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The Optimal Inflation Rate and Firm-Level Productivity Growth

Henning Weber[1]

Abstract:

Empirical data show that firms tend to improve their ranking in the productivity distribution over time. A sticky-price model with firm-level productivity growth fits this data and predicts that the optimal long-run inflation rate is positive and between 1.5% and 2% per year. In contrast, the standard sticky-price model cannot fit this data and predicts optimal long-run inflation near zero. Despite positive long-run inflation, the Taylor principle ensures determinacy in the model with firm-level productivity growth, and optimal inflation stabilization policies are standard. In a two-sector extension of this model, the optimal long-run inflation rate weights the sector with the stickier prices more heavily.

Keywords: Optimal monetary policy, indeterminacy, heterogenous firms, firm entry and exit.

JEL classification: E31, E32, E52, E61

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1 Motivation

Many central banks around the globe target a long-run inflation rate between 1% and 3% per year, as they deem this rate to be the optimal rate.[1] For instance, the European Central Bank interprets its price-stability mandate as a mandate to target a growth rate in the harmonized index of consumer prices of less than, but close to, 2% per year. However, many academics in the field of monetary economics identify a very different range for the optimal long-run inflation rate, namely from minus the real interest rate to zero.[2] These opposing views leave plenty of room for disagreement, and the recent nosedive in nominal short-term interest rates in the wake of the financial crisis has moved the classical debate about the optimal long-run inflation rate back to center stage (e.g., Blanchard, Dell'Ariccia, and Mauro (2010)).

In this paper, I analyze the optimal long-run inflation rate under plausible assumptions about the growth rate in firm-level productivity, and make the case for a moderately positive long-run inflation rate. In firm-level data, firms move systematically through the productivity distribution over time. A firm tends to have below-average productivity upon market entry, and its productivity tends to grow thereafter. Thus, the firm improves its ranking in the productivity distribution over time. Accounting for the positive growth rate in firm-level productivity, I find that the optimal long-run inflation rate can be as high as 2% per year. The finding helps to justify the sign and, at least to some degree, the magnitudes of the long-run inflation rates that many central banks actually target.

The optimal long-run inflation rate is often analyzed in monetary models with sticky nominal prices, like the New Keynesian model. In this model, a firm sets the nominal price of its product on the basis of its marginal costs and, thereafter, maintains this price for an extended period of time. A positive long-run inflation rate thus erodes the actual real price of the firm, whereas the firm's real marginal costs, which are not directly affected by the long-run inflation rate, stay constant. In the New Keynesian model, this misalignment between the actual real price and the real marginal costs, which is caused by having a

[1] See Kuttner (2004), Table 2, and Schmitt-Grohe and Uribe (2010), Table 1.

[2] See Schmitt-Grohe and Uribe (2010) for a literature review and independent results.

positive instead of a zero long-run inflation rate, represents a major source of social loss.

While I also work with a fairly basic monetary model with sticky nominal prices, I modify it to account for the positive growth rate in firm-level productivity and for the entry and exit of firms. Like in the New Keynesian model, a positive long-run inflation rate erodes the real product price of a firm in my model. However, unlike in the New Keynesian model, the firm's real marginal costs decline in my model, as a result of the positive growth rate in the firm-level productivity. The main result of this paper is that in my model with the positive growth rate in firm-level productivity, the optimal long-run inflation rate is positive and equal to the growth rate in firm-level productivity. This long-run inflation rate erodes the actual real price of a firm at the same pace as the pace at which the firm's real marginal costs decline and, therefore, aligns the firm's actual real price with its real marginal costs. This alignment yields the first-best resource allocation.

The model with firm-level productivity growth shows that sticky nominal prices alone do not constitute a compelling reason for an optimal long-run inflation rate near zero. Instead, if one accepts the idea that there are disaggregate factors that systematically shift a firm's real marginal costs over time, such as gains in firm-level productivity, then sticky nominal prices actually are consistent with a positive optimal long-run inflation rate. This insight challenges the prominent zero-inflation or price-stability finding (e.g., Goodfriend and King (2001), Khan, King, and Wolman (2003)), which relies on combining sticky nominal prices with constant real marginal costs. However, the price-stability finding is in conflict with the fact that many central banks target a positive long-run inflation rate, as Schmitt-Grohe and Uribe (2010) emphasize.

I also quantify the optimal long-run inflation rate using a model with two sectors, thus accounting for varying growth rates in firm-level productivity, varying degrees of price stickiness, and varying firm entry and exit rates. I show that each sector has its own optimal long-run inflation rate and that the government faces an important policy tradeoff when selecting the aggregate long-run inflation rate. Using only the aggregate long-run inflation rate as its policy instrument, the government generally cannot target the optimal long-run inflation rate in both sectors at the same time. I show that the

optimal aggregate long-run inflation rate minimizes the weighted distance with respect to the long-run inflation rate that is optimal in each sector and that using calibrated parameters, a reasonable estimate of the optimal aggregate long-run inflation rate lies between 1.5% and 2% per year.

The government tilts the optimal aggregate long-run inflation rate towards the optimal long-run inflation rate in the sector with the more sticky prices because thereby it shifts the price adjustment to the sector with the more flexible prices, where it is least distortive. It is natural to extrapolate this finding to the case of a monetary union. In this case, the optimal union-wide long-run inflation rate weighs the optimal long-run inflation rate in the member state with stickier prices more than the optimal long-run inflation rate in the member state with more flexible prices. Benigno (2004) has obtained a similar finding with respect to the optimal inflation *stabilization* policy. Both findings obey the "stickiness principle" (Goodfriend and King (1997)), i.e., to weight the state (sector) with the stickier prices more heavily.

A recent literature examines the relationship between the magnitude of the long-run inflation rate and the inflation stabilization policy in the New Keynesian model, and my analysis also relates to this literature, e.g., Ascari (2004), Hornstein and Wolman (2005), Ascari and Ropele (2007), Kiley (2007), Ascari and Ropele (2009), and Kobayashi and Muto (2011). These authors have found that a suboptimally positive long-run inflation rate dramatically changes the aggregate dynamics of the actual inflation rate and output, shrinks the determinacy region of the simple interest rate rules and, at least in some cases, renders optimal stabilization policy indeterminate. They conclude that neither positive nor normative predictions of the New Keynesian model extrapolate to the realistic case of a positive long-run inflation rate.

However, I find none of these unpleasant consequences of a positive long-run inflation rate in my one-sector model in which the positive long-run inflation rate is optimal. Instead, I find that in the neighborhood of the optimal positive long-run inflation rate, this model behaves like the New Keynesian model in the neighborhood of a zero long-run inflation rate. Thus, both models generate the same aggregate dynamics for the actual

inflation rate and output, fulfill the same determinacy conditions, e.g., the Taylor principle, and yield the same optimal inflation stabilization policies, independently of whether the government acts discretionarily or with commitment.

A main feature of my analysis is the heterogenous productivity across firms, which arises because firms are of different ages as a result of firm entry and exit and because older firms are more productive than younger firms as a result of firm-level growth. Accordingly, my analysis is related to the literature on the role of heterogenous firms for the business cycle dynamics (e.g., Ottaviano (2011)), the open-economy macroeconomics (e.g., Ghironi and Melitz (2005)), and the new trade theory (e.g., Melitz (2003), Burstein and Melitz (2011)). Unlike these papers, however, this paper analyzes optimal monetary policy. Bergin and Corsetti (2008), Bilbiie, Ghironi, and Melitz (2008), Faia (2009), and Bilbiie, Fujiwara, and Ghironi (2011) also analyze optimal monetary policy in models with firm entry and exit. However, while these authors highlight models with aggregate productivity growth and homogenous firms, I highlight a model with firm-level productivity growth and heterogenous firms.

This paper is also related to the literature on the optimal long-run inflation rate. This literature has examined a long list of factors, and I have left many of them out of my analysis in order to focus it on a lean model.[3] In contrast to this literature, which focuses mostly on the role of aggregate factors, my analysis focuses on the role of disaggregate factors. Few other papers examine disaggregate factors, and their mechanisms and results differ from the ones obtained here. Wolman (2011) examines the role of sector-specific productivity growth in a two-sector model with sticky prices and finds that the government obeys the stickiness principle and that mild deflation is socially optimal. Schmitt-Grohe and Uribe (2011) examine quality bias in the officially measured inflation rate in one and two-sector models and recover the price-stability finding if non-quality adjusted prices

[3]Among these factors are monetary and transaction frictions (e.g., Friedman (1969), Aruoba and Schorfheide (2011), Berentsen, Menzio, and Wright (2011)), the zero lower bound on nominal interest rates (e.g., Billi (2011), Coibion, Gorodnichenko, and Wieland (forthcoming)), downwardly rigid nominal wages (e.g., Kim and Ruge-Murcia (2009)), or a positive trend growth rate in the aggregate productivity (e.g., Amano, Moran, Murchison, and Rennison (2009)).

are sticky. Finally, Janiak and Monteiro (2011) examine the entry and exit of heterogenous firms in a model with a cash-in-advance constraint, but without sticky prices, and show that the long-run inflation rate affects welfare through the level of the aggregate productivity.

This paper continues as follows. Section 2 briefly reviews the evidence on firm-level productivity growth and describes the one-sector model. Section 3 derives the optimal long-run inflation rate in this model, and Section 4 extends the model to two sectors and incorporates sectoral asymmetries. Section 5 derives the optimal long-run inflation rate in the two-sector model, illustrates the government's policy tradeoff, and shows how to resolve it optimally. Section 6 examines the consequences of the positive long-run inflation rate for aggregate dynamics and inflation stabilization policy.

2 Model

This section describes a monetary model with a positive growth rate in firm-level productivity and with exogenous firm entry and exit. The model features sticky nominal prices and represents a cashless economy. A special case of the model is the basic New Keynesian model derived in, e.g., Woodford (2003) or Gali (2008). Before describing the model, I briefly review the evidence on the positive growth rate in firm-level productivity to motivate the model setup.

2.1 Review of empirical evidence

The reviewed evidence on firm-level productivity growth can be split into the magnitude of the marginal productivity, i.e., the productivity of a new firm over the average productivity of incumbent firms, and into the post-entry growth rate of surviving firms. The evidence on the marginal productivity suggests that new firms or new plants have 75% to 95% of the productivity of incumbents.[4] A marginal productivity of this magnitude reemerges across different time periods and countries. Among others, Baily, Hulten, Campbell, Bresnahan,

[4]Geroski (1995), Caves (1998), and Bartelsman and Doms (2000) survey the evidence from longitudinal micro data.

and Caves (1992), Baldwin (1996), Foster, Haltiwanger, and Krizan (2001), Aw, Chen, and Roberts (2001), Farias and Ruano (2005), Baldwin and Gu (2006) and Wagner (2010) examine the marginal productivity in the manufacturing sector. Foster, Haltiwanger, and Krizan (2001) also examine it in the services sector.

The evidence on the post-entry growth rate in the productivity of firms or plants suggests that the productivity of surviving manufacturing firms grows between 2% and 3% per year. Foster, Haltiwanger, and Krizan (2001) estimate that the productivity of U.S. manufacturing firms grows at 2% per year, Huergo and Jaumandreu (2004) estimate that the productivity of Spanish manufacturing firms grows at 3% per year, and Baldwin and Gu (2006) estimate that the productivity of Canadian manufacturing firms grows at roughly 2% per year. Baily, Hulten, Campbell, Bresnahan, and Caves (1992) estimate that U.S. manufacturing plants that were established between 1972 and 1977 and that were initially of below-average productivity attained above-average productivity by 1987. This time period of one decade or so is consistent with a post-entry growth rate in productivity of 3% per year and a marginal productivity of 75%. The remainder of this section describes a model that is consistent with this firm-level evidence.

2.2 Firms

In order to set up the model, I index firms by $j \in [0, 1]$ and let each firm produce a single product variety. The technology of firm j needs labor ℓ_{jt} as the sole input to produce output y_{jt}:

$$y_{jt} = a_t g^{s_{jt}} \ell_{jt} \ .$$

The integer variable $s_{jt} = 0, 1, 2, \ldots$ indicates the firm's age. The growth rate in firm-level productivity, $g \geq 1$, is the same for all firms and independent of firm size. When g exceeds unity, established firms are more productive than new firms, in line with the evidence and with the models used in Melitz (2003) or Burstein and Melitz (2011). That the growth rate g is independent of a firm's size is Gibrat's law and a first approximation to the

data.[5] In the special case when g equals unity, all firms are equally productive, as in the basic New Keynesian model.

The positive growth rate g in firm-level productivity represents the productivity gains that firms in the real world tend to achieve over their lifetime, e.g., through learning by doing, economies of scale, process innovation, or through changes in the product mix. I treat these productivity gains as exogenous with respect to the firm's pricing problem, which I consider below. The other productivity component in the firm's technology, a_t, is common to all firms and a stationary exogenous stochastic process with a constant mean $a > 0$. I abstract from trend growth in a_t, which is distinct from a positive growth rate in firm-level productivity and analyzed in, e.g., Amano, Moran, Murchison, and Rennison (2009).

Firms enter and exit the economy continuously. At the beginning of a period, $\delta \in [0, 1)$ new firms enter the economy, while at the end of a period, δ firms exit the economy.[6] The exit of firms occurs randomly and, therefore, firms with various levels of productivity are equally exposed to exit. In reality, a firm with high productivity may exit because a major shift in consumer taste occurs, a new regulation is passed, or product liability legislation is changed; because a new firm crowds the established firm out of the market by supplying a close substitute; or because the established firm starts exporting and stops selling at home.

Furthermore, Baily, Hulten, Campbell, Bresnahan, and Caves (1992) find that highly productive firms frequently exit at the industry level because they optimize their product mix and, therefore, switch to different industries. Bernard, Redding, and Schott (2010)

[5]Pfaffermayr (2007) and Rogers, Helmers, and Koch (2010), for example, demonstrate the tendency of small firms to grow faster than large firms and, therefore, to catch up with the large firms. One interesting refinement of my model would allow it to capture this catch-up process. While it would be possible to incorporate this catch-up process into a model with flexible prices, incorporating it into one with sticky prices would require using a considerably more complicated model that no longer appears to admit analytical aggregation.

[6]If I were to consider endogenous firm entry, as in, e.g., Bilbiie, Fujiwara, and Ghironi (2011), the number of firms evolves according to $N_t = (1 - \delta)[N_{t-1} + N_{Et-1}]$. In this case, the steady-state fraction of new over all firms, N_E/N, also depends on only the exit rate, $N_E/N = \delta/(1 - \delta)$. This suggests that endogenous firm entry adds little to my results on the optimal long-run inflation rate derived for exogenous firm entry.

analyze the product-switching activities of U.S. manufacturing firms and find that product switching enhances firms' efficiency. Along these lines, my assumption of nonselective exit is best interpreted as capturing both firms that switch industries and firms that die.

When a new firm enters the economy, it sets a price for its product. In subsequent periods, the firm resets its price with probability $(1-\alpha)$, $\alpha \in [0,1)$, each period until exit. The firm j sets its nominal price P_{jt} to solve

$$\max_{P_{jt}} \; E_t \sum_{i=0}^{\infty} \; \kappa^i \; \Omega_{t,t+i} \left[P_{jt} - W_{t+i}/(a_{t+i} g^{s_{jt+i}}) \right] y_{jt+i} \qquad \text{s.t.} \qquad y_{jt+i} = \left(\frac{P_{jt}}{P_{t+i}} \right)^{-\theta} y_{t+i} \; . \quad (1)$$

$\Omega_{t,t+i}$ discounts nominal payoffs, and $\kappa = \alpha(1-\delta)$ is the probability to produce at current prices in the next period. When the firm sets its price, it accounts for the positive growth rate in firm-level productivity. The constraint in the firm's problem is the household demand for product j, derived below, and P_t, W_t, and y_t denote the aggregate price level, the nominal wage, and the aggregate output, respectively. Wages are identical across the firms because firms hire labor in a perfectly competitive labor market, as in, e.g., Melitz (2003).

The optimal price of firm j equates the expected discounted sum of marginal revenues to the expected discounted sum of marginal costs. I rearrange this condition using the fact that s_{jt+i} is equal to $i + s_{jt}$, since both the firm's age s_{jt+i} and the index i are integers:

$$P^{\star}_{jt} \; g^{s_{jt}} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{i=0}^{\infty} (\kappa/g)^i \; \Omega_{t,t+i} P^{\theta}_{t+i} \left(y_{t+i}/a_{t+i} \right) W_{t+i}}{E_t \sum_{i=0}^{\infty} \kappa^i \; \Omega_{t,t+i} P^{\theta}_{t+i} y_{t+i}} \; . \quad (2)$$

It follows from this equation that for any two firms j and j', their optimal prices at date t are proportional to each other:

$$P^{\star}_{jt} = g^{(s_{j't} - s_{jt})} \; P^{\star}_{j't} \; , \quad (3)$$

where j denotes the new firm and j' the established firm, such that $s_{j't} > s_{jt}$, and where g exceeds unity. The equation states that the optimal price of the new firm exceeds the optimal price of the established firm. The proportionality is related to the growth rate

g in firm-level productivity because the new firm is less productive than the established firm and, hence, sets a higher optimal price.

2.3 Household

The representative household maximizes expected discounted lifetime utility:

$$\max_{\{\ell_t, c_{jt}, Q_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(\ell_t)\right] \ , \quad 0 < \beta < 1 \ , \tag{4}$$

where E_0 is the expectation operator conditional on the information available at date zero, c_t is aggregate consumption, and ℓ_t is aggregate labor. The function u is twice continuously differentiable, increasing, and concave. The function h is twice continuously differentiable, increasing, and convex. The household is subject to the budget constraint

$$E_t[\Omega_{t,t+1} Q_{t+1}] + \int_0^1 P_{jt} c_{jt} \ dj \ \ \leq Q_t + (1 - \tau_L) W_t \ell_t + D_t + T_t \ . \tag{5}$$

It selects a financial portfolio of nominal claims with random payoff Q_{t+1}. The price of this portfolio at date t is $E_t[\Omega_{t,t+1} Q_{t+1}]$, where $\Omega_{t,t+1}$ is the unique discount factor, to be determined by complete financial markets. The household spends on consumption and receives $(1 - \tau_L) W_t \ell_t$ as labor income net of taxes. While the labor income tax τ_L is not essential for the main results, it will facilitate characterizing them analytically. The household also receives profits D_t from the ownership of firms and a lump-sum transfer T_t from the government. Terminal conditions (not shown) require household solvency. The household's preference for intermediate products is $c_t = (\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} \ dj)^{\frac{\theta}{\theta-1}}$, with $\theta > 1$. The household's optimization yields the product demand $c_{jt}/c_t = (P_{jt}/P_t)^{-\theta}$, the cost-minimal price $P_t = (\int_0^1 P_{jt}^{1-\theta} \ dj)^{\frac{1}{1-\theta}}$ of aggregate consumption, and $P_t c_t = \int_0^1 P_{jt} c_{jt} \ dj$.

2.4 Equilibrium and aggregation

In the decentralized equilibrium, firms set prices according to equation (2); the household maximizes the lifetime utility (4) subject to the budget constraint (5) and the definition of aggregate consumption c_t; product markets clear at $y_{jt} = c_{jt}$; the labor market clears at

$\ell_t = \int_0^1 \ell_{jt}\, dj$; financial markets clear at $Q_t = 0$; the resource constraint $y_t = c_t$ holds; and the government sets τ_L, ensures $T_t = \tau_L W_t \ell_t$, and also controls the nominal short-term interest rate i_t, which is the payoff to a one-period nominal bond, $(1 + i_t)^{-1} = \beta E_t \Omega_{t,t+1}$.

In the model with the firm-level productivity growth, aggregating product prices to the price level is non-trivial because firms differ from one another in two dimensions, namely, in the level of their productivity and in the length of their price spell. Differences in the first dimension arise from firm entry and from assuming that the firm-level productivity grows over the lifetime of a firm, whereas differences in the second dimension arise from the staggered pricing of firms.

To aggregate product prices, I replace the firm index j by two new indices, n and k, each representing one dimension of heterogeneity, and denote the price P_{jt} as[7]

$$P_{jt} = P^{\star}_{t-(n+k),t-k} \ , \quad n = 0, 1, 2, \ldots, \ k = 0, 1, 2, \ldots \ .$$

The first subscript $t - (n + k)$ indicates the date of market entry. The second subscript $t - k$ indicates the date of the last price change. Thus, index k denotes the length of the price spell, and index n denotes the time between market entry and last price change.

The price level P_t comprises the prices of all cohorts of firms. For the moment, I consider the cohort that entered $s \geq 0$ periods ago, at date $t - s$, and normalize its mass to unity. At date t, the weighted average price of this cohort, $\Lambda_t(s)$, is

$$\Lambda_t(s) = (1 - \alpha) \sum_{k=0}^{s-1} \alpha^k (P^{\star}_{t-s,t-k})^{1-\theta} + \alpha^s (P^{\star}_{t-s,t-s})^{1-\theta} \ , \tag{6}$$

if $s \geq 1$, and $\Lambda_t(s) = (P^{\star}_{t,t})^{1-\theta}$ if $s = 0$. Upon entry ($s = 0$), all firms in a cohort s set the same optimal price. At subsequent dates ($s \geq 1$), some firms change their prices, while others keep their price, and therefore the price distribution of the cohort s fans out over time. At date t, the mass of the cohort s is equal to $(1 - \delta)^s \delta$ because firm exit diminishes

[7]This aggregation approach is related to Dotsey, King, and Wolman (1999). Unlike my approach, however, they consider a finite-dimensional state vector of prices and firms with homogenous productivity.

the cohort's mass over time. Summing over all cohorts s yields the unit mass of firms that underlies the price level: $1 = \sum_{s=0}^{\infty}(1-\delta)^s\delta$.

After weighting each cohort price $\Lambda_t(s)$ by the mass $(1-\delta)^s\delta$ of its cohort, the price level $P_t^{1-\theta} = \int_0^1 P_{jt}^{1-\theta}\, dj$ obtains as the sum of the weighted cohort prices of all cohorts:

$$P_t^{1-\theta} = \sum_{s=0}^{\infty}(1-\delta)^s\delta\Lambda_t(s) \ . \tag{7}$$

I rearrange this equation using the equations (3) and (6), the definitions $\gamma = g^{\theta-1}$ and $n_\gamma = \delta/[1-(1-\delta)\gamma]$, and imposing $(1-\delta)\gamma < 1$. This yields (see Appendix A.1):

$$P_t^{1-\theta} = \left\{n_\gamma(1-\kappa\gamma)(P_{t,t}^\star)^{1-\theta}\right\} + \kappa P_{t-1}^{1-\theta} \ . \tag{8}$$

The term in curly brackets differs from what is obtained in the basic New Keynesian model. To simplify the interpretation of this term, I rewrite the price level as (see Appendix A.1):

$$P_t \quad = \quad \left\{\delta\left(\frac{y_{t,t}}{y_t}\right)P_{t,t}^\star + (1-\alpha)\sum_{s=1}^{\infty}(1-\delta)^s\delta\left(\frac{y_{t-s,t}}{y_t}\right)P_{t-s,t}^\star\right\} + \kappa\pi_t^\theta P_{t-1} \ . \tag{9}$$

The curly brackets contain the optimal prices of the δ firms that are new at date t and of the $1-\alpha$ incumbent firms that adjust their price at date t. All optimal prices are weighted by the relative output. The change in the price level is denoted by the aggregate inflation rate π_t.

The aggregation of the model with firm-level productivity growth also involves combining the technology of firms to the aggregate technology. To this end, I combine the technology of firms, the labor-market clearing condition, and the product demand. This yields:

$$y_t = a_t\ell_t/\Delta_t \ , \tag{10}$$

where the endogenous aggregate productivity Δ_t is defined as

$$\Delta_t = \int_0^1 g^{-s_{jt}} \left(\frac{P_{jt}}{P_t}\right)^{-\theta} dj \ ,$$

and summarizes two effects. The aggregate-productivity effect, which is absent in the basic New Keynesian model, is captured by the term $g^{-s_{jt}}$ and arises from aggregating the firm-level productivity. Positive firm-level productivity growth increases the level of aggregate productivity, a_t/Δ_t, because the productivity of incumbent firms grows faster the higher g is, while new firms continue to start production with a level of productivity equal to unity. The price-dispersion effect in Δ_t, which also occurs in the basic model, is captured by the term $(P_{jt}/P_t)^{-\theta}$ and arises from the cross-sectional dispersion of prices. Price dispersion implies that the household consumes an uneven distribution of products, substituting expensive for less expensive products, and this reduces aggregate output.

The aggregate-productivity effect is constant over time because both the rate of firm turnover and the growth rate in firm-level productivity are constant. In contrast, the price-dispersion effect varies over time because firms set prices based on the time-varying state of the economy. However, with firm-level productivity growth, the price dispersion arises not only from staggered price setting, but also from the firm-specific levels of productivity. Therefore, prices will differ from one another even if they are fully flexible; this is distinct from the price dispersion in Yun (2005), which arises exclusively from the staggered pricing of firms. This consequence of firm-level productivity growth helps to improve the model's fit to the large amount of price dispersion observed in micro data.

Going through steps analogous to those used when aggregating the price level yields the recursive representation $\Delta_t = n_\gamma(1-\kappa\gamma)\left(p_t^\star\right)^{-\theta} + (\kappa/g)\pi_t^\theta \Delta_{t-1}$, with $p_t^\star = P_{t,t}^\star/P_t$. The decentralized equilibrium consists of this representation, the price level (8), the aggregate technology (10), the household's optimality conditions $u_c(y_t) = \beta E_t u_c(y_{t+1})(1+i_t)/\pi_{t+1}$ and $(1-\tau_L)w_t = h_\ell(\ell_t)/u_c(y_t)$, the pricing equation (2), and the government's policy rules.

2.5 Steady state

In the steady state, the aggregate shock a_t is equal to its unconditional mean while firm-level shocks to firm turnover and staggered pricing continue to operate. Aggregate variables are constant in the steady state because two polar forces, an expanding force and a contracting force, balance each other.[8] The expanding force is growth in firm-level productivity, which makes the output of the average firm per cohort grow. The contracting force is firm entry and exit. The sample of exiting firms is randomly drawn and, therefore, exhibits the average level of productivity in the economy. This sample is replaced by the sample of new firms with below-average productivity, and this keeps average productivity constant.

In the steady state, the decentralized equilibrium consists of the aggregate technology (10) and the intratemporal household optimality condition, respectively:

$$y = R(\pi)\frac{a\ell}{\Delta^e} \ , \qquad \frac{h_\ell(\ell)}{u_c(y)} = \left(\frac{1-\tau_L}{\mu(\pi)}\right)\frac{a}{\Delta^e} \ . \tag{11}$$

Here, I define the relative price distortion, $R(\pi) = \Delta^e/\Delta$, the average markup distortion, $(1-\tau_L)/\mu(\pi)$, and the average markup, $\mu(\pi) = a/(w\Delta^e)$. While the relative price distortion arises from the staggered pricing of firms, the markup distortion arises from the monopolistic competition among firms. Appendix A.2 derives $R(\pi)$ and $\mu(\pi)$ as functions of only π.

The parameter Δ^e derives from the planner's solution that consists of two equations, which are similar to the decentralized equilibrium (see Appendix A.3):

$$y^e = \frac{a\ell^e}{\Delta^e} \ , \qquad \frac{h_\ell(\ell^e)}{u_c(y^e)} = \frac{a}{\Delta^e} \ . \tag{12}$$

The planner exhausts the aggregate technology and sets the marginal rate of substituting labor for consumption equal to the marginal rate of transformation. Furthermore, the

[8] Aggregate output y is constant under two common assumptions, i.e., aggregate labor does not grow, $\ell_t = \ell$, and the long-run inflation rate is constant, $\pi_t = \pi$. Using equation (8) and expressing it in terms of π_t, the constant long-run inflation rate implies a constant relative price $p^\star$ of new firms. Using the recursive representation of Δ_t, π and $p^\star$ imply that Δ_t is constant. Using equation (10), it follows that y_t is constant.

planner resolves an important tradeoff at the firm level: while established firms can produce a given amount of a product with less labor than new firms, the household prefers to consume an even distribution of all products instead of only established products. The parameter $(\Delta^e)^{-1} = (\int_0^1 g^{(\theta-1)s_{jt}} dj)^{1/(\theta-1)}$ arises from resolving this tradeoff optimally.

3 The optimal long-run inflation rate

My approach to derive the optimal long-run inflation rate is to consider an optimizing government that uses a restricted set of policy instruments to maximize steady-state welfare. This restricted set comprises the long-run inflation rate and, in the cases in which I derive analytical results, may also comprise the labor income tax. A restricted set of policy instruments is realistic from a central bank's perspective. In Proposition 1, I establish the optimal long-run inflation rate and labor income tax by comparing the decentralized equilibrium of the model with firm-level productivity growth to the planner's solution.

Proposition 1: The optimal long-run inflation rate is equal to the growth rate in firm-level productivity,

$$\pi = g \geq 1 \ ,$$

and the optimal labor income tax is equal to $\tau_L = -1/(\theta-1)$. In this case, the decentralized equilibrium (11) coincides with the planner's solution (12) and, therefore, is first best.

Proof: See Appendix A.4.

To understand the first main result of this paper, i.e., that π equal to g is first best, the key equation is the firms' pricing equation (2). For a new firm and in the steady state, this equation can be rearranged as

$$0 = \sum_{i=0}^{\infty} (\kappa\beta\pi^{\theta})^i \left[\frac{p^\star}{\pi^i} - \frac{\theta}{\theta-1} \frac{w}{ag^i} \right] \ . \tag{13}$$

The square brackets contain the difference between the (constrained) optimal real price $p^\star/\pi^i$ and the desired real price $\frac{\theta}{\theta-1}w/(ag^i)$, which is equal to the static markup times the real marginal costs. The equation shows that the difference between the optimal and the desired real price evolves over time depending on the difference between the long-run inflation rate π and the firm-level productivity growth rate g. For the case in which π is equal to g, the optimal and the desired real price in the equation (13) are always equal to one another because in this case the long-run inflation rate π erodes $p^\star$ at the same pace at which the growth rate g reduces the real marginal costs.[9] Therefore, when the firm can adjusts its price, it has no reason to actually change its price, and this prevents any distortions in relative prices. Furthermore, the firm continuously maintains the static markup and, therefore, the optimal labor income tax remedies the markup distortion.

In this optimal decentralized equilibrium, the positive long-run inflation rate arises from the nominal price set by the new firms. They set their nominal price to above the average price level because their productivity is below the productivity of the average incumbent firm. In contrast, the incumbent firms do not create any inflation because they keep their nominal prices constant.

Proposition 1 is independent of assuming the time-dependent Calvo pricing instead of assuming, e.g., the state-dependent pricing used in Dotsey, King, and Wolman (1999). Effectively, using different assumptions about what makes nominal prices sticky will change the discount factor that multiplies the term in square brackets in the pricing equation (13). But the government's ability to recover the planner's solution does not hinge on the discount factor because by selecting $\pi = g$, it restores a decentralized equilibrium in which the term in square brackets continues to be zero and, therefore, both the average markup distortion and the relative price distortion also continue to be zero.

[9]The positive growth rate in firm-level productivity thus provides a simple rationale for why it can be suboptimal to index product prices perfectly to the long-run inflation rate.

3.1 Average markup and relative price distortion

Panel A in Figure 1 contains the average markup $\mu(\pi)$ for two different values of the growth rate in firm-level productivity, $g > 1$ and $g = 1$. The panel shows that, regardless of the exact value of g, the average markup exceeds the static markup for long-run inflation rates π below g and for π sufficiently above g.[10] This behavior is due to two effects that impinge on the average markup: a price-adjustment effect and a price-continuation effect.[11] The price-adjustment effect dominates the average markup for long-run inflation rates above g, while the price-continuation effect dominates it for long-run inflation rates below g.

The price-adjustment effect works through firms that adjust their price and I illustrate it using the pricing equation (13). This equation implies

$$0 = \sum_{i=0}^{\infty} (\kappa\beta\pi^{\theta}/g)^{i} \left[\frac{p^{\star}}{w/a} \left(\frac{g}{\pi} \right)^{i} - \frac{\theta}{\theta - 1} \right] .$$

When the long-run inflation rate is above g, $p^{\star}$ is eroded faster than the marginal costs decline. Therefore, the future markups of adjusting firms are compressed to below the static markup. The adjusting firms anticipate this and counterbalance the markup compression in future periods by initially using elevated markups that drive up the average markup (Panel A).

The price-continuation effect works through firms that do not adjust their price. When the long-run inflation rate is below g, this effect implies that the average markup exceeds the static markup in Panel A. This happens because the real marginal costs of the non-adjusting firms have declined at the rate that is equal to the growth rate g in firm-level productivity, whereas the real prices of these firms have declined only at the rate that is equal to the long-run inflation rate. Consequently, the markups of the non-adjusting firms exceed the static markup and, therefore, drive up the average markup.

Panel B in Figure 1 shows the relative price distortion $R(\pi)$. Evidently, most values of the long-run inflation rate disperse relative prices in excess of what is efficient and, therefore, reduce the aggregate output. Excessive price dispersion arises whenever π differs

[10] The average markup is below the static markup when π is only slightly above g.

[11] King and Wolman (1999) describe similar effects in the basic New Keynesian model.

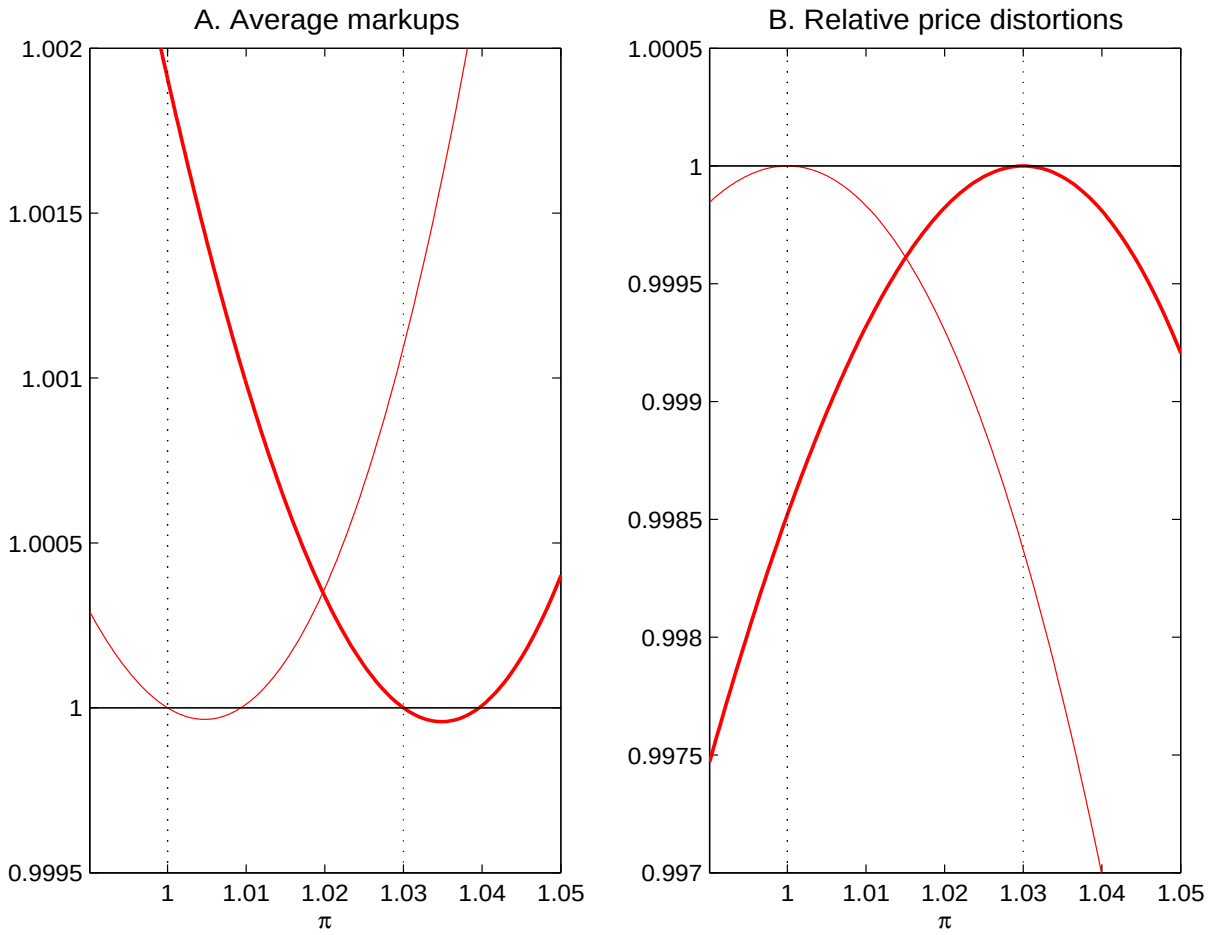

Figure 1: Panel A shows the average markup μ and Panel B shows the relative price distortion R. Both μ and R are functions of the long-run inflation rate π (annualized). Bold lines correspond to the model with firm-level productivity growth in which $g = 1.03^{1/4}$. Thin lines correspond to the basic New Keynesian model without firm-level productivity growth in which $g = 1$. Markups are normalized by $1 - \tau_L$.

from g. In this case, firms do not manage to continuously realize the static markup when their nominal price is kept fixed. Therefore, the firms adjust their price whenever they can, and this disperses relative prices because only a subset of firms adjust their price in each period.

4 The model with sectoral asymmetries

The optimal long-run inflation rate and labor income tax recover the first-best resource allocation because these policy instruments are sufficient to fully eliminate both the average markup distortion and the relative price distortion at the same time. Underlying this

result is a symmetry assumption, namely, that firm-level productivity grows at the same rate across firms. I now incorporate sectoral asymmetries into my model by extending it to a two-sector model. The firms in one sector differ from the firms in the other sector in terms of their firm-level productivity growth, their degree of price stickiness, and their likelihood to survive. Such asymmetries are not only a realistic feature, which allows me to better quantify the optimal long-run inflation rate, but the literature has also shown that they can imply important policy tradeoffs.

4.1 Firms

As stated above the model now has two sectors, $z = 1, 2$, and each sector contains many firms that produce intermediate products. Firms in a sector z enter and exit continuously at the rate $\delta_z \in [0, 1)$, and exiting firms are drawn randomly. Firm $j \in [0, 1]$ in a sector z uses the technology $y_{zjt} = a_{zt} g_z^{s_{zjt}} \ell_{zjt}$, where g_z denotes the growth rate in firm-level productivity in this sector and s_{zjt} the firm's age. The exogenous productivity a_{zt} is common to the firms in a sector and is stationary. Firm j hires labor ℓ_{zjt} in an economy-wide, competitive labor market.

Firm j's pricing problem is analogous to the one in equation (1), after incorporating the sectoral asymmetries, one of which is the probability to produce tomorrow at current prices, $\kappa_z = \alpha_z(1 - \delta_z)$. The firm's pricing problem is subject to the household's demand for product j in a sector z, which I derive below. It follows from the optimality condition of this problem that for any two firms j and j', their optimal prices at date t are proportional to one another, $P^\star_{zjt} = g_z^{(s_{zj't} - s_{zjt})} \; P^\star_{zj't}$.

4.2 Household

The household uses the preference $c_t = c_{1t}^{\psi} \, c_{2t}^{1-\psi}$, with $\psi \in (0, 1)$, to combine the consumption in a sector z, c_{zt}, to the aggregate consumption c_t. The demand function for sectoral

consumption and the aggregate price level P_t are

$$c_{zt} = \psi_z \left(\frac{P_{zt}}{P_t}\right)^{-1} c_t \ , \quad P_t = \left(\frac{P_{1t}}{\psi}\right)^{\psi} \left(\frac{P_{2t}}{1-\psi}\right)^{1-\psi} \ , \tag{14}$$

respectively, where P_{zt} is the price level in a sector z, $\psi_1 = \psi$, and $\psi_2 = 1 - \psi$. Further, the household uses the preference $c_{zt} = (\int_0^1 c_{zjt}^{\frac{\theta-1}{\theta}} \ dj)^{\frac{\theta}{\theta-1}}$, with $\theta > 1$, to combine the intermediate products to the consumption in a sector z. The demand function for intermediate products is $c_{zjt} = (P_{zjt}/P_{zt})^{-\theta} c_{zt}$, and the price level in a sector z corresponds to $P_{zt} = (\int_0^1 P_{zjt}^{1-\theta} \ dj)^{\frac{1}{1-\theta}}$. The household also solves an intertemporal problem, and this problem corresponds to the one described in Section 2.3.

4.3 Equilibrium and aggregation

In the decentralized equilibrium, intermediate firms in a sector z, with $z = 1, 2$, set their prices optimally; the household maximizes the lifetime utility (4) subject to the budget constraint (5), the definitions of aggregate consumption c_t and consumption c_{zt} in a sector z; product markets clear at $y_{zjt} = c_{zjt}$; financial markets clear; and the labor market clears at $\ell_t = \ell_{1t} + \ell_{2t}$ and $\ell_{zt} = \int_0^1 \ell_{zjt} \ dj$, where ℓ_{zt} denotes the amount of labor in a sector z. The resource constraints, $y_{zt} = c_{zt}$, and the aggregate resource constraint, $y_t = c_t$, hold, and the setup of the government is as described in Section 2.4.

Aggregating the product prices in a sector z proceeds along similar lines as aggregating the product prices in the one-sector model because the optimal prices of the firms that are of different ages in a sector z are still proportional to one another. The recursive representation of P_{zt} that follows from the aggregation yields $1 = n_z(1-\kappa_z\gamma_z)(p_{zt}^{\star}/p_{zt})^{1-\theta} + \kappa_z \pi_{zt}^{\theta-1}$, where $p_{zt}^{\star}$ denotes the relative price of a new firm in a sector z, $p_{zt} = P_{zt}/P_t$ the relative price in this sector, and $\pi_{zt} = P_{zt}/P_{zt-1}$ the inflation rate in this sector. I also use $\gamma_z = g_z^{\theta-1}$ and $n_z = \delta_z/[1-(1-\delta_z)\gamma_z]$, and impose the condition $(1-\delta_z)\gamma_z < 1$ to obtain a finite price level in a sector z. Furthermore, I denote the change in the aggregate price level P_t as the aggregate inflation rate π_t. Using these definitions, the aggregate price level in (14) yields $\pi_t = \pi_{1t}^{\psi} \pi_{2t}^{1-\psi}$.

The aggregation in the two-sector model also involves deriving the technology in a sector z, and I use the technology of firms in a sector z, the clearing condition $\ell_{zt} = \int_0^1 \ell_{zjt}\, dj$, and the demand for intermediate products in a sector z, obtaining

$$\ell_{zt} = \left(\frac{\Delta_{zt}}{a_{zt}}\right) y_{zt} \ , \tag{15}$$

where the endogenous productivity $\Delta_{zt} = n_z(1-\kappa_z\gamma_z)(p^\star_{zt}/p_{zt})^{-\theta} + (\kappa_z/g_z)\pi^\theta_{zt}\Delta_{zt-1}$.

To obtain the aggregate technology, I use the technology (15) and the clearing condition $\ell_t = \ell_{1t} + \ell_{2t}$. This yields $\ell_t = (\Delta_{1t}/a_{1t})y_{1t} + (\Delta_{2t}/a_{2t})y_{2t}$. Furthermore, in this equation, I replace the output in a sector z using the household's demand $y_{zt} = \psi_z p_{zt}^{-1} y_t$. Rearranging the result yields the aggregate technology:

$$y_t = \left[\psi\left(\frac{\Delta_{1t}}{a_{1t}}\right) p_{1t}^{-1} + (1-\psi)\left(\frac{\Delta_{2t}}{a_{2t}}\right) p_{2t}^{-1}\right]^{-1} \ell_t \ . \tag{16}$$

The term in square brackets is the inverse aggregate level of productivity, which is a weighted average of the exogenous productivity in a sector z, a_{zt}, the price dispersion in a sector z as captured by Δ_{zt}, and the price dispersion between the sectors as captured by p_{zt}^{-1}.

4.4 Steady state

In the steady state, the long-run inflation rates in both sectors are equal to one another and equal to the aggregate long-run inflation rate (see Appendix B.1):

$$\pi_1 = \pi_2 = \pi \ . \tag{17}$$

Supposing instead that $\pi_1 \neq \pi_2$ and, therefore, the relative price $\bar{P}_{1t}/\bar{P}_{2t}$ is trending (bars denote the steady state), absent a trend in a_1/a_2, then the household continuously increases consumption in the sector with the decreasing relative price, and labor continuously moves into this sector to meet the increasing demand. This situation is inconsistent with a steady state in which variables grow at a constant rate, because eventually one

sector will disappear.

Along the lines of the one-sector model, the decentralized equilibrium in the two-sector model consists of the aggregate technology, the intratemporal household optimality condition, and two aggregate distortions that are indexed by the aggregate long-run inflation rate. Rearranging the aggregate technology (16) and the intratemporal household optimality condition yields (see Appendix B.1):

$$y = R(\pi)A^e\ell \ , \qquad \frac{h_\ell(\ell)}{u_c(y)} = \left(\frac{1-\tau_L}{\mu(\pi)}\right) A^e \ . \tag{18}$$

$R(\pi)$ denotes the aggregate relative price distortion, $(1-\tau_L)/\mu(\pi)$ denotes the aggregate markup distortion, and $\mu(\pi)$ denotes the aggregate average markup.

The parameter A^e is the efficient level of the aggregate productivity derived from the planner's solution. The planner's solution consists of the two equations (see Appendix B.2)

$$y^e = A^e\ell^e \ , \qquad \frac{h_\ell(\ell^e)}{u_c(y^e)} = A^e \ . \tag{19}$$

As in the one-sector model, the planner exhausts the aggregate technology and sets the marginal rate of substituting labor for consumption equal to the marginal rate of transformation. In the two-sector model, however, the efficient level of the aggregate productivity, $A^e = \psi^\psi(1-\psi)^{(1-\psi)}\,(a_1/\Delta_1^e)^\psi\,(a_2/\Delta_2^e)^{1-\psi}$, is a weighted geometric mean of the productivity in a sector z, a_z/Δ_z^e. The parameter Δ_z^e is defined as $(\Delta_z^e)^{-1} = (\int_0^1 g_z^{(\theta-1)s_{zjt}}\,dj)^{1/(\theta-1)}$.

The decentralized equilibrium (18) differs from the planner's solution (19) by the two aggregate distortions $R(\pi)$ and $(1-\tau_L)/\mu(\pi)$. They are functions of the sectoral relative price distortion $\rho_z(\pi)$ and the sectoral average markup $\mu_z(\pi)$, with $z = 1, 2$:

$$R(\pi) = \left[\psi\left(\frac{\mu_2(\pi)}{\mu_1(\pi)}\right)^{1-\psi}\rho_1(\pi)^{-1} + (1-\psi)\left(\frac{\mu_1(\pi)}{\mu_2(\pi)}\right)^{\psi}\rho_2(\pi)^{-1}\right]^{-1} , \tag{20}$$

$$\mu(\pi) = \mu_1(\pi)^\psi\mu_2(\pi)^{1-\psi} \ . \tag{21}$$

The aggregate average markup $\mu(\pi)$ is a weighted geometric mean of the sectoral aver-

age markup defined as $\mu_z(\pi) = p_z/(w\Delta_z^e/a_z)$. Furthermore, the aggregate relative price distortion $R(\pi)$ is a weighted mean of the sectoral relative price distortion defined as $\rho_z(\pi) = \Delta_z^e/\Delta_z$. The weights depend on ψ and $1-\psi$ and on the ratio of sectoral average markups, which is related to the relative price p_2/p_1 according to

$$\frac{p_2}{p_1} = \left(\frac{a_1/\Delta_1^e}{a_2/\Delta_2^e}\right)\frac{\mu_2(\pi)}{\mu_1(\pi)}\ , \tag{22}$$

where I have divided $\mu_z(\pi) = p_z/(w\Delta_z^e/a_z)$, with $z = 1, 2$, by one another. Uneven sectoral average markups distort the relative price p_2/p_1 of sectoral consumption and, therefore, the allocation of the household's expenditure across sectors. This source of the aggregate relative price dispersion is absent in the one-sector model.

Finally, the sectoral relative price distortion and the sectoral average markup can be expressed in terms of the aggregate long-run inflation rate (see Appendix B.1):

$$\rho_z(\pi) = \left(\frac{1-\kappa_z\pi^\theta/g_z}{1-\kappa_z g_z^{\theta-1}}\right)\left(\frac{1-\kappa_z g_z^{\theta-1}}{1-\kappa_z\pi^{\theta-1}}\right)^{\frac{\theta}{\theta-1}}\ , \tag{23}$$

$$\mu_z(\pi) = \frac{\theta}{\theta-1}\left(\frac{1-\kappa_z\beta\pi^{\theta-1}}{1-\kappa_z\beta\pi^\theta/g_z}\right)\left(\frac{1-\kappa_z\pi^{\theta-1}}{1-\kappa_z g_z^{\theta-1}}\right)^{\frac{1}{\theta-1}}\ , \tag{24}$$

where $z = 1, 2$. If the government sets the aggregate long-run inflation rate π equal to the growth rate g_z in a sector z, it eliminates both the relative price distortion $\rho_z(\pi)$ and the gap between the average markup $\mu_z(\pi)$ and the static markup in this sector. Essentially, this finding recovers the logic underlying Proposition 1, which applies to the one-sector model. In the two-sector model, however, the growth rates in firm-level productivity differ from one another, $g_1 \neq g_2$. Therefore, the government faces a policy tradeoff because eliminating the distortions in one sector will prevent the government from also eliminating them in the other sector.

5 The optimal aggregate long-run inflation rate with sectoral asymmetries

Selecting the aggregate long-run inflation rate in the model with sectoral asymmetries involves resolving an important policy tradeoff. This tradeoff is incorporated into how the aggregate long-run inflation rate affects the aggregate distortions that push the decentralized equilibrium away from the first-best resource allocation. As it will turn out, the tradeoff prevents the government from recovering this allocation because the government's policy instruments are generally not able to eliminate the distortions in both sectors at the same time.

5.1 Analytical results

To obtain analytical results, I first derive the optimal aggregate long-run inflation rate that maximizes steady-state welfare, in the limiting case in which the discount factor β approaches unity. Maximizing steady-state welfare in this case is equivalent to minimizing only one of the two aggregate distortions in the decentralized equilibrium (18) because these distortions are equal to one another. Minimizing only one aggregate distortion instead of maximizing steady-state welfare simplifies deriving analytical results.

To show that the aggregate distortions are the same in the limiting case, I use the aggregate average markup (21) to rewrite the aggregate relative price distortion (20) as

$$R(\pi) = \left(\frac{\psi}{\rho_1(\pi)\mu_1(\pi)} + \frac{1-\psi}{\rho_2(\pi)\mu_2(\pi)} \right)^{-1} \mu(\pi)^{-1} \ .$$

In the limit $\beta \to 1$, equations (23)–(24) imply that $\rho_z(\pi)\mu_z(\pi) = \theta/(\theta-1)$, where $z = 1, 2$. Thus, the term in round brackets collapses to the static markup, and this yields:

$$R(\pi) = \left(\frac{\theta/(\theta-1)}{\mu(\pi)} \right) \ . \tag{25}$$

Without loss of generality, I assume that the labor income tax perfectly offsets the static

markup, i.e., $1-\tau_L = \theta/(\theta-1)$.[12] Equation (25) thus states that the aggregate distortions are equal to one another. The second main result in this paper follows from minimizing $\mu(\pi)$ (or maximizing $R(\pi)$) and shows how the government selects the aggregate long-run inflation rate to resolve the policy tradeoff from sectoral asymmetries optimally.

Proposition 2: In the limiting case $\beta \to 1$, the optimal aggregate long-run inflation rate that maximizes steady-state welfare solves

$$0 = \omega(\pi)\left(\frac{\pi - g_1}{g_1}\right) + (1-\omega(\pi))\left(\frac{\pi - g_2}{g_2}\right) . \tag{26}$$

The weight fulfills the condition that $\omega(\pi) \in [0,1]$ and depends on the optimal aggregate long-run inflation rate and the parameters:

$$\omega(\pi) = \left[1 + \frac{1-\psi}{\psi}\left(\frac{\kappa_2}{\kappa_1}\right)\left(\frac{1-\kappa_1\pi^{\theta}/g_1}{1-\kappa_2\pi^{\theta}/g_2}\right)\left(\frac{1-\kappa_1\pi^{\theta-1}}{1-\kappa_2\pi^{\theta-1}}\right)\right]^{-1} , \tag{27}$$

where $\kappa_z = \alpha_z(1-\delta_z)$ and $z = 1, 2$.

Proof: See Appendix B.3.

A natural interpretation of Proposition 2 is that the growth rate g_z represents the optimal long-run inflation rate in a sector z, and that the optimal aggregate long-run inflation rate π is a weighted average of the optimal long-run inflation rates in both sectors.[13] The weight $\omega(\pi)$ depends on the aggregate long-run inflation rate and, beyond this, on the growth rate in firm-level productivity in a sector z, the price stickiness in a sector z, the probability to survive in a sector z, and the relative sector size. In line with this interpretation, Proposition 2 shows that what matters for the steady-state welfare is the distance between the aggregate long-run inflation rate and the optimal long-run inflation rate in a sector z, $(\pi - g_z)/g_z$, instead of only the aggregate long-run inflation rate.

[12] In the limit $\beta \to 1$, I obtain the same optimal π if τ_L is selected optimally.

[13] Alternatively, using equation (26), π can also be expressed as a weighted harmonic average:

$$\pi = \left(\frac{\omega(\pi)}{g_1} + \frac{1-\omega(\pi)}{g_2}\right)^{-1} .$$

In contrast to the Proposition 1 in the one-sector model, the optimal aggregate long-run inflation rate in the Proposition 2 generally cannot recover the first-best planner's solution in the two-sector model. The reason for this is the policy tradeoff that arises from a lack of policy instruments that work at the sectoral level. Namely, while the government can use the aggregate long-run inflation rate to fully offset the distortions in either sector 1 or sector 2, this instrument is not able to fully offset the distortions in both sectors at the same time.

To illustrate this policy tradeoff, I consider the case in which the firms in sector 1 grow more slowly than the firms in sector 2, $g_1 < g_2$, while all firms are subject to the same amount of price stickiness, $\kappa_1 = \kappa_2 = \kappa$ with $\kappa > 0$, and both sectors are of equal size. Proposition 2 shows that in this case, the optimal aggregate long-run inflation rate obeys $g_1 < \pi < g_2$ and weights g_1 higher than g_2 because $\omega(\pi) > 0.5$. This is optimal because firms in sector 1 that adjust their price anticipate that π will erode their price at a rate that is above g_1. Therefore, they elevate their markup and, hence, the average markup in sector 1, to above the static markup. In contrast, the adjusting firms in sector 2 rather compress their markup and, hence, the average markup in this sector, to below the static markup because π will erode their price at a rate that is below g_2. However, even though the optimal aggregate long-run inflation rate π weights g_1 higher than g_2 and, therefore, compresses the elevated average markup in sector 1, π cannot be set to align the average markup and the static markup in both sectors at the same time.[14]

In the general case in which firms are subject to various amounts of price stickiness across sectors, Proposition 2 yields that the optimal aggregate long-run inflation rate weights the sector with the stickier prices more heavily. Figure 2 shows how the weight $\omega(\pi)$ depends on the probabilities α_1 and α_2 of not adjusting the price. For a particular value of α_1, reducing the value of α_2 increases the weight on sector 1. This phenomenon is

[14]One special case in which the policy tradeoff disappears and the decentralized equilibrium is first best arises when firms in both sectors grow at the same rate g. In this case, Proposition 2 yields $\pi = g$, as in the one-sector model. Accordingly, this case generalizes Proposition 1, for the limit $\beta \to 1$, to a model with asymmetric price stickiness. Another special case in which the policy tradeoff disappears arises when firms in sector 2, say, have flexible prices. In this case, Proposition 2 yields $\pi = g_1$.

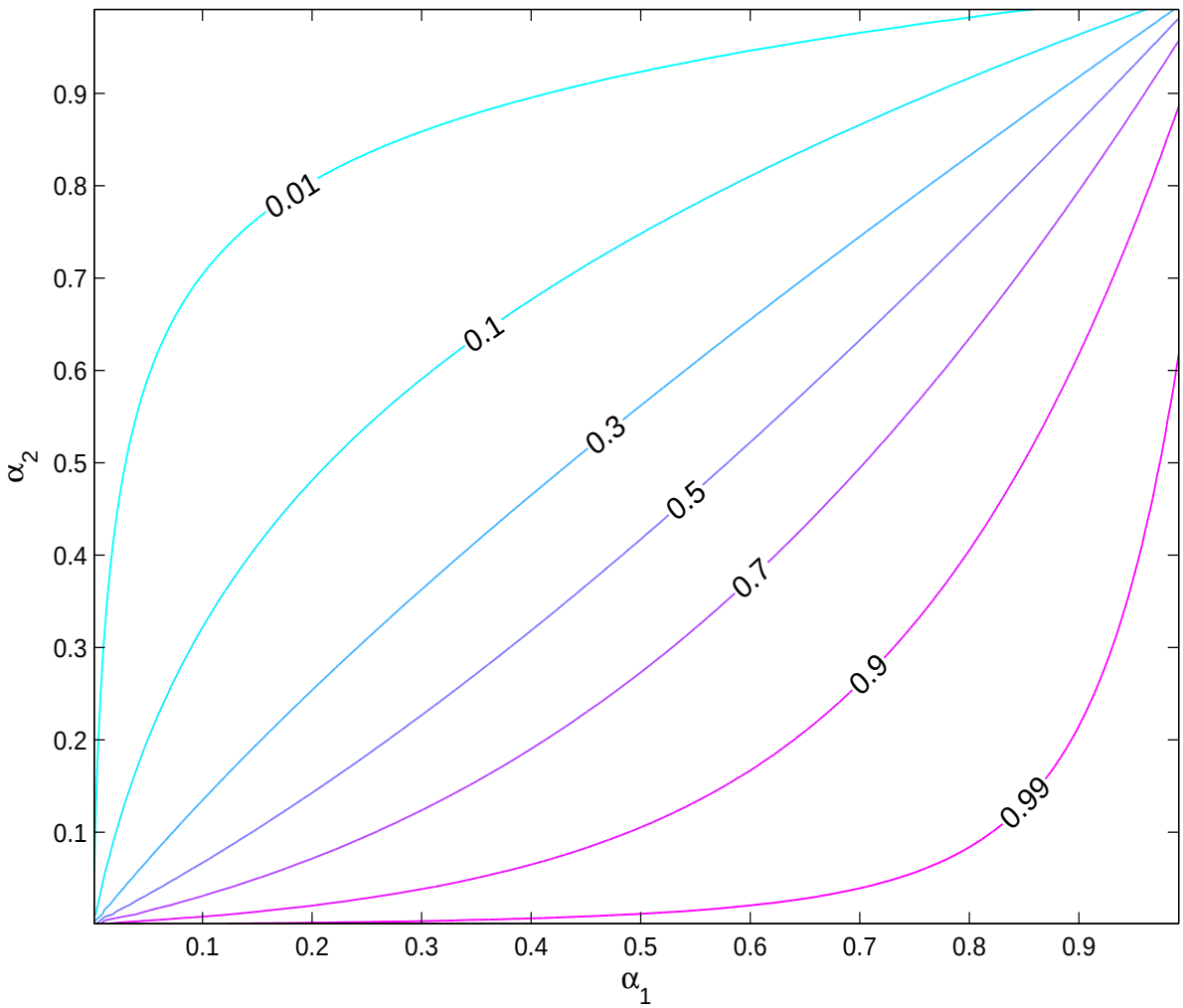

Figure 2: Weight $\omega(\pi)$ as a function of the degree of price stickiness α_1 and α_2 in the two sectors. Lines indicate the combinations of α_1 and α_2 that yield a particular value of $\omega(\pi)$. The growth rates in firm-level productivity are equal to $g_1 = 1.01^{1/4}$ and $g_2 = 1.03^{1/4}$, and the aggregate long-run inflation rate is the one in Proposition 2. The calibration is described in Section 5.2.

known as the "stickiness principle" in the literature on the optimal inflation *stabilization* policy (e.g., Benigno (2004) and Eusepi, Hobijn, and Tambalotti (2011)). An important consequence of Proposition 2 is that this principle applies equally to the choice of the optimal aggregate *long-run* inflation rate.

The optimal aggregate long-run inflation rate obeys the stickiness principle because the expected duration for a firm to keep its price unchanged increases when its price becomes more sticky.[15] Therefore, the firm adjusts more sensitively to a gap between the aggregate

[15]Increasing α_z or reducing the probability of firm exit δ_z has the same qualitative

long-run inflation rate and the growth rate in firm-level productivity because it expects the suboptimal erosion of its price to occur over a longer period of time. This adjustment exacerbates the distortions in the sector with the stickier prices to above their levels in the sector with the less sticky prices, and the optimal aggregate long-run inflation rate weights the sector with the stickier prices more heavily because thereby it partly undoes the distortions in this sector.

As a result of the government's policy tradeoff and unlike in Proposition 1, the optimal aggregate long-run inflation rate in Proposition 2 depends on assuming Calvo pricing. However, two reasons suggest that the quantitative effects of alternative pricing assumptions are small. First, alternative pricing assumptions affect the degree of price erosion that follows from a positive aggregate long-run inflation rate. However, unlike in the basic New Keynesian model in which any positive inflation rate is detrimental to welfare, in the model with firm-level productivity growth a positive inflation rate is only detrimental to welfare to the extent to which it differs from the positive growth rate in firm-level productivity. Second, what matters for resolving the government's policy tradeoff is not the absolute amount of price stickiness, but the relative amount of price stickiness across sectors, and the effect of the alternative pricing assumptions on this relative stickiness is likely to be small.

5.2 Quantifying the optimal aggregate long-run inflation rate

Calibration. Two key parameters are the growth rates g_1 and g_2 in firm-level productivity. I take a quarter as one period in the model and calibrate the growth rate in sector 1 to 1% per year and the growth rate in sector 2 to 3% per year. The growth rate in sector 2, which I now call the goods sector, is consistent with the evidence on surviving manufacturing firms, which I reviewed in Section 2.1. The growth rate in sector 1, which I now call the services sector, is related to the evidence on services and retailing firms.

effect on $\omega(\pi)$ because the degree of price stickiness that matters for $\omega(\pi)$ accounts for the probability of firm exit, $\kappa_z = \alpha_z(1 - \delta_z)$. At the firm level, both increasing α_z and reducing δ_z imply that a firm that adjusts its price discounts the future less and, hence, attributes a greater role to a gap between π and g_z than otherwise.

Unfortunately, there is considerably less evidence on services firms than on manufacturing firms because firm-level data on services are generally less comprehensive and of poorer quality than firm-level data on manufacturing.[16] To overcome these data problems, I use two additional statistics, which in the model depend predominantly on the growth rate in firm-level productivity.

The first statistic is the marginal productivity, i.e., the productivity of a new firm relative to the average productivity of incumbent firms. In the model, the marginal productivity is 94% in the services sector and 80% in the goods sector (see Appendix B.4). These numbers are broadly consistent with the empirical evidence. Foster, Haltiwanger, and Krizan (2002) report a marginal productivity of 94% in U.S. firm-level data on retailing, and the evidence reviewed in Section 2.1 suggests that the marginal productivity in manufacturing is generally lower than in services and similar to what the model predicts.

The second statistic is the marginal size, i.e., the employment size of a new firm relative to the average employment size of incumbent firms. In the model, the marginal size is 75% in the services sector and 33% in the goods sector (see Appendix B.4). In Canadian firm-level data, Baldwin and Gu (2011) find that the marginal size is 65% in services and 44% in manufacturing.[17] Thus, the match between the model and the data again is fairly close.

Turning to the firm entry and exit rates, I set the rate δ_1 in the services sector to 11% per year and the corresponding rate in the goods sector to 10%.[18] These numbers imply that the expected lifetime of a firm is about 7 years in the services sector and

[16] I use firm-level data instead of sectoral data to calibrate the model because sectoral data conflates the growth rate in firm-level productivity of surviving firms with selection and reallocation effects that occur at the firm level.

[17] While it appears as a general pattern in, e.g., the U.S., Canada, Germany, Portugal, and the Netherlands that new firms are relatively smaller in services than in manufacturing, the absolute magnitudes of the marginal size vary considerably across countries. For instance, Bartelsman, Scarpetta, and Schivardi (2003) show that in these countries, the marginal size in services varies between roughly 10% and 40%. In light of this evidence, my calibration of g_1 is rather conservative because reducing the marginal size in the services sector in the model requires a higher value of g_1 and, thus, will yield a higher optimal aggregate long-run inflation rate.

[18] When I convert annual into quarterly rates, I solve $0.11 = \delta_1 \sum_{s=0}^{3}(1-\delta_1)^s$ to account for firm exit throughout the year.

about 6.3 years in the goods sector. Annual firm entry and exit rates of close to 10% are common in firm-level data across countries, with services experiencing slightly higher rates than manufacturing, see, e.g., Bartelsman, Scarpetta, and Schivardi (2005), Foster, Haltiwanger, and Krizan (2006), or Baldwin and Gu (2011).

Furthermore, I set the probability α_1 for a firm in the services sector not to adjust its price equal to 0.828 and the probability α_2 in the goods sector to 0.656, using data from Nakamura and Steinsson (2008) (see Appendix B.4). These probabilities imply that services prices are stickier than goods prices, with a median price duration of 5.3 and 2.3 quarters, respectively. In Wolman (2011), services prices are also stickier than goods prices, but the price durations are shorter, i.e., 3 quarters for services prices and 2 quarters for goods prices, than those used here. However, it is the relative, not the absolute, amount of price stickiness that matters most for the optimal aggregate long-run inflation rate in Proposition 2.

The remaining parameters are calibrated as follows. I set the relative size of the services sector to 38.5%, $\psi = 0.385$, using data from Nakamura and Steinsson (2008). I set the static markup to 30%, which yields that θ is equal to $4\frac{1}{3}$. I set the discount factor β to 0.995 and I parameterize the period utility function as $u(c) = (c^{1-1/\sigma} - 1)/(1 - 1/\sigma)$ and $h(\ell) = \eta\ell^{1+\nu}/(1+\nu)$. Further, I set σ to unity, which corresponds to the log-utility of consumption. Finally, I set the labor-supply elasticity ν to 0.25 and η to 3.

Quantitative results. Using the calibrated parameters, I quantify the optimal aggregate long-run inflation rate in the model with sectoral asymmetries and with discounting, $\beta < 1$. In the case with discounting, I use the equations (18), (20), (21), (23), and (24) to compute aggregate output and labor in steady state for a given aggregate long-run inflation rate. The optimal aggregate long-run inflation rate minimizes the overall welfare effect of the aggregate average markup distortion and the aggregate relative price distortion, although when $\beta < 1$, these distortions are no longer proportional to one another.

Proposition 3: For the calibrated parameter values and with $\tau_L = -1/(\theta - 1)$, the optimal aggregate long-run inflation rate is 1.53% per year.

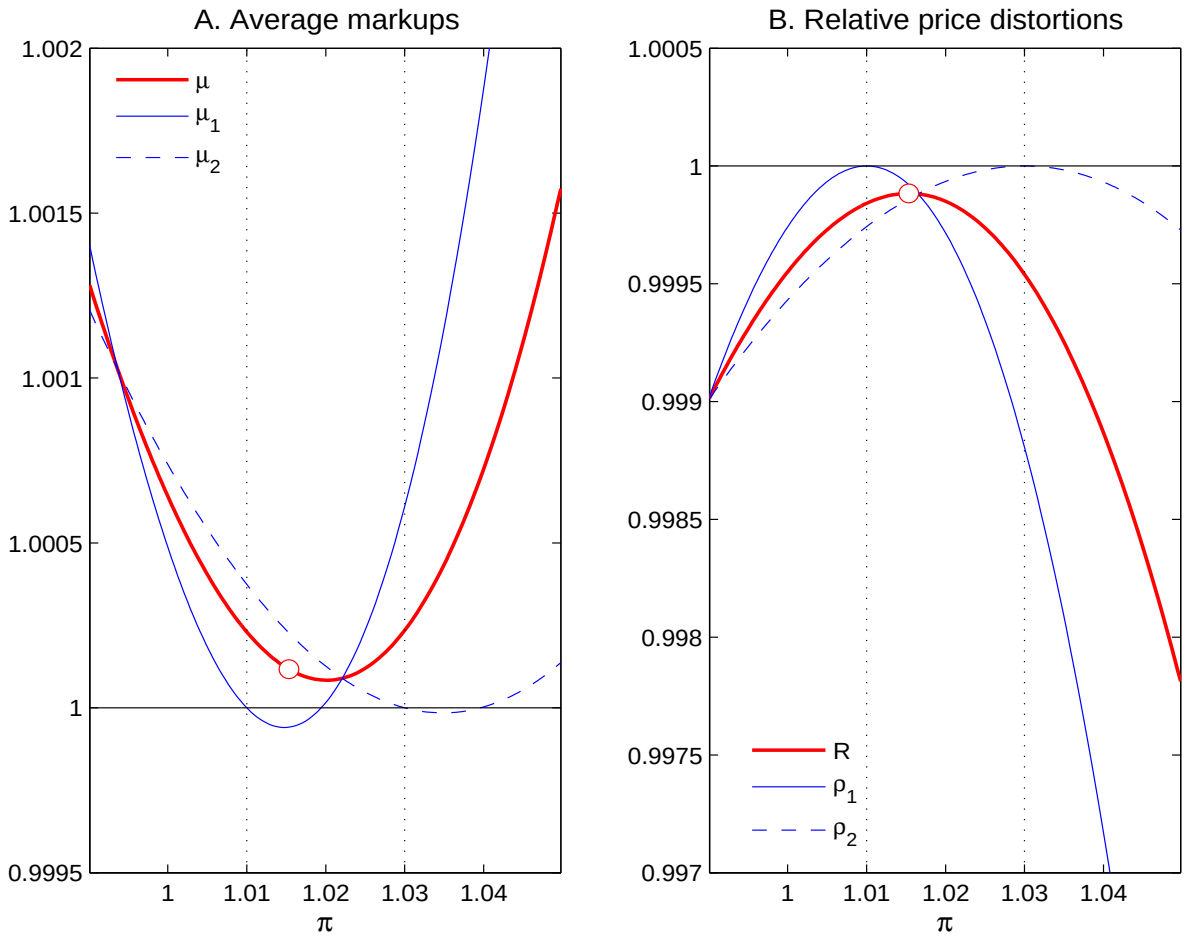

Figure 3: Panel A shows the aggregate markup μ and the sectoral average markups μ_1 and μ_2 as functions of the aggregate long-run inflation rate π (annualized). Markups are normalized by $1 - \tau_L$. Panel B shows the aggregate relative price distortion R and the sectoral relative price distortions ρ_1 and ρ_2 as functions of π. Circles indicate the optimal aggregate long-run inflation rate.

The aggregate long-run inflation rate affects the aggregate and the sectoral average markups as shown in Panel A in Figure 3, and the aggregate and the sectoral relative price distortions as shown in Panel B. In both panels, the optimal aggregate long-run inflation rate in Proposition 3 is marked by circles. Furthermore, the vertical lines mark the values of the aggregate long-run inflation rate that would fully eliminate the distortions in either the services sector, $\pi = g_1$, or the goods sector, $\pi = g_2$.

Figure 3 shows that the optimal aggregate long-run inflation rate leans towards the growth rate g_1 in firm-level productivity in the services sector mainly because services prices are stickier than goods prices, even after accounting for the censoring of price

Table 1: Optimal aggregate long-run inflation rate

	$\beta \to 1$	$\beta = 0.995$	$\beta = 0.975$
$\tau_L = 0$	1.53	1.62	1.94
$\tau_L = -1/(\theta - 1)$	1.53	1.53	1.53

Notes: All values rounded, annualized, and in percent.

spells caused by firm exit. Thus, any distance between π and g_1 distorts the decentralized equilibrium more than the same distance between π and g_2, as is evident by the fact that in the Figure 3, μ_1 and ρ_1 are more bended than μ_2 and ρ_2. It is also evident from this figure that no value of the aggregate long-run inflation rate fully eliminates the aggregate distortions (bold lines) because they continue to differ from unity for each value of π. This illustrates the policy tradeoff that the government faces if $g_1 \neq g_2$ and the prices in both sectors are sticky.

5.3 Robustness of the quantitative results

To demonstrate the robustness of my quantitative results, the Table 1 shows the optimal aggregate long-run inflation rate π that I obtain when I vary the magnitude of the labor income tax τ_L and the discount factor β. When the labor income tax is zero, then reducing β from unity to 0.975 increases π from 1.53% to close to 2% per year. A reduced β gives the government more leverage to use π to erode the average markup to below the static markup and, thus, increases π. The reason for this is that when β is reduced, the firms that adjust their price respond less sensitively to π because they discount the erosion of their future markups that is caused by π more heavily. Therefore, they select a smaller current markup than they would do otherwise, and thereby they reduce the average markup. Furthermore, independently of the magnitude of β, increasing π erodes the current markups of the non-adjusting firms, and this also reduces the average markup.

Table 1 also shows that when the labor income tax perfectly offsets the static markup, the magnitude of β essentially does not affect the optimal aggregate long-run inflation rate. This happens because when the static markup is offset, the government has no incentive to use the aggregate long-run inflation rate to erode the average markup to below the static

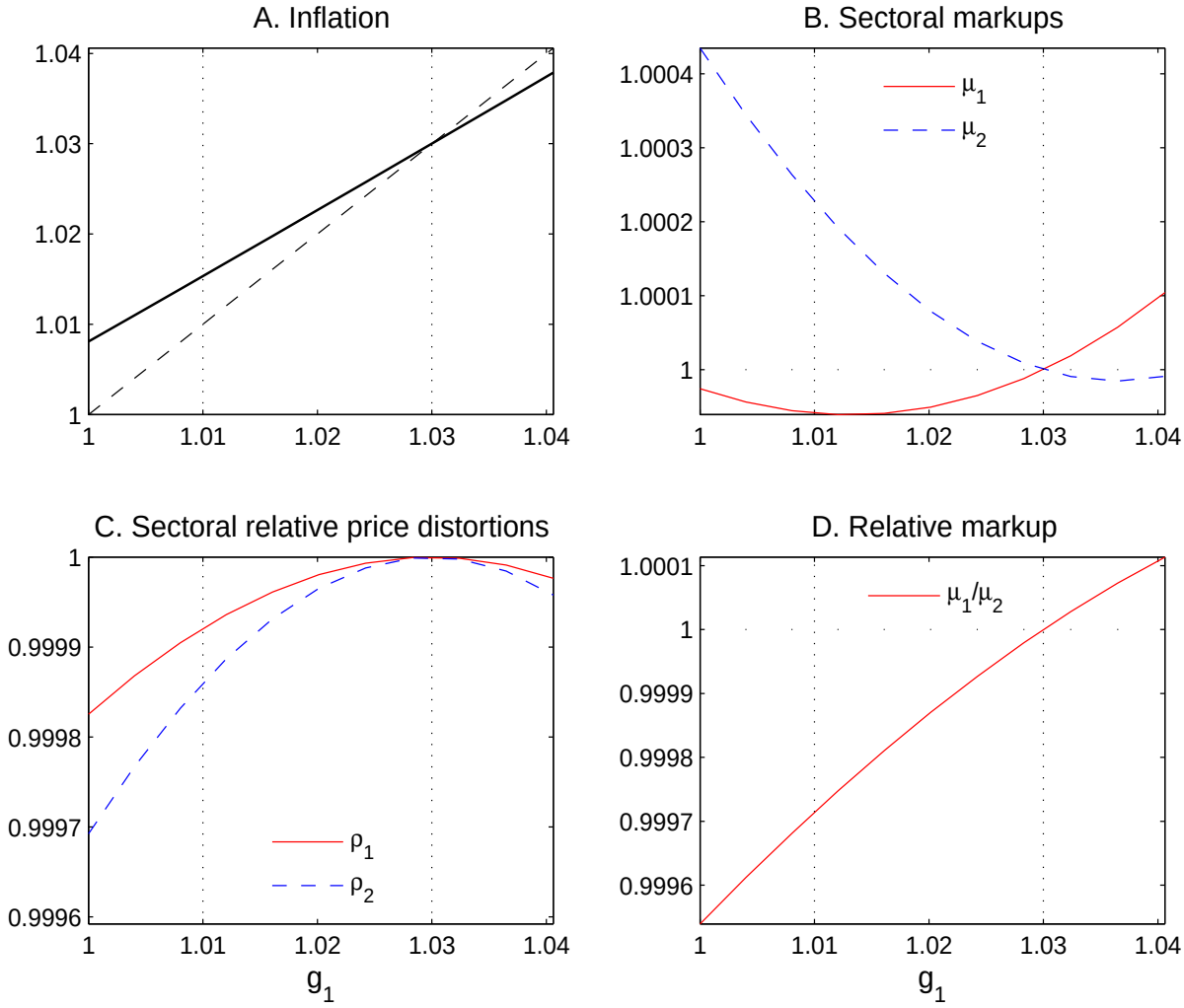

Figure 4: Panel A shows the optimal aggregate long-run inflation rate π (annualized) as function of the (annualized) growth rate g_1 in firm-level productivity in the services sector. The dashed line is the 45 degree line. Sectoral average markups in Panel B are normalized by $1 - \tau_L$. Vertical lines indicate the benchmark values of g_1 and g_2, see Section 5.2.

markup. This finding is in line with the finding in King and Wolman (1999), namely, that the optimal long-run inflation rate is higher in a model with a positive static markup than in a model without a static markup.

To further demonstrate the robustness of my quantitative results, I vary the growth rate g_1 in firm-level productivity in the services sector since calibrating this growth rate is difficult, given the scarcity of respective data. Panel A in Figure 4 shows that for values of g_1 below g_2, the optimal aggregate long-run inflation rate exceeds the 45 degree line and thus g_1, because the government faces a tradeoff between stabilizing the distortions in one sector versus the other. Remarkably, even if the growth rate g_1 in the services sector

is zero, a value that is most likely too low, the optimal aggregate long-run inflation rate is still positive and close to 1% per year.

When g_1 approaches g_2, the government no longer faces a tradeoff because the optimal aggregate long-run inflation rate approaches $\pi = g_1 = g_2$ and, in this case, is equal to 3% per year. When g_1 approaches g_2, the sectoral average markups in Panel B, the sectoral relative price distortions in Panel C, and the relative average markup in Panel D all approach unity.[19] As a result, the aggregate relative price distortion R (not shown) and the aggregate markup distortion $(1 - \tau_L)/\mu$ (not shown) also approach unity and, therefore, the decentralized equilibrium recovers the first-best planner's solution. Overall, my quantitative results suggest that after accounting for the positive growth rates in firm-level productivity, a reasonable estimate of the optimal aggregate long-run inflation rate is between 1.5% and 2% per year.

6 Aggregate dynamics and inflation stabilization

The finding that the optimizing government targets a positive long-run inflation rate relates to the literature that argues that one unpleasant consequence of a government targeting a positive instead of a zero long-run inflation rate is that it dramatically changes the aggregate dynamics of the actual inflation rate and output. This literature also argues that two further unpleasant consequences of a government targeting a positive long-run inflation rate are that it shrinks the set of parameters in simple interest rate rules, which yield a determinate equilibrium, and that the optimal policy for stabilizing the actual inflation rate and output may no longer yield a determinate local equilibrium.

When I account for the positive growth rate in firm-level productivity, these unpleasant consequences of a positive long-run inflation rate for the aggregate dynamics and the inflation stabilization policy disappear. I derive this finding by showing that in the neighborhood of a positive long-run inflation rate, the one-sector model with a positive growth rate in firm-level productivity behaves like the basic New Keynesian model without this

[19] With τ_L set such that it offsets the static markup, the fact that the average markup μ_1 in the services sector is below unity for $g_1 < g_2$ represents an inefficiency, see Panel B.

feature in the neighborhood of a zero long-run inflation rate.

6.1 Aggregate dynamics

To derive the aggregate dynamics of the actual inflation rate and output, I use the one-sector model with the positive growth rate in firm-level productivity, i.e., the FIP model, and the basic New Keynesian model, which obtains as the special case $g = 1$ of the FIP model, and linearize each model at its efficient steady state. The efficient steady state in the FIP model is derived in Proposition 1 and features a positive long-run inflation rate, $\pi = g$. As is well known, the efficient steady state in the basic New Keynesian model features a zero long-run inflation rate, $\pi = 1$.[20] After linearizing the FIP model at $\pi = g$ and the basic New Keynesian model at $\pi = 1$, I obtain the following equivalence result:

Proposition 4: The dynamics of the aggregate variables in the linearized FIP model and in the linearized basic New Keynesian model are equivalent to one another if the slope of the New Keynesian Phillips curve is parameterized directly.

Proof: Calculating the FIP model to the first order at its efficient steady state yields (see Appendix C.1):

$$\begin{aligned}
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \zeta(\pi) x_t \\
x_t &= E_t x_{t+1} - s_c \sigma(\hat{i}_t - E_t \hat{\pi}_{t+1} - \hat{r}_t^f) \\
\left[\nu + (\sigma s_c)^{-1}\right] \hat{y}_t^f &= (\sigma s_c)^{-1} s_q \hat{q}_t + (1+\nu)\hat{a}_t \\
s_c \sigma \hat{r}_t^f &= -E_t(1 - L^{-1})[\hat{y}_t^f - s_q \hat{q}_t] \ ,
\end{aligned} \tag{28}$$

where x_t denotes the output gap $\hat{y}_t - \hat{y}_t^f$, $\hat{y}_t^f$ denotes the natural level of output, and $\hat{r}_t^f$ denotes the natural real interest rate. Furthermore, a hat on top of a variable denotes the percentage deviation from the steady state, L denotes the lag operator, s_c denotes the steady-state share of private consumption over output, and $s_q = 1 - s_c$.[21]

[20] Proposition 1 also comprises the basic New Keynesian model in the special case $g = 1$.

[21] I extend the one-sector model to include also exogenous government consumption q_t. This extension involves replacing the government budget constraint by $P_t q_t + T_t \leq \tau_L W_t \ell_t$

To show that both linearized models are equivalent, I consider the FIP model stated by the equations in (28) and the basic New Keynesian model, which obtains as the special case $\pi = g = 1$ of the FIP model. It follows from the equations in (28) that both models are equivalent except for the slope $\zeta(\pi) = \frac{(1-\kappa\pi^{\theta-1})(1-\kappa\pi^{\theta-1}\beta)}{\kappa\pi^{\theta-1}}[\nu + (\sigma s_c)^{-1}]$ of the New Keynesian Phillips curve (NKPC), which is the first equation in (28). In the case in which the slope of the NKPC is parameterized directly and, hence, is the same in both models, they are equivalent. This completes the proof.

One consequence of Proposition 4 is that after accounting for the firm-level productivity growth, the long-run inflation rate is irrelevant for the aggregate dynamics of the actual inflation rate and output in the FIP model. This consequence differs from the findings in the literature. Ascari (2004), for example, shows that when the basic New Keynesian model with Calvo pricing is linearized at a positive instead of a zero long-run inflation rate, the aggregate dynamics in this model change dramatically because they suddenly depend on the relative price distortion. Hornstein and Wolman (2005) provide another example and show that the magnitude of the long-run inflation rate also plays an important role for the aggregate dynamics in a model with firm-specific capital and Taylor-type pricing.

The reason for why in the linearized FIP model the positive long-run inflation rate π is irrelevant for the aggregate dynamics is related to the positive growth rate g in firm-level productivity. Namely, when the FIP model is calculated to the first order at $\pi = g$, the aggregate dynamics do not depend on the relative price distortion, unlike in, e.g., Ascari (2004). This is apparent from Panel B in Figure 1, which shows the relative price distortion (bold line) as a function of π. In the figure, the relative price distortion takes its maximum value at $\pi = g$ and, hence, is insensitive to marginal changes in π at $\pi = g$. It follows that when the relative price distortion is calculated to the first order at $\pi = g$, it is equal to zero throughout and, therefore, is irrelevant for the aggregate dynamics.

Another consequence of Proposition 4 is that the long-run inflation rate is also irrele-

and the aggregate resource constraint by $y_t = c_t + q_t$.

vant for the determinacy conditions of simple interest rate rules. The reason for this is that the same determinacy conditions apply to both the linearized FIP model with the positive long-run inflation rate and the linearized basic New Keynesian model with the zero long-run inflation rate. In particular, Bullard and Mitra (2002) show in their Proposition 1 that the Taylor principle, i.e., $\zeta(f_\pi - 1) + (1 - \beta)f_x > 0$, is the determinacy condition in the model stated by the equations in (28) when the government pursues the interest rate rule $\hat{i}_t = f_\pi \hat{\pi}_t + f_x x_t$. Thus, it follows from their proposition and from my Proposition 4 that the Taylor principle applies to both the FIP model and the basic New Keynesian model and, therefore, does so irrespectively of the long-run inflation rate.

This other consequence of Proposition 4 also differs from the results obtained in the literature. Kiley (2007) and Ascari and Ropele (2009), for example, show that a moderately positive long-run inflation rate undermines the Taylor principle in the basic New Keynesian model with Taylor-type pricing (Kiley) or Calvo pricing (Ascari and Ropele).[22] Hornstein and Wolman (2005) provide another example and show that in a model with firm-specific capital, the long-run inflation rate also has a large impact on the determinacy conditions of interest rate rules.

A main assumption underlying my Proposition 4 is that the slope of the NKPC is the same in the FIP model and the basic New Keynesian model. This is reasonable because both models yield the same slope estimate if the slope is estimated using macro data, as it is done frequently in the literature.[23] One alternative to assuming that the slope is the same in both models is to compute it from the deep parameters that are calibrated using micro data. In this case, the two models do make different predictions because π differs across the models and the slope of the NKPC depends on π. Supplement D.3 quantifies these differences, but they turn out to be small.

[22]Based on that a positive long-run inflation rate undermines the Taylor principle, Coibion and Gorodnichenko (2011) show how to explain the U.S. Great Moderation.

[23]For example, Altig, Christiano, Eichenbaum, and Linde (2011) shows that in terms of the aggregate dynamics, a model with firm-specific capital is equivalent to a nested model with homogenous capital, and estimate these models using macro data.

6.2 Inflation stabilization

The equivalence of the FIP model and the basic New Keynesian model, shown in Proposition 4, extends to beyond the first order and encompasses the welfare-based loss function, which is the second-order expansion of the lifetime utility (4) of the representative household. I show this in the following proposition.

Proposition 5: The welfare-based loss function calculated to the second order and at the efficient steady state,

$$\mathcal{L} \propto E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \hat{\pi}_t^2 + [\zeta(\pi)/\theta] x_t^2 \right\} \ , \tag{29}$$

is equivalent in the FIP model and in the basic New Keynesian model if the slope $\zeta(\pi)$ of the New Keynesian Phillips curve is parameterized directly.

Proof: See Appendix C.2.

The finding that the welfare-based loss function (29) applies to both the FIP model and the basic New Keynesian model if ζ is parameterized directly is consistent with the fact that the optimal long-run inflation rate differs across the models. This can be seen by rearranging the definition $\hat{\pi}_t \equiv \log(\pi_t/\pi)$ as $\hat{\pi}_t = (\pi_t - \pi)/\pi$, which is accurate to the first order, to obtain the welfare-based loss function in a period t,

$$((\pi_t - \pi)/\pi)^2 + [\zeta(\pi)/\theta] x_t^2 \ . \tag{30}$$

That is, while the government in the FIP model should stabilize the actual inflation rate at the positive long-run inflation rate $\pi = g$, the government in the basic New Keynesian model should stabilize the actual inflation rate at the zero long-run inflation rate $\pi = 1$.

The key consequence of Proposition 5, jointly with Proposition 4, is that after accounting for the firm-level productivity growth, the optimal stabilization policy is independent of the long-run inflation rate. This policy derives from a linear-quadratic policy problem with the welfare-based loss (29) as the objective function and the equilibrium conditions (28) as constraints (including shock processes and initial conditions). Propositions 4 and

5 show that all parts of this policy problem are the same in both models.[24] Consequently, the optimal stabilization policy is the same in both models and, hence, is independent of the long-run inflation rate. This key consequence arises independently of whether the government selects its optimal stabilization policy discretionarily or with commitment.

That the optimal stabilization policy is independent of the long-run inflation rate differs from the results obtained in the literature. Ascari and Ropele (2007), for example, posit a government that aims to stabilize the actual inflation rate at a positive long-run inflation rate and analyze the optimal stabilization policy in the basic New Keynesian model linearized at the positive long-run inflation rate.[25] They show that the optimal stabilization policy under discretion, which yields a determinate equilibrium in the case with a zero long-run inflation rate, yields an indeterminate equilibrium in their case with a positive long-run inflation rate.

In contrast, in the FIP model analyzed here, the optimal stabilization policy under discretion yields a determinate equilibrium in spite of the positive long-run inflation rate. A key difference between their analysis and my analysis is that in their analysis of the basic New Keynesian model, the positive long-run inflation rate that the government targets is suboptimal in terms of welfare. However, in my analysis of the FIP model, the positive long-run inflation rate that the government targets is optimal, as I have shown in Proposition 1.

In another example from the literature, Coibion, Gorodnichenko, and Wieland (forthcoming), the optimal stabilization policy also depends on the long-run inflation rate. They derive the welfare-based loss function in the New Keynesian model for the case in which the long-run inflation rate is positive.[26] In this case, the optimal stabilization policy depends on the positive long-run inflation rate through the weight for the output gap in their welfare-based loss function. The positive long-run inflation rate affects this weight

[24]Solution details for the policy problem are in Woodford (2003), Proposition 7.5, or in Clarida, Gali, and Gertler (1999), Section 3.

[25]They posit a loss function with the same functional form as equation (29), but treat the weight for the output gap as a primitive parameter. Here, this weight is determined as a function of the model parameters.

[26]They solve for the optimal value of the long-run inflation rate, which, in their model, is positive because the zero lower bound on nominal interest rates is occasionally binding.

because in their model, the excessive price dispersion from a positive long-run inflation rate increases the welfare loss that is caused by variation in the actual inflation rate. In the FIP model used here, however, the weight for the output gap in the loss function (29) does not display this effect because with $\pi = g$, the price dispersion is efficient instead of excessive.

7 Conclusion

In micro data, it appears that firms move systematically within the productivity distribution over time. A firm tends to have below-average productivity upon market entry, and its productivity tends to grow thereafter. The purpose of this paper is to analyze the consequences of this positive growth rate in firm-level productivity for macroeconomic policy choices and, particularly, for choosing the optimal long-run inflation rate and the optimal inflation stabilization policy.

My analysis incorporates a positive growth rate in firm-level productivity into a stylized monetary model with sticky prices that admits the heterogenous firms to be aggregated analytically. My baseline result is that this positive growth rate justifies an optimizing government in targeting a positive long-run inflation rate of between 1.5% and 2% per year. As a result of the positive growth rate in firm-level productivity, a firm's real marginal costs decline over time. Therefore, the positive long-run inflation rate erodes the firm's sticky nominal price at exactly the right pace for the firm's actual real price to track the firm's real marginal costs; this is socially optimal.

Many central banks around the globe pursue positive long-run inflation rates, and the model used here employs a positive growth rate in firm-level productivity to suggest this is optimal. A key difference between this model and the basic New Keynesian model, which predicts an optimal long-run inflation rate near zero, is the behavior of the real marginal costs at the firm level. While they remain constant in the steady state of the basic New Keynesian model, they decline over time in the steady state of the model used here. The generic conclusion that can be derived from this discrepancy is that sticky nominal prices

alone do not constitute a compelling reason for the government to target a zero long-run inflation rate.

In the wake of the recent financial turmoil, Williams (2009), Blanchard, Dell'Ariccia, and Mauro (2010), and McCallum (2011), among others, discuss some of the consequences of raising inflation targets to above their current levels in order to provide central banks with more leeway to cope with large adverse shocks. My results contribute to this discussion by demonstrating that the welfare costs caused by a moderately positive long-run inflation rate derived in, e.g., Coibion, Gorodnichenko, and Wieland (forthcoming), represent fairly conservative estimates if one also accounts for the positive growth rate in firm-level productivity, as I have done here.

When I extend my analysis to a model with two sectors in which each sector has its own optimal long-run inflation rate, the optimal aggregate long-run inflation rate minimizes a weighted distance with respect to the optimal long-run inflation rate in each sector. Further, the weights obey the stickiness principle, i.e., the sector with the stickier prices is weighted more, as in Benigno (2004). Extrapolating this result to the case of a monetary union suggests that the long-run inflation rate that is optimal in the member state with stickier prices should receive more weight than the long-run inflation rate in the member state with less sticky prices in determining the union-wide long-run inflation rate.

The literature warns that a government that targets a positive long-run inflation rate may render the economic equilibrium indeterminate if it follows otherwise standard policy prescriptions. My model, which accounts for the positive growth rate in firm-level productivity, does not confirm this warning. Instead, my model obeys the same determinacy conditions and predicts the same optimal stabilization policies as the basic New Keynesian model, but at the same time is consistent with the positive average inflation rate observed in the data.

There are at least two interesting ways to extend my analysis in future work. First, while my analysis finds a positive optimal long-run inflation rate in a cashless economy, the literature emphasizes that the costs arising from holding money imply a negative optimal long-run inflation rate, and future work could incorporate these costs. Second, in

line with the evidence, my analysis emphasizes a supply-side factor, i.e., productivity, and shows how it affects the firm-level marginal costs and the optimal long-run inflation rate. Yet, Foster, Haltiwanger, and Syverson (2008) have recently suggested that demand-side factors are another important distinction across the firms, and future work could analyse models that also include them.

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