Kiel
Working Papers

Kiel Institute
for the World Economy

Insurance Demand under Prospect Theory: A Graphical Analysis

by Ulrich Schmidt

No. 1764| March 2012

Web: www.ifw-kiel.de

Kiel Working Paper No. 1764|March 2012

Insurance Demand under Prospect Theory: A Graphical Analysis

Ulrich Schmidt

Abstract:
Many people do not take up desaster insurance for their houses even though premiums are subsidized. A well-known example is flood insurance. At the same time they buy highly loaded insurance contracts for modest risk. Examples here are extended warranties or cellular phone insurance. This paper shows that unlike traditional decision theory prospect theory can explain this evidence provided the reference point is chosen in the right way

Keywords:
insurance demand, prospect theory, flood insurance, diminishing sensitivity, loss aversion

JEL classification: D14, D81, G21

Prof. Dr. Ulrich Schmidt
Kiel Institute for the World Economy
24100 Kiel, Germany
Telephone:
E-mail: Ulrich.schmidt@ifw-kiel.de

Insurance Demand under Prospect Theory: A Graphical Analysis

Ulrich Schmidt*

Kiel Institute for the World Economy & Dept. of Economics, University of Kiel

Abstract:
This paper analyzes insurance demand under prospect theory in a simple model with two states of the world and fair insurance contracts. We argue that two different reference points are reasonable in this framework, state-dependent initial wealth or final wealth after buying full insurance. Applying the value function of Tversky and Kahneman (1992), we find that for both reference points subjects will either demand full insurance or no insurance at all. Moreover, this decision depends on the probability of the loss: the higher the probability of the loss, the higher is the propensity to take up insurance. This result can explain empirical evidence which has shown that people are unwilling to insure rare losses at subsidized premiums and at the same time take-up insurance for moderate risks at highly loaded premiums.

JEL codes: D14, D81, G21

Key words: insurance demand, prospect theory, flood insurance, diminishing sensitivity, loss aversion

1 Introduction

A major puzzle in insurance economics is the fact that people underinsure low-probability events with high losses and overinsure moderate risks. It is well documented that many people do not take up disaster insurance even though premiums for such insurance contracts are often subsidized (Kunreuther et al., 1978; Kunreuther and Pauly, 2004). A very prominent example for this type of behavior is flood insurance in the USA. At the same time, for modest risk people do often buy insurance with premiums exceeding expected losses substantially (Pashigian et al.,1966; Drèze, 1981; Cutler and Zeckhauser, 2004; Kunreuther and Pauly, 2006; Sydnor, 2010). Examples here are demand for low deductibles and markets for extended warranties or cellular-phone insurance. Beside the cited evidence from the field, also several experimental studies indicate that – holding loading factor and expected loss constant

* I thank Peter P. Wakker for helpful comments. Address for correspondence: Ulrich Schmidt, Kiel Institute for the World Economy and University of Kiel, Düsternbrooker Weg 120, 24105 Kiel, Tel.: +49 4318801400, Fax: +49 4318804621, email: ulrich.schmidt@ifw-kiel.de.

– the rate of insurance take-up increases with the probability of the loss (Slovic et al., 1977; McClelland et al., 1993; Ganderton et al., 2000; but see also the contrary results of Laury et al., 2008).

The standard theory of decision making under risk, expected utility (EU) theory, is not able to explain these phenomena. Under EU a subject will buy full insurance if and only if premiums are fair, i.e. equal expected losses. This excludes not taking up subsidized flood insurance or buying highly loaded cellular-phone insurance. Also fitting the demand for low deductibles to EU leads to implausible high degrees of risk aversion (Sydnor, 2010). While EU is primarily a normative theory of decision making under risk, also prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992) – currently one of the most prominent descriptive theory of decision making under risk – has not yet been successfully employed to organize the evidence. In fact, there exist only very few studies applying prospect theory (PT) to insurance demand. One reason for this may be the fact that it is not obvious what the right reference point for such an analysis should be. Usually the status quo is taken as reference point under PT. Accordingly, Wakker et al. (1997) and Sydnor (2010) take initial wealth – i.e. the wealth if no loss occurs – as reference point. The main problem, as argued by Sydnor (2010), is the fact that the decision to take up insurance is then determined entirely in the loss domain, where subjects are according to PT in general risk seeking. With overweighting of small probabilities, risk aversion for improbable losses can occur under PT, but also in this case the high demand for insuring modest risks cannot be explained.

In the present paper we argue that it is questionable whether initial wealth is the right choice of the reference point for analyzing insurance problems with PT. Consider a simple insurance problem where the subject has initial wealth w but in one state of the world a loss L might occur such that final wealth equals w – L in this state. Wakker et al. (1997) and Sydnor (2010) take w as reference point. However, in fact the status quo is state-dependent in this example (it equals either w or w – L) and a new variant of PT proposed by Schmidt et al. (2008) allows for analyzing state-dependent reference points. Consequently, it seems reasonable to take w as reference point for the state without loss and w – L as reference point for the state in which the loss L occurs. Doing so implies that keeping the status quo (i.e. not taking up insurance) leads to neither gains nor losses in both states. The consequences for insurance demand with this reference point will be analyzed in section 3.1.

Taking the status quo as reference point is, however, not necessarily the right choice from an empirical point of view. For instance, Hershey and Schoemaker (1985) and Bleichrodt, et al. (2001) found that people take one of the alternatives (usually a safe option) as their reference point and evaluated the outcomes of the other alternative relative to this reference point (see also Stalmeier and Bezembinder 1999, Morrison 2000, Robinson, Loomes, and Jones-Lee 2001, van Osch et al. 2004). Since a safe alternative is available if subjects take up full insurance, final wealth after buying full insurance is also a reasonable reference point which will be explored in section 3.2.

Before analyzing insurance demand we will introduce PT in the next section. As we wish to analyze insurance demand graphically, we particularly investigate the properties of PT´s indifference curves in two-outcome diagrams. We will also motivate why we refrain from taking into account probability weighting in the present analysis. Schmidt and Zank (2007) characterized insurance demand under PT with probability weighting and a piecewise linear value function. Contrary to the present analysis they take a reference point which lies between w and final wealth after buying full insurance. Apart from the already mentioned papers the only other paper analyzing insurance demand under PT is Eckles and Volkman Wise (2011). Here the reference point is different for each alternative, i.e. the reference point depends on the chosen level of insurance coverage. Although they analysis is behaviorally convincing, their model is indeed not compatible with PT as under PT all alternatives are evaluated under the same reference point.

Even in our simple model with fair insurance contracts, only two states of the world and no probability weighting PT in general does not allow to derive clear-cut implications for insurance demand. For both reference points we analyze optimal insurance demand is determined by trading off gains in one state with losses in the other state. As the value function of PT is concave for gains and convex for losses, it is not guaranteed that this trade-off has an interior solution. Therefore, the bulk of our analysis is based on the specific functional form of the value function proposed by Tversky and Kahneman (1992) which is also employed in most empirical applications of PT. For this value function we show that no interior solution exists, i.e. subjects either buy full insurance or no insurance at all. In line with the evidence presented above, the propensity to take up insurance is increasing with the loss probability.

2 Prospect Theory

PT was developed by Kahneman and Tversky (1979) and Tversky and Kahneman (1992) in order to accommodate empirical observed violations of EU. Compared to EU, PT introduces two main innovations, reference dependence and probability weighting. Insurance problems like disaster insurance are often concerned with rare events and both, the theory and the evidence for weighting probabilities of rare events is not entirely conclusive. In original PT for instance very small probabilities are either overweighted or rounded to zero. In order to circumvent this ambiguity and to keep the analysis simple we refrain from probability weighting in the present paper.

For the graphical analysis we consider two-outcome diagrams with possible consequences x_1 and x_2. If p is the probability of the first state, the PT value for any lottery in the two-outcome diagram is given by

(1) $V = pv(x_1) + (1 - p)v(x_2).$

The value function v with $v(0) = 0$ is defined on gains and losses measured as deviations from the reference point and not on final wealth positions as the utility function in EU. It is generally assumed that v satisfies diminishing sensitivity and loss aversion. Diminishing sensitivity means that marginal value is decreasing if one moves away from the reference point in either direction which implies that the value function is concave in the domain of gains and convex in the domain of losses. Loss aversion holds if the value function is steeper for losses than for gains, i.e. for $x > 0$ we have $v'(-x) > v'(x)$.

Empirical applications of PT often employ the following functional form of the value function proposed by Tversky and Kahneman (1992):

$$(1) \qquad v(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0 \\ -\lambda |x|^{\alpha} & \text{if } x < 0. \end{cases}$$

This value function, in the sequel referred to as K&T value function, exihibts diminishing sensitivity for $\alpha < 1$ and loss aversion for $\lambda > 1$. Median parameters observed in the experimental study of Tversky and Kahneman (1992) correspond to $\alpha = 0.88$ and $\lambda = 2.25$.

For a graphical analysis of PT preferences we obviously need to know the curvature of indifference curves. As far as we know, this has not been analyzed yet in the literature. The slope of indifference curves can be approximated by

$$(2) \qquad \frac{dx_1}{dx_2} = -\frac{(1-p)v'(x_2)}{pv'(x_1)} < 0.$$

It is well known from expected utility theory that indifference curves are convex if v is concave. However, for PT v is only concave in the gain domain but convex in the loss domain. Therefore, the curvature of indifference is ambiguous if one outcome is a gain and the other is a loss. To see this, suppose that $x_1 > 0$ and $x_2 < 0$. If we move along an indifference curve by increasing x_2 and decreasing x_1, both $v'(x_1)$ and $v'(x_2)$ are increasing as in both cases we move closer to the reference point. Therefore, the total effect on the slope in (2) is unknown without further assumptions on the shape of the value function. For the K&T value function we can get however more concrete results.

Lemma 1:
Consider a x_1-x_2 diagram and prospect theory with the K&T value function with $\alpha < 1$ and $\lambda > 1$. Then the indifference curve for a utility level V in the quadrant with $x_1 > 0$ and $x_2 < 0$ is convex (resp. linear, resp. concave) if V < (=, >) 0.

Proof: See Appendix.

3 Insurance Demand

In order to allow for an analysis in the two-outcome diagram we focus on a model with only two states of the world, in one of which a loss L will occur. Initial wealth is given by w such that in the absence of an insurance contract state-dependent wealth equals either w or w – L. We assume that the subject can buy coverage C, $0 \leq C \leq L$, for the fair premium pC where p equals the probability of the loss. If insurance is taken up, final wealth equals w – pC if no loss occurs and w – L + (1 – pC) if the loss occurs.

2.1 Status Quo as Reference Point

In this section we analyze insurance demand with a state-dependent reference point given by the status quo, i.e. it equals either w or w – L. Therefore, if the subject refrains from taking up insurance there is a gain of zero in both states, i.e. the utility level equals zero (note that $v(0) = 0$ is always required in PT). For full insurance there is a gain of $w - pL - (w - L) = (1 - p)L$ with probability p and a loss of $w - pL - w = -pL$ with probability $1 - p$. Table 1 gives an overview over the model.

Table 1: Status Quo as Reference Point

State	No Loss	Loss
Probability	1 – p	p
Reference Point (Initial Wealth)	w	w – L
Final Wealth	w – pC	w – L + (1 – p)C
Gain/Loss	– pC	(1 – p)C

Utility when taking up insurance is given by

(3) $V = pv((1 - p)C) + (1 - p)v(-pC)$,

which leads to the following first-order condition

(4) $p(1 - p)v'((1 - p)C) - (1 - p)pv'(-pC) = 0$.

Rearranging yields the condition for optimal coverage:

(5) $v'((1 - p)C) = v'(-pC)$.

This condition shows that an interior solution might not exist as in the case of diminishing sensitivity both $v'((1 - p)C)$ and $v'(-pC)$ are decreasing in C. Consistent with this observation the second-order condition

(6) $(1 - p)v''((1 - p)C) + pv''(-pC) < 0$

is not necessarily satisfied as $v''(-pC) > 0$.

In order to be able to derive concrete results we focus in the following on the K&T value function. According to equation (3), when deriving optimal insurance demand, the subject trades off a gain in one state with a loss in the other state. Lemma 1 shows that the sign of V determines the curvature of indifference curves for such cases. Inserting the K&T value function in (3) yields

(7) $V = p((1-p)C)^{\alpha} - \lambda(1-p)|-pC|^{\alpha}$.

For any C > 0 this implies

(8) $V > (=, <)\ 0 \Leftrightarrow p > (=, <) \dfrac{\lambda^{1/(1-\alpha)}}{1+\lambda^{1/(1-\alpha)}}$.

Obviously, the subject will demand no insurance and attain a utility level of zero if C > 0 leads to V < 0. Therefore, we have C = 0 for $p < \lambda^{1/(1-\alpha)}/(1+\lambda^{1/(1-\alpha)})$. This case is depicted in the left panel of Figure 1. According to Lemma 1 the indifference curve for a utility level of zero is linear here. The right panel shows the case where $p > \lambda^{1/(1-\alpha)}/(1+\lambda^{1/(1-\alpha)})$. Here the indifference curve is concave since V > 0 and, therefore, full coverage is optimal. Note that when the indifference curve crosses the x_1-axis from the left there is a kink due to loss aversion and it becomes convex since the value function is now concave for both outcomes. We can summarize our results in the following proposition.

Proposition 1: For the K&T value function with $\alpha < 1$ and $\lambda > 1$ individuals who take the status quo as reference point will demand either full insurance or no insurance at all. Full insurance will be taken up if $p > \lambda^{1/(1-\alpha)}/(1+\lambda^{1/(1-\alpha)})$, no insurance is optimal for $p < \lambda^{1/(1-\alpha)}/(1+\lambda^{1/(1-\alpha)})$.

Proposition 1 shows that the model is in principle compatible with the evidence that people do not take up insurance for low loss probabilities. However, for realistic values of α and λ the condition in Proposition 1 leads to quite high probabilities. If we take the values of Tversky and Kahneman (α = 0.88 and λ = 2.25) we get for instance p > 0,999, which means that subjects would take up insurance only for losses which are nearly certain.

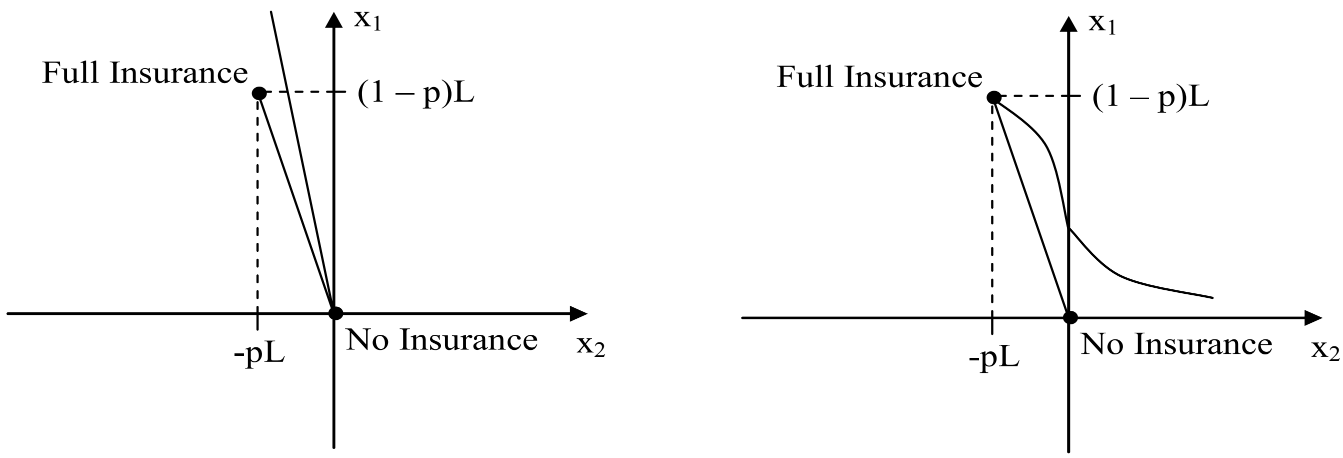

Figure 1: Optimal insurance demand for status quo as reference point. Left panel: No coverage is optimal, right panel: full coverage is optimal

2.2 Full Insurance as Reference Point

As argued in in the introduction it is plausible that individuals take a safe option as reference point whenever it is available. For insurance demand the safe option is taking up full insurance. In this case $w - pL$ is the final wealth and also the reference point in both states. When an individual decides upon optimal coverage she trades off gaining $p(L - C)$, i.e. the saved premium, in the state without loss with loosing $(1 - p)(L - C)$, i.e. the reduced compensation minus the saved premium, in the state with loss. Table 2 gives an overview over the model.

Table 2: Full Insurance as Reference Point

State	No Loss	Loss
Probability	$1 - p$	p
Reference Point	$w - pL$	$w - pL$
Final Wealth	$w - pC$	$w - L + (1 - p)C$
Gain/Loss	$p(L - C)$	$-(1 - p)(L - C)$

Utility when taking up insurance is given by

(9) $\quad V = pv(-(1 - p)(L - C)) + (1 - p)v(p(L - C))$

which leads to the following condition for optimal coverage

(10) $\quad v'(-(1 - p)(L - C)) = v'(p(L - C)).$

Again, an interior solution might not exist as both sides are increasing in C. As in the preceding section also the second order condition

(11) $\quad (1 - p)v''(-(1 - p)(L - C)) + pv''(p(L - C)) < 0$

is not necessarily satisfied as the first summand is positive.

In order to derive concrete results we have to go back to the K&T value function. For $C = L$, V obviously equals zero. If $C < L$ is optimal we must have $V > 0$, i.e.

(12) $\quad V = -\lambda p| (-(1 - p)(L - C)|^{\alpha} + (1 - p)(p(L - C))^{\alpha} > 0.$

Rearranging yields for any $C < L$

$$(13)\qquad V > (=,<)\, 0 \Leftrightarrow p < (=,>) \frac{1}{1+\lambda^{1/(1-\alpha)}}.$$

Since according Lemma 1 indifference curves are concave for V > 0, C = 0 must be optimal in this case. Figure 2 clarifies these results. In the left panel we assume $p > 1/(1+\lambda^{1/(1-\alpha)})$. This implies V < 0 for any positive coverage and hence full insurance is optimal. In the right panel we have $p < 1/(1+\lambda^{1/(1-\alpha)})$ which yields V > 0. According to Lemma 1 indifference curves are concave for V > 0 which implies that optimal coverage is given by zero.

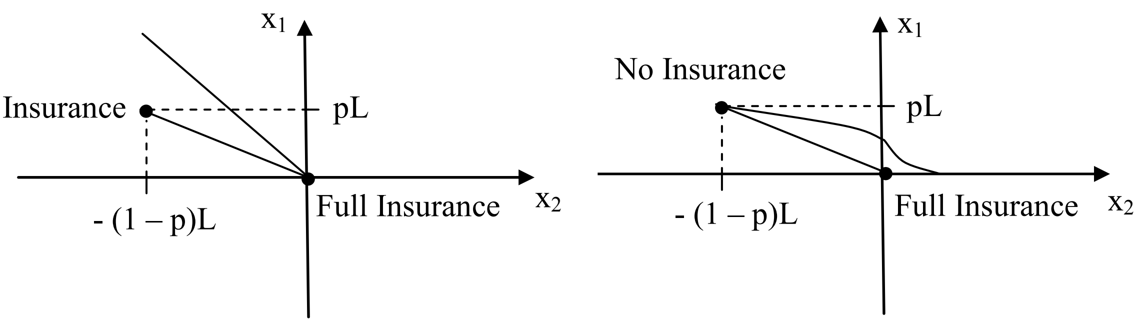

Figure 1: Optimal insurance demand for full insurance as reference point.
Left panel: full coverage is optimal. Right panel: no coverage is optimal

The results of this section can be summarized in the following proposition.

Proposition 2: For the K&T value function with $\alpha < 1$ and $\lambda > 1$ individuals who take final wealth after buying full insurance as reference point will demand either full insurance or no insurance at all. Full insurance will be taken up if $p > 1/(1+\lambda^{1/(1-\alpha)})$, no insurance is optimal for $p < 1/(1+\lambda^{1/(1-\alpha)})$.

Applying the values of Tversky and Kahneman (α = 0.88 and λ = 2.25) the condition for taking up insurance in Proposition 2 becomes p > 0,0012, i.e. only very rare risks will be uninsured. This value for p is however very sensitive to the value of α. For e.g. α = 0.5, which is also a quite realistic value, the condition would change to p > 0.165. Altogether, we can conclude that PT with full insurance as reference point provides a rather realistic accommodation of the evidence presented in the introduction.

4. Conclusions

The present paper has considered insurance demand under PT in a simple model with fair insurance contracts and only two states of the world. In one variant of the model we considered the status quo as reference point which is state-dependent in the case of insurance problems. In a second variant we have argued that according to the evidence of Hershey and Schoemaker (1985) and Bleichrodt, et al. (2001) it is realistic that individuals may take final wealth after buying full insurance as their reference point. Employing the K&T value function we can show that in both variants of the model subjects do either buy full insurance or no insurance at all. Moreover, in both cases the propensity to take up insurance is increasing with the loss probability. Therefore, PT is in principle able to accommodate the evidence that people are unwilling to insure rare losses at subsidized premiums and at the same time take-up insurance for moderate risks at highly loaded premiums. The second variant of the model seems to be however more realistic as insurance will be taken up here already for relatively low probabilities.

Appendix: Proof of Lemma 1

An arbitrary utility level V in the two outcome diagram with $x_1 > 0$ and $x_2 < 0$ is given by

(A1) $V = px_1^{\alpha} - \lambda(1-p)|x_2|^{\alpha}$.

Solving for x_1 yields

(A2) $x_1 = [(V + \lambda(1-p)|x_2|^{\alpha}) / p]^{1/\alpha}$.

For constant V the slope of the corresponding indifference curve is given by the first derivative:

(A3) $dx_1 / dx_2 = -\lambda[(1-p)/p]\,|x_2|^{\alpha-1}[(V + \lambda(1-p)|x_2|^{\alpha}) / p]^{(1-\alpha)/\alpha}$,

and the curvature equals

(A4) $$d^2x_1 / (dx_2)^2 = (\alpha-1)\lambda[(1-p)/p]\,|x_2|^{\alpha-2}[(V + \lambda(1-p)|x_2|^{\alpha}) / p]^{(1-\alpha)/\alpha} + (1-\alpha)[\lambda[(1-p)/p]\,|x_2|^{\alpha-1}]^2\,[(V + \lambda(1-p)|x_2|^{\alpha}) / p]^{(1-2\alpha)/\alpha}.$$

This implies

(A5) $$d^2x_1 / (dx_2)^2 < (=, >)\ 0 \Leftrightarrow |x_2|^{-1} - \lambda[(1-p)/p]\,|x_2|^{\alpha-1}\,[(V + \lambda(1-p)|x_2|^{\alpha}) / p]^{-1} > (=, <)\ 0,$$

which yields

(A6) $d^2x_1 / (dx_2)^2 < (=, >)\ 0 \Leftrightarrow \lambda[(1-p)/p]\,|x_2| < (=, >)\ |x_2|^{1-\alpha}\,[(V + \lambda(1-p)|x_2|^{\alpha})] / p$.

Hence we get

(A7) $d^2x_1 / (dx_2)^2 < (=, <)\ 0 \Leftrightarrow \lambda(1-p) < (=, >)\ |x_2|^{-\alpha}\,V + \lambda(1-p)$,

which implies

(A8) $d^2x_1 / (dx_2)^2 < (=, >) 0 \Leftrightarrow V > (=, <) 0$.

References

Bleichrodt, H., Pinto, J. L., and Wakker, P. P. (2001), “Making Descriptive Use of Prospect Theory to Improve the Prescriptive Use of Expected Utility,” Management Science 47, 1498-1514.

Drèze, J. H. (1981), “Inferring Risk Tolerances from Deductibles in Insurance Contracts,” Geneva Papers on Risk and Insurance 6, 48–52.

Cutler, D. M., and Zeckhauser, R. (2004), “Extending the Theory to Meet the Practice of Insurance,” in: *Brookings-Wharton Papers on Financial Services*, edited by R. E. Litan and R. Herring, 1–47. Washington, DC: Brookings Institution.

Eckles, D. L., and Volkman Wise, J. (2011), Propsect Theory and the Demand for Insurance,” Working Paper.

Ganderton, P. T., Brookshire, D. S., McKee, M., Steward, S., and Thurston, H. (2000), “Buying Insurance for Disaster-Type Risks: Experimental Evidence”, Journal of Risk and Uncertainty 20, 271-89.

Hershey, J. C., and Schoemaker, P. J. H. (1985), “Probability versus Certainty Equivalence Methods in Utility Measurement: Are They Equivalent?” Management Science 31, 1213-1231.

Kahneman, D., and Tversky, A. (1979), “Prospect Theory: An Analysis of Decision Under Risk,” Econometrica 47, 263-291.

Kunreuther, H., Ginsberg, R., Miller, L., Sagi, P., Slovic, P., Borkan, B. and Katz, N., (1978), *Disaster Insurance Protection: Public Policy Lessons*, New York: Wiley.

Kunreuther, H., and Pauly, M. (2004), “Neglecting Disaster: Why Don´t People Insure Against Large Losses?,” Journal of Risk and Uncertainty 28, 5-21.

Kunreuther, H,, and Pauly, M. (2006), *Insurance Decision-Making and Market Behavior*, Hanover, MA: now Publishers Inc.

Laury, S. K., McInnes, M. M., and Swarthout, J. T. (2008), “Insurance Purchase for Low-Probability Losses,” Working Paper.

McClelland, G. H., Schulze, W. D., and Coursey, D. L. (1993), “Insurance for Low-Probability Hazards: A Bimodal Response to Unlikely Events,” Journal of Risk and Uncertainty 7, 95-116.

Pashigian, B. P., Schkade, L. L., and Menefee, G. H. (1966), “The Selection of an Optimal Deductible for a Given Insurance Policy,” Journal of Business 39, 35–44.

Schmidt, U., Starmer, C., and Sugden, R. (2008), "Third-Generation Prospect Theory," Journal of Risk and Uncertainty 36, 203-223.

Slovic , P. Fischhoff , B., Lichtenstein , S., Corrigan , B., and Combs, B. (1977), "Preferences for Insuring Against Probable Small Losses: Insurance Implications," Journal of Risk and Insurance 44, 237-257.

Sydnor, J. (2010), "(Over)insuring Modest Risks," American Economic Journal: Applied Economics 2, 177-199.

Tversky, A. and Kahneman, D. (1992), "Advances in Prospect Theory: Cumulative Representation of Uncertainty," Journal of Risk and Uncertainty 5, 297-323.

Wakker, P. P., Thaler, R. H., and Tversky, A. (1997), "Probabilistic Insurance", Journal of Risk and Uncertainty 15, 7-28.