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by **Mewael F. Tesfaselassie**

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JEL classification: E4; E5

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# Trend Growth and Learning About Monetary Policy Rules

Mewael F. Tesfaselassie\*

## Abstract

The paper examines the effect of trend productivity growth on the determinacy and learnability of equilibria under alternative monetary policy rules. It shows that under a policy rule that responds to current period inflation and the output gap a higher trend growth rate relaxes the conditions for determinacy and learnability. Results are mixed for other policy rules. Under the expectations-based rule, trend growth reduces the scope for determinacy but it relaxes the conditions for learnability. Under the lagged-data-based rule trend growth reduces the scope for determinacy and learnability.

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## 1 Introduction

Business cycle models with forward-looking expectations may be prone to two types of problems. The first is real indeterminacy—the possibility that a unique, stationary rational expectations equilibrium does not exist. The second is expectational instability or E-instability under private sector learning (see, e.g., Evans and

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Honkapohja, 2001).<sup>1</sup> In sticky price models in which monetary policy constitutes one of the building blocks that determine macroeconomic outcomes one may wonder what sorts of policy rules may lead the economy into indeterminacy and/or E-instability, so that policymakers can avoid using such undesirable policy rules. Bullard and Mitra (2002) were among the first to analyze determinacy and learnability of rational expectations equilibria in the standard New Keynesian model of inflation and the output gap. They evaluate the performance of various forms of Taylor-type rules for setting the nominal interest rate. One of the key results of their analysis is that following the so-called Taylor principle, where the central bank adjusts the nominal interest rate more than one-for-one with changes in inflation, is desirable both from determinacy and learnability point of view. Another is that, in general determinacy does not imply learnability of rational expectations equilibria.<sup>2</sup>

While Bullard and Mitra (2002) make an important contribution in showing how the specification of monetary policy rules matter for determinacy and learnability of rational expectations equilibria, subsequent research has extended their analysis in several directions. For instance, Evans and Honkapohja (2003) show how the problems of instability and indeterminacy identified by Bullard and Mitra (2002) can be overcome if the central bank can observe private agents' expectations while Honkapohja and Mitra (2005) examine the implications of heterogeneity in forecasting by the central bank and private agents for determinacy and learnability. In a model with money in the utility function Kurozumi (2006) analyzes how the timing of money balances matters for determinacy and learnability of Taylor type rules. Bullard and Schaling (2009) show how an open economy framework modifies the conditions for determinacy and learnability of equilibria depending on the exchange rate regime.

This paper makes a contribution to the expanding literature on determinacy and learnability by extending the Bullard and Mitra (2002) framework to allow for trend productivity growth (in short, trend growth). The aim is to analyze whether and how changes in trend growth affects the performance of Taylor-type rules. Regarding the structure of the economy, we show that trend productivity growth changes the

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<sup>1</sup>In what follows we use the words learnability and E-stability interchangeably.

<sup>2</sup>See McCallum (2007) for a detailed analysis of the connections between the determinacy and learnability criteria and Evans and Honkapohja (2008) for a survey of the learning literature).

slope and position of the Phillips curve, one of the key structural equations of the New Keynesian model. In particular, the sensitivity of actual inflation to expected inflation is lower while its sensitivity to output gap is higher the higher is trend growth. The paper then examines how trend growth changes the performance of three alternative forms of the policy rule in terms of determinacy and learnability. It shows that for a policy rule that responds to current period inflation and the output gap higher trend growth relaxes the conditions for determinacy and learnability of equilibria. Results are mixed for policy rules that respond to expectations and lags of inflation and the output gap. Under the expectations-based rule, trend growth reduces the scope for determinacy but it increases the scope for learnability. Under the lagged-data-based rule trend growth reduces the scope for both determinacy and learnability.

The paper is organized as follows. Section 2 lays out a New Keynesian model that incorporates trend productivity growth, where the effects of trend growth on the short-run dynamics of the model are discussed. Then, Section 3 shows the results pertaining to trend growth changes and the determinacy and learnability properties of the model. Finally, Section 4 gives concluding remarks.

## 2 The model

We introduce exogenous productivity growth into the standard New Keynesian model in a way that is consistent with balanced growth (see, e.g., King, Plosser and Rebelo, 1988; Basu and Kimball, 2002).<sup>3</sup> In the presence of productivity growth and in a balanced growth path output, consumption, and real wages grow at the same rate as productivity, while aggregate hours is constant.

Let  $A_t$  denote productivity, which follows an exogenous process given by  $A_{t+i} = \alpha_{t+i}\tilde{A}_{t+i}$ , where  $\alpha_t$  is an i.i.d stationary shock and  $\tilde{A}_t$  is the trend component. The latter is given by  $\tilde{A}_{t+i} = \Gamma\tilde{A}_{t+i-1} = \Gamma^i\tilde{A}_t$ , where  $\Gamma$  is one plus the trend growth rate of productivity  $\gamma$ .

Next we derive the structural equations of the model by solving the households'

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<sup>3</sup>For a detailed discussion of the New Keynesian model see, e.g., Woodford (2003).

consumption and labor supply decisions and firms pricing decisions.

## 2.1 Households

There is a representative household which consumes a continuum of differentiated goods on the unit interval, which are transformed into a Dixit-Stiglitz composite good  $C_t$  as follows

$$C_t = \left( \int_0^1 C_{j,t}^{1/\mu} dj \right)^\mu, \quad (1)$$

where each good is indexed by  $j$ ,  $\mu = \frac{\theta}{\theta-1}$  and  $\theta$  is the elasticity of substitution between any two differentiated goods. We first solve for the household's consumption allocation across all goods for a given level of  $C_t$ . Minimizing total expenditure  $\int_0^1 P_{j,t} C_{j,t} dj$  subject to (1) gives the demand for each good  $j$

$$C_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\theta} C_t, \quad (2)$$

where  $P_t$  is the aggregate price index (or the price level), which is defined as

$$P_t = \left( \int_0^1 P_{j,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \quad (3)$$

Household utility is a function of consumption and leisure. It take the same form as in King, Plosser and Rebelo (1988), which allows for non-separable utility in household preferences, and which is consistent with recent empirical evidence on the consumption Euler equation (see, e.g., Basu and Kimball (2002) and Kiley (2010)).<sup>4</sup>

The household's intertemporal utility is given by

$$E_t \sum \beta^i \frac{(C_t e^{-v(N_t)})^{1-\sigma}}{(1-\sigma)}, \quad (4)$$

where  $\sigma > 0$ , and the flow budget constraint is given by

$$C_t + \frac{B_t}{P_t} = \frac{W_t N_t}{P_t} + \frac{R_{t-1} B_{t-1}}{P_t} + \frac{D_t}{P_t}. \quad (5)$$

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<sup>4</sup>where parameter restrictions are imposed consistent with balanced growth facts.

Here,  $\beta$  is the discount factor,  $R_t$  is the nominal interest rate on bond holdings  $B_t$ ,  $N_t$  is the number of hours worked,  $W_t$  is the nominal wage,  $D_t$  is the nominal profit income from all firms. Finally,  $v(N_t) = N_t^{\eta+1}/(\eta + 1)$ , where  $\eta > 0$  and  $v'(N_t) = N_t^\eta > 0$ .

The household's problem is to maximize (4) subject to (5). The first order condition with respect to intertemporal allocation of consumption leads to the familiar Euler equation

$$1 = \beta(1 + i_t)E_t \left( \frac{C_{t+1}^{-\sigma} e^{(\sigma-1)v(N_{t+1})} P_t}{C_t^{-\sigma} e^{(\sigma-1)v(N_t)} P_{t+1}} \right), \quad (6)$$

where, due to non-separability in the utility function, consumption growth depends on hours. Moreover, optimal labor supply choice implies that the real wage is equal the marginal rate of substitution between consumption and work,

$$\frac{W_t}{P_t} = C_t N_t^\eta. \quad (7)$$

Rewriting (6) and (7) in stationary variables we have, respectively,

$$1 = \beta \Gamma^{-\sigma} (1 + i_t) E_t \left( \frac{c_{t+1}^{-\sigma} e^{(\sigma-1)v(N_{t+1})} P_t}{c_t^{-\sigma} e^{(\sigma-1)v(N_t)} P_{t+1}} \right), \quad (8)$$

and

$$\frac{W_t}{\tilde{A}_t P_t} = c_t N_t^\eta, \quad (9)$$

where  $c_t = C_t/\tilde{A}_t$  is detrended consumption.

## 2.2 Firms

There is a continuum of monopolistically competitive firms over the unit interval sharing identical technology. Firm  $j$ 's production function is of the form  $Y_{jt} = A_t N_{jt}$ . While firms set prices, output is demand determined according to equation (2), which in turn pins down each firm's labor demand. Pricing decisions in the goods market

is subject to Calvo-type price staggering, where in any given period a fraction  $\omega$  of firms cannot reset their prices optimally. It follows that for each firm  $j$  in period  $t$ , its nominal price  $P_{j,t}$  is set such that  $P_{j,t} = P_t^*$  if set optimally and  $P_{j,t} = P_{j,t-1}$  otherwise. Price staggering turns out to be important for trend growth to have meaningful effects on the short-run dynamics of the model.

Firm  $j$  maximizes the expected lifetime profit  $E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} (P_{j,t}/P_{t+i} - \phi_{t+i}) Y_{j,t+i}$  where  $Q_{t,t+i} = \beta^i C_{t+i}^{-\sigma} e^{(\sigma-1)v(N_{t+i})} / (C_t^{-\sigma} e^{(\sigma-1)v(N_t)})$  is the stochastic discount factor and  $\phi_t = \phi_{j,t} = W_t/(A_t P_t)$  is the real marginal cost, which is identical across all firms. Note here that the stochastic discount factor depends negatively on trend consumption growth if  $\sigma > 1$ , that is, if consumption and labor are complementary in household utility.

Substituting the demand function for good  $j$  (2) into the profit function (??) and differentiating the latter with respect to  $P_{j,t} = P_t^*$  (as all optimizing firms face identical maximization problem) gives

$$\frac{P_t^*}{P_t} = \frac{\mu E_t \sum (\beta \omega)^i C_{t+i}^{1-\sigma} e^{(\sigma-1)v(N_{t+i})} \phi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^\theta}{E_t \sum (\beta \omega)^i C_{t+i}^{1-\sigma} e^{(\sigma-1)v(N_{t+i})} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1}}, \quad (10)$$

where  $P_t^*/P_t$  is the optimal relative price. Next, expressing (10) in detrended form leads to

$$\frac{P_t^*}{P_t} = \frac{\mu E_t \sum (\beta \omega \Gamma^{1-\sigma})^i c_{t+i}^{1-\sigma} e^{(\sigma-1)v(N_{t+i})} \phi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^\theta}{E_t \sum (\beta \omega \Gamma^{1-\sigma})^i c_{t+i}^{1-\sigma} e^{(\sigma-1)v(N_{t+i})} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1}}. \quad (11)$$

Note that the term  $\Gamma^{1-\sigma}$  captures the discounting-like effect of trend growth. The higher is trend growth the lower is the effective discount factor  $\beta \omega \Gamma^{1-\sigma}$ .

Finally, the Calvo-type price staggering among firms implies that equation (3), which determines the price level, can be rewritten as a weighted average of  $P_t^*$  and  $P_{t-1}$ ,

$$P_t = \left( (1 - \omega)(P_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta} \right)^{\frac{1}{1-\theta}}. \quad (12)$$



## 2.3 Aggregate output and marginal cost

Using the optimal labor supply choice of households (9), the aggregate real marginal cost  $\phi_t$  can be rewritten as

$$\phi_t = \alpha_t^{-1} c_t N_t^\eta. \quad (13)$$

Next, from firm's production function we get the aggregate hours worked

$$N_t = \alpha_t^{-1} y_t \delta_t, \quad (14)$$

where  $y_t = Y_t/\tilde{A}_t$  is detrended output and  $\delta_t = \int_0^1 (P_{j,t}/P_t)^{-\theta} dj$  is a measure of price dispersion across firms. Using equation (14) and the goods market clearing condition,  $c_t = y_t$ , to substitute out  $c_t$  and  $N_t$  in equation (13) we get

$$\phi_t = \delta_t^\eta \left( \frac{y_t}{\alpha_t} \right)^{1+\eta}. \quad (15)$$

## 2.4 Linearized system

The nonlinear system is linearized around a zero inflation steady state so that the resulting equations generalize those of Bullard and Mitra (2002). First combining the linearized versions of equations (11) and (12) leads to a dynamic equation relating inflation  $\pi_t$ , expected inflation  $E_t\pi_{t+1}$  and the real marginal cost  $\hat{\phi}_t$ ,

$$\pi_t = \beta\Gamma^{1-\sigma} E_t\pi_{t+1} + \frac{(1-\omega)(1-\beta\Gamma^{1-\sigma}\omega)}{\omega} \hat{\phi}_t. \quad (16)$$

where the sign  $\hat{\cdot}$  denotes deviations from the steady state level. Under flexible prices,  $\phi_t = \mu^{-1}$ , so that detrended output in the flexible price equilibrium is  $y_t^f = \mu^{-1/(1+\eta)}\alpha_t$  implying  $\hat{y}_t^f = \hat{\alpha}_t$  and  $\hat{\phi}_t = (1+\eta)(\hat{y}_t - \hat{\alpha}_t) = (1+\eta)x_t$ , where  $x_t \equiv \hat{y}_t - \hat{y}_t^f$  is the output gap.<sup>5</sup> Then equation (16) can be rewritten as

$$\pi_t = \tilde{\beta} E_t\pi_{t+1} + \tilde{\lambda} x_t, \quad (17)$$

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<sup>5</sup>As is standard we have made use the fact that around the zero inflation steady state,  $\delta_t = 1$  (see, e.g., Woodford, 2003).

where  $\tilde{\beta} \equiv \beta\Gamma^{1-\sigma}$  and  $\tilde{\lambda} \equiv (1 + \eta)(1 - \omega)(1 - \beta\Gamma^{1-\sigma}\omega)/\omega$ . Equation (17) is the familiar New Keynesian Phillips curve augmented to allow for trend growth. Note that trend growth affects both the slope and the position of the Phillips curve. Given  $\sigma > 1$ , we have  $\partial\tilde{\beta}/\partial\Gamma > 0$  and  $\partial\tilde{\lambda}/\partial\Gamma > 0$ , so that the sensitivity of actual inflation to expected inflation is lower while its sensitivity to output gap is higher the higher is trend growth.

Next, expressing the Euler equation (8) in linearized form around zero steady state inflation, we have

$$0 = -\sigma E_t \hat{c}_{t+1} + \sigma \hat{c}_t - (1 - \sigma)N^{1+\eta} E_t \hat{N}_{t+1} + (1 - \sigma)N^{1+\eta} \hat{N}_t + \hat{i}_t - E_t \pi_{t+1}, \quad (18)$$

where  $N$  is the steady state level of aggregate hours. Furthermore, using the production function and the goods market clearing condition to substitute out  $\hat{N}$  and  $\hat{c}$  in equation (18) and rearranging gives

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\tilde{\sigma}} (\hat{i}_t - E_t \pi_{t+1}), \quad (19)$$

where  $\tilde{\sigma} \equiv \sigma + (1 - \sigma)N^{1+\eta}$ . As  $0 < N < 1$  it follows that  $\sigma > \tilde{\sigma} > 1$  provided  $\sigma > 1$ .

Finally, equation (19) can be rewritten in terms of the output gap

$$x_t = E_t x_{t+1} - \frac{1}{\tilde{\sigma}} (\hat{i}_t - E_t \pi_{t+1} - \hat{r}_t^f), \quad (20)$$

where  $\hat{r}_t^f \equiv -\tilde{\sigma} (E_t \hat{y}_{t+1}^f - \hat{y}_t^f)$  is the natural rate of interest.

Equation (20) is the familiar New Keynesian IS curve. However, due to the non-separable utility, the interest rate sensitivity of aggregate demand  $\tilde{\sigma}^{-1}$  differs from the inverse of the elasticity of intertemporal substitution in consumption  $\sigma^{-1}$ , as long as  $0 < N < 1$ .

Having shown how the introduction of trend growth and the assumption of non-separable utility affect the key structural equations of the New Keynesian model we now turn to the analysis of determinacy and learnability.

### 3 Determinacy and Learnability

The linearized equations (17) and (20) capture aggregate private sector behavior, for a given interest rate  $\hat{l}_t$  set by monetary policy. They generalize equations (1) and (2) of Bullard and Mitra (2002) by allowing for the effects of trend productivity growth in the presence of non-separable utility. For the analysis of the model under learning, we follow the standard approach in the learning literature and replace rational expectations by subjective expectations ( $\pi_{t+1}^e$  and  $x_{t+1}^e$ ) in the Phillips curve and the IS curve,

$$\pi_t = \tilde{\beta}\pi_{t+1}^e + \tilde{\lambda}x_t \quad (21)$$

and

$$x_t = x_{t+1}^e - \frac{1}{\tilde{\sigma}}(\hat{l}_t - \pi_{t+1}^e - \hat{r}_t^f). \quad (22)$$

The model is closed by specifying a monetary policy rule. As in Bullard and Mitra (2002) we provide the conditions for determinacy and E-stability of minimum state variables (MSV) equilibria under alternative monetary policy rules. In particular, we consider (i) a current-data-based rule, where  $\hat{l}_t$  responds to  $\pi_t$  and  $x_t$ , (ii) a lagged-data-based rule, where  $\hat{l}_t$  responds to  $\pi_{t-1}$  and  $x_{t-1}$  and (iii) an expectations-data-based rule, where  $\hat{l}_t$  responds to  $\pi_{t+1}^e$  and  $x_{t+1}^e$ . In what follows we discuss how those conditions are affected by the presence of trend productivity growth.

#### 3.1 Current-data-based rule

The policy rule that responds to current period inflation and the output gap is given by

$$\hat{l}_t = \varphi_\pi\pi_t + \varphi_x x_t, \quad (23)$$

where  $\varphi_\pi, \varphi_x \geq 0$ . As far as determinacy is concerned, the question is whether the system of equations given by (21), (22) and (23) has a unique, stationary rational expectations equilibrium.

The issue of learning is more subtle, especially when there are multiple rational expectations equilibria, as there are many ways of specifying a forecasting rule for the private sector. Following Bullard and Mitra (2002) much of the literature focuses on learnability of a Minimum State Variables (MSV) solution (in the sense of McCallum, 1983). For the model at hand the only state variable is  $r_t^n$  so that under rational expectations the MSV solution is of the form

$$z_t = a + cr_t^n, \tag{24}$$

where  $z_t = (x_t, \pi_t)'$ . Under rational expectations any MSV solution will necessarily have  $a = 0$ .

Under learning about the MSV solution, equation (24) becomes the private agents' (reduced form) perceived law of motion (PLM) of  $z_t$ . In every period they use historical data and least squares to get estimates for  $a$  and  $c$  and subsequently update their inflation and output gap forecasts using those estimates. The actual law of motion (ALM) is then determined by the interaction of the inflation and output gap forecasts and the structural equations (21), (22) and (23). As the process is recursive the issue is then whether under learning the ALM converges to the MSV solution (under rational expectations). As Bullard and Mitra (2002) provide the technical details regarding the learnability of MSV solutions, we spare the reader from these details and in what follows, we make use of Bullard and Mitra (2002) results on the necessary and sufficient conditions for determinacy and learnability. We have the following proposition regarding the effect of trend growth under the current-data-based rule.

**Proposition** *Under the current-data-based rule, trend growth relaxes the conditions for determinacy and E-stability of MSV equilibria.*

*Proof:* From a version of Propositions 1 and 2 of Bullard and Mitra (2002) the necessary and sufficient for determinacy as well as E-stability of MSV equilibria is

$$(1 - \tilde{\beta})\varphi_x + \tilde{\lambda}(\varphi_\pi - 1) > 0$$

The borderline that separates the determinate and indeterminate regions is given by  $\varphi_x = -H_1(\varphi_\pi - 1)$ , where  $H_1 = \tilde{\lambda}/(1 - \tilde{\beta}) > 0$ . The smaller is  $H_1$  (i.e., the flatter is the borderline) the larger is the size of the determinate and E-stable region. It remains to show that  $\partial H_1/\partial \Gamma < 0$  (i.e., the borderline becomes flatter the higher is the trend growth). From the definition of  $H_1$  and given our assumption that  $\sigma > 1$ ,  $\partial H_1/\partial \Gamma < 0$ . Thus, we conclude that under a current-data-based rule, a higher trend growth rate relaxes the condition for determinacy of equilibria and E-stability of MSV solutions.

One of the important findings of Bullard and Mitra (2002) is that their results regarding the current-data-based rule carry over to the case where the central bank has imperfect information about current period inflation and the output gap. In particular, they show that a policy rule of the form

$$\hat{h}_t = \varphi_\pi \pi_t^e + \varphi_x x_t^e, \quad (25)$$

where  $\pi_t^e$  and  $x_t^e$  are forecasts of current period inflation and the output gap, respectively, leads to exactly the same regions of determinate and learnable equilibria as those implied by the policy rule (23). By implication our proposition above also applies to the case where the policy rule is of the form (25). This is important because, as Bullard and Mitra (2002) note, central banks usually do not have precise information on data pertaining to current quarter inflation and the output gap and are thus more likely to respond to their own forecasts of these variables.

### 3.2 Lagged-data-based rule

The policy rule that responds to lags of inflation and the output gap is given by

$$\hat{h}_t = \varphi_\pi \pi_{t-1} + \varphi_x x_{t-1}. \quad (26)$$

In this case the system of equations is given by (21), (22) and (26) and there are three state variables— $\pi_{t-1}$ ,  $x_{t-1}$  and  $r_t^n$ . Thus under rational expectations the MSV solution is of the form

$$z_t = a + bz_{t-1} + cr_t^n. \quad (27)$$

Equation (27) is also the PLM under private sector learning. From a version of Proposition 3 of Bullard and Mitra (2002) a set of sufficient conditions for determinacy under the lagged-data-based rule are

$$(1 - \tilde{\beta})\varphi_x + \tilde{\lambda}(\varphi_\pi - 1) > 0 \quad (28)$$

and

$$(1 + \tilde{\beta})\varphi_x + \tilde{\lambda}(\varphi_\pi - 1) < 2\tilde{\sigma}(1 + \tilde{\beta}). \quad (29)$$

Provided that condition (28) (i.e., the Taylor principle) is satisfied, condition (29) requires that  $\varphi_x$  and  $\varphi_\pi$  should not be too high. When  $\varphi_\pi < 1$ , the above two conditions imply

$$-H_1(\varphi_\pi - 1) < \varphi_x < -H_2(\varphi_\pi - 1) + 2\tilde{\sigma},$$

where  $H_2 = \tilde{\lambda}/(1 + \tilde{\beta}) > 0$ . On the other hand, when  $\varphi_\pi > 1$ , the two conditions imply

$$0 < \varphi_x < -H_2(\varphi_\pi - 1) + 2\tilde{\sigma},$$

provided  $H_2(\varphi_\pi - 1) \leq 2\tilde{\sigma}$ .

However, conditions (28) and (29) are not necessary for determinacy. In particular, determinacy also obtains if<sup>6</sup>

$$(1 - \tilde{\beta})\varphi_x + \tilde{\lambda}(\varphi_\pi - 1) < 0 \quad (30)$$

and

$$(1 + \tilde{\beta})\varphi_x + \tilde{\lambda}(\varphi_\pi - 1) > 2\tilde{\sigma}(1 + \tilde{\beta}). \quad (31)$$

Thus, even if the Taylor principle is not satisfied, (30) and (31) imply that determinacy obtains if  $-H_2(\varphi_\pi - 1) + 2\tilde{\sigma} < \varphi_x < -H_1(\varphi_\pi - 1)$ .

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<sup>6</sup>This would correspond to region II of Figure 1 (shown below).

Under learning, private agents conjecture that the law of motion of  $z_t$  is of the form (27) and use least squares to get estimates for  $a$ ,  $b$  and  $c$ . As Bullard and Mitra (2002) remark the analysis of learning under the lagged-data-based rule does not yield analytical results. They illustrate their findings quantitatively using a calibrated version of the model. They show four possible outcomes associated with a particular parametrization of the policy rule: (I) determinate and E-stable equilibria, (II) determinate and E-unstable equilibria, (III) indeterminate and E-unstable equilibria and (IV) explosive MSV equilibria (see figure 1, which reproduces figure 2 of Bullard and Mitra (2002)). Interestingly, the policy rule satisfies the Taylor principle under outcome (I) but not under outcomes (II) and (III).

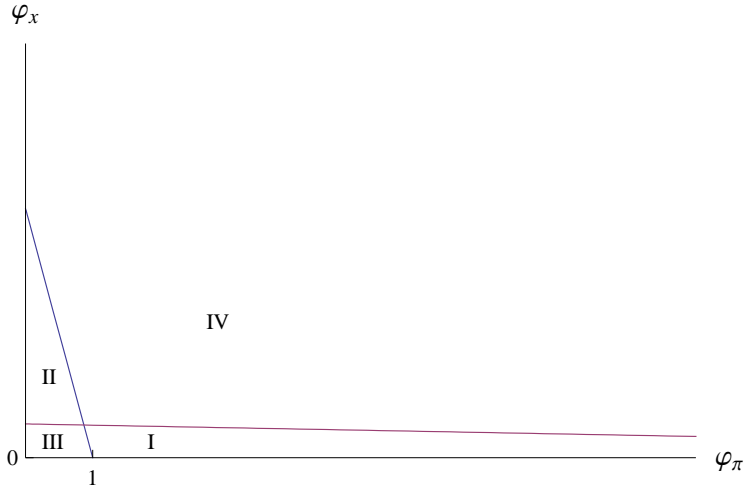


Figure 1: Determinacy and learnability under the lagged-data-based rule. I: Determinate and E-stable, II: Determinate and E-unstable, III: Indeterminate and E-unstable, IV: Explosive MSV solutions.

As far as trend growth is concerned, since  $\tilde{\beta}$  is smaller and  $\tilde{\lambda}$  is larger the larger is  $\Gamma$ , the effect of trend growth on the size of the determinate region is a priori ambiguous. To see this the two borderlines that separate the regions corresponding to the four outcomes are

$$\varphi_x = -H_1(\varphi_\pi - 1)$$

and

$$\varphi_x = -H_2(\varphi_\pi - 1) + 2\tilde{\sigma},$$

where  $H_1 > H_2$ ,  $\partial H_1/\partial\Gamma < 0$  and  $\partial H_2/\partial\Gamma > 0$ .

First, the size of the determinate and E-stable region (where the policy rule does satisfy the Taylor principle) is

$$Area_I = \frac{(1 + \tilde{\beta})^2 \tilde{\sigma}^2}{\tilde{\beta} \tilde{\lambda}}. \quad (32)$$

In this case  $\partial Area_I/\partial\Gamma < 0$  if and only if  $\Gamma^{\sigma-1} < \beta(1+2\omega)$ ; that is, if trend growth is not too high, a condition that is easily satisfied for realistic values of the parameter space. Using our standard calibration,  $\partial Area_I/\partial\Gamma < 0$  provided  $\Gamma < 2.31$  (i.e., growth rate is less than 500%, annualized).

Next, the size of the determinate and E-unstable region (where the policy rule does not satisfy the Taylor principle) is

$$Area_{II} = \frac{(\tilde{\beta} \tilde{\lambda} - \tilde{\sigma}(1 - \tilde{\beta}^2))^2}{(1 - \tilde{\beta}^2) \tilde{\beta} \tilde{\lambda}}. \quad (33)$$

It is straightforward to show that  $\partial Area_{II}/\partial\Gamma < 0$  if either both  $\tilde{\beta} \tilde{\lambda} - \tilde{\sigma}(1 - \tilde{\beta}^2)$  and  $\tilde{\lambda}(1 + \tilde{\beta}^2) \partial \tilde{\beta}/\partial\Gamma + \tilde{\beta}(1 - \tilde{\beta}^2) \partial \tilde{\lambda}/\partial\Gamma$  are positive or both are negative. However the signs of both terms are unclear a priori. We thus evaluate the sign of  $\partial Area_{II}/\partial\Gamma$  numerically by calibrating all parameters, other than  $\Gamma$ . In particular, we set  $\beta = 0.99$ ,  $\omega = 2/3$ ,  $N = 1/3$ ,  $\eta = 1$  and  $\sigma = 2$  (see, e.g. Basu and Kimball (2002)).

In Figure 2 we plot  $Area_{II}$  as function of  $\Gamma$ , where  $\Gamma \in [1, 1.02]$ ; that is, the trend growth rate lies between zero and 8%, annualized. This interval is large enough to include the average trend growth rate in advanced countries over the period 1980-2000, which is in the order of 2 – 4% (see, e.g., OECD, 2003). As can be seen from the figure, region  $Area_{II}$  decreases monotonically with  $\Gamma$ . Thus, the higher the trend growth rate the smaller the sizes of the indeterminate region and the determinate region that does not satisfy the Taylor principle.



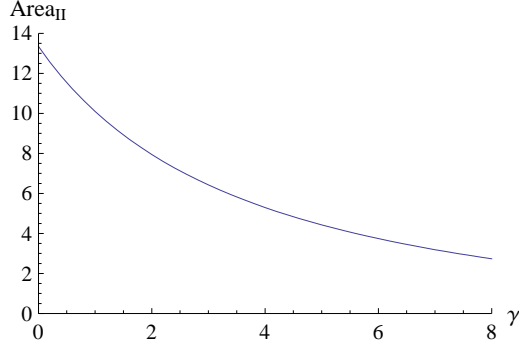


Figure 2: The effect of trend growth on the size of the determinate and E-unstable region under the lagged-data-based rule.

Finally, the size of the indeterminate and E-unstable region is given by

$$Area_{III} = 2\tilde{\sigma} + \frac{\tilde{\lambda}}{2 + 2\tilde{\beta}} + \frac{(-1 + \tilde{\beta}^2)\tilde{\sigma}^2}{\tilde{\beta}\tilde{\lambda}}. \quad (34)$$

In that case  $\partial Area_{III}/\partial\Gamma < 0$  if and only if

$$\left( -\frac{\tilde{\lambda}}{2(1 + \tilde{\beta})^2} + \frac{(1 + \tilde{\beta}^{-2})\tilde{\sigma}^2}{\tilde{\lambda}} \right) \frac{\partial\tilde{\beta}}{\partial\Gamma} + \left( \frac{1}{2(1 + \tilde{\beta})} + \frac{(1 - \tilde{\beta}^2)\tilde{\sigma}^2}{\tilde{\beta}\tilde{\lambda}^2} \right) \frac{\partial\tilde{\lambda}}{\partial\Gamma} < 0. \quad (35)$$

In condition (35)  $\partial\tilde{\beta}/\partial\Gamma$  is negative, while  $\partial\tilde{\lambda}/\partial\Gamma$  and the term in the second parenthesis are both positive. However the sign of the term in the first parenthesis is ambiguous a priori. As the condition is not amenable to further analytical manipulation we evaluate (34) numerically. In Figure 3 we plot  $Area_{III}$  as a function of  $\Gamma$  fixing the values of other parameters. We see from the figure that  $Area_{III}$  decreases monotonically with  $\Gamma$ . Thus, the higher the trend growth rate the smaller the sizes of the indeterminate and E-unstable region.

Combining the results of Bullard and Mitra (2002) with ours, we conclude that the higher is trend growth, the smaller is the determinate and E-stable region, the smaller is the determinate and E-unstable region, and the smaller is the indeterminate and E-unstable region. These results in turn imply that, the higher is trend growth, the larger is the region with explosive MSV solutions.

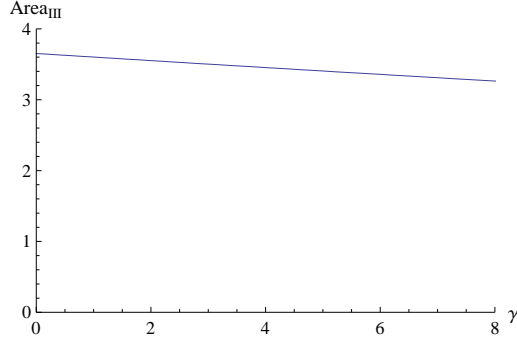


Figure 3: The effect of trend growth on the size of the indeterminate and E-unstable region under the lagged-data-based rule.

### 3.3 Expectations-data-based rule

The policy rule that responds to expectations of next period inflation and the output gap is given by

$$\hat{l}_t = \varphi_\pi \pi_{t+1}^e + \varphi_x x_{t+1}^e, \quad (36)$$

so that the system of equations is given by (21), (22) and (36). As in the case of the current-data-based rule, the only state variable is  $r_t^n$  so that under rational expectations the MSV solution is of the form (24).

By a version of Proposition 4 of Bullard and Mitra (2002) a set of necessary and sufficient conditions for determinacy are

$$(1 - \tilde{\beta})\varphi_x + \tilde{\lambda}(\varphi_\pi - 1) > 0, \quad (37)$$

$$(1 + \tilde{\beta})\varphi_x + \tilde{\lambda}(\varphi_\pi - 1) < 2\tilde{\sigma}(1 + \tilde{\beta}) \quad (38)$$

and

$$\varphi_x < \tilde{\sigma}(1 + \tilde{\beta}^{-1}). \quad (39)$$

As Bullard and Mitra (2002) remark, unlike the other policy rule specifications, the magnitude of  $\varphi_x$  is crucial for determinacy. In particular, large values of  $\varphi_x$  lead to

indeterminacy, regardless of  $\varphi_\pi$ . Regarding the effect of trend growth, we first note that the three borderlines defined by conditions (37), (38) and (39) all intersect at  $\varphi_\pi^* = 1 - \tilde{\sigma} (1 - \tilde{\beta}^2) / (\tilde{\beta}\tilde{\lambda})$ . Moreover, for  $\varphi_\pi > \varphi_\pi^*$  condition (39) is redundant.

It follows that, our result regarding the effect of trend growth on the size of the determinate and E-stable (where the policy rule satisfies the Taylor principle) under the lagged data rule (see equation (32)) carries over. That is,  $\partial Area_I / \partial \Gamma < 0$  if and only if  $\beta(1 + 2\omega) > \Gamma^{\sigma-1}$ . As we remarked above, higher trend growth shrinks the determinate and E-stable region for realistic values of the parameter space.

Under learning, by a version of Proposition 5 of Bullard and Mitra (2002) the necessary and sufficient condition for an MSV solution to be E-stable is  $(1 - \tilde{\beta})\varphi_x + \tilde{\lambda}(\varphi_\pi - 1) > 0$ , which is the same condition as that under current-data-based rule. It follows that, under expectations based rule higher trend growth increases the scope for E-stability of MSV equilibria.

We conclude that under the expectations-based rule, changes in trend growth lead to a tradeoff between determinacy and learnability of equilibria. In particular, the higher is trend growth the lower the scope for determinacy but the higher the scope for learnability of MSV equilibria.

## 4 Concluding remarks

The paper extends the basic New Keynesian model to allow for trend productivity growth. It shows that the sensitivity of actual inflation to expected inflation is lower while its sensitivity to output gap is higher the higher is trend productivity growth. It then examines how these changes in the slope and position of the Phillips curve affect the determinacy and learnability properties of different monetary policy rules.

The paper shows that for a policy rule that responds to current period inflation and the output gap a higher trend growth rate relaxes the conditions for determinacy and learnability of equilibria. Results are mixed for policy rules that respond to expectations and lags of inflation and the output gap. Under the expectations-data-based rule, a higher trend growth rate tightens the conditions for determinacy but it relaxes the conditions for learnability. Under the lagged-data-based rule rule a

higher trend growth rate reduces the scope for both determinacy and learnability.

The paper focuses on the performance of simple policy rules and learnability of MSV solutions, so as to stay close to the framework of Bullard and Mitra (2002). However, the model with trend productivity growth could be used to study the performance of interest rate rules that are derived from optimal monetary policy, as well as learnability of non-MSV rational expectations equilibria.

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