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JEL-Code: G21, G01, E42

Keywords: interbank market, network models, scale-free networks, powerlaw

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On Assortative and Disassortative Mixing in Scale-Free Networks: The Case of Interbank Credit Networks.[†]

Daniel Fricke[‡§] Karl Finger[¶] Thomas Lux[‡§‖]

This version: February 2013

Abstract

Networks constructed from credit relationships in the interbank market have been found to exhibit disassortative mixing together with a scale-free degree distribution, in contrast to most social networks that are assortative and not necessarily scale-free. This provokes the question whether generating mechanisms for scale-free networks have enough flexibility to generate both assortative and disassortative structures depending on their parametrization. Using Monte-Carlo simulations, we show that scale-free networks with a small tail exponent tend to be disassortative. However, the simulations indicate also that the level of disassortativity is sensitive to changes in the scaling exponent and the density. A given combination of disassortativity, scaling of the degree distribution, and density in an empirical data set, might be hard or impossible to obtain from any of the known generating mechanisms for scale-free networks.

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1 Introduction and Existing Literature

The global financial crisis (GFC) in 2007/08 moved interbank networks into the focus of academia, since the linkages created by liabilities among banks and other financial institutions played a crucial role and yet are poorly understood. An intriguing example is the bankruptcy of Lehman in September 2008, where the immense effects on the system as a whole were dramatically underestimated by the authorities. As a consequence, investigations of complex systems in terms of their network properties gain more and more attention in economics following the lead of other disciplines in which network analyses have already a long tradition. One remarkable finding is that many complex[1] real world networks share some apparently universal features.[2] A prominent example is a highly skewed degree distribution, where the degree of a node is the number of (incoming/outgoing) links.[3]

This paper focuses on the relationship between the degree distribution and another aspect of the network topology, namely the concept of degree assortativity, which measures whether nodes prefer links to other nodes with similar degree. The main finding in the network literature in this respect is that social networks tend to be assortative, meaning that low (high) degree nodes form links to other low (high) degree nodes (Newman, 2002, 2003). König *et al.* (2010) argue that capacity constraints in social networks lead to assortative networks and introduce a network formation algorithm taking this into account. A counterexample to this rule of assortativity of social networks is reported by Holme *et al.* (2004). They find disassortativity together with a skewed degree distribution in an internet dating community which, thus, appears to share some characteristics with networks constructed from interbank credit relations. On the other hand, technological and biological networks tend to be disassortative. Economic networks can be seen as a hybrid case sharing features of both depending on the context (Jackson, 2008). The question to which class the banking network belongs in this respect is crucial, since the stability of the whole network is affected by it. For instance, Newman (2003) found that assortative networks are more resilient, whereas disassortative networks are particularly vulnerable against directed attacks on high degree nodes (hubs). In the existing empirical literature Soramäki *et al.* (2006) for the Fedwire payments network, Bech and Atalay (2010) for the Federal funds network and Iori *et al.* (2008) for the Italian overnight money market find banking networks to be disassortative.

[1]Complex in this regard refers to the fact that the networks are neither purely random nor regular.

[2]See e.g. Reka and Barabási (2002).

[3]See e.g. Newman (2010), ch. 8.

This paper takes as its empirical starting point interbank networks derived from the credit extended via the e-MID (electronic market for interbank deposits) trading platform for overnight loans from 1999 to 2010, which is a privately owned Italian company and currently the only electronic broker market for interbank deposits. The focus of the analysis will be on Italian banks, since the participation of foreign banks in the e-MID had been very volatile and ceased almost completely after 2008. Our results support the earlier findings that banking networks are disassortative and this result is true for the directed and the undirected case.[4] The strongest result for the directed case is that banks with a high in-degree tend to have links to banks with a small out-degree, which holds also for the out-in case but to a lesser degree. This robust finding of interbank networks to be disassortative emphasizes the importance to identify the so-called 'systemically important' banks, which would adversely affect large parts of the network in case of their failure. For instance, Craig and von Peter (2010) and Fricke and Lux (2012) estimated a core-periphery model for the German and the Italian interbank network to identify these important components.

However, the finding of interbank networks to be scale-free (SF) and disassortative at the same time are hard to combine with the analytical results of Newman (2002), showing that SF networks (based on the preferential attachment, or PA, algorithm from Barabasi and Albert, 1999) with a typical scaling exponent of $\alpha = 3$ are unassortative, i.e. neither assortative nor disassortative, at least in the limit of very large networks. Therefore, the evidence for a SF distribution together with disassortativity motivates our investigation of whether SF networks are unassortative in general or only for specific parametrizations, and whether the results are different for a finite size of the network. Monte-Carlo simulations show that Erdös-Renyi (ER) (1959) random networks are unassortative, independent of the system parameters. In contrast, SF random networks with relatively small scaling exponents tend to be disassortative. However, the degree of dissasortativity is still stronger for the observed networks than for SF networks with the scaling exponent of 2.3 reported in the literature. Another finding is that even small changes in the scaling exponent can lead to substantially different mixing patterns.

The remaining part of this paper is organized as follows. Section 2 gives a brief introduction into interbank networks. Section 3 introduces the data set obtained from the e-MID trading system. Section 4 presents the empirical analysis of the observed networks. Section 5 contrasts this with Monte-Carlo simulations of various generating mechanisms for SF networks and section 6

[4] Directed networks take the direction of the liability into account and therefore links do not have to be mutual.

concludes.

2 Networks

A network consists of a set of N nodes that are connected by M edges (links). Taking each bank as a node and the interbank positions between them as links, the interbank network can be represented as a square matrix of dimension $N \times N$ (data matrix, denoted $\mathbf{D}$). An element d_{ij} of this matrix represents a gross interbank claim, the total value of credit extended by bank i to bank j within a certain period. The size of d_{ij} can thus be seen as a measure of link intensity. Row (column) i shows bank i's interbank claims (liabilities) towards all other banks. The diagonal elements d_{ii} are zero, since a bank will not trade with itself.[5] Off-diagonal elements are positive in the presence of a link and zero otherwise.

Interbank data usually give rise to directed, sparse and valued networks.[6] However, much of the extant network research ignores the last aspect by focusing on binary adjacency matrices only. An adjacency matrix $\mathbf{A}$ contains elements a_{ij} equal to 1, if there is a directed link from bank i to j and 0 otherwise. Since the network is directed, both $\mathbf{A}$ and $\mathbf{D}$ are asymmetric in general.

The degree of a node gives the total number of links that a bank has with all other banks and can thus be seen as a measure for the importance of individual nodes. Undirected networks imply symmetric adjacency matrices. In this case bank i's total degree k_i is simply the number of relationships bank i has with other banks, i.e.

$$k_i = \sum_{j \neq i} a_{ij}^u. \tag{1}$$

For directed networks, we differentiate between incoming links (bank i borrows money from other banks) and outgoing links (i lends money to other banks), and define the in- and out-degree of i (k_i^{in} and k_i^{out}) as

$$\begin{aligned} k_i^{in} &= \sum_{j \neq i} a_{ji} \\ k_i^{out} &= \sum_{j \neq i} a_{ij}, \end{aligned} \tag{2}$$

[5]This is of course only true when taking banks as consolidated entities.

[6]Directed means that $d_{ij} \neq d_{ji}$ in general. Sparse means that at any point in time the number of links is only a small fraction of the $N(N-1)$ possible links. Valued means that interbank claims are reported in monetary values as opposed to 1 or 0 in the presence or absence of a claim, respectively.

respectively. In our case the diagonal elements are zero, so in- and out-degree can be calculated by simply taking column- and row-sums of the adjacency matrix, respectively.

As usual, some data aggregation is necessary to represent the system as a network. In the following, we define interbank networks by aggregating over daily as well as quarterly lending activity.

3 The Italian Interbank Market (e-MID)

The Italian electronic market for interbank deposits (e-MID) is a screen-based platform for trading of unsecured money-market deposits in Euros, US-Dollars, Pound Sterling, and Zloty operating in Milan through e-MID SpA.[7] The market is fully centralized and very liquid; in 2006 e-MID accounted for 17% of total turnover in the unsecured money market in the Euro area. Average daily trading volumes were 24.2 bn Euro in 2006, 22.4 bn Euro in 2007 and dropped to only 14 bn Euro in 2008 as a consequence of the financial crisis. We should mention that researchers from the European Central Bank have repeatedly stated that the e-MID data is representative for the interbank overnight activity, cf. Beaupain and Durré (2012).

Detailed descriptions of the market and the corresponding network properties can be found in Finger *et al.* (2012).[8] In this paper we used all registered trades in Euro in the period from January 1999 to December 2010. For each trade we know the banks' ID numbers (not the names), their relative position (aggressor and quoter), the maturity and the transaction type (buy or sell). The majority of trades is conducted overnight and due to the GFC markets for longer maturities essentially dried up. We will focus on the overnight trades conducted on the platform. From these we take the trades conducted within the subset of Italian banks as foreign banks withdrew more or less completely from this market after 2008. Another reason to focus on the subnetwork formed by Italian banks only is that they are likely to use the e-MID market as their main source of funding over the entire period and are in general more homogenous. This leaves a total number of 1,215,759 trades.

A network is defined in the following by the binary adjacency matrix obtained from quarterly aggregated data, i.e. a link exists if at least one transaction has been taking place between the banks i and j within a quarter (the number of transaction or their volume would define the entries of a valued network data matrix). Aggregation of quarterly data had been chosen

[7]The vast majority of trades (roughly 95%) is conducted in Euro.

[8]See also the e-MID website http://www.e-mid.it/.

because many existing links might be dormant at higher frequencies, while for even higher levels of time aggregation structural changes of the network (entry and exit of participants) might interfere with the requirement of a stable structure. Finger *et al.* (2012) indeed show that a certain saturation of the density and stabilization of the structure happens at our chosen level of aggregation.

4 Assortativity Patterns in the Interbank Market

The concept of assortativity is concerned with the similarity, in terms of some attribute, of connected nodes. Here we are interested in assortative mixing by degree, i.e. how similar the degrees of connected nodes are. A network shows assortative mixing, if high-degree nodes tend to have many connections with other high-degree nodes. There exist several measures of assortativity in the literature. In the following we define the assortativity coefficient r as the Pearson correlation coefficient of degree between pairs of linked nodes, see Newman (2002). Hence, positive values of r indicate a correlation between nodes of similar degree, while negative values indicate relationships between nodes of different degrees. Thus, r lies between -1 and 1, with $r = 1$ ($r = -1$) corresponding to perfectly assortative (disassortative) mixing patterns. The assortativity coefficient is most often reported for the undirected version of a network. Empirically, social networks tend to display assortative mixing patterns, while technological and biological networks are usually characterized by disassortative mixing patterns. Interbank networks have been reported to display disassortative mixing patterns as well, so in this respect they are closer to most technological and biological networks than to most social networks.[9]

Figure 1 indicates that the Italian interbank network indeed displays disassortative mixing patterns, since we find negative values for r over the complete sample period (blue line). Note the positive trend in the coefficient over time indicates that the network evolves into a more unassortative state over time. It is not quite clear why this is the case.

We also calculated the assortativity coefficient for the directed version of the Italian interbank network.[10] The calculation works as follows: consider for example, the case where we want to calculate the assortativity coefficient for the combination of in- and out-degree of connected nodes (In-Out). For

[9] See Soramäki *et al.* (2006), Bech and Atalay (2010), and Iori *et al.* (2008).

[10] See Newman (2002) and Piraveenan *et al.* (2010) as well.

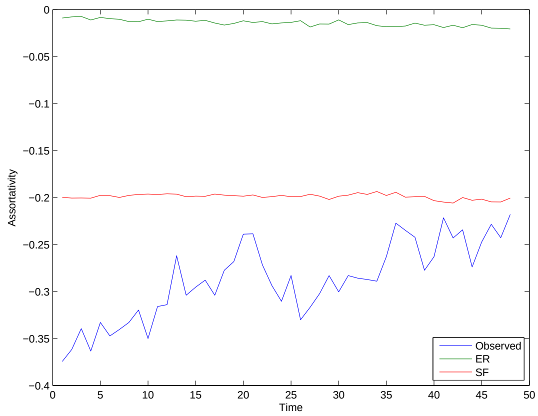

Figure 1: Assortativity coefficient r of observed (blue) and random networks for ER (green), and SF networks (red) for the undirected version of the network over time. The densities in the SF and random networks match the observed ones for the network of Italian banks. A scaling exponent of 2.3 was used for the in- and out-degrees in the SF networks. Results for random and SF networks are the average of 100 simulations.

each link we collect the in-degree for the source node and the out-degree of the target node, and then calculate the correlation between the two vectors. Figure 2 shows that the findings are comparable to the undirected case. For example, for the assortativity coefficient for the In-Out case, we see negative values for the complete sample period. Thus, it is likely for a bank with high in-degree to have outgoing links to nodes with small out-degree. Similarly, the values for the Out-In case are also negative over the complete sample period, albeit on a significantly smaller absolute level closer to the unassortative case. Thus, a bank with high out-degree may have banks of any kind of in-degree as counterparty. These findings are in line with the results in Fricke and Lux (2012), where the Italian interbank market is shown to display a hierarchical core-periphery structure. The set of highly connected core banks tends to lend money to other core banks and a large number of loosely connected periphery banks, which in turn tend to lend money to a small number of selected core banks, but appear to trade relatively scarcely among themselves. The large (absolute) values of the In-Out combination are thus an indicator of core banks (who have high in-degrees in general), to lend money to a large number of periphery banks (who have small out-degrees in general). The same is true for the Out-In combination, but on a smaller scale, given that core banks' out-degrees tend to exceed their in-degrees. The results for the In-In and Out-Out combinations are less clear, since both coefficients are close to zero over the complete sample period. The Out-Out combination even shows positive values for most of the sample period. Here we should keep in mind the small (at times even negative) correlation between single banks' in- and out-degrees. Thus, a bank with a high in-degree does not necessarily have a high out-degree, and vice versa.

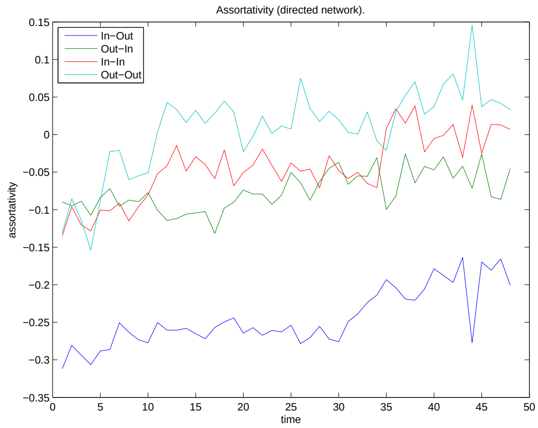

Figure 2: Assortativity coefficient over time for the directed version of the network using Italian banks only.

Overall, these findings indicate that high-degree banks tend to connect to low-degree banks and vice versa (with a certain tendency towards less disassortative mixing over time). This finding can be seen as an indicator of the tiered structure of the interbank market[11] and is also useful for understanding the spread of contagion in the interbank network.[12]

5 Assortativity in Finite-Size Scale-Free Networks

Interestingly, Newman (2002) shows that both ER networks and SF networks (based on the Barabasi-Albert, 1999, PA mechanism, i.e. with tail parameter of 3) are unassortative for large network sizes, i.e. display no correlation between the degrees of connected nodes. Since interbank networks were reported to display smaller scaling parameters (typically around 2.3), we

[11]See Fricke and Lux (2012).

[12]In assortative networks, diseases targeting high-degree individuals are likely to spread to other high-degree nodes. In disassortative networks, specifically targeted vaccination strategies for high-degree nodes can quickly stop the epidemic in the network.

checked the assortativity of synthetic SF networks in a Monte-Carlo exercise. For this purpose, we generated (directed) SF random networks using the algorithm of Goh *et al.* (2001), which allows to choose N, M, and the scaling exponent(s) α freely. We should note that this algorithm belongs to the class of static SF networks, working as follows: each of the N vertices is indexed by an integer value i ($i = 1, \cdots, N$). For each node we assign two weights, $p_i = i^{-\gamma_{\text{out}}}$ and $q_i = i^{-\gamma_{\text{in}}}$ to each vertex for outgoing and incoming edges, respectively. The control parameters γ_{out} and γ_{in} are in the interval $[0, 1)$. At each leg, we select two vertices with probability $p_i / \sum_k p_k$ and $q_j / \sum_k q_k$, and a link from i to j is added (ignoring self-links and multiple links). This repeats until the network has the desired number of links. Goh *et al.* (2001) show that the out- and in-degrees are power-law distributed with parameters $\alpha_{\text{out}} = (1 + \gamma_{\text{out}})/\gamma_{\text{out}}$ and $\alpha_{\text{in}} = (1 + \gamma_{\text{in}})/\gamma_{\text{in}}$. In the following, we only show results using the same tail exponent for both in- and out-degrees, i.e. assuming symmetric in- and out-degree distributions such that $\alpha_{\text{in}} = \alpha_{\text{out}} = \alpha$. In a first step we compare the disassortativity for the observed network of Italian banks only with synthetic ER and SF networks. For each quarter, we take the observed values of N and M, and for the SF networks we additionally assume a constant α of 2.3.

Figure 1 shows the average results for 100 random networks in each quarter. As expected the ER networks are unassortative, even though the average r is not exactly equal to zero. Quite interestingly, the SF networks yield relatively constant values for r around -.2 for all quarters. Thus, SF networks with $\alpha = 2.3$ are not too different from the observed ones in terms of disassortativity, even though the absolute level of the observed values for r is somewhat smaller.

Given that we expected the SF networks to be close to the unassortative state, we performed an additional Monte-Carlo experiment with varying density and tail exponent (while fixing $N = 100$), cf. Figure 3. Quite interestingly, we find a U-shaped relationship between the assortativity coefficient and the density, i.e. r is smallest for intermediate densities. Note that the minimum is usually found for densities around .2, which is close to the observed density of our quarterly networks. Clearly, a higher density makes interesting mixing patterns less likely, since most nodes are connected. Similarly, for low densities only few nodes are connected, resulting in small values for r. Additionally, we find that r tends to be positively related to the scaling exponent, approaching the unassortative state for larger values. However, we see that the assortativity coefficient of the synthetic SF networks with $\alpha = 3$ is not zero in general, indicating that the analytical result of Newman (2002) is only valid in the limit of very large networks (in particular with densities around 0.2). We also varied the size of the network, but the findings are remarkably robust (unreported result).[13] This shows that SF networks may indeed display disassortative mixing patterns, but the extent of their disassortativity crucially depend both on the density and the exponent.

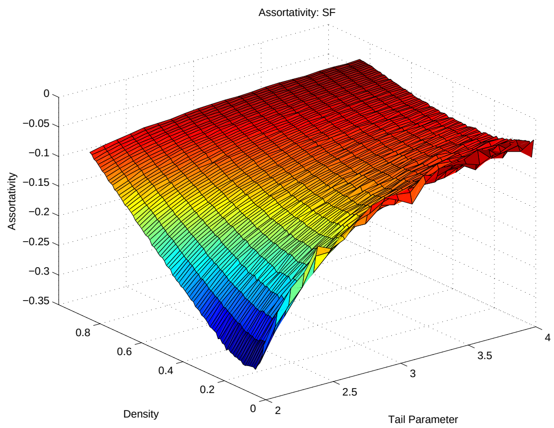

Figure 3: Assortativity coefficient r in scale-free networks with $N = 100$, varying both the density and the tail exponent. The plot shows average results from a Monte-Carlo simulation with 100 runs for each parameter setting. SF networks were generated using the algorithm of Goh *et al.* (2001).

To see how general these findings are, we performed a number of robustness checks using alternative algorithms for generating SF networks. From the class of static SF networks, we used the fitness-based algorithm of De Masi *et al.* (2006) and found qualitatively very similar results, cf. Figure 4.[14] The major difference lies in a somewhat higher level of noise, but the general shape is very similar. We also checked the results using standard PA approaches. Typically, PA networks are constructed on the basis of growing networks, since nodes are added sequentially to the network until it has the desired size. Each newly added node creates a certain (fixed) number of links to existing nodes, where high-degree nodes have a higher probability of attracting additional links. Here we used the approaches of Barabasi

[13] We found very similar results using networks with up to 2,500 nodes. The calculations are computationally too demanding for even larger networks.

[14] To be precise, we used the probability function $P(f_i, f_j) = \left(\frac{f_i}{f_{\max}}\right)^{\beta_1}\left(\frac{f_j}{f_{\max}}\right)^{\beta_2}$, where f is a 'fitness' measure for the individual banks, and $f_{\max}$ is the maximum fitness. We employed the basic parametrization of Montagna and Lux (in progress), that had been calibrated to roughly fit the link distribution of the e-MID data. Hence, f follows a power-law distribution on the interval $[5, 100]$ with scaling exponent 2. The β parameters are related to the scaling exponents of the degree distributions as follows: $\alpha_{\text{out}} = (1 + \beta_1)/\beta_1$ and $\alpha_{\text{in}} = (1 + \beta_2)/\beta_2$.

and Albert (1999) and Xie *et al.* (2008) for constructing the synthetic networks.[15] We should stress the difficulty in comparing the results to those from the static SF networks, as the tail exponent typically cannot be varied in the PA approaches. Still, we find that also the PA networks tend to be disassortative, with r approaching zero for larger densities. In contrast to the static approaches, we also found the size of the network to play a role for the structural properties of the growing network models, since larger PA networks are closer to the unassortative state for any density (unreported).

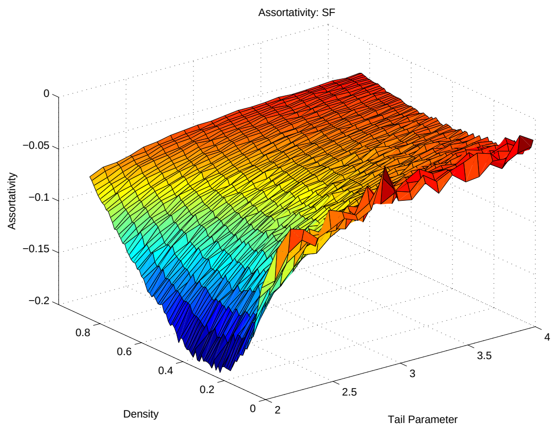

Figure 4: Assortativity coefficient r in scale-free networks with $N = 100$, varying both the density and the tail exponent. The plot shows average results from a Monte-Carlo simulation with 100 runs for each parameter setting. SF networks were generated using the fitness-based algorithm, with parameters taken from Montagna and Lux (in progress). See footnote 14 for details.

To shed light on the underlying structure of the SF networks, Figures 5 and 6 (left) show examples for adjacency matrices of scale-free networks with $\alpha = 2.0$ and 3.0, respectively, again based on the Goh *et al.* (2001) algorithm. In addition, we show their most assortative (center) and disassortative (right) counterparts, which were obtained using an adjusted version of the rewiring algorithm of Xulvi-Brunet and Sokolov (2004) for the case of directed networks.[16] We should stress that the rewiring algorithm ensures that, besides

[15]Strictly speaking, the approach of Xie *et al.* (2008) is not based on growing networks, but rewires existing links in accordance with PA.

[16]The basic idea of the algorithm is as follows: we can vary the level of assortativity in the network by rewiring observed links. At each iteration, we choose two random links connecting four unique nodes. With a certain probability we change the ends of the nodes, without changing the sources, to generate assortative or disassortative mixing patterns. In

keeping the total number of links constant, the original in-degree and out-degree sequences remain unaffected. For $\alpha = 2.0$, the left panel of Figure 5 shows a relatively dense core on the top left, which contains the most highly connected nodes, whereas moving to the bottom right shows a large number of loosely connected periphery nodes. In contrast, the center panel shows the most assortative network (after rewiring), where nodes with similar degree tend to form clusters along the main diagonal. Note that this network displays disassortative mixing patterns due to the high degree nodes that are connected with practically all other nodes, i.e. also with the low degree nodes on the bottom right. The right panel shows the most disassortative case. Again, the highly connected core persists, but the remaining links are not in clusters along the main diagonal, but rather form a concave shape starting from the bottom left to the bottom right. Given that the nodes are sorted by their degrees, this indicates that highly dissimilar nodes are connected to each other. In contrast, Figure 6 shows the results for $\alpha = 3.0$. In this case, there are fewer highly connected nodes, which allows to rewire the links into a truly assortative state (center). The right panel shows the most disassortative case, leaving only relatively few links among the most highly connected nodes, but displaying a similar value of r as for $\alpha = 2.0$. Thus, SF networks with small scaling exponents will always be disassortative, whereas this is not necessarily true for larger values of α.

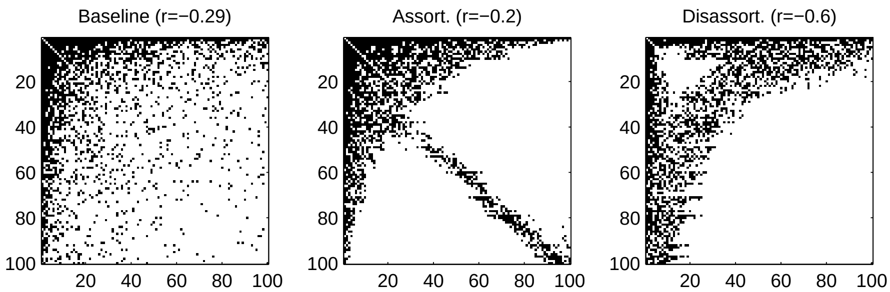

Figure 5: Network matrix for scale-free networks with $N = 100$, $M = 1,980$, $\alpha = 2.0$, and different levels of assortativity. Left: scale-free network, center: most assortative case, right: most unassortative case. Black dots indicate links. Assortativity coefficient in brackets.

the former case, we connect the higher degree nodes with each other and similarly for the lower degree nodes. In the latter case, the node with the two high-degree nodes connect to the two lower degree nodes. For a sufficient number of iterations and different rewiring probabilities, this algorithm generates networks with different levels of r.

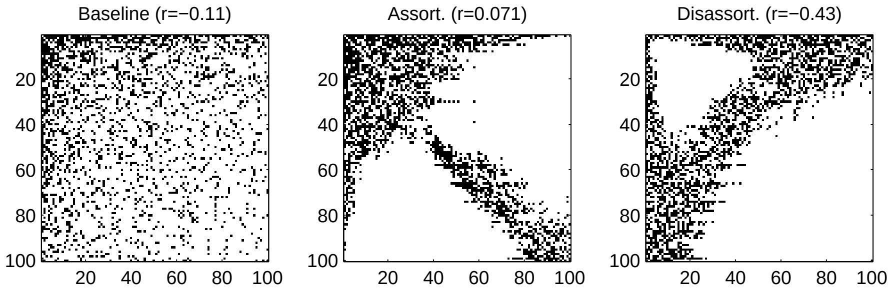

Figure 6: Network matrix for scale-free networks with $N = 100$, $M = 1,980$, $\alpha = 3.0$, and different levels of assortativity. Left: scale-free network, center: most assortative case, right: most unassortative case. Black dots indicate links. Assortativity coefficient in brackets.

Our above experiments have shown that small changes in the tail exponent can lead to substantially different mixing patterns, at least in networks of finite size. Fricke and Lux (2013) show that the scaling exponents of the degree distribution for the Italian interbank network tend to be substantially larger than those considered in this exercise (provided the data are well described by a power-law at all). With these values ($\alpha > 5$), however, it will be impossible to replicate disassortative mixing patterns. In the end, our findings show hardly any evidence in favor of practically all generating mechanisms for scale-free networks, including preferential attachment, as possible underlying models for the Italian interbank network. As this is the most popular generating mechanism for scale-free networks with power-law degree distributions, this inability to explain the financial network formation with a single mechanism should also warrant a closer look at the distributional features of the data.

6 Conclusions

This paper finds evidence for interbank networks having disassortative mixing patterns with respect to their degree, i.e. high-degree nodes tend to connect to low-degree nodes and vice versa. In the case of the Italian interbank market this result is true for directed networks and undirected networks over the whole sample period from 1999-2010. The vulnerability of disassortative networks to the failure of central nodes motivates us to search for network formation mechanism that could provide an explanation for the particular combination of features we encounter in the interbank data.[17] Such generating mechanisms could then be used for conducting realistic simulations of contagion effects via credit relationships between banks.

Our finite size Monte-Carlo simulations show that for tail exponents smaller than $\alpha = 3$, SF networks become disassortative. The level of disassortativity is in general highly sensitive to changes of the scaling exponent. In addition, the density affects the degree of assortativity, with the strongest level of disassortativity for densities around .2, which is close to the observed values of the Italian interbank network. Hence, the frequent finding of a power-law distribution of degrees and disassortativity of interbank networks would be in line with our results. However, it is not clear whether interbank networks are in fact always characterized by a power-law distribution with low α. While disassortativity and a low density seem to be uniform features of all data investigated so far, the power-law distribution of degrees seems less clear-cut. Findings of high α or inappropriateness of a power-law to describe the degree distribution (Fricke and Lux, 2013) would both imply that SF generating mechanisms are not consistent with the structure of interbank data. For a high scaling coefficient, known SF generating algorithms would not be consistent with disassortative behavior, while rejection of a scale-free distribution of the degrees would invalidate the very statistical foundation of SF networks. Therefore, our results indicate that new models of interbank networks formation have to be considered, but for that purpose a more solid behavioral foundation of the link formation between financial institutions is needed.

[17] See for more details Basel Committee on Banking Supervision (2011).

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