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Keywords: trend growth, open economy, monetary policy rules, determinacy, learning.

JEL classification: E58, E61, F31, F41.

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Abstract

Available evidence supports the view that growth is faster in more open economies. In order to analyze the implications of openness and growth on determinacy and learnability of worldwide rational expectations equilibria we develop a two-country New Keynesian model with growth. We analyze these issues for contemporaneous data and expectations-based monetary policy rules. Our results highlight how growth matters for the overall effect of opening an economy to more trade, as we find that (i) under the contemporaneous data policy rule the conditions for determinacy and learnability become more stringent on account of openness but less stringent on account of growth, so that growth weakens the effect of openness, (ii) under the expectations-based policy rule the conditions for determinacy and learnability also become more stringent on account of openness while on account growth the conditions for determinacy become *more* stringent (thus reinforcing the effect of openness) but those for learnability become *less* stringent (thus weakening the effect of openness). As in Bullard and Schaling (2009) the elasticity of intertemporal substitution is key to our result but within a framework that is consistent with long-run labor supply and balanced growth facts.

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1 Introduction

In sticky price New Keynesian models monetary policy constitutes one of the building blocks that determine macroeconomic outcomes, including whether the economy is subject to indeterminacy and/or learning instability (Evans and Honkapohja (2001)). Bullard and Mitra (2002) were among the first to analyze determinacy and learnability of rational expectations equilibria in the standard New Keynesian model of inflation and output. They evaluate the performance of various forms of Taylor-type rules for setting the nominal interest rate. They find that following the so-called Taylor principle, where the central bank adjusts the nominal interest rate more than one-for-one with inflation, is desirable both from a determinacy and learnability point of view. Moreover, in general determinacy does not imply learnability and vice versa.[1]

Subsequent research has extended Bullard and Mitra (2002) in several directions. For example, Evans and Honkapohja (2003) show how the problems of instability and indeterminacy identified by Bullard and Mitra (2002) can be overcome if the central bank can observe private agents' expectations while Honkapohja and Mitra (2005) examine the implications of heterogeneity in forecasting by the central bank and private agents for determinacy and learnability. In a model with money in the utility function Kurozumi (2006) analyzes how the timing of money balances matters for determinacy and learnability of Taylor type rules. Ascari and Ropele (2009) and Coiboin and Gorodnichenko (2011) study the effects of positive trend inflation for the determinacy properties of the New-Keynesian model.

Closer to our paper, Bullard and Schaling (2009) derive determinacy and learnability conditions in the two-country world economy setup of Clarida, Gali and Gertler (2002), while Tesfaselassie (2011) studies these conditions in an extension of the closed economy model of Bullard and Mitra (2002) with trend productivity growth. The present paper shares elements of both of these papers due to the fact that we extend the two-country, New Keynesian model of Clarida, Gali and Gertler (2002) to allow for trend productivity growth. Unlike Clarida, Gali and Gertler (2002) and following Tesfaselassie (2011) we assume that household utility is non-separable, which is consistent with the recent empirical evidence on the consumption Euler equation on a balanced growth path (see, e.g., Basu and Kimball (2002) and Guerron-Quintana (2008)).

[1]See McCallum (2007) for a detailed analysis of the connections between the determinacy and learnability criteria and Evans and Honkapohja (2008) for a survey of the learning literature.

We evaluate the performance of alternative specifications of simple policy rules (corresponding to the information set of the central banks), in terms of achieving determinacy and E-stability. In particular we examine the performance of what Bullard and Mitra (2002) call *contemporaneous data* policy rule, (where each central bank responds to current period domestic inflation and the output gap), and *a forward expectations* policy rule, (where each central bank responds to private sector expectations of domestic inflation and the output gap). We focus on the performance of these two policy rules, as both are analytically tractable.[2] We stay close to Bullard and Mitra (2002) in focusing on the performance of simple rules while Bullard and Schaling (2009) also analyze the performance of optimal policy rules. Unlike Bullard and Mitra (2002) and the current paper Bullard and Schaling (2009) do not consider the case of simple rules based on forward expectations.

We obtain the following results. First, non-separable utility changes the international transmission of productivity and markup shocks relative to Clarida, Gali and Gertler (2002) who assume separable utility. Bullard and Schaling (2009) show that in the Clarida, Gali and Gertler (2002) model and under the contemporaneous data policy rule the conditions for determinacy and learnability become more stringent, compared to a closed economy benchmark, provided the elasticity of intertemporal substitution (in consumption) is larger than one.However, empirical evidence suggests this elasticity is smaller than one. In line with this we find that the conditions for determinacy and learnability become more stringent with openness provided the elasticity of intertemporal substitution is smaller than one and, furthermore, this result extends to the case of forward expectations in the policy rule.This suggests that the Bullard and Schaling (2009) results are not in line with stylized facts.

Second, when growth considerations are taken into account we find that (i) under the contemporaneous data in the policy rule the conditions for determinacy and learnability become *less* stringent, compared to a no-growth economy benchmark, and (ii) under the forward expectations in the policy rule the conditions for determinacy become *more* stringent while the conditions for learnability become *less* stringent. We note that Tesfaselassie (2011) finds similar results in a closed economy New Keynesian model, implying that the effect of trend growth on determinacy and learnability is robust, at least for the policy rules under consideration, to the degree of openness.

[2] As shown by Bullard and Mitra (2002) simple rules based on lagged data turn out to be intractable and can be analyzed only numerically.

Our results highlight that productivity growth matters for the overall effect of opening an economy to more trade, as available evidence supports the view that open economies experience faster productivity growth (see, e.g., Edwards (1998) and Alcala and Ciccone (2004)).[3] Our main qualitative result can be stated as follows. If one starts with a no growth and closed economy benchmark, (i) under the contemporaneous data in the policy rule positive growth weakens the effect of openness, as positive growth increases the scope for determinacy and learnability while openness decreases it, and (ii) under forward expectations in the policy rule positive growth strengthens the effect of openness, as both positive growth and openness decrease the scope for determinacy and learnability.

2 Model

We have a two-country open economy model as in Clarida, Gali and Gertler (2002) (henthforth CGG). The two countries are labeled H and F. The home country H has a mass of households $1-\gamma$ (a household is indexed by $h \in [0, 1-\gamma]$) and the foreign country F has a mass of households γ. Otherwise they are symmetric regarding household preferences and production technology. foreign economy variables are indicated with superscript "$*$".

Let C_t be an index of a home produced good $C_{H,t}$ and an imported good $C_{F,t}$

$$C_t = C_{H,t}^{1-\gamma} C_{F,t}^{\gamma}, \tag{1}$$

where $P_{H,t}$ and $P_{F,t}$ are the respective producer price indices, and let P_t be the consumption price index (that follows from cost minimization)

$$P_t = k^{-1} P_{H,t}^{1-\gamma} P_{F,t}^{\gamma} = k^{-1} P_{H,t} S_t^{\gamma}, \tag{2}$$

where $S_t \equiv P_{F,t}/P_{H,t}$ is the terms of trade and $k \equiv (1-\gamma)^{(1-\gamma)}\gamma^{\gamma}$.

We allow for non-separable utility and trend productivity growth as in the seminal work of King, Plosser and Rebelo (1988) (henthforth KPR).[4] KPR show that household preferences need to be restricted so as to be consistent with balanced

[3]For example, Edwards (1998) finds that more open countries (in terms of the share of trade in GDP) experienced faster productivity growth throughout the decades 1960 to 1990.

[4]As in CGG, we abstract from capital accumulation.

growth facts.[5]

Let $g_r \equiv A_t/A_{t-1}$ denote trend productivity growth and α_t temporary variation in technology, which is normally distributed with mean equal to one. Except for notational differences, the introduction of a temporary technology shock and trend productivity is analogous to that in KPR. In particular, as shown further below, our production function is similar to equation (2.6) of KPR but with no capital. In the presence of productivity growth and on a balanced growth path output, consumption, and real wages grow at the same rate as A_t, while aggregate hours are constant.

Trend productivity growth in conjunction with non-separable utility modifies several behavioral equations of the CGG model, namely, consumption and labor supply by households, the international risk sharing condition, profit maximization by firms under price rigidity, and the relationship between domestic marginal cost and foreign output. Below we show these key behavioral equations of our model, leaving the details of our derivations to Appendix A.

On the demand side, utility takes the same form as in KPR, who allow for non-separable utility in consumption C_t and hours N_t. To be specific

$$U(C_t, N_t) = \frac{(C_t e^{-v(N_t)})^{1-\sigma}}{1-\sigma}, \tag{3}$$

where $1/\sigma$ is the elasticity of intertemporal substitution in consumption, which, consistent with recent empirical evidence on the consumption Euler equation, is assumed to be smaller than one (see, e.g., Basu and Kimball (2002)). Without loss of generality we specify $v(N_t) = N_t^{1+\phi}/(1+\phi)$, where $\eta \geq 0$ and $v'(N_t) = N_t^{\phi} > 0$.[6] Under complete asset markets, the first order condition with respect to the intertemporal allocation of consumption leads to

$$Q_{t,t+1} = \beta \frac{C_{t+1}^{-\sigma} e^{(\sigma-1)v(N_{t+1})}}{C_t^{-\sigma} e^{(\sigma-1)v(N_t)}} \Pi_{t+1}^{-1}, \tag{4}$$

where $Q_{t,t+1}$ is the stochastic discount factor associated with (random payoffs of) the household's asset portfolio and $\Pi_{t+1} \equiv P_{t+1}/P_t$ is consumer price inflation.

[5] Recent empirical studies on the consumption Euler equation also impose parameter restrictions pertaining to long-run labor supply in line with with balanced growth facts (see, e.g., Basu and Kimball (2002)).

[6] CGG assume $U = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t(h)^{1-\phi}}{1-\phi}$ implying U_{c_t} is independent of N_t. Note that when $\sigma = 1$ both (3) and CGG's utility function reduce to the log-separable utility $U(C_t, N_t) = \log C_t - v(N_t)$.

Equation (5) shows that when utility is non-separable, consumption growth depends on aggregate hours. We assume the presence of a one-period discount bond with (risk free) gross nominal yield R_t. Then taking expectations (as of period t) of equation (4) we arrive at the familiar Euler equation

$$1 = \beta R_t E_t \left(\frac{C_{t+1}^{-\sigma} e^{(\sigma-1)v(N_{t+1})}}{C_t^{-\sigma} e^{(\sigma-1)v(N_t)}} \Pi_{t+1}^{-1} \right), \tag{5}$$

where $R_t^{-1} = E_t Q_{t,t+1}$ is the price of the discount bond.

Next, under a flexible wage setting, optimal wage setting by monopolistically competitive households and symmetric equilibrium (so that one can drop the index h) leads to

$$\frac{W_t}{P_t} = (1 + \mu_t^w) C_t N_t^{\phi} \tag{6}$$

where μ_t^w is the optimal wage markup and $C_t N_t^{\phi}$ is the marginal rate of substitution between consumption and hours. Moreover, full international risk sharing implies the equalization of the marginal utility of consumption across home and foreign households

$$C_t^{-\sigma} e^{(\sigma-1)v(N_t)} = C_t^{*-\sigma} e^{(\sigma-1)v(N_t^*)} \Leftrightarrow C_t^* = H_t C_t \tag{7}$$

where $H_t \equiv e^{(1-\sigma)(v(N_t)-v(N_t^*))/\sigma} = e^{B\left(N_t^{1+\phi} - N_t^{*1+\phi}\right)}$, with $B \equiv (1-\sigma)/(\sigma(1+\phi))$. Here $\sigma > 1 \Rightarrow B < 0$. Thus, under non-separable utility home and foreign consumption levels are not necessarily equalized, which is in contrast to CGG, and H_t can be thought of as a *wedge* between C_t and C_t^* when the two countries face asymmetric shocks that lead to differences in employment levels.[7]

On the supply side, the key behavioral equation relates to optimal price setting by domestic intermediate firms,

$$\begin{aligned} \frac{P_{H,t}^0}{P_{H,t}} &= \frac{(1+\mu^p) E_t \sum (\theta\beta)^i Y_{t+i} C_{t+i}^{-\sigma} e^{(\sigma-1)v(N_{t+i})} MC_{t+i} \left(\frac{P_{H,t+i}}{P_{H,t}}\right)^{\xi}}{E_t \sum (\theta\beta)^i Y_{t+i} C_{t+i}^{-\sigma} e^{(\sigma-1)v(N_{t+i})} \left(\frac{P_{H,t+i}}{P_{H,t}}\right)^{\xi-1}} \\ &= \frac{(1+\mu^p) E_t \sum (\theta\beta g_r^{1-\sigma})^i y_{t+i} c_{t+i}^{-\sigma} e^{(\sigma-1)v(N_{t+i})} MC_{t+i} \left(\frac{P_{H,t+i}}{P_{H,t}}\right)^{\xi}}{E_t \sum (\theta\beta g_r^{1-\sigma})^i y_{t+i} c_{t+i}^{-\sigma} e^{(\sigma-1)v(N_{t+i})} \left(\frac{P_{H,t+i}}{P_{H,t}}\right)^{\xi-1}} \end{aligned} \tag{8}$$

[7] Here $H_t = 1$ if $\sigma = 1$ (i.e., the case of separable utility).

where θ is the fraction of firms not able to set their price optimally in a given period, μ^p is steady state price markup and ξ is the elasticity of substitution between the differentiated intermediate goods, $P^0_{H,t}/P_{H,t}$ is the optimal domestic relative price, Y_t is (demand determined) domestic output and $MC_t = (1-\tau)W_t/(\alpha_t A_t P_{H,t})$ is real domestic marginal cost, with τ representing the wage subsidy rate, as in CGG. The intuition for why trend growth matters is straightforward. Due to price rigidity, $P^0_{H,t}/P_{H,t}$ in (8) is set so as to maximize the expected discounted value of current and future profits. Along the balanced growth path both C_t and Y_t grow at the rate g. Here, on the one hand, the presence of trend consumption growth reduces the stochastic discount factor (this follows from the Euler equation), so that future profits are *less* important (relative to current profits) for price setting (i.e., firms become less forward-looking relative to the case with no trend growth). On the other hand, the presence of trend output growth implies aggregate demand growth for intermediate goods (this follows from goods demand), so that future profits are *more* important (relative to current profits) for price setting (i.e., firms become more forward-looking). Since we have non-separable utility with $\sigma > 1$, the overall effect of trend productivity growth is to make firms *less* forward-looking.

Next, under the assumption of symmetric preferences across home and foreign households, producer currency pricing and complete pass-through consumer price index-based purchasing power parity (PPP) holds (i.e., $\varepsilon_t P^*_t = P_t$, where ε_t is the nominal exchange rate, defined as the price of one unit of the foreign currency in terms of home currency). The market clearing conditions for home and imported goods imply the following trade balances for the home and foreign countries

$$P_{H,t}Y_t = P_t[(1-\gamma)C_t + \gamma C^*_t] \tag{9}$$

$$P^*_{F,t}Y^*_t = P^*_t[(1-\gamma)C_t + \gamma C^*_t] \tag{10}$$

Then taking note of the PPP condition, (9) and (10) imply

$$S_t \equiv P_{F,t}/P_{H,t} = Y_t/Y^*_t. \tag{11}$$

Using this result and substituting equation (7) in equation (9) we get

$$C_t = \frac{kS_t^{-\gamma}Y_t}{1-\gamma+\gamma H_t}. \tag{12}$$

As in CGG, the aggregate domestic production function under price rigidity is given by

$$N_t = V_t Y_t / (\alpha_t A_t), \tag{13}$$

with $V_t \equiv \int_0^1 (P_{H,t}(f)/P_{H,t})^{-\xi} \, df$ being the domestic price dispersion in the intermediate goods sector.

Using equations (12) and (13) we can rewrite domestic marginal cost MC_t as

$$\begin{aligned} MC_t &= (1-\tau)(1+\mu_t^w)\frac{k^{-1}C_t N_t^{\phi} S_t^{\gamma}}{A_t} \qquad (14) \\ &= (1-\tau)(1+\mu_t^w)\frac{V_t^{\phi}(y_t/\alpha_t)^{1+\phi}}{1-\gamma+\gamma H_t} \qquad (15) \end{aligned}$$

which are the counterparts of equations (34) and (35) of CGG. From equation (14) we can see that Y_t^* affects MC_t in two ways: the terms of trade effect, as S_t is negatively related to Y_t^*, (see equation (11), which is identical to that in CGG), and the wealth effect, as C_t is related to Y_t^* due to international risk sharing. Note, however, that the wealth effect differs from that in CGG due to the presence of H_t owing to our assumption of non-separable utility. From equation (12) the presence of H_t dampens the overall effect of Y_t^* on C_t, (compared to CGG). From the international risk sharing condition $\partial H_t/\partial Y_t^* > 0$ given our assumption $\sigma > 1$. This implies that if Y_t^* increases given Y_t, (equivalently N_t^* increases given N_t) then the ratio C_t/C_t^* decreases (foreign consumption increases relative to home consumption). The reason is that, when foreign employment increases given domestic employment, the marginal utility of consumption abroad increases relative to the marginal utility of consumption at home.

Equation (15) shows the net effect of Y_t^* on MC_t, in which only the effect via H_t matters. Unlike CGG (see p. 887, paragraph 1 of CGG) foreign output matters for domestic marginal cost only because it creates a wedge between domestic and foreign consumption. Using the definition of H_t, we see that $\partial MC_t/\partial Y_t^* = (\partial MC_t/\partial H_t)(\partial H_t/\partial Y_t^*) < 0$ provided $\sigma > 1$, so that domestic marginal cost decreases as foreign output increases and vice versa.

In what follows we linearize the model around the *domestic* flexible price equilibrium, which arises when prices are flexible at home, given foreign output.[8]

[8] The *domestic* flexible price equilibrium is distinct from what Clarida, Gali and Gertler (2002)

3 Linearized model

As shown in Appendix A, the linearized home economy Phillips curve is

$$\pi_t = \beta_g E_t \pi_{t+1} + \lambda_{g,o} \tilde{y}_t + u_t \tag{16}$$

where π_t is domestic (producer) price inflation, $\tilde{y}_t$ is the *domestic* output gap (i.e., the gap between actual output and the *domestic* natural level), u_t is a composite cost-push shock (a function of a time-varying wage markup—$u_t \equiv \delta_g \widehat{\mu_t^w}$. The coefficients are $\beta_g \equiv \beta g_r^{1-\sigma}$, $\lambda_{g,o} \equiv \delta_g \kappa$, where $\delta_g = (1-\theta)(1-\beta_g\theta)/\theta$, $\kappa \equiv 1 + \phi - \kappa_o$, $\kappa_o \equiv \gamma v$ and $v \equiv \sigma^{-1}(1-\sigma)N^{1+\phi}$.[9] Note that trend growth affects both the slope and the position of the Phillips curve. Given $\sigma > 1$, $\frac{\partial \beta_g}{\partial g_r} < 0$ and $\frac{\partial \lambda_{g,o}}{\partial g_r} > 0$, so that the sensitivity of actual inflation to expected inflation is lower while its sensitivity to the domestic output gap is higher the higher is trend growth. Unlike trend growth, openness affects only the slope of the Phillips curve.

The home IS curve is

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \sigma_o^{-1}(r_t - E_t \pi_{t+1} - \overline{rr}_t), \tag{17}$$

where $\sigma_o \equiv \sigma d_o$ and $d_o \equiv 1 - \gamma(1-\sigma^{-1}) + v - \kappa_o$. The term $\overline{rr}_t \equiv \sigma(q_1 E_t \Delta \widehat{\overline{y}}_{t+1} + q_2 E_t \Delta \hat{y}^*_{t+1} + q_3 E_t \Delta \hat{\alpha}^*_{t+1} - v E_t \Delta \hat{\alpha}_{t+1})$ is the natural rate of interest, where $q_1 \equiv 1 + v - \gamma(1-\sigma^{-1})$, $q_2 \equiv \gamma(1-\sigma^{-1}) - q_3$, and $q_3 \equiv \kappa_o(1 + \kappa_o/\kappa)$. If $\gamma = 0$, then $d_o = q_1 = 1 + v$ and $q_2 = q_3 = 0$ so that $\overline{rr}_t$ does not depend on foreign variables.

The corresponding linearized foreign economy New Keynesian Phillips curve is

$$\pi^*_t = \beta_g E_t \pi^*_{t+1} + \lambda^*_{g,o} \tilde{y}^*_t + u^*_t, \tag{18}$$

and the foreign IS curve is

$$\tilde{y}^*_t = E_t \tilde{y}^*_{t+1} - \sigma_o^{*-1}(r^*_t - E_t \pi^*_{t+1} - \overline{rr}^*_t) \tag{19}$$

call the *world* flexible price equilibrium, in which prices are flexible worldwide. This distinction is emphasized in Clarida, Gali and Gertler (2002), who study optimal monetary policy under Nash (i.e., non-cooperative) and cooperative equilibria. It is not so obvious what a cooperative equilibria entails for simple policy rules. One possibility is that a pair of simple rules are jointly evaluated in terms of the stability of the system that involves deviations of variables from world flexible price equilibrium. In that case, one would have to evaluate determinacy and learnability over a four-dimensional parameter space (corresponding to the home and foreign policy rule coefficients). We leave the issue of 'cooperation' for future research.

[9] The subscript g stands for trend growth while the subscript o stands for openness.

where the foreign country parameters are defined analogously to their home economy counterparts, with γ replaced by $1-\gamma$. The linearized Phillips and IS equations summarize aggregate private sector behavior given the setting of monetary policy. The model is thus closed by specifying simple policy rules for the home and foreign economies. We consider two alternatives (i) a rule based on contemporaneous data, where home and foreign policymakers respond to contemporaneous domestic inflation and the output gap and (ii) a rule based on forward expectations, where the policymakers respond to one-period-ahead expectations of domestic inflation and the output gap. In what follows we discuss how those conditions are affected by the presence of trend productivity growth.

4 Determinacy and learnability

For the analysis of the model under learning, we follow the standard approach in the learning literature and replace rational expectations by subjective expectations in the Phillips and IS curves. As in Bullard and Schaling (2009) we provide the necessary and/or sufficient conditions for determinacy of equilibria and E-stability of minimum state variables (MSV) equilibria under alternative monetary policy rules. Before showing our results on the determinacy and learnability of alternative monetary policy rules we briefly discuss the concepts of determinacy and E-stability.[10] For this purpose one may substitute a given policy rule into the IS curve and write the structural equations in the canonical form

$$z_t = AE_t z_{t+1} + e_t \tag{20}$$

where z_t is a vector of endogenous variables of the model, A is a conformable matrix and e_t is a vector of exogenous variables, which are not relevant for the issue of (in)determinacy.

One can rewrite the structural equation (20) as

$$z'_t = A' E_t z'_{t+1} + Bs_t \tag{21}$$

where z'_t includes only free (i.e., non-predetermined) endogenous variables and s_t is a vector of state variables (predetermined endogenous variables and exogenous variables, e_t).

[10] Here the main reference for the issues of determinacy and E-stability are Blanchard and Kahn (1980) and Evans and Honkapohja (2001), respectively.

We employ well known methods for examining the determinacy and learnability properties of linear rational expectations models. Regarding determinacy we use Proposition 1 of Blanchard and Kahn (1980), which provides the necessary and sufficient conditions for equilibrium determinacy of models of the form (20). In particular, the model is determinate if and only if the number of eigenvalues of matrix A that are inside the unit circle is equal to the number of free endogenous variables.

Under learning we replace rational expectations $E_t z'_{t+1}$ in (21) by subjective expectations z'^e_{t+1} and apply Proposition 10 of Evans and Honkapohja (2001), which provides the necessary and sufficient conditions for learnability of a given rational expectations equilibrium. Take for instance the *minimum state variables* (MSV) solution of the system (21) under rational expectations, which takes the form $z'_t = a^{RE} + b^{RE} s_t$, where a^{RE} is a vector of constants and b^{RE} is a conformable matrix.[11] Since we have rewritten the model in terms of percentage deviations from its steady state, it follows that $a^{RE} = 0$. Under learning, the MSV solution is taken as the (reduced form) perceived law of motion (PLM) for z'_t

$$z'_t = a + bs_t \tag{22}$$

where a and b generally differ from their rational expectations counterparts.[12] While private sector forecasts z'^e_{t+1} are based on (22) the actual law of motion (ALM) of the system is determined by the interaction of these forecasts and the structural equation (21). Given the beliefs a and b, the ALM will be of the form

$$z'_t = T_a(a, b, A') + T_b(b, A', B)s_t \tag{23}$$

and this gives a mapping from (a, b) to $(T_a(a, b, A'), T_b(b, A', B))$.[13] The issue under learning the MSV solution is thus whether (a, b) converges to (a^{RE}, b^{RE}). Evans and Honkapohja (2001) show that under fairly general conditions the local convergence of real time learning using recursive least-squares and closely related algorithms is

[11]See McCallum (1983) and McCallum (2007) for a detailed discussion of the MSV solution approach.

[12]Note here that, as in Bullard and Mitra (2002) and much of the subsequent research on E-stability of monetary policy rules, the information set under learning is assumed not to include current endogenous variables z_t. As noted by Evans and Honkapohja (2001) this assumption avoids the simultaneity problem. See for e.g., Evans and Honkapohja (2001) and McCallum (2007) for discussions of alternative information sets under learning.

[13]Under rational expectations a^{RE} and b^{RE} are solutions to the fixed point problem $a = T_a(a, b, A')$ and $b = T_b(b, A', B)$, respectively.

governed by what they call the *E-stability principle* related to the local convergence of a differential equation system in notional time

$$
\begin{aligned}
\frac{da}{dt'} &= T_a(a, b, A') - a \\
\frac{db}{dt'} &= T_b(b, A', B) - b
\end{aligned}
\tag{24}
$$

where t' represents notional time. If the above system is locally asymptotically stable around (a^{RE}, b^{RE}), then the MSV is said to be E-stable (see also p.232 of Evans and Honkapohja (2001)).

Below we discuss how those conditions are affected by the presence of trend productivity growth.

4.1 Contemporaneous data in the policy rules

In the case where home and foreign policy rules respond to the respective current period inflation and output gap,

$$
\begin{aligned}
r_t &= \overline{rr}_t + \varphi_\pi \pi_t + \varphi_y \tilde{y}_t \\
r_t^* &= \overline{rr}_t^* + \varphi_\pi^* \pi_t^* + \varphi_y^* \tilde{y}_t^*
\end{aligned}
\tag{25}
$$

where the coefficients are all non-negative. As in Bullard and Schaling (2009) we include the natural rate of interest in the respective policy rules. By doing so we allow each central bank to insulate the domestic economy from foreign output, as the latter matters for domestic variables only by affecting the natural rate of interest. In this case openness matters because it changes the sensitivity of domestic inflation to changes in the domestic output gap and the sensitivity of domestic output gap to changes in the rate of interest. Moreover, the policy rules are evaluated in terms of stability or instability of domestic inflation and the domestic output gap. This reflects the presence of these two variables in the policy rules. By doing so, and similarly for a policy rule that responds to forward expectations of domestic variables, we stay close to Bullard and Mitra (2002), who note that "a practitioner wishing to find an optimal policy rule in this set could then postulate an objective criterion for the central bank and use it to locate the best rule."

Under rational expectations the MSV solution is of the form

$$z_t = a + c\overline{rr}_t^v \tag{26}$$

where $z_t = (\tilde{y}_t, \pi_t, \tilde{y}_t^*, \pi_t^*)'$ and $\overline{rr}_t^v = (\overline{rr}_t, u_t, \overline{rr}_t^*, u_t^*)'$. Under rational expectations, private agents have full knowledge of the MSV solution. Thus any MSV solution will necessarily have $a = 0$. Under learning, we follow the standard approach (see, e.g., Bullard and Mitra (2002)), where one endows the private agents with a law of motion for z_t that is of the MSV form and use least squares to get estimates for a and c and update their inflation and output gap forecasts. Under learning equation (26) is the PLM. The issue is then whether the system under learning converges to the MSV solution under rational expectations. The matrix system (26) is block diagonal so that the determinacy and learnability properties can be studied country by country.[14] As each block is isomorphic to the closed economy counterpart, the necessary and sufficient conditions for determinacy and E-stability of MSV equilibria are analogous to those in Bullard and Mitra (2002),

$$\lambda_{g,o}(\varphi_\pi - 1) + (1 - \beta_g)\varphi_y > 0 \tag{27}$$

and

$$\lambda_{g,o}^*(\varphi_\pi^* - 1) + (1 - \beta_g)\varphi_y^* > 0. \tag{28}$$

Conditions (27) and (28) are the open economy versions of the so-called Taylor principle, (see also Tesfaselassie (2011)), which in our case requires each central bank to adjust the nominal interest rate more than one-for-one with changes in long-run domestic inflation. Here the Taylor principle should be fulfilled country by country (see Bullard and Schaling (2009) p. 1591–92 for a discussion). Note that the determinacy and E-stability conditions for the two economies are similar but not identical (even under symmetric Taylor rules) except in the special case with $\gamma = 1/2$ (i.e., equally open economies).

We have the following propositions regarding the effect of trend growth and openness on the size of the regions of (in)determinacy and and E-(in)stability under our assumption of non-separable utility (in particular, $\sigma > 1$).

[14]If either of the policy rules responds to foreign variables the system will not be block diagonal. As the policy rules are evaluated in terms of stability or instability of domestic inflation and the domestic output gap, we focus on rules that respond to these variables.

Proposition 1 *Under the rule based on contemporaneous data higher trend growth expands the determinate and E-stable region in the policy space* (φ_π, φ_y) *and* $(\varphi_\pi^*, \varphi_y^*)$.

Proof: From condition (27) the frontier that defines the determinate region for the home economy is given by $\varphi_y = -m_1(\varphi_\pi - 1)$, where $m_1 \equiv \lambda_{g,o}/(1-\beta_g) = \delta_g \kappa/(1-\beta_g) > 0$. Then $\partial m_1/\partial g_r < 0$, provided $\sigma > 1$, so that the determinacy frontier is flatter (with a pivot at $\varphi_\pi = 1$) the larger is g_r. The implication of condition (28) follows analogously.

Note that the effect of higher trend growth on the determinate and E-stabile region is larger the larger is σ (i.e., the smaller is $1/\sigma$—the elasticity of intertemporal substitution).

Proposition 2 *Under the rule based on contemporaneous data greater openness shrinks the determinate and E-stable region in the policy space* (φ_π, φ_y) *and* $(\varphi_\pi^*, \varphi_y^*)$.

Proof: From condition (27) the frontier that defines the determinate region for the home economy is given by $\varphi_y = -m_1(\varphi_\pi - 1)$. Then $\partial m_1/\partial \gamma > 0$, provided $\sigma > 1$, so that the determinacy frontier is steeper (with a pivot at $\varphi_\pi = 1$) the larger is γ. The implication of condition (28) follows analogously.

Note again the requirement that, for Proposition 2 to hold, the elasticity of intertemporal substitution must be smaller than one. As a matter of comparison Bullard and Schaling (2009) show that in the Clarida, Gali and Gertler (2002) model greater openness shrinks the determinate and E-stable region provided the elasticity of intertemporal substitution is larger than one. As we remarked above, these differing implications are the results of differences in the international spillover effects under separable and non-separable utility.

4.2 Forward expectations in the policy rules

Next we consider the case where home and foreign policy rules respond to domestic expected inflation and domestic expected output gap

$$\begin{aligned} r_t &= \overline{rr}_t + \varphi_\pi \pi_{t+1}^e + \varphi_y \tilde{y}_{t+1}^e \\ r_t^* &= \overline{rr}_t^* + \varphi_\pi^* \pi_{t+1}^{*e} + \varphi_y^* \tilde{y}_{t+1}^{*e} \end{aligned} \tag{29}$$

so that the system of equations is given by (16), (18), (17), (19) and (29). The MSV solution is of the same form as in the case of contemporaneous data in the policy rule, which is given by (26).

As the Phillips curve and the IS curve under our setup are isomorphic to those in Bullard and Mitra (2002), by a version of Proposition 4 of Bullard and Mitra (2002) a set of necessary and sufficient conditions for determinacy are (27), (28),

$$(1+\beta_g)\varphi_y + \lambda_{g,o}(\varphi_\pi - 1) < 2\sigma_o(1+\beta_g), \tag{30}$$

$$\varphi_y < \sigma_o(1+\beta_g^{-1}), \tag{31}$$

$$(1+\beta_g)\varphi_y^* + \lambda_{g,o}^*(\varphi_\pi^* - 1) < 2\sigma_o^*(1+\beta_g), \tag{32}$$

and

$$\varphi_y^* < \sigma_o^*(1+\beta_g^{-1}). \tag{33}$$

Under learning, by a version of Proposition 5 of Bullard and Mitra (2002) conditions (27) and (28) are necessary and sufficient for an MSV solution to be E-stable. In section 4.1 we show that under the policy rules with contemporaneous data higher trend growth increases the scope for determinacy. By analogy, under the policy rules with forward expectations higher trend growth increases the scope for E-stability of MSV equilibria.

As Bullard and Mitra (2002) point out, the conditions for determinacy in the case of forward expectations in the policy rules are more stringent than those in the case of contemporaneous data in the policy rules. In particular, for the model with forward expectations in the policy rules to be determinate the coefficients in the home and foreign policy rules should not be too large. In other words, the Taylor principle is necessary but not sufficient for determinacy. As an illustration, consider the two special cases: (i) $\varphi_y = \varphi_y^* = 0$ so that the central banks respond only to expectations of domestic inflation and (ii) $\varphi_\pi = \varphi_\pi^* = 0$ so that the central banks respond only to expectations of the domestic output gap. Under special case (i) the determinacy conditions simplify to become $1 < \varphi_\pi < 1 + 2\sigma_o(1+\beta_g)/\lambda_{g,o}$ and $1 < \varphi_\pi^* < 1 + 2\sigma_o^*(1+\beta_g)/\lambda_{g,o}^*$, which contrast with the corresponding conditions under the policy rules with contemporaneous data, namely, $\varphi_\pi, \varphi_\pi^* > 1$. Here $(1+\beta_g)/\lambda_{g,o}$ and $(1+\beta_g)/\lambda_{g,o}^*$ both decrease as g increases, so that in this special case higher trend growth shrinks the size of the determinate region.

Let $m_2 \equiv \lambda_{g,o}/(1+\beta_g)$. Under special case (ii) the determinacy conditions become $m_1 < \varphi_y < m_2 + 2\sigma_o$ or $m_1 < \varphi_y < \sigma_o(1+\beta_g^{-1})$, whichever of the two upper bounds of these intervals binds, and analogously for the foreign economy. For the first and second interval to have nonzero measure, it must be that $\sigma_o > \beta_g \lambda_{g,o}/(1-\beta_g^2)$ and $\sigma_o > m_1/(1+\beta_g^{-1})$, respectively (this is the case, if, for instance, the degree of non-separability in utility is strong enough; contrast the left and right panels in the figure below. In either case, since m_1 and β_g decrease while m_2 increases as g increases, higher trend growth enlarges the size of the determinate region, in contrast to the first special case ($\varphi_y = \varphi_y^* = 0$) where higher trend growth shrinks the size of the determinate region.

In the general case with both coefficients in the policy rules allowed to be nonzero, the figure below illustrates the effects of trend growth and the degree of openness on determinacy and learnability.[15] The four panels in the figure correspond to four case associated with $\gamma \in \{0, 0.5\}$ and $g \in \{0\%, 3\%\}$, where g is the annualized trend growth rate given by $g \equiv (g_r - 1) \times 100\% \times 4$.

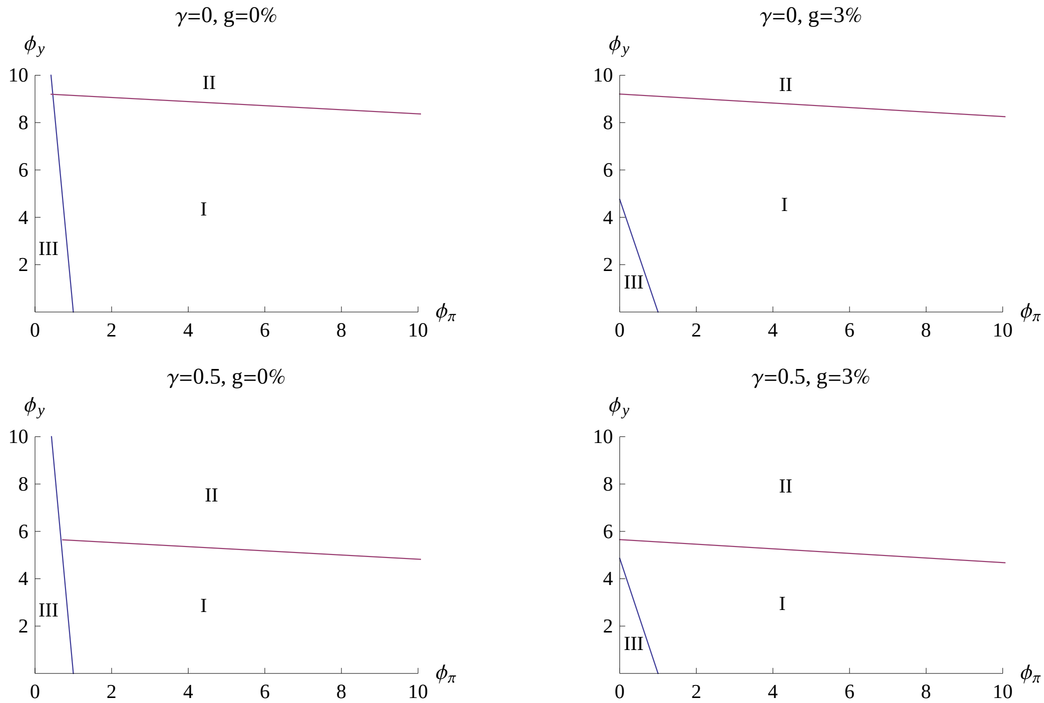

Figure: The effects of openness and trend growth on determinacy and E-stability under the policy rule with forward expectations. Region *I*: Determinate and E-stable; region *II*: Determinate and E-unstable; region *III*: Indeterminate and E-unstable.

[15] Our calibration is standard: $\beta = 0.99, \theta = 0.75$ and we set $\sigma = 5$ as in Linnemann (2006) and Matsumoto (2007), which is within the range of values reported in the empirical literature. For instance Guerron-Quintana (2008) reports σ as large as 6.33. Finally, $N = 1/3$ following Linnemann (2006).

Note that openness affects the inequality condition (30) mainly by decreasing the φ_y-intercept of the downward sloping frontier associated with this condition, which is $\varphi_y = -m_2(\varphi_\pi - 1) + 2\sigma_o$.[16] Moreover, openness affects the inequality condition (27) by steepening the downward sloping frontier associated with this condition. By contrast, the effect of trend growth on determinacy and E-stability comes via the inequality condition (27). In section 4.1 we showed that trend growth flattens the downward sloping frontier associated with this condition.

Note here that when region *I* shrinks, implying either region *II* or *III* expands, there is less scope for determinacy and when region *III* shrinks, implying either region *I* or *II* expands, there is a greater scope for learnability. We have the following propositions regarding the effect of trend growth and openness on the size of the regions of (in)determinacy and and E-(in)stability under our assumption of non-separable utility (in particular, $\sigma > 1$).

Proposition 3 *Under the forward expectations in the policy rule higher trend growth decreases the size of the*

i) determinate and E-stable region in the policy space (φ_π, φ_y) *if and only if*

$$g_r < (\beta(1 + 2\theta))^{1/(\sigma-1)}$$

ii) indeterminate and E-unstable region.

Proof: see Appendix B.

Proposition 4 *Under the forward expectations in the policy rule a higher degree of openness*

i) decreases the size of the determinate and E-stable region in the policy space (φ_π, φ_y) *and*

ii) increases the size of the indeterminate and E-unstable region.

Proof: see Appendix C.

Note that, regarding Proposition 3.i, the condition $g < (\beta(1 + 2\theta))^{1/(\sigma-1)}$ is easily satisfied for standard values of the parameter space ($\beta = 0.99, \theta = 0.75$). For instance, under $\sigma = 2$ (relatively weak degree of nonseparability) the condition

[16] Openness also makes this frontier flatter although this effect is less visible to the eye.

is satisfied provided $g_r < 2.48$ (i.e., $g < 600\%$), while under $\sigma = 5$ (relatively strong degree of nonseparability) the condition is satisfied provided $g_r < 1.25$ (i.e., $g < 102\%$). In both cases, the condition is easily fulfilled by empirical standards, given that in advanced countries the magnitude of observed productivity growth rates are of single digit.

To summarize, under the contemporaneous data policy rule the conditions for determinacy and learnability become more stringent on account of openness but less stringent on account of positive trend growth. Therefore, here, positive trend growth weakens the effect of openness. Under the forward expectations policy rule the conditions for determinacy and learnability also become more stringent on account of openness while on account positive trend growth the conditions for determinacy become *more* stringent (thus reinforcing the effect of openness) but those for learnability become *less* stringent (thus weakening the effect of openness).

5 Concluding remarks

This paper extends the standard two-country New Keynesian model to allow for the effect of trend productivity growth and examines its effect on the determinacy and learnability properties of alternative monetary policy rules. We show that irrespective of the policy rule under consideration the conditions for determinacy and learnability become more stringent with openness for realistic values of the elasticity of intertemporal substitution. When growth considerations are taken into account we show that (i) under the contemporaneous data in the policy rule the conditions for determinacy and learnability become less stringent, compared to a no-growth economy benchmark, and (ii) under the forward expectations in the policy rule the conditions for determinacy become more stringent while the conditions for learnability become less stringent.

Our results highlight that trend growth matters for the overall effect of opening an economy to more trade, as available evidence supports the view that productivity growth is faster in more open economies. We remark that, in the case of determinacy under the forward expectations policy rule openness and growth reinforce each other so that the overall effect is clear cut. In other cases—determinacy and learnability under the contemporaneous data policy rule as well as learnability under the forward expectations policy rule—by contrast, growth weakens the effect of openness so that the overall effect is ambiguous and any quantitative analysis depends on a realistic calibration of the sensitivity of growth to openness, which, admittedly, is not a trivial matter.

Appendices

A A two-country model with trend growth

The derivation in this appendix follows Clarida, Gali and Gertler (2002) (henthforth CGG) but allowing for non-separable utility and trend growth as in the seminal work of King, Plosser and Rebelo (1988), although we abstract from capital accumulation.

Households

The two countries are labeled H and F. The home country H has a mass of households $1-\gamma$ (a household is indexed by $h \in [0, 1-\gamma]$) and the foreign country F has a mass of households γ. Otherwise they are symmetric.

Let C_t be an index of a domestic domestically produced good $C_{H,t}$ and an imported good $C_{F,t}$

$$C_t = C_{H,t}^{1-\gamma} C_{F,t}^{\gamma}, \tag{A.1}$$

where $P_{H,t}$ and $P_{F,t}$ are the respective producer price indices, and let P_t be the consumption price index (that follows from cost minimization)

$$P_t = k^{-1} P_{H,t}^{1-\gamma} P_{F,t}^{\gamma} = k^{-1} P_{H,t} S_t^{\gamma}, \tag{A.2}$$

where $S_t \equiv P_{F,t}/P_{H,t}$ is the terms of trade and $k \equiv (1-\gamma)^{(1-\gamma)}\gamma^{\gamma}$.

The representative household in the home country maximizes its intertemporal utility (3) subject to the budget constraint

$$P_t C_t + E_t(Q_{t,t+1} D_{t+1}) = W_t(h) N_t(h) + D_t - T_t + \Gamma_t. \tag{A.3}$$

Here, β is the subjective discount factor, D_{t+1} is the (random) payoff in period $t+1$ of the portfolio purchased at t, with $Q_{t,t+1}$ the corresponding stochastic discount factor, $N_t(h)$ is the number of hours worked, $W_t(h)$ is the corresponding nominal wage, T_t is lump sum tax and Γ_t is profit from intermediate firms.[17]

Aggregate demand for labor type h is given by

$$N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\eta_t} N_t, \tag{A.4}$$

while the wage index W_t is given by

$$W_t = \left(\int_0^1 W_t(h)^{1-\eta_t} dh\right)^{\frac{1}{1-\eta_t}}. \tag{A.5}$$

The first-order condition with respect to $W_t(h)$ is $W_t(h)/P_t = (1+\mu_t^w)N_t(h)^\phi C_t$. In equilibrium $W_t(h) = W_t$ (as well as $N_t(h) = N_t$), implying

$$\frac{W_t}{P_t} = (1+\mu_t^w)N_t^\phi C_t. \tag{A.6}$$

Note here that, on a balanced growth path the aggregate real wage increases at the same rate as aggregate consumption, while aggregate hours is constant.[18]

Next the first order condition with respect to intertemporal allocation of consumption gives the consumption Euler equation (where we dropped the index h)

$$\beta \frac{U_c(C_{t+1}, N_{t+1})P_t}{U_c(C_t, N_t)P_{t+1}} = Q_{t,t+1}, \tag{A.7}$$

where $U_c(C_t, N_t)$ is the marginal utility of consumption. As in CGG, let R_t be the gross nominal yield of a one-period discount bond. Then taking expectations of equation (A.7) and rearranging gives equation (5) in the main text.

Next, the household's optimal consumption allocation across home and imported goods for a given level of C_t gives the demand equations

$$C_{H,t} = (1-\gamma)\frac{P_t}{P_{H,t}}C_t \tag{A.8}$$

[17] As is standard, we assume the existence of complete asset markets so that the optimal risk sharing condition involves $U_{C(j)} = U_{C(j')}$ for any two households j and j' (domestic or foreign). Under symmetric initial conditions, in particular, zero net asset holdings, and given that preferences and productivity are identical across domestic households, and that wages are flexible, we have $C_t(h) = C_t(h') = C_t$ and similarly for the foreign economy $C_t(h^*) = C_t(h^{*'}) = C_t^*$ (see also Guerron-Quintana (2008)).

[18] We assume the wage markup μ_t^w is stationary.

and

$$C_{F,t} = \gamma \frac{P_t}{P_{F,t}} C_t. \tag{A.9}$$

A symmetric set of conditions holds for the representative household in the foreign country

$$\beta E_t \left(\frac{U_{c^*}(C^*_{t+1}, N^*_{t+1})\varepsilon_t P^*_t}{U_{c^*}(C^*_t, N^*_t)\varepsilon_{t+1} P^*_{t+1}} \right) = Q_{t,t+1}, \tag{A.10}$$

where $U_{c^*}(C^*_t, N^*_t) = C_t^{*-\sigma} e^{(\sigma-1)v(N^*_t)}$. Using the PPP relation $P_t/P^*_t = \varepsilon_t$ in (A.10) and comparing the resulting equation with the domestic Euler equation gives the international risk sharing condition

$$\frac{C_{t+1}^{-\sigma} e^{(\sigma-1)v(N_{t+1})}}{C_t^{-\sigma} e^{(\sigma-1)v(N_t)}} = \frac{C_{t+1}^{*-\sigma} e^{(\sigma-1)v(N^*_{t+1})}}{C_t^{*-\sigma} e^{(\sigma-1)v(N^*_t)}}, \tag{A.11}$$

With a symmetric initial condition we get

$$C_t^{-\sigma} e^{(\sigma-1)v(N_t)} = C_t^{*-\sigma} e^{(\sigma-1)v(N^*_t)}, \tag{A.12}$$

for all t.

Firms

Final good sector

The final good Y_t is a Dixit-Stiglitz composite of a continuum of intermediate goods over the unit interval (expressed in per capita)

$$Y_t = \left(\int_0^1 Y_t(f)^{\frac{\xi-1}{\xi}} df \right)^{\frac{\xi}{\xi-1}}, \tag{A.13}$$

where an intermediate good is indexed by f and ξ is the elasticity of substitution between any two differentiated goods. Cost minimization by final goods firms, for a given level of Y_t, gives the demand for each good

$$Y_t(f) = \left(\frac{P_{H,t}(f)}{P_{H,t}} \right)^{-\xi} Y_t, \tag{A.14}$$

where $P_{H,t}$ is given by

$$P_{H,t} = \left(\int_0^1 P_{H,t}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}}. \tag{A.15}$$

Intermediate goods sector

Each intermediate goods firm's production function is of the form $Y_t(f) = \alpha_t A_t N_t(f)$, where

$$N_t(f) = \left((1-\gamma)^{-1} \int_0^{1-\gamma} N_t(h)^{(\eta_t - 1)/\eta_t} dh \right)^{\eta_t/(\eta_t - 1)} \tag{A.16}$$

and aggregate labor demand is $N_t = \int N_t(f)df$. Intermediate firms are wage takers in the labor market. Cost minimization by firms leads to the aggregate demand for labor and the wage index shown above.

While firms set prices, output is demand determined according to equation (A.14), which in turn pins down each firm's labor demand. Pricing decisions in the goods market are subject to Calvo-type price staggering, where in any given period a fraction θ of firms cannot reset their prices optimally. It follows that for each firm f in period t, its nominal price $P_{H,t}(f)$ is set such that $P_{H,t}(f) = P^0_{H,t}$ if set optimally and $P_{H,t}(f) = P_{H,t-1}(f)$ otherwise. When firm f gets a chance to reset its price do so in order to maximizes the expected lifetime profit

$$E_t \sum_{i=0}^{\infty} \theta^i Q_{t,t+i} \left(P^0_{H,t} - P_{H,t+i} MC_{t+i} \right) Y_{t+i}(f), \tag{A.17}$$

where the term $\left(P^0_{H,t} - P_{H,t+i} MC_{t+i} \right) Y_{t+i}(f)$ is nominal profit and θ^i reflects the probability that $P^0_{H,t}$ still holds in period $t+i$. As firms are owned by households, future profits are discounted using the stochastic discount factor $Q_{t,t+i}$.

Detrending (A.7) gives

$$Q_{t,t+i} = \frac{(\beta g^{-\sigma})^i c_{t+i}^{-\sigma} e^{(\sigma-1)v(N_{t+i})} P_{t+i}}{c_t^{-\sigma} e^{(\sigma-1)v(N_t)} P_t}, \tag{A.18}$$

where $c_t = C_t / A_t$.

Each firm receives a subsidy of τ percent of its wage bill, implying that the real marginal cost is $MC_t = MC_t(f) = (1-\tau)W_t/(A_t P_{H,t})$, which is identical across all firms. Substituting the demand for good f in the profit function and differentiating with respect to $P^0_{H,t}$ (because all optimizing firms face identical maximization problem) gives the first order condition

$$E_t \sum_{i=0}^{\infty} \theta^i Q_{t,t+i} \left(\frac{P^0_{H,t}}{P_{H,t+i}} \right)^{-\xi} Y_{t+i} \left(P^0_{H,t} - (1+\mu^p) P_{H,t+i} MC_{t+i} \right) = 0, \tag{A.19}$$

where $\mu^p = 1/(\xi - 1)$ is the price markup. We can rewrite (A.19), taking note of equation (A.18) and detrending aggregate output Y_t (so that $y_t = Y_t/A_t$), as equation (8) of the main text.

Price staggering among firms implies that the price level (A.15) can be rewritten as

$$P_{H,t} = \Big((1-\theta)(P^0_{H,t})^{1-\xi} + \theta P^{1-\xi}_{H,t-1}\Big)^{\frac{1}{1-\xi}}. \tag{A.20}$$

Equilibrium

From goods market clearing in the home and foreign economies we get[19]

$$(1-\gamma)Y_t = (1-\gamma)C_{H,t} + \gamma C^*_{H,t} \tag{A.21}$$

and

$$\gamma Y^*_t = (1-\gamma)C_{F,t} + \gamma C^*_{F,t}. \tag{A.22}$$

Moreover, under the assumption of producer currency pricing and complete pass-through PPP holds (i.e., $\varepsilon_t P^* = P_t$). Then the PPP condition together with equations (A.21) and (A.22) lead to equations (9) and (10) of the main text.[20]

Next, equation (A.12) can be rewritten as

$$C^*_t = H_t C_t, \tag{A.23}$$

where

$$H_t \equiv e^{B\left(N_t^{1+\phi} - N_t^{*1+\phi}\right)}, \tag{A.24}$$

and $B \equiv (1-\sigma)/(\sigma(1+\phi))$. Here $\sigma > 1 \Rightarrow B < 0$. Using (A.23) to substitute out C^*_t and using (A.2) to substitute out $P_{H,t}/P_t$ in equation (9) we get

$$C_t = \frac{kS_t^{-\gamma}Y_t}{1-\gamma+\gamma H_t}. \tag{A.25}$$

[19] In the goods market clearing conditions total domestic output is equal to the per capita domestic output Y_t times the size of the domestic economy $1-\gamma$. The same holds for total domestic demand and total foreign demand and analogously for goods market clearing in the foreign economy.

[20] To see this for the domestic goods market clearing, say, substitute the demand equations for $C_{H,t}$ and $C^*_{H,t}$ in the goods market clearing condition (25) of CGG, and use the law of one price, $P_{H,t} = \varepsilon_t P^*_{H,t}$, and PPP, $P_t = \varepsilon_t P^*_t$.

where the terms of trade can be rewritten as $S_t \equiv P_{F,t}/P_{H,t} = Y_t/Y_t^*$.

On the supply side, aggregate employment is given by

$$N_t = \frac{V_t Y_t}{A_t} \equiv \frac{V_t y_t}{\alpha_t}, \tag{A.26}$$

where $V_t = \int \left(\frac{P_{H,t}(f)}{P_{H,t}}\right)^{-\xi} df$ is the price dispersion.

Using (A.25) and (A.26) in the domestic real marginal cost gives equation (15) of the main text.

Under domestic flexible price equilibrium, $\overline{MC}_t = 1/(1+\mu^p)$ and $\bar{V}_t = 1$ (no price dispersion) so that output in the flexible price equilibrium $\bar{y}_t$ (setting $\mu_t^w = \mu^w$) is given by the implicit function

$$1/(1+\mu^p) = (1-\tau)(1+\mu^w)\frac{(\bar{y}_t/\alpha_t)^{1+\phi}}{(1-\gamma)+\gamma\bar{H}_t}, \tag{A.27}$$

where $\bar{H}_t = H(\bar{y}_t, y_t^*, \alpha_t, \alpha_t^*)$.

Linearization

The nonlinear system is linearized around the domestic flexible price equilibrium with zero steady state domestic inflation (see also Clarida, Gali and Gertler (2002)). For any variable Z_t, (i) $\bar{Z}_t$ is the value of Z_t under flexible home prices, (ii) $\widehat{\bar{Z}}_t$ is the percentage deviation of $\bar{Z}_t$ from its steady state value and (iii) $\hat{Z}_t$ is the percentage deviation of Z_t from its steady state value.

Combining the optimal relative price and the price level leads to an equation relating domestic inflation to the domestic real marginal cost

$$\pi_t = \beta_g E_t \pi_{t+1} + \delta_g \widehat{MC}_t, \tag{A.28}$$

where $\beta_g \equiv \beta g_r^{1-\sigma}$ and $\delta_g \equiv (1-\theta)(1-\theta\beta_g)/\theta$. Next we rewrite $\widehat{MC}_t$ in terms of the domestic output gap $\tilde{y}_t \equiv \hat{y}_t - \widehat{\bar{y}}_t$ and the domestic cost-push shock. First, the aggregate production function (A.26) and its foreign economy counterpart are given by $\widehat{N}_t = \hat{y}_t - \hat{\alpha}_t = \tilde{y}_t + \widehat{\bar{y}}_t - \hat{\alpha}_t$ and $\widehat{N}_t^* = \hat{y}_t^* - \hat{\alpha}_t^*$. Under flexible prices (A.27) can be written in percentage deviation around a symmetric steady state (i.e., $H = 1$) as

$\gamma\widehat{\widehat{H}}_t = (1+\phi)(\widehat{\widetilde{y}}_t - \hat{\alpha}_t)$, where $\widehat{\widehat{H}}_t = v(\widehat{\widehat{N}}_t - \widehat{N}_t^*)$, $\widehat{\widehat{N}}_t = \widehat{\widetilde{y}}_t - \hat{\alpha}_t$ and $v \equiv \sigma^{-1}(1-\sigma)N^{1+\phi}$. Solving for $\widehat{\widetilde{y}}_t$ gives $\widehat{\widetilde{y}}_t = \hat{\alpha}_t - v'(\hat{y}_t^* - \widehat{\alpha}_t^*)$, where $v' \equiv \gamma v/(1+\phi-\gamma v)$ and the foreign output level is taken as given. Note that $\gamma = 0 \Rightarrow v' = 0$.

Next, writing MC_t in percentage deviation from steady state, and using $\widehat{H}_t = v(\widehat{N}_t - \widehat{N}_t^*) = v(\tilde{y}_t - (1+v')(\hat{y}_t^* - \widehat{\alpha}_t^*))$, gives

$$\widehat{MC}_t = \widehat{\mu^w}_t + (1+\phi)(\hat{y}_t - \hat{\alpha}_t) - \gamma\widehat{H}_t = \widehat{\mu^w}_t + (1+\phi-\gamma v)\tilde{y}_t \tag{A.29}$$

Using equation (A.29) to substitute for $\widehat{MC}_t$ in equation (A.28) we get equation (16) of the main text.

Limiting case I: if $\gamma = 0$ then

$$\pi_t = \beta_g E_t \pi_{t+1} + \lambda_g \tilde{y}_t + u_t,$$

where $\lambda_g \equiv \delta_g(1+\phi)$. In this case domestic inflation depends only on domestic conditions and trend productivity growth.

Limiting case II: if $\sigma = 1$ (i.e., log utility in consumption) then $v' = v = 0$ and

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \tilde{y}_t + u_t,$$

where $\lambda \equiv \delta(1+\phi)$ and $\delta \equiv (1-\theta)(1-\theta\beta)/\theta$. This is the standard closed economy New Keynesian Phillips curve. In this case domestic inflation does not depend on openness and trend productivity growth.

Next, expressing the Euler equation in linearized form around zero steady state inflation, we have

$$\hat{c}_t = E_t\hat{c}_{t+1} + v(E_t\widehat{N}_{t+1} - \widehat{N}_t) - \sigma^{-1}(r_t - E_t\pi_{t+1} - \gamma E_t \Delta\hat{S}_{t+1}), \tag{A.30}$$

where r_t is the nominal rate of interest. Using the production function, the domestic goods market clearing condition and the terms of trade equation to substitute out $\widehat{N}$, $\hat{c}$ and s_t in equation (A.30), and rearranging so as to express the equation in terms of the output gap we get equation (17) of the main text.[21]

It is straight forward to show that the foreign economy Phillips curve is

$$\pi_t^* = \beta_g E_t \pi_{t+1}^* + \lambda_{g,o}^* \widetilde{y}_t^* + u_t^*,$$

[21] From (A.25) $\hat{y}_t = \hat{c}_t + \gamma\hat{S}_t + \gamma\widehat{H}_t$ where $\hat{S}_t = \hat{y}_t - \hat{y}_t^* = \tilde{y}_t + \widehat{\widetilde{y}}_t - \hat{y}_t^*$.

where $u_t^* \equiv \delta_g \widehat{\mu^w}_t^*$, while the foreign economy IS curve is

$$\tilde{y}_t^* = E_t \tilde{y}_{t+1}^* - \sigma_o^{*-1}(r_t^* - E_t \pi_{t+1}^* - \overline{rr}_t^*).$$

The composite parameters with superscript "$*$" in the foreign Phillips and IS curves correspond to their home economy counterpart except that γ is replaced with $1-\gamma$.

B Proof of Proposition 2

Proof of Proposition 3.i

First the borderlines $\varphi_y = -H_1(\varphi_\pi - 1) \equiv f_1(\varphi_\pi)$ and $\varphi_y = 2\sigma_o - H_2(\varphi_\pi - 1) \equiv f_2(\varphi_\pi)$, which define region I, intersect at $\varphi_\pi = \varphi_\pi^* \equiv 1 - \sigma_o \left(1 - \beta_g^2\right) / (\beta_g \lambda_{g,o})$. Then the area of region I is given by

$$\begin{aligned} Area_I &= \int_{\varphi_\pi^*}^{t} f_2(\varphi_\pi) d\varphi_\pi - \int_{\varphi_\pi^*}^{1} f_1(\varphi_\pi) d\varphi_\pi \\ &= \frac{(1+\beta_g)^2 \sigma_o^2}{\beta_g \lambda_{g,o}}, \end{aligned} \tag{B.1}$$

where t is the horizontal intercept of f_2. Then

$$\frac{\partial Area_I}{\partial g} = -\frac{\sigma_o^2 (1+\beta_g)(1-\beta_g(1+2\theta))}{\beta_g^2 \lambda_{g,o}(1-\theta\beta_g)} \frac{\partial \beta_g}{\partial g}$$

It can be easily checked that, provided $\sigma > 1$, (which implies that $\partial \beta_g / \partial g < 0$), $\partial Area_I / \partial g < 0$ if and only if $1 - \beta_g(1+2\theta) < 0$. This condition is equivalent to that in the main text.

Proof of Proposition 3.ii

The area of region III is given by

$$\begin{aligned} Area_{III} &= \int_0^1 f_1(\varphi_\pi) d\varphi_\pi \\ &= \frac{\lambda_{g,o}}{2(1-\beta_g)} \end{aligned} \tag{B.2}$$

so that

$$\frac{\partial Area_{III}}{\partial g} = \frac{(1+\phi-\gamma v)(1-\theta)^2}{2\theta(1-\beta_g)^2}\frac{\partial \beta_g}{\partial g} < 0$$

provided $\sigma > 1$.

C Proof of Proposition 3

Proof of Proposition 4.*i*

From equation (B.1)

$$\frac{\partial Area_I}{\partial \gamma} = \frac{(1+\beta_g)^2}{\beta_g}\frac{\sigma_o}{\lambda_{g,o}^2}\left(2\lambda_{g,o}\frac{\partial \sigma_o}{\partial \gamma} - \sigma_o\frac{\partial \lambda_{g,o}}{\partial \gamma}\right) < 0,$$

provided $\sigma > 1$, which implies that $\partial\sigma_o/\partial\gamma < 0$ and $\partial\lambda_{g,o}/\partial\gamma > 0$.

Proof of Proposition 4.*ii*

From equation (B.2)

$$\frac{\partial Area_{III}}{\partial \gamma} = \frac{1}{2(1-\beta_g)}\frac{\partial \lambda_{g,o}}{\partial \gamma} > 0$$

provided $\sigma > 1$.

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