma\omega)\tau} d\tau + \frac{S_0}{1+\omega} - \frac{\omega(R_0 + W_0)}{1+\omega} \right) + \frac{\omega K_0}{1+\omega}. \tag{34}$$

By taking the derivative with respect to time at t_{cr} where $S(t_{cr}) = \bar{S}$ one obtains

$$\frac{\gamma\omega R(t_{cr}) - q(t_{cr})}{\gamma + \gamma\omega} = \bar{S} - \frac{\omega K_0}{1+\omega}, \tag{35}$$

from which follows that a lower amount of extraction in Scenario *Ceil* at t_{cr} also implies a lower fossil fuel resource stock. □

The ceiling is not just approached later due to the lower extraction rate in Scenario *Ceil* but also to the higher cumulative oceanic carbon uptake compared to Scenario *Ext*: $W^{Ceil}(t_{cr}) = K_0 - \bar{S} - R_{cr}^{Ceil} > W^{Ext}(t_{cr}) = K_0 - \bar{S} - R_{cr}^{Ext}$.

Note that the cumulative oceanic uptake with the two-box model at t_{cr}, does not necessarily differ from the cumulative uptake obtained with the proportional decay assumption, $W(t) = \gamma_d \int_0^t S(t) + W_0$, if the decay parameter γ_d is chosen appropriately:

$$\gamma_d = \frac{e^{-\gamma\omega t}\gamma \int_0^t S^{Ceil}(\tau)e^{\gamma\omega\tau}d\tau + W_0(e^{-\gamma\omega t} - 1)}{\int_0^t S_d^{Ceil}(\tau)d\tau}. \tag{36}$$

Obviously, γ_d varies with time, so that the endogenous oceanic carbon uptake in the two-box model description also alters the optimal extraction path before the ceiling has been approached compared to the proportional decay description. If the ceiling is approached at the same point in time with both the proportional decay and the two-box model description, then the initial extraction must be larger with the latter description as the net transfer into the deep ocean is not only influenced by the atmospheric carbon stock but also by the saturation of the ocean with anthropogenic carbon. Therefore, the dynamics of λ_S are influenced by the oceanic saturation, as it can be seen from (16) and (17). The initial level of λ_S is not only influenced by S_0 but also by W_0, where an initial lower oceanic saturation level implies a lower value for λ_S for a given value of S_0.

3.3. Convex sequestration costs and oceanic carbon storage

We include now the option of capturing carbon and injecting it into the deep ocean. The implications for the optimal solution depend crucially on the behavior of the costs associated with such an activity. According to Assumption 6, we distinguish between $A' = a_c$, as is assumed in Chakravorty et al. (2006) and Lafforgue et al. (2008) and $A' > 0$ with $A'' > 0$. To clarify the difference, we apply first the proportional decay description for the global carbon cycle without endogenous oceanic carbon uptake as is done in those papers and investigate the Scenarios AC and AX.[15] Note that with the proportional decay description, $W(t)$ does not affect the objective function so that it does not have to be included in the Hamiltonian function. Consequently, the necessary conditions for an optimal solution are only described by (12) to (16) and (18) to (22), where λ_W vanishes in conditions (13), (15), and (22). Considering ocean sequestration with a proportional decay description is equivalent to considering geological storage without scarcity with respect to storage volume. Furthermore, we presuppose that the Scenarios Ext and $Ceil$ can be accordingly defined for the proportional decay description.

In Scenario AC, it follows from conditions (12) and (13) that only the total amount of fossil fuel energy consumption $x^{AC} = q^{AC} + a^{AC}$ is determined and that positive sequestration requires $\lambda_S = a_c$. However, even if the latter condition were fulfilled, it would be beneficial to provide total energy consumption only by q^{AC} because $a^{AC} > 0$ would imply additional costs in the objective function. This changes if q^{AC} is

[15] From Assumption 4 follows that the initial value S_0 has to be normalized to be zero for the proportional decay description.

determined by the binding ceiling to be $q^{AC} = \gamma_d \bar{S}$, implying that $a^{AC} = x - \gamma_d \bar{S}$, as pointed out by Chakravorty et al. (2006) and Lafforgue et al. (2008). From $a_c > 0$ and $U'(\gamma_d \bar{S}) - C(R(t_{cl}) - \lambda_R(t_{cl}) = 0$ follows that there exists a point in time $t_a^{AC} < t_{cl}^{AC}$ where $U'(\gamma_d \bar{S}) - C(R) - \lambda_R - \lambda_S - a_c = 0$, so that sequestration does not take place over the entire period at the ceiling. The price continuity condition implied by (12) requires that $q^{AC}(t_{cr}^{AC}) = \gamma_d \bar{S} + a^{AC}(t_{cr}^{AC})$.[16] Consequently, at t_{cr}^{AC}, q^{AC} jumps down from x^{AC} to $\gamma_d \bar{S}$ and a^{AC} jumps up from zero to $x^{AC} - \gamma_d \bar{S}$. The costate variable λ_S^{AC} is increasing, constant, decreasing, and zero for $t \,\epsilon\, (0, t_{cr}^{AC})$, (t_{cr}^{AC}, t_a^{AT}), (t_a^{AC}, t_{cl}^{AC}), and (t_{cl}^{AC}, ∞), respectively, implying that λ_S^{AC} stays at its maximum value for $t \,\epsilon\, (t_{cr}^{AC}, t_a^{AC})$, which is equal to a_c (Chakravorty et al., 2006; Lafforgue et al., 2008).

In Scenario AX, the regularity condition for q and a is fulfilled, so that the control variables are continuous apart from the switch to the backstop technology that is assumed to take place after the ceiling has been left. Conditions (12) and (13) determine not only the total optimal amount of energy consumption but also its composition because $A'(a^{AX}) = \lambda_S$. From the continuity in the control variables follows that $q^{AX}(t_{cr}^{AX}) = \gamma \bar{S}$. The costate variable λ_S^{AX} is increasing, decreasing, and zero for $t \,\epsilon\, (0, t_{cr}^{AX})$, $(t_{cr}^{AX}, t_{cl}^{AX})$, and (t_{cl}^{AX}, ∞), respectively, implying that λ_S^{AX} approaches its maximum value at t_{cr}^{AX}. If $A'(0) = 0$, sequestration will be used for $t \,\epsilon\, (0, t_{cl}^{AX})$; if $A'(0) > 0$, sequestration will be used for $t \,\epsilon\, (t_{as}^{AX}, t_{ae}^{AX})$ with $0 \leq t_{as}^{AX} < t_{cr}^{AX} < t_{ae}^{AX} < t_{cl}^{AX}$. From $\lambda_{S0}^{AX} > 0$ follows that even if $A'(0) > 0$ holds, sequestration can start at $t = 0$, but must end before t_{cl}^{AX} because $\lambda_S^{AX}(t_{cl}^{AX}) = 0$.

Proposition 5. *If sequestration costs are constant, sequestration is used only at the ceiling and the ceiling is approached earlier than without sequestration. If sequestration costs are convex, sequestration is also used before the ceiling is approached and the ceiling can also be approached later than without sequestration.*

Proof. In both scenarios, AC and AX, energy consumption x is larger at t_{cr} than in Scenario $Ceil$ if sequestration is used, requiring the right hand side in (12) to be smaller. If $t_{cr}^{AC} = t_{cr}^{AX} = t_{cr}^{Ceil}$, $\lambda_S(t_{cr})$ is lower in scenario AC and AX. From $\lambda_S(t) = \lambda_0 e^{(\rho+\gamma)t}$ follows for both scenarios that also the initial values are lower, so that then $x(t) > x^{Ceil}(t)$ for $t \,\epsilon\, (0, t_{cr})$ extraction is monotonically decreasing. Consequently, in Scenario AC $x^{AC} = q^{AC}(t) > x^{Ceil} = q^{Ceil}(t)$ holds for $t \,\epsilon\, (0, t_{cr})$. From $S(t_{cr}) = \bar{S} = \int_0^{t_{cr}} q(\tau) e^{-\gamma(t_{cr}-\tau)} d\tau$ follows that $t_{cr}^{AC} < t_{cr}^{Ceil}$. In Scenario AX, it follows from

$$q^{AX}(t) = U'^{-1}(C(R) + \lambda_R + \lambda_S) - A'^{-1}(\lambda_S) \tag{37}$$

that $q^{AX}(t) \lesseqgtr q^{Ceil}(t)$ for $t \,\epsilon\, (0, t_{cr})$ is possible because $\frac{\partial q^{AX}}{\partial \lambda_S} = (U'^{-1})'_{\lambda_S} - (A'^{-1})'_{\lambda_S} < 0$. Even though $\int_0^{t_{cr}} x^{AX}(\tau) d\tau = \int_0^{t_{cr}} q^{AX}(\tau) + a^{AX}(\tau) d\tau > \int_0^{t_{cr}} q^{Ceil}(\tau) d\tau$ needs to be fulfilled, it is possible that $\int_0^{t_{cr}} q^{AX}(\tau) d\tau \lesseqgtr \int_0^{t_{cr}} q^{Ceil}(\tau) d\tau$, so that from $S(t_{cr}) = \bar{S} = \int_0^{t_{cr}} q(\tau) e^{-\gamma(t_{cr}-\tau} d\tau$ follows $t_{cr}^{AX} \lesseqgtr t_{cr}^{Ceil}$. □

[16] This can also be seen by the carbon balance equation: $\int_0^{t_{cr}} q^{AC}(\tau) d\tau + \gamma \bar{S}\,(t_{cl} - t_{cr}) + \int_{t_{cr}}^{t_a} a^{AC}(\tau) d\tau + \int_{t_{cl}}^{t_b} q^{AC}(\tau) d\tau = R_0 - R_b$. From derivation with respect to t_{cr} follows $q^{AC}(t_{cr}) = a^{AC}(t_{cr}) + \gamma_d \bar{S}$.

With respect to the point in time when the ceiling is reached, Chakravorty et al. (2006) argue that if sequestration is costless, the energy consumption from Scenario Ext is realized and therefore the ceiling is approached earlier than in Scenario $Ceil$, while $q^{Ext}(t) - \gamma\bar{S}$ is sequestered once the ceiling is reached with $\lambda_S^{AC}(t) = 0$ for $t\epsilon(0, \infty)$. But one could also argue that if sequestration is costless, Condition (12) and (13) coincide and only total energy consumption x is determined to be equal to q^{Ext}. Consequently, it is also possible to use sequestration for total energy consumption so that the atmospheric carbon stock would remain unchanged and never reach the ceiling. Assuming very small sequestration costs, then in both Scenarios AC and AX, almost the energy consumption of Scenario Ext would be realized. However, in Scenario AC, sequestration is only realized at the ceiling, which is approached only slightly later than in Scenario Ext, whereas in in Scenario AX, a substantial fraction of energy consumption involves sequestration before and at the ceiling, so that the ceiling is approached substantially later than in Scenario Ext and probably also later than in Scenario $Ceil$.[17]

In Scenarios AC and AX, it is assumed that sequestration is determined only by the associated costs and not by the availability of appropriate storage sides. The case where a scarcity of the carbon storing capacity of geological reservoirs exists, is investigated for constant sequestration costs by Lafforgue et al. (2008). If the cumulative stored amount of carbon for Scenario AC or AX, denoted by, e.g., $A(t) = \int_0^t a(\tau)d\tau$, exceeds the capacity of the geological reservoir, denoted by, e.g., $\bar{A}$, an additional costate variable measuring this scarcity, e.g., λ_A, has be to included in the optimization problem. The amount of sequestration is determined not only by the sequestration costs and the scarcity of atmospheric storing capacity with respect to the ceiling but also by the scarcity of the storing capacity: $A'(a) = \lambda_S - \lambda_A$. Irrespective of whether sequestration takes place before the atmospheric ceiling has been reached, as in Scenario AX, or once the ceiling has been reached, as in Scenario AC, the overall period of sequestration shrinks, so that $\int_{t_1}^{t_2} a(\tau)d\tau = \bar{A}$. Even if $A'(0) = 0$ holds in Scenario AX, sequestration ends before t_{cl} because λ_S is decreasing at the ceiling whereas $\lambda_A(t) = \lambda_{A0}e^{\rho t}$ is monotonically increasing until the storing limit has been reached. Obviously, even if sequestration is costless, the extraction path of Scenario Ext is not regained. Lafforgue et al. (2008) distinguish between the case where only one or the case where many geological reservoirs exists. The former case implies that sequestration cost are equal for all geological reservoirs and only the overall storing capacity has to be considered. The latter case requires that sequestration costs differ with respect to the geological reservoirs. They show that the reservoirs are used for sequestration in ascending order with respect to their costs without the simultaneous use of two reservoirs. This results requires, apart from the sequestration unit cost being constant, that there is no kind of regeneration of the reservoirs, e.g., due to chemical processes or leakage. This can easily been seen by thinking about the overall optimization problem of storing carbon

[17]The fraction of total energy consumption that involves sequestration in Scenario AX is relative to $\frac{\lambda_S^{AX}}{C(R)+\lambda_R^{AX}+\lambda_S^{AX}}$, so that the share of sequestration is largest at t_{cr}^{AX}.

in reservoirs with limited capacity, where the atmosphere is the cheapest storing reservoir that regenerates naturally. The case of multiple reservoirs n with different costs shows that even if overall storing capacity is not limited, e.g., $\int_0^{t_{cl}} a(\tau)d\tau < \bar{A} = \sum_1^n \bar{A}_n$, a shadow value must be associated with the $n-1$ reservoirs, where the storing capacity of the nth reservoir will not entirely be used (Lafforgue et al., 2008).

We also consider possible scarcity issues related to the storage side, but with respect to ocean sequestration instead of geological sequestration (Scenario $SeqC$ and $SeqX$). In contrast to the scenarios AC and AX, sequestration is determined not only by the scarcity of the atmospheric storing capacity but by the difference between this scarcity and the scarcity of oceanic storing capacity:

$$A'(a) = \lambda_S - \lambda_W = \lambda_S^W, \tag{38}$$

As explained in Section 3.2 and above, the difference between both scarcities increases until t_{cr}, is constant until t_a, and then decreases until t_{cl}, to zero, if sequestration unit costs are constant, or it directly decreases to zero from t_{cr} onwards if sequestration costs are convex, whereby $\lambda_S(t) \geq 0$ and $\lambda_W(t) \geq 0$ for $t \,\epsilon\, (0, t_{cl})$. Note neither the storing capacity of the atmosphere nor that of the ocean is scarce by itself if the ceiling is such that $\bar{S} > S_\infty$ so that $\bar{S}/\omega > W_\infty$. Nevertheless, scarcity arises if the inertia of the carbon cycle to move carbon into the deep ocean and the decreasing oceanic buffer capacity result in an atmospheric peak concentration with $S_p > \bar{S}$. The benefit of ocean sequestration arises from overcoming this inertia and using the oceanic buffer capacity. This can be understood by considering instantaneous equilibration between atmospheric and oceanic carbon stocks without including oceanic buffer capacity ($\gamma = \omega = 1$), implying that both carbon stocks would be monotonically increasing to their steady state values ($S_\infty = W_\infty$). If the ceiling is binding with respect to the steady state values ($\bar{S} < S_\infty$), both costate variables are equal ($\lambda_S = \lambda_W$) and ocean sequestration is of no benefit, as it increases both stocks equally. Factoring in either the inertia ($\gamma < 1$) or the oceanic buffer capacity ($\omega < 1$) implies that ocean sequestration causes the atmospheric carbon stock to increase by only γa or ωa, respectively. As a result, the difference between the scarcities becomes positive and ocean sequestration is of benefit if $A'(0) < \lambda_S - \lambda_W$. With both the inertia and the buffer capacity included, ocean sequestration causes the atmospheric carbon stock to increase by only $\gamma\omega a$ so that the difference between the scarcities increases further.

Because the two-box model description includes the oceanic buffer capacity, it is able to demonstrate that ocean sequestration does not only economically but also physically influence the amount of extraction. In the two-box model, extraction at the ceiling is given by $q^{Seq} = \gamma(\bar{S} - \omega W)$, where W is influenced by ocean sequestration. Therefore, Proposition 3 needs to be modified:

Proposition 6. *When atmospheric carbon concentration is at the ceiling, extraction must be monotonically decreasing at a faster rate if sequestration is used.*

Proof. From $\dot{S}^W = 0 = q - (\gamma + \gamma\omega)\bar{S} + \gamma\omega(K_0 - R(t))$ follows that $\frac{\dot{q}}{q+a} = -\gamma\omega$, whereas without sequestration $\frac{\dot{q}}{q} = -\gamma\omega$ holds. □

Proposition 6 implies that we observe a kink in the extraction path at the ceiling at the point in time when extraction ends in Scenario *SeqC* and in Scenario *SeqX* given $A'(0) > 0$. For a given value of the resource stock and therefore also the oceanic carbon stock at t_{cr} $(K_0 - \bar{S} - R(t_{cr}) = W(t_{cr}))$, the period at the ceiling is shorter for Scenario *SeqC* and *SeqX* than for Scenario *Ceil* because from (33) follows that the oceanic carbon stock is unique for all three scenarios at t_{cl}. Consequently, the path from t_{cl} onwards until the switching point t_b is the same for all three scenarios. However, as implied by Proposition 4 and 5, the initial values at t_{cr} are not unique for the three scenarios. In Scenario *SeqC*, the ceiling is approached earlier and therefore with a higher resource stock and a lower oceanic carbon stock than in Scenario *Ceil*. If in Scenario *SeqX* the ceiling is approached later than in Scenario *Ceil*, the resource stock is lower and the oceanic carbon stock is higher, so that initial extraction at the ceiling is lower than in Scenario *Ceil*. Note, even if the ceiling is reached at the same point in time or earlier than in Scenario *Ceil*, the oceanic carbon stock can be higher due to positive sequestration. However, that would again imply lower initial extraction at the ceiling, so that it seems more likely that in an optimal solution the ceiling is approached later than in Scenario *Ceil*.

To illustrate the implications of the various scenarios, we provide a numerical example.[18] Figure 1 shows the optimal paths for extraction, sequestration, total energy consumption by fossil fuels, and backstop technology in the left column and the atmospheric carbon concentration in the right column for the scenarios *Ext*, *Ceil*, *SeqC*, and *SeqX*. The parameter values are chosen so that assumptions 3 to 6 are fulfilled. In the unconstrained Scenario *Ext*, atmospheric carbon stock approaches its peak concentration before energy consumption switches to the backstop technology, whereby the ceiling is crossed twice. Accordingly, in Scenario *Ext*, t_{cr} and t_{cl}, denote when the ceiling is crossed, whereas in the other scenarios they denote the start and end of the ceiling period. In Scenario *Ext*, the amount of extraction at t_{cl} is actually lower than in the other scenarios. This can be seen from the shorter period between t_{cl}^{Ext} and t_b^{Ext}, compared to other scenarios where this period is equal for Scenario *Ceil*, *SeqC*, and *SeqX*, as explained above. However, the period at the ceiling varies between these three latter scenarios. In line with Proposition 5, the ceiling is approached earlier in Scenario *SeqC* than in Scenario *Ceil* due to sequestration. Even though the ceiling is left earlier in Scenario *SeqC*, the overall period at the ceiling is extended. Consequently, the effect of approaching the ceiling earlier because of higher initial extraction overcompensates the faster decline in fossil fuel energy consumption at the ceiling. For the chosen parameter values, the ceiling is reached later and left

[18] The utility function is $U(q) = b_1 * q - b_2 * q^2$, the stock-dependent extraction cost function is $c_1 - c_2 R$, the ocean sequestration cost function for Scenario *SeqX* is $A(a) = a_x * a^2$ and for Scenario *SeqC* $A(a) = a_c * a$. The parameter values are $b_1 = 6$, $b_2 = 6/20$, $c_1 = 6$, $c_2 = 1/10$, $a_x = a_c = 2/10$, $c_y = 5.6$, $\gamma = 1/10$, $\omega = 1/10$, $\rho = 3/100$, and the initial values are $R_0 = 50$, $S_0 = 20$, and $W_0 = 200$. The ceiling is $\bar{S} = \frac{45}{28} S_0 = 32.1429$ where 280 ppm is the preindustrial atmospheric carbon stabilization level and 450 ppm is a ceiling that could possibly be used to comply with the 2°C temperature limit discussed above.

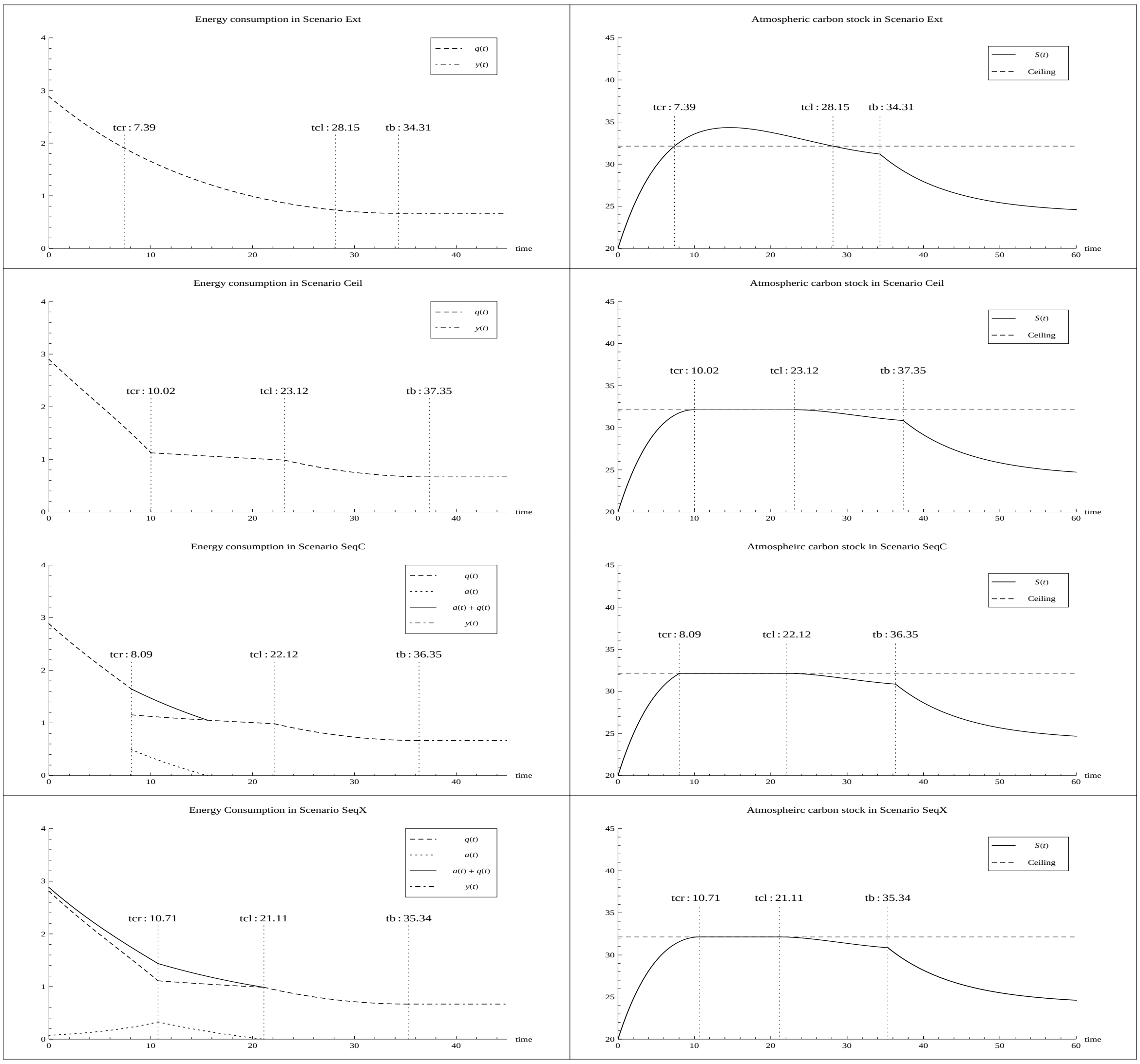

Figure 1: Energy consumption and atmospheric carbon stock in scenarios Ext, $Ceil$, $SeqC$, and $SeqX$

earlier in Scenario $SeqX$ than in Scenario $Ceil$ because of continuous sequestration. Scenario $SeqX$ has the shortest period at the ceiling, Scenario $SeqC$ the longest period. In both scenarios $SeqC$ and $SeqX$, the option of sequestration allows an path of energy consumption to be reached which looks more like the energy consumption path in Scenario Ext. Note if the initial values S_0 and W_0 are chosen such that Assumption 4 is violated but such that $S_0 > \omega W_0$ and Assumption 5 are still satisfied, the period until the ceiling is reached shrinks and the path for atmospheric carbon concentration until the ceiling could be U-shaped.

It remains to briefly discuss the implications of the change in path of the oceanic carbon stock caused by ocean sequestration. As explained above, the oceanic carbon stock $W(t_{cl})$ is unique for the scenarios $Ceil$, $SeqX$, and $SeqC$. Leaving the ceiling earlier with ocean sequestration implies that the oceanic carbon stock has increased faster than without ocean sequestration. Consequently, ocean sequestration influences the rate of ocean acidification even though the total level of ocean acidification is not affected, as it follows from the unaffected steady state values ($\bar{S} > S_\infty$). A ceiling for the oceanic carbon stock or a damage function capturing the social costs of an increasing oceanic carbon stock could be included in the objective function. Both of these possibilities would have a similar effect as the oceanic carbon stock is monotonically increasing to its steady state value. Therefore, λ_W and, in turn λ_S, would be positive for $t \,\epsilon\, (0, \infty)$ and total fossil fuel consumption would shrink.[19] The amount of ocean sequestration would also shrink or even become zero in the period until t_{cl}, as the tax difference decreases. After the ceiling has been left, the tax difference would become negative, implying that it would be beneficial to "pump" carbon back from the ocean to atmosphere ($\theta_2 > 0$). Furthermore, the positive costate variable λ_S for $t \to \infty$ implies that fossil fuels and the backstop technology would be used simultaneously from t_b onwards.

4. Conclusion

In this paper I investigate the optimal role of carbon sequestration from a social planner's perspective while the atmospheric carbon concentration is constrained by a ceiling. In contrast to existing analyses, we include stock-dependent extraction costs, describe the carbon cycle using a two-box model, assume carbon sequestration costs to be convex, and consider oceanic instead of geological carbon storage.

The inclusion of stock-dependent extraction costs does not influence the optimal sequestration decision as discussed in the literature. However, their inclusion can imply that not the entire stock of fossil resources is extracted, as is shown by Dasgupta and Heal (e.g., 1979) and Tahvonen (1997). The description of the carbon cycle using a two-box model allows the ongoing saturation of the ocean with anthropogenic carbon to be taken into account. Consequently, extraction at the ceiling is not constant, as is the case with a proportional decay description, but is monotonically decreasing. This implies that the simultaneous use of fossil fuels and a backstop technology could start at some point at the ceiling which is not possible with the proportional decay description without also using sequestration. The non renewable description of the carbon cycle provided by the two-box model implies positive atmospheric and oceanic carbon stock stabilization values that have to be larger in sum than the initial values as a consequence of the anthropogenic release of carbon into the cycle. Therefore, it is possible that the ceiling limits total carbon accumulation in the cycle, where with the proportional decay description, the ceiling is a temporary problem per definition. When the ceiling is permanently binding in the two-box model, the stock of fossil resources left in the ground must increase, compared to a when a ceiling is only temporarily binding regardless of whether extraction costs are stock-dependent or not.

[19] In the case of a damage function instead of a ceiling the steady state values for λ_S and λ_W are: $\lambda_S(\infty) = \frac{D'(W)\gamma}{\rho(\gamma+\rho+\gamma\omega)}$ and $\lambda_W(\infty) = \frac{D'(W)(\gamma+\rho)}{\rho(\gamma+\rho+\gamma\omega)}$.

Convex carbon sequestration costs imply that sequestration is used in a continuous manner, increasing on the path towards the ceiling and decreasing at the ceiling. Constant sequestration costs imply that sequestration is used in a discontinuous manner, jumping from zero to its maximum value and decreasing at the ceiling (Chakravorty et al., 2006; Lafforgue et al., 2008). Both cost assumptions result in a total energy consumption path that looks more like the one observed in the unconstrained solution. However, the path resulting from convex sequestration costs seems to be more realistic with respect to physical and investment requirements related to the implementation of such a technology. This is confirmed by simulation results that project an increasing share of carbon sequestration for various temperature stabilization targets (e.g., IEA, 2010). Furthermore, constant sequestration costs imply that in an optimal solution, the atmospheric ceiling will be approached earlier if sequestration is used at the ceiling. Convex sequestration also allows the atmospheric ceiling to be approached later. Our ability to profoundly determine a ceiling on atmospheric carbon concentration that can be regarded as safe with respect to climate change is restricted by our still limited understanding of the earth's climate system, in particular with respect to tipping points. Accordingly, at the meeting of the parties in Cancun, it was recognized that the limit for temperature increase to 2°C need to revised to a limit of 1.5°C because of new scientific knowledge (UNFCCC, 2010). Therefore, technologies allowing society to approach an agreed ceiling later and to gain time to learn more about the consequences of climate change can be regarded as preferable. Results based on linear sequestration costs, suggest that carbon sequestration would not be a recommend option to deal with the atmospheric carbon accumulation problem, whereas results based on convex sequestration costs suggest that such technology could be an important option.

Ocean sequestration and therefore oceanic carbon storage allows the kinetic constraints of natural oceanic carbon uptake to be overcome. The atmospheric carbon accumulation problem is crucially influenced by the inertia of the global and in particular the marine carbon cycle in balancing anthropogenic disturbances to the carbon cycle. About 80 percent of the anthropogenic carbon emitted to the atmosphere is expected to be taken up by the ocean on timescales of several hundred years (e.g., Archer et al., 1997; Körtzinger and Wallace, 2002; Sarmiento and Gruber, 2006). Ocean sequestration takes advantage of the oceanic buffer capacity and the slow turnover speed of the natural adjustment process. Therefore, the optimal amount of ocean sequestration is determined by the difference between the scarcities of the atmospheric carbon storing capacity and of the oceanic carbon storing capacity. Whereas the former results from the temporarily binding ceiling, the latter results from its negative feedback effect on natural oceanic carbon uptake. Ocean sequestration contributes to the saturation of the ocean with anthropogenic carbon, implying that emissions released into the atmosphere at the ceiling have to decline at a faster rate than without ocean sequestration. If the oceanic carbon storing capacity becomes scarce by itself, e.g., by accounting for ocean acidification, ocean sequestration is less beneficial. Furthermore, after the ceiling has been left, the optimal amount of fossil fuel extraction would no longer be limited by the atmospheric carbon stock but rather by the ongoing ocean acidification. With respect to ocean acidification, geological storage would be more beneficial, as it allows overall atmospheric and oceanic stabilization levels to be decreased, whereas ocean sequestration does not. Apart from ocean acidification, geological storage does not affect the buffer capacity of the ocean, so that, if costs for geological and oceanic carbon sequestration are equal, the geological storage capacity will be fully used, irrespective of whether simultaneous use is implied by convex sequestration costs or successive use is implied by linear sequestration costs. However, if carbon sequestration is applied on a large scale, it will be probably more expensive to inject carbon into suitable geological storage sites which each require specific investments with respect to pipelines and injection facilities, than to inject carbon into the ocean. Therefore, the interesting case for future research arises from considering various storage options that can be ranked according to the associated injection costs but also according to the associated environmental costs, which would probably result in different ranking orders.

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A. Necessary and sufficient optimality conditions

For the three control variable constraints, $g_1(q,a,y) = -q \leq 0$, $g_2(q,a,y) = -a \leq 0$, and $g_3(q,a,y) = -y \leq 0$, and the state variable constraint, $h(S(t)) = S(t) - \bar{S} \leq 0$, the constraint qualification is fulfilled if the matrix

$$m = \begin{pmatrix} \frac{\partial g_1}{\partial q} & \frac{\partial g_1}{\partial a} & \frac{\partial g_1}{\partial y} & g_1 & 0 & 0 & 0 \\ \frac{\partial g_2}{\partial q} & \frac{\partial g_2}{\partial a} & \frac{\partial g_2}{\partial y} & 0 & g_2 & 0 & 0 \\ \frac{\partial g_3}{\partial q} & \frac{\partial g_3}{\partial a} & \frac{\partial g_3}{\partial y} & 0 & 0 & g_3 & 0 \\ \frac{\partial \frac{\partial h}{\partial t}}{\partial q} & \frac{\partial \frac{\partial h}{\partial t}}{\partial a} & \frac{\partial \frac{\partial h}{\partial t}}{\partial y} & 0 & 0 & 0 & h \end{pmatrix} \tag{A.1}$$

has the full row rank (Feichtinger and Hartl, 1986, p.165, 6.17), which can be seen to be fulfilled from

$$m = \begin{pmatrix} -1 & 0 & 0 & -q & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -a & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & -y & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & S(t) - \bar{S} \end{pmatrix} \tag{A.2}$$

The concavity of the maximized Hamiltonian can be shown by proving that the Hessian Matrix of the Hamiltonian is at least negative semi-definite (Feichtinger and Hartl, 1986, p.37, Remark 2.4). To do so, we use the $Redux^W$ system, where the oceanic carbon stock is dropped. The corresponding Current Value Hamiltonian is $H^c = U(q+a+y) - A(a) - C(R)(q+a) - c_y y + \lambda_R \dot{R} + \lambda_S \dot{S}$, so that the corresponding Hessian matrix is:

$$\begin{pmatrix} H_{SS} & H_{SR} & H_{Sq} & H_{Sa} & H_{Sy} \\ H_{RS} & H_{RR} & H_{Rq} & H_{Ra} & H_{Ry} \\ H_{qS} & H_{qR} & H_{qq} & H_{qa} & H_{qy} \\ H_{aS} & H_{aR} & H_{aq} & H_{aa} & H_{ay} \\ H_{yS} & H_{yR} & H_{yq} & H_{ya} & H_{yy} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -C''(R)(q+a) & -C'(R) & -C'(R) & 0 \\ 0 & -C'(R) & U'' & U'' & U'' \\ 0 & -C'(R) & U'' & U'' - A'' & U'' \\ 0 & 0 & U'' & U'' & U'' \end{pmatrix}. \tag{A.3}$$

The negative semi definiteness can easily be seen by checking whether all the leading principal minors are zero.

The regularity of the Hamiltonian follows from the strict concavity of the Hamiltonian in the control variables (Feichtinger and Hartl, 1986, p.167). If the optimization problem is restricted to the control variables q and a and sequestration costs are defined by Case X in Assumption 6, this strict concavity is fulfilled:

$$Det \begin{pmatrix} H_{qq} & H_{qa} \\ H_{aq} & H_{aa} \end{pmatrix} = -U''A'' > 0. \tag{A.4}$$

If sequestration costs are defined by Case C in Assumption 6 and/or the backstop technology is included, we see from (A.4) and the lower right $3x3$ matrix in the Hesse matrix (A.3) that the regularity condition is not fulfilled because the Hamiltonian is not strictly concave in the control variables anymore.