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by Ignat Stepanok

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JEL classification: O41, J63, F16.

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# Creative Destruction and Unemployment in an Open Economy Model*

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March 6, 2013

**Abstract**

I develop a model of endogenous economic growth and search and matching frictions in the labor market. I study the effect of trade liberalization between two identical economies on unemployment. I solve for two versions of the growth model, the first one where trade liberalization has only a temporary effect on growth, a semi-endogenous growth model. In the second version trade liberalization has a permanent effect on growth, a fully endogenous growth model. I show that in both versions trade liberalization has a steady state effect on unemployment that can be either negative or positive depending on parameters.

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# 1 Introduction

This paper studies the effect of trade liberalization on long run unemployment in a model of endogenous growth, where growth is the result of a creative destruction process. Firm turnover, job creation and job destruction are therefore endogenous. Unemployment comes from search and matching frictions in the labor market and is affected by trade openness through the economy-wide distribution of resources dedicated to R&D and production of goods for consumption. I study two versions of the growth model: in the first one trade liberalization has only a temporary effect on growth, a semi-endogenous growth model. In the second version trade liberalization has a permanent effect on growth, a fully endogenous growth model. Both versions do not exhibit scale effects, in the sense that growth is not affected by the size of the population. I find that trade liberalization has a long run effect on unemployment. I show that trade liberalization can lead to both higher or lower unemployment levels depending on parameters in both the semi-endogenous and fully-endogenous version of the model.

One can easily think of other channels connecting openness and unemployment levels: models with firm heterogeneity as in Felbermayr et. al. (2011a), with asymmetric countries which have different factor endowments as in Davidson et. al. (1999). A growth model is however particularly useful due to the endogenous firm turnover rate that is of importance for job creation and job destruction.

The empirical evidence on the long run connection between trade liberalization and aggregate unemployment is somewhat scarce. It is widely considered that trade openness has no effect on long run unemployment. Nevertheless some recent evidence shows the opposite. Felbermayr et. al. (2011b) find that greater trade openness leads to lower levels of structural unemployment. Kim (2010) finds that in the presence of rigid labor market institutions trade liberalization can lead to higher unemployment or it may reduce unemployment for economies with flexible labor markets. Dutt et. al. (2009) provide evidence that a more open trade policy leads to lower unemployment.[1]

[1] A short run connection between trade and firm turnover clearly exists with periods of trade liberalization usually being followed by a higher exit rate of firms and productivity gains due to market redistribution to the more productive firms within industries. Evidence is provided by Gibson and Harris (1996), Pavcnik (2002), Gu, Sawchuk and Rennison (2003), Trefler (2004). Firm entry and exit leads to unemployment and retraining for the workers that have to switch jobs or occupations. Trefler (2004) documents that aggregate unemployment rose in the short run as a result of NAFTA. In this version of the model I do not look at

The first model to address the question of growth and unemployment was that of Aghion and Howitt (1994). They find that within an exogenous growth model, growth increases unemployment, but the effect is non-linear, for very high values of growth unemployment decreases. After endogenizing growth, they show that unemployment is neutral to an increase in the frequency of innovation, which in itself increases steady state growth. Increasing the size of the innovation step on the other side, causes higher growth but is accompanied by higher unemployment.

Mortensen (2005) builds a Schumpeterian growth model where unemployed workers are indifferent between being unemployed or doing R&D, effectively assuming away frictions in the R&D labor market. He studies the effects of employee bargaining power, payroll taxes and employment protection policies on growth and unemployment.

Both of the above models are closed economy models and exhibit scale properties, in the sense that increasing the labor force would increase economic growth. This is not a desirable property. As shown in Jones (1995a), there has been no increase in the productivity growth of France, Germany, Japan and the US since 1950, while there has been a steady increase in their populations. A number of growth models that take this fact into consideration have been subsequently developed starting with Jones (1995b).

On the connection between growth and unemployment, there does not seem to be a consensus or a clear relationship. Both variables are endogenous and affect each other through a number of channels. A positive correlation is documented in Muscatelli and Tirelli (2001) and Caballero (1993). Bean and Pissarides (1993) look at unemployment, average labor productivity growth and total factor productivity growth. They do not find a clear and significant cross-country correlation for the OECD. In a regression of unemployment on growth of GDP again for the OECD, Aghion and Howitt (1992) find an inverse U-shaped relationship: increasing growth would increase unemployment for lower growth rates and would decrease it for the highest growth rates.

The model with international trade, growth and unemployment closest to mine is the one developed in Sener (2001). Sener introduces population growth and removes the scale property mentioned above. He assumes an exogenous duration for how long it takes firms to find workers, which yields an exogenous arrival rate of workers to vacancies. Sener also solves for a semi-endogenous and a fully endogenous growth version of the model. In both versions

transitional dynamics and therefore do not aim to contribute to the debate on the short run effect of trade on unemployment.

unemployment depends on the rate at which workers are matched to vacancies, the birth rate of people and the innovation rate. In the semi-endogenous version, unemployment of the workers subject to search and matching frictions in the labor market remains unchanged by trade liberalization in the long run. In my model with an endogenous rate of workers being matched to jobs, I find that unemployment either increase or decreases depending on parameters.

In the fully endogenous version of the model in Sener (2001) trade liberalization increases the innovation rate, which in turn increases unemployment of the workers subject to search and matching. In my model this channel is present. However due to the endogenous rate of workers finding a job, the effect on unemployment can be reversed again depending on exogenous parameters.

In the model developed in this paper economic growth is described as a process of creative destruction like in Grossman and Helpman (1991). Population grows and the size of the economy scale effect on growth is removed as in Segerstrom (1998) and in Dinopoulos and Thompson (1996). Unemployment is generated by search and matching frictions in the labor market. A departure from the standard search and matching literature is that posting a vacancy is costless, similar to Mortensen (2005). The hiring process for the firm is not costless however. The time it takes to find workers is time it could have used to produce and sell its product. In order to keep the model tractable one does have to avoid describing firms as waiting to first hire R&D workers and later workers for production. I assume that R&D is done with final consumption goods. In equilibrium unemployment depends on the innovation rate, the population growth rate and the rate at which workers find a job.

The contribution of the current paper is twofold: first, it integrates labor market frictions in the R&D sector. Sener (2001) assumes no labor market frictions in the R&D sector and in Mortensen (2005) R&D is done by people who are indifferent between remaining unemployed or doing R&D. In my model final goods are the input to the R&D process, which final goods are in turn produced by workers that are subject to search frictions when looking for a job. The second contribution lies in endogenizing the rate at which workers arrive to vacancies in a model without a growth scale effect. This yields results that differ from the ones in Sener (2001), showing that trade liberalization has an effect on steady state unemployment in a semi-endogenous growth model and that it can decrease unemployment in a fully endogenous growth model.

The next section lays out the model and shows the steady state equilibrium, section three discusses its properties, section four concludes. Some of the more involving calculations are presented in the appendix.

# 2 The Model

There are two symmetric countries Home and Foreign. Consumer preferences are represented by a Cobb-Douglas utility function. Consumers either work and receive wages or are unemployed. Labor is the only input to production and grows at a constant rate $n$. There is a continuum of products of mass one, each denoted by $\omega$. Half of the products originate from Home, the other half from Foreign. Each product has quality levels denoted by $j \in \mathbb{N}$. If $j^*$ is the state-of-the art quality, then all lower qualities are open for production by any firm in both countries. The state-of-the-art quality however is protected by a patent and can be produced only by the firm that discovered it.

A basket of all goods sold on the market serve as the input to R&D. Firms invest in discovering higher qualities of existing products. When a state-of-the-art quality of a product is discovered the firm holding the patent announces the vacancies it needs to cover demand in both Home and Foreign. Announcing vacancies is costless and no other firm can announce a vacancy for the same quality of that specific product. It takes a certain amount of time to find the workers, which time is a function of the aggregate number of vacancies announced and the number of unemployed workers available for hiring in the economy. I assume that workers that have a job do not search for one. After a firm has the workers to start producing it immediately enters both its home and the foreign market, where it sells until it gets replaced by another firm that holds a blueprint for a higher quality level of the same product and has found the workers to produce. There are iceberg trade costs $\tau > 1$ for shipping goods between countries, meaning that $\tau > 1$ units have to be produced in order for one to arrive and be sold on the foreign market.

## 2.1 Consumers

There is a fixed number of households in the economy and each household grows at a rate $n$. The number of consumers in the economy at time $t$ therefore equals $L_t = L_0 e^{nt}$, where $L_0$ is the number of consumers at the initial period. Each consumer is infinitely lived and is either unemployed, during which period she does not receive any wages, or employed and earning wages $w$. Households consist of infinitely many members, which share their income. Any uncertainty related to individual unemployment is therefore taken away. The representative household maximizes present discounted utility

$$U \equiv \int_0^\infty e^{-(\rho-n)t} \left(\ln[\upsilon_t] + u_t k\right) dt.$$

where $\rho$ is the consumer's subjective discount rate, $u_t$ is the share of unemployed people within the household and $k$ is the utility from home production or leisure. The individual's static utility defined over all available products and product qualities at time $t$ is

$$\upsilon_t \equiv \int_0^1 \sum_j \lambda^j d(j,\omega,t) d\omega \qquad (1)$$

where $d(j,\omega,t)$ is the amount consumed of product $\omega$, quality $j$ at time $t$ and $\lambda > 1$ is the step-size of each innovation.

There are three steps of optimizing consumer utility. If $p(j,\omega,t)$ is the price of quality $j$ of product $\omega$ at time $t$, then the consumer buys that quality level, which offers the lowest quality adjusted price $p(j,\omega,t)/\lambda^j$. If the quality adjusted price for two versions of the same product is the same, I assume that consumers buy the higher quality version. Let $p(\omega,t)$ be price of that quality level that offers the lowest quality adjusted price. Demand for all other quality levels is zero. The second optimization step yields demand for product $\omega$, given per capita expenditure $c_t$ at time $t$:

$$d(\omega,t) = \frac{c_t}{p(\omega,t)},$$

where $d(\omega,t)$ is demand for that quality $j$ of product $\omega$, which has the lowest quality adjusted price.

The third optimization step determines the consumer expenditure path $\dot{c}_t/c_t = r_t - \rho$, which is the familiar Euler equation. I follow Mortensen (2005) in choosing consumer expenditure to be the numeraire, thus $c_t = 1$ for all $t$, from which follows that $r_t = \rho$ for all $t$.

## 2.2 R&D Races

Firms do innovative R&D to improve on the state-of-the-art qualities of products. Home firms do not improve on Foreign originating products and vice versa. Leaders do not find it optimal to improve on their own products because they have strictly less to gain than a follower firm.

Let $I_i(\omega, t)$ be the Poisson arrival rate of improved products from follower $i$'s investment in R&D trying to improve on product $\omega$ at time $t$. A basket of goods is used as an input to the R&D process. I depart from the usual assumption that labor is the only input to R&D in order to simplify the labor market interactions. The R&D sector will still be influenced by search and matching frictions between firms and workers due to the fact that the goods needed to do R&D are produced by workers that are subject to those frictions. Let $l_i(\omega^*, \omega, t)$ denote the number of units of product $\omega^*$ used for the R&D process of firm $i$ trying to improve on the quality of product $\omega$ at time $t$. The R&D technology takes as input the goods and their quality versions in the same way as the Cobb-Douglas utility function. Higher quality products have greater significance in the R&D function. For every product $\omega$ it is the quality $j$ offering the lowest quality adjusted price that is used. In equilibrium this happens to be the highest quality available.

$$I_i(\omega, t) = a_F \frac{\int_0^1 \lambda^{j(\omega^*)} l_i(\omega^*, \omega, t) d\omega^*}{X(\omega, t) \int_0^1 \lambda^{j(\omega^*)} d\omega^*}. \tag{2}$$

The parameter $a_F > 0$ is exogenous. The variable $X(\omega, t)$ changes with time and has a specific value for every product $\omega$. If it increases, R&D becomes more costly, which means that more resources have to be invested in order to keep the arrival rate of new qualities constant. Details on the exact nature of $X(\omega, t)$ will be provided below. It suffices to say for now that it is key in removing the growth scale effect present in Aghion and Howitt (1994) and Mortensen (2005). The expression $\int_0^1 \lambda^{j(\omega^*)}$ denotes that as the average quality of products in the economy grows, R&D becomes more difficult. Due to the unit-elastic nature of the R&D technology through which individual products enter, innovating firms spend an equal amount of resources on every good at a given period $t$. An alternative and equivalent in its implications approach would be to introduce a final good as in Mortensen (2005), which would be produced by intermediate inputs. Those intermediate goods would be the set of all product varieties available and individual product qualities would matter for the productivity of the final good technology.

A number of follower firms invest in improving the existing state-of-the-art quality of a given product $\omega$. When taking into consideration all their efforts, the aggregate investment in R&D within a particular product variety $\omega$ would be $I(\omega, t) \equiv \sum_i I_i(\omega, t)$, also $l(\omega, t) = \sum_i l_i(\omega, t)$. I solve for an equilibrium where the arrival rates of new product qualities are independently distributed over time, across firms and products $\omega$, that is $I(\omega, t) = I$.

The reason to depart from the usual assumption that the input to R&D is labor is to avoid modeling several hiring stages for a firm. If one models the search first for finding R&D workers and then production workers the setup becomes much more involving. Mortensen (2005) simplifies by making the unemployed indifferent between staying unemployed or doing R&D, thus making employment in research yield strictly lower return in comparison to production. Sener (2001) simplifies by dividing the labor force into skilled and unskilled workers. The former are the ones doing R&D and are hired in a frictionless market, thus not suffering from unemployment throughout their lifetime. He cites empirical evidence showing that skilled workers suffer from substantially lower unemployment rates relative to unskilled workers: Nickel and Bell (1995). The only workers subject to the search and matching frictions and therefore to unemployment are unskilled workers. Although leading to greater tractability of the model, by assuming no labor market frictions for the skilled workers, one may overstate the incentive of the economy to redirect resources towards the R&D process and lead either to a higher steady state or transitional growth rate, depending on the assumptions on the R&D function employed.

In my model, using goods as input to the R&D sector indirectly makes that sector dependent on labor market frictions. The goods used for R&D are produced by workers who are not hired immediately and face the risk of being unemployed.

## 2.3 Product Markets

The production technology has constant returns to scale, it takes one unit of labor to produce one unit of any quality level of any product. The marginal cost of producing one unit is therefore equal to the wage rate $w$. Firms set prices, where the optimal price both at home and abroad is the limit price $p = \lambda w_{CF}$, where $w_{CF}$ is the wage a hypothetical competitive fringe firm would pay its workers. Due to the fact that the competitive fringe firm would be selling at marginal cost, $w_{CF}$ would also be the price at which it would sell if it were to operate. Note that $w \neq w_{CF}$. I assume that it is costless to post a vacancy and that how to produce the one step lower quality of the product is common knowledge. Given those two assumptions, the quality leader would not price above $\lambda w_{CF}$. If the price were to be higher, infinitely many firms can announce a vacancy to produce the one step lower quality of the same product. One of those vacancies would be filled immediately and by pricing at marginal cost $w_{CF}$ the competitive fringe firm could sell at a lower quality adjusted price than the leader, thus stealing away the entire market. Since quality leaders will price so that they would keep the entire market, in equilibrium there will be no competitive fringe firms selling and every product will be sold in its state-of-the-art version by the same quality leader in both markets Home and Foreign.

Given the identical prices of all goods, and the fact that R&D firms spend an equal amount of resources on every good, it follows that they purchase identical amounts of every good $\omega$ available on the market. Using this, I can rewrite the numerator of the R&D technology in (2) as $l_i(\omega,t)\int_0^1 \lambda^{j(w^*)}$ and reduce equation (2) to the following:

$$I_i = \frac{a_F l_i(\omega,t)}{X(\omega,t)},$$

where $l_i(\omega,t)$ denote the units of that basket, consisting of an equal amount of every product (its highest quality) available on the market, that are invested in the R&D process for improving on the quality of good $\omega$ at time $t$.

Profits of a producer are determined by what it sells at home and abroad. In order to export a firm needs to ship $\tau > 1$ units of a good to sell one unit abroad. Demand for a product is determined by what is being sold and used for consumption, $d(\omega,t)L_t$ and what is being sold and used for R&D, $l(\omega,t) = \int_0^1 l_i(\omega,\omega^*,t)d\omega^*$. Profits from selling at home and abroad after substituting for total demand are

$$\pi(\omega,t) = (2\lambda w_{CF} - \tau w - w)\left(\frac{1}{\lambda w_{CF}} + \frac{Ix_t}{a_F}\right)L_t, \tag{3}$$

where $x_t \equiv X_t/L_t$ denotes per capita R&D difficulty. Expression (3) takes into consideration not only demand for consumption of good $\omega$, which equals $\frac{1}{\lambda w_{CF}}$ (keeping in mind that per capita expenditure is the numeraire $c = 1$ and is constant in steady state equilibrium) but also the amount of that good that goes into the R&D process of all innovating firms $\frac{Ix_t}{a_F}$. Since prices both at home and abroad are the same, this combined demand is identical both at home and abroad. Summing the markups at home $\lambda w_{CF} - w$ and abroad $\lambda w_{CF} - \tau w$ yields $2\lambda w_{CF} - \tau w - w$ in the brackets above.

## 2.4 Value Functions and the R&D Equation

Follower firms make a decision on how much to invest in learning how to produce the next quality level of a product $\omega$. The investment in learning how to produce the next quality level

$j$, starts immediately after quality level $j-1$ has been discovered. The Bellman equation of a follower that does R&D is

$$rv_F(j) = \max_{l_i} - \lambda w_{CF} l_i + I_i v_L(j+1), \tag{4}$$

where $v_L$ is the value of a firm that holds a blueprint for a state-of-the-art product, but has not hired workers to start producing yet. $l_i$ is the optimal number of units of every product $\omega$ the follower firm $i$ will invest in the R&D process, $\lambda w_{CF}$ is the price of those products and $I_i$ the instantaneous probability with which it discovers the state-of-the-art quality. In the analysis below, firm values and profits are all function of $j$, which I will omit for brevity.

The new local technological leader has to find workers in order to be able to produce and start selling. Until this happens the old incumbent continues producing. The larger the demand for the product, the higher the number of workers needed to be hired. Firms know exactly how much labor they would need in order to cover demand. The value of the technological leader searching for workers to start production is

$$v_L = e^{-r/q} \left( \int_0^\infty Ie^{-I\eta} \int_0^\eta \pi(\omega, t) e^{-(r-n)s} ds d\eta \right). \tag{5}$$

For now it suffices to say that $1/q$ signifies the duration required for a vacancy to be filled. For simplicity I will treat it as a fixed endogenous variable. With an instantaneous probability $I$, the firm loses its position of a technological leader and $\eta$ is the duration of the incumbency of an innovation, where $\eta$ is drawn from a distribution with a probability density function equal to $Ie^{-I\eta}$. The newest quality level of the product will not be sold for a period $1/q$, since the new leader will also have to search for workers first.

The value of a follower firm $v_F$ equals zero. From (4) I obtain

$$v_L = \frac{X(\omega, t)}{\lambda w_{CF} a_F}.$$

This expression determines the value of the investment in R&D for a follower firm. It must equal the expected gain from holding a patent for a state-of-the-art quality expressed in (5). When I solve the integral in (5) and substitute for profits from (3), I obtain:

$$\frac{x}{\lambda w_{CF} a_F} = e^{-r/q} \frac{2\lambda w_{CF} - \tau w - w}{r + I - n} \left( \frac{1}{\lambda w_{CF}} + \frac{Ix}{a_F} \right). \tag{6}$$

Equation (6) is the R&D equation and is one of the key equations in the model. For it to hold in steady state it must be the case that $x_t = x$ for all $t$. The R&D equation describes the incentive of firms to do R&D. More research leads to a higher relative R&D difficulty $x$. A higher level of R&D investments (left-hand-side of the equation) would be justifiable only in the presence of higher demand, which consists of demand for both consumption and R&D (right-hand side of the equation).

## 2.5 Finding the Labor Equation

Workers are either employed in production or unemployed. The workers employed in production service the demand for consumption goods and for goods used in the R&D process. Since both consumption and R&D make use of the full array of goods available in a given market, every good produced at Home will cover demand both at Home and abroad. Total labor used for production is

$$L_{Pt} = \frac{1+\tau}{2}\left(\frac{1}{\lambda w_{CF}} + \frac{Ix_t}{a_F}\right)L_t.$$

If $u_t$ is the number of unemployed workers, then the following must hold $(1-u_t)L_t = L_{Pt}$. This equation means that everyone who is employed must be working in the production sector. Substituting for $L_{Pt}$ and dividing both sides by $L_t$ yields

$$1 - u = \frac{1+\tau}{2}\left(\frac{1}{\lambda w_{CF}} + \frac{Ix}{a_F}\right). \tag{7}$$

This is the labor equation. I know that $x$ is constant in steady state. For the labor equation to hold in steady state it must be the case that $u$ is also constant, which is why I drop the time subscript. The intuition behind the labor equation is very straightforward. All employed resources on the left-hand side must be divided between production intended for consumption $\frac{1+\tau}{2}\frac{1}{\lambda w}$ and production intended for R&D $\frac{1+\tau}{2}\frac{Ix}{a_F}$. Higher consumption would mean less resources devoted to R&D, which translates into a lower R&D difficulty parameter $x$.

## 2.6 Unemployment

What remains is to find the number of vacancies and unemployed people. I will use a standard constant returns to scale matching function $m_t(U_t, V_t) = U_t^{\gamma}V_t^{1-\gamma}$ that depends on the number of unemployed people $U_t = uL_t$ and the number of announced vacancies $V_t = vL_t$. Once a worker finds a job he or she does not continue to search, there is no on-the-job search. I define $q \equiv m(U_t, V_t)/V_t$ to be the rate at which a vacancy is filled and $p \equiv m(U_t, V_t)/U_t$ the rate at which a worker finds a job. For simplicity I assume that the times it takes to fill a vacancy $1/q$ and for a worker to be matched with a job $1/p$ are deterministic.

The flow of vacancies is determined by several factors. First it is the announcements of technological leaders that have just discovered a state-of-the-art product. They only have a blueprint for a product, but no workers to produce. The arrival rate of those announcements is equal to the arrival rate of innovations $I$. Those vacancies announced at time $t$ will have to cover demand at home and abroad at time $t+1/q$. Due to growing population demand for a product grows constantly at a rate $n$. Firms are able to anticipate this increase and timely announce the vacancies in order to be able to cover the increase in demand, a period $1/q$ from the time of the announcement. Lastly, the pool of vacancies is reduced by the matches between vacancies and unemployed workers.

$$\begin{aligned} \dot{V}_t &= I\frac{1+\tau}{2}\left(\frac{1}{\lambda w_{CF}} + \frac{Ix}{a_F}\right) L_{t+1/q} \\ &\quad + n\frac{1+\tau}{2}\left(\frac{1}{\lambda w_{CF}} + \frac{Ix}{a_F}\right) L_{t+1/q} - m(U_t, V_t). \end{aligned}$$

The dynamics of the number of unemployed people is characterized by the following equation

$$\dot{U}_t = \dot{L}_t - m(U_t, V_t) + I\frac{1+\tau}{2}\left(\frac{1}{\lambda w_{CF}} + \frac{Ix}{a_F}\right) L_t.$$

Newborn enter the pool of the unemployed immediately. The number of matches announced at time $t-1/q$ reduces unemployment at time $t$. At the rate $I$ local leaders go out of business, which means unemployment for all their workers. This last inflow into unemployment is due to innovations that happened a period $1/q$ earlier.

I divide by $L_t$ and use $m(U_t, V_t) = pU_t$. With the help of the labor equation (7) I rewrite per capita production as $1-u$ to obtain in the end an expression for unemployment:

$$u = \frac{n+I}{p+n+I}. \tag{8}$$

This is a key equation in the model that determines the rate of unemployment in the economy. Aghion and Howitt (1994) find that in their model of endogenous growth, the frequency of innovations is neutral to unemployment and that the size of the innovation step is positively related to unemployment. On the contrary, I find that the frequency of innovation $I$ is positively related to unemployment and the innovation step is neutral. This is similar to what is found in Sener (2001).

Using the definition of the matching function I can write $v = uq^{-\frac{1}{\gamma}}$. I derive an expression for $v$ from the equation for the evolution of vacancies using also (7). I substitute that in $v = uq^{-\frac{1}{\gamma}}$ and for $u$ from (8) to find $p = q^{1-\frac{1}{\gamma}}$.

## 2.7 The Wage Bargaining Problem

In order to find wages $w$, I start from the bargaining problem for a competitive fringe firm. Although in steady state equilibrium there will be no operational competitive fringe firms it is their potential entry and pricing behavior that influences the price that quality leaders charge. Everyone knows how to produce qualities of products lower than the state-of-the-art. Let us denote the price charged by a competitive fringe firm by $w^*_{CF}$. Therefore if a quality leader chooses to raise its price above $\lambda w^*_{CF}$ a home competitive fringe firm can enter and cover the home market, while another foreign one would enter and cover the foreign market by selling at a lower quality adjusted price than the market leader. Both competitive fringe firms would price at marginal cost $w^*_{CF}$.

Let the bargaining power of a firm be $0 < \beta < 1$ and that of a worker be $1 - \beta$. This is a noncooperative bargaining game of the Rubinstein type (described in Binmore et. al. 1986), where the parties do not search while negotiating. The firm would charge $w^*_{CF}$. If trying to price higher another competitive fringe firm can enter. The worker would gain her wage and would enjoy the value of leisure $k$ if the bargaining process is protracted:

$$w_{CF} = \arg\max(w^*_{CF} - w_{CF})^{\beta}(w_{CF} - k)^{1-\beta}.$$

The solution yields $w_{CF} = k$. This is how much the competitive fringe would pay its workers if it were to enter. This is also the price it would charge.

With this information in mind I move on to the bargaining problem of a quality leader firm with its workers. Let the wage paid by a quality leader be denoted by $w$. The bargaining power parameter $\beta$ remains the same. The bargaining game of the Rubinstein type is defined by the following maximization problem

$$w = \arg\max \left(2\lambda k - (\tau + 1)w\right)^{\beta} (w - k)^{1-\beta}.$$

Both parties do not search while negotiating. Protracting the bargaining process has no value for the firm, while the potential worker can receive their value of leisure $k$. Solving yields the wage:

$$w = (1-\beta)\frac{2\lambda k}{\tau+1} + \beta k. \tag{9}$$

This equation pins down the wage as a function of exogenous variables only. It is clear that trade liberalization $\tau \downarrow$ increases the wage $w \uparrow$, which occurs due to the increasing profits that can be shared after shipping goods to the other country becomes cheaper.

# 3 The Steady State Equilibrium and Its Properties

Before defining the equilibrium I need to define the equation which defines the evolution of R&D difficulty $X(\omega, t)$. Following Sener (2001), I will explore two versions of the evolution of R&D difficulty $X(\omega, t)$. In the first one it depends on the innovation rate $I$ and leads to a so called TEG model, where tariffs have a temporary effect on growth, as introduced in Segerstrom (1998). In the second version R&D difficulty depends on population size and gives rise to a model where tariffs have a permanent effect on growth (PEG). It was introduced in Dinopoulos and Thompson (1996). Both of those versions of the R&D technology take care of the population scale effect and allow for a balanced growth path in an economy with a growing population. This happens due to the steadily increasing R&D difficulty $X(\omega, t)$, which means that more resources have to be dedicated to R&D in order to preserve the same product quality innovation rate $I$ in steady state.

To calculate the economy's steady state growth rate I use the expression for the individual's static utility (1) and substitute for demand. Differentiating with respect to time and solving the integral yields the economic growth rate:

$$g \equiv \frac{\dot{\upsilon}_t}{\upsilon_t} = I \ln \lambda.$$

It increases in the rate of innovation $I$ and the step-size of each innovation $\lambda$.

## 3.1 The TEG Version of the Model

In the TEG version of the model, the R&D difficulty parameter evolves according to the following process $\frac{\dot{X}(\omega,t)}{X(\omega,t)} = \mu I(\omega, t)$. The parameter $\mu > 0$ is exogenous and as previously mentioned, in equilibrium $I(\omega, t)$ is the same for every $\omega$. This technology has the property that R&D becomes increasingly more difficult as more R&D is done on a product. From the fact that relative R&D difficulty $x$ is constant in steady state, it follows that $\frac{\dot{X}(\omega,t)}{X(\omega,t)} = \frac{\dot{L}_t}{L_t} = n$.

This also means that the innovation rate is pinned down by the population growth rate and the R&D difficulty parameter $I = \frac{n}{\mu}$.

The steady state equilibrium in this model is defined by the set ($x$, $w$, $u$, $q$), which can be solved for, using the R&D equation (6), the labor equation (7), the expression for unemployment (8) and the equation that pins down the wage (9).

I substitute for the wage from (9) in the R&D equation (6) and then for $\frac{1}{\lambda k} + \frac{Ix}{a_F}$ from the labor equation (7) to obtain an expression for $x$. I then substitute this expression for $x$ back into the labor equation to obtain an equation in one unknown $q$. I am able to show that if

$$\frac{I\lambda e^{-r/q}\beta k^2}{r+I-n}2\lambda > 1 \tag{10}$$

holds, then trade liberalization $\tau\downarrow$ leads to a lower rate at which workers arrive to vacancies $q$, which in turn means a higher steady state value of $p$. If (10) des not hold then the effect on $p$ is reversed. Inequality (10) is clearly not a simple expression in exogenous variables only. The arrival rate of workers to vacancies $q$ can be solved for numerically.

Turning to the unemployment equation (8), it is clear that the unemployment rate $u$ is inversely related to $p$ and any changes to $u$ would come from changes in $p$, given an innovation rate $I$ that is constant in steady state and an exogenous population growth rate $n$. The steady state growth rate $g$ remains unaffected by trade liberalization as is standard for this type of an endogenous growth model. I summarize the results in the following proposition:

**Proposition 1** *In the TEG model trade liberalization represented by a bilateral reduction in the iceberg trade cost $\tau\downarrow$:*

*a) increases the rate at which workers find a job $p\uparrow$ if (10) holds and decreases it otherwise.*

*b) reduces unemployment $u\downarrow$ if (10) holds and increases it otherwise.*

*c) has no effect on the rate of innovation $I$ or on the long run economic growth rate $g$.*

It is interesting to contrast the result on unemployment $u$ with the one in Sener (2001). Sener separates labor into two categories, skilled and unskilled. A simplifying assumption is that there are no search frictions for the skilled workers and they are the ones that do R&D. Due to the exogenous labor market tightness parameter, $p$ and $q$ are constants. The unemployment rate of the unskilled workers that are subject to search and matching frictions in their labor market is therefore independent of $\tau$ in a TEG world.

In Sener (2001) aggregate unemployment, which consists of the number of unskilled unemployed workers divided by all workers, skilled and unskilled, decreases as a result of trade liberalization. This happens because more workers choose to become skilled and in steady state they are not subject to search and matching frictions. In a model with labor market frictions in both the production and R&D sector and with an exogenous labor market tightness parameter, unemployment would not respond to changes in $\tau$ however. Endogenizing labor market tightness gives the long run effect of trade liberalization on unemployment.

To illustrate the results in the above proposition I solve the model numerically as well. The benchmark parameters I use are the following: $\rho = 0.04$, $n = 0.018$, $\lambda = 1.4$, $\gamma = 0.5$, $\beta = 0.5$, $a_F = 1$, $\mu = 0.5$, $k = 1.1$ and $\tau$ changes from 1.3 to 1. The discount rate $\rho$ is set to match the 4% real interest rate as in McGrattan and Prescott (2005). According to Kremer (1993) the world population growth rate during the 1980 was 1.8%. The markup of price over marginal cost from selling at home $\lambda$, is 40%. The markup for selling abroad $\lambda/\tau$ is between 7% and 40%, which are numbers within the range reported in Morrison (1990). Regardless of the choice of $k$, a consumer would always strictly prefer to work than to enjoy home production because the endogenous wage is always higher than the value of leisure $w > k$, which can be seen in the equation for the wage (9) since $\frac{2\lambda k}{\tau+1} > 1$. The choice of $k$ has to also be such that the labor equation (7) holds. Roughly speaking, the left-hand side can not exceed one, since $0 < u < 1$. Given a world in which $\tau = 1$ aggregate demand for a product can not exceed one, i.e. $\frac{1}{\lambda k} + \frac{Ix}{a_F} < 1$, which means that $k > \frac{1}{\lambda}$. The estimate of year 2000 tariff-equivalent costs to trade between the US and Canada was 25% and between the US and Mexico 33% according to Novy (2011). I therefore solve the model for a change in variable trade costs from $\tau = 1.3$ to $\tau = 1$. Given the choice of the bargaining parameter $\beta$ the left-hand side of (10) varies between 1.329 for $\tau = 1.3$ and 1.203 for $\tau = 1$. Given the chosen parameters inequality (10) holds, and we would expect for trade liberalization to lead to an equilibrium where workers find jobs faster (higher $p$) and unemployment is lower.

| $\beta = 0.5$ | $x$ | $I$ | $w$ | $p$ | $u$ | $v$ | $g$ |
|---|---|---|---|---|---|---|---|
| $\tau = 1.3$ | 5.61 | 0.036 | 1.21 | 2.54 | 0.020 | 0.13 | 0.012 |
| $\tau = 1.2$ | 6.82 | 0.036 | 1.25 | 3.46 | 0.015 | 0.18 | 0.012 |
| $\tau = 1.1$ | 8.09 | 0.036 | 1.28 | 4.31 | 0.012 | 0.23 | 0.012 |
| $\tau = 1.0$ | 9.44 | 0.036 | 1.32 | 5.04 | 0.010 | 0.26 | 0.012 |

Table 1.

This is indeed what happens, unemployment decreases from 2% for $\tau = 1.3$ to 1% for the free trade case $\tau = 1$. The innovation rate $I$ and the growth rate $g$ remain constant in steady state, however the economy shifts resources to R&D on the transitional path because in steady state relative R&D difficulty $x$ is higher. This means that in equilibrium more of the economy's resources are dedicated to the R&D sector.

To show the case of increasing unemployment as a result of trade liberalization, I change the bargaining parameter $\beta$ from 0.5 to 0.1. The left-hand side of (10) varies between 0.2921 for $\tau = 1.3$ and 0.2929 for $\tau = 1$, which given (10) would predict higher unemployment in a more open economy.

| $\beta = 0.1$ | $x$ | $I$ | $w$ | $p$ | $u$ | $v$ | $g$ |
|---|---|---|---|---|---|---|---|
| $\tau = 1.3$ | 0.99 | 0.036 | 1.31 | 0.20 | 0.21 | 0.008 | 0.012 |
| $\tau = 1.2$ | 1.20 | 0.036 | 1.37 | 0.17 | 0.23 | 0.007 | 0.012 |
| $\tau = 1.1$ | 1.42 | 0.036 | 1.43 | 0.15 | 0.26 | 0.006 | 0.012 |
| $\tau = 1.0$ | 1.64 | 0.036 | 1.49 | 0.13 | 0.29 | 0.005 | 0.012 |

Table 2.

In addition to the increasing unemployment brought about by the lower rate at which unemployed workers are matched with jobs $p$, trade liberalization increases relative R&D difficulty $x$ as in the previous parameterization. Wages unambiguously increase as previously found.

## 3.2 The PEG Version of the Model:

The PEG version of the model is based on the following function for the evolution of R&D difficulty: $X(\omega, t) = mL_t$. The parameter $m > 0$ determines how much more costly it becomes to do R&D with a growing population. It is now the size of the population as opposed to the innovation rate as in the TEG version of the model that determines R&D difficulty. This changes the implications of the model significantly. Relative R&D difficulty $x \equiv X_t/L_t = m$ is immediately pinned down in steady state equilibrium. The equilibrium is defined by the set $(I, w, u, q)$. Given that the economic growth rate $g$ depends on $I$, it would be affected by changes in the exogenous policy variables as well. Trade has a permanent effect on steady state growth. I solve using the same equations: the R&D equation (6), the labor equation (7), the unemployment equation (8) and the equation that determines the wage (9).

I substitute for unemployment from the unemployment equation (8) into the labor equation (7) and solve for $p$. I then substitute for $x = m$ in the R&D equation (6) and solve for $I$. The expression for $I$ depends on $q$ which I can substitute for using $p = q^{1-\frac{1}{\gamma}}$ and the previously found expression for $p$, which is a function of one other endogenous variable, the innovation rate $I$. That way I arrive at one equation in one unknown $I$. I am able to show that $\frac{\partial I}{\partial \tau} < 0$, which means that trade liberalization $\tau \downarrow$ leads to a higher innovation rate $I \uparrow$. The details are spelled out in the appendix.

It is clear from the unemployment equation (8) that the effect of trade liberalization on unemployment will depend on the changes in $p$ and the innovation rate $I$. Since the innovation rate always increases with trade liberalization, unemployment will increase through that channel. The higher firm turnover rate leads to more layoffs and thus more frequent vacancy announcements, which need time to be filled. In order to know the final effect of trade liberalization on steady state unemployment however, one needs to know not only the sign but also the magnitudes of $\frac{\partial I}{\partial \tau}$ and $\frac{\partial p}{\partial \tau}$. The exercise of finding them and the sign of $\frac{\partial p}{\partial \tau}$ is not tractable, which is why in order to show how trade liberalization affects unemployment I solve the model numerically. For now I summarize the analytical results in the following proposition:

**Proposition 2** *In the PEG model trade liberalization represented by a bilateral reduction in the iceberg trade cost $\tau \downarrow$ leads to:*

*a) an unchanged relative R&D difficulty $x$.*

*b) a higher rate of innovation $I \uparrow$ and a higher long run economic growth rate $g \uparrow$.*

The benchmark parameters I use are the same as the ones used in the numerical simulation of the TEG version of the model with a few exceptions: $m = 0.5$ instead of $\mu = 0.5$, where $m$ is the new variable of the PEG R&D difficulty process but with a similar meaning as $\mu$. To be consistent with the evidence presented in Jones (2005) where the average US GDP per capita growth rate for the period 1950 to 1994 is reported to be 2%, I set the technology parameter $a_F = 0.2$ and the bargaining parameter $\beta = 0.3$. I again solve for a change in $\tau$

from 1.3 to 1. The growth rate $g = I \ln \lambda$ is between 1.8% for $\tau = 1.3$ and 3.9% for $\tau = 1$.

| $\beta = 0.3$ | $I$ | $w$ | $q$ | $p$ | $u$ | $v$ | $g$ |
|---|---|---|---|---|---|---|---|
| $\tau = 1.3$ | 0.05 | 1.26 | 1.378 | 0.72 | 0.092 | 0.048 | 0.018 |
| $\tau = 1.2$ | 0.07 | 1.31 | 0.891 | 1.12 | 0.077 | 0.097 | 0.025 |
| $\tau = 1.1$ | 0.09 | 1.35 | 0.602 | 1.66 | 0.064 | 0.178 | 0.032 |
| $\tau = 1.0$ | 0.11 | 1.40 | 0.432 | 2.31 | 0.055 | 0.296 | 0.039 |

Table 3.

As expected trade liberalization leads to a higher arrival rate of innovations $I$. As a result of this the growth rate increases. The unemployment rate increases through that channel as well. At the same time however, the rate at which unemployed people find a job $p$ increases strongly enough to overturn the effect of the higher innovation rate and leads to an overall lower rate of unemployment decreasing from 9.2% at $\tau = 1.3$ to 5.5% at $\tau = 1$.

Reducing the bargaining power of firms to $\beta = 0.2$ changes the relation between $\tau$ and unemployment $u$.

| $\beta = 0.2$ | $I$ | $w$ | $q$ | $p$ | $u$ | $v$ | $g$ |
|---|---|---|---|---|---|---|---|
| $\tau = 1.3$ | 0.026 | 1.29 | 4.93 | 0.20 | 0.178 | 0.007 | 0.008 |
| $\tau = 1.2$ | 0.037 | 1.34 | 3.96 | 0.25 | 0.181 | 0.011 | 0.012 |
| $\tau = 1.1$ | 0.050 | 1.39 | 3.30 | 0.30 | 0.185 | 0.017 | 0.017 |
| $\tau = 1.0$ | 0.064 | 1.45 | 2.81 | 0.35 | 0.188 | 0.023 | 0.021 |

Table 4.

As previously, the innovation rate $I$ and the wage $w$ are higher in a more open economy. The rate at which unemployed people are matched to vacancies $p$ also increases but this time not sufficiently to overturn the effect of the innovation rate. Unemployment as a result of this increases. It is clear from the expression for steady state growth $g = I \ln \lambda$ that a higher innovation rate results in higher growth.

In a model with an exogenous labor market tightness and therefore an exogenous arrival rate of workers to vacancies as in Sener (2001), the unemployment rate would respond to trade liberalization only through the innovation rate, since it is only the innovation rate that responds to changes in $\tau$, see equation (8). Given an increasing innovation rate unemployment would unambiguously increase as is the case in Sener (2001) for unskilled workers in whose labor market there are search and matching frictions.[2] I show however that endogenizing the rate at which workers find a job $p$ can reverse the effect of trade liberalization on unemployment.

# 4 Conclusion

This paper builds a model of endogenous growth where newer qualities of products replace old ones and make them obsolete. The process of creative destruction introduces an endogenous entry and exit rate of firms that have to search for workers before they can start operating. R&D is performed using a basket of all consumption goods available within the economy, thus incorporating the labor market frictions into the R&D sector as well. I solve for two versions of the evolution of the R&D difficulty process. The first one, where R&D difficulty increases depending on the innovation rate, yields a semi-endogenous growth model without scale effects where trade liberalization has no permanent effect on steady state growth. Unemployment however is affected in steady state by trade liberalization. It can both increase or decrease depending on parameter values. In a model with an exogenous labor market tightness, steady state unemployment would remain unaffected by openness.

In a second version of the evolution of R&D difficulty, where it depends on the level of population, the growth rate is affected by openness in steady state. This yields a so-called fully endogenous growth model also without a scale effect. Openness makes the economy switch resources towards the R&D sector and leads to a higher innovation rate. This in turn increases steady state unemployment. Due to the higher relative number of vacancies however people tend to find work faster, which in turn decreases unemployment. The strength of the two channels and the net effect of openness on unemployment depends on parameters of the model.

[2] In Sener (2001) due to the presence of skilled workers and the fact that they do not suffer unemployment, a more open economy which tends to shift resources towards the R&D sector, would mean that fewer people are subject to search and matching frictions and aggregate unemployment defined by the number of the unskilled unemlpoyed divided by the sum of all skilled and unskilled workers might decrease depending on parameters of the model.

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# Appendix

# The Wage Bargaining

The quality leader firm's bargaining problem with workers is defined by:

$$w = \arg\max \left(2\lambda k - (\tau+1)w\right)^{\beta} \left(w-k\right)^{1-\beta}.$$

Solving yields

$$\begin{aligned}
(\tau+1)\beta \left(2\lambda k - (\tau+1)w\right)^{\beta-1} \left(w-k\right)^{1-\beta} &= (1-\beta)\left(w-k\right)^{-\beta} \left(2\lambda k - (\tau+1)w\right)^{\beta} \\
(\tau+1)\beta \left(w-k\right) &= (1-\beta)2\lambda k - (1-\beta)(\tau+1)w \\
w &= (1-\beta)\frac{2\lambda k}{\tau+1} + \beta k.
\end{aligned}$$

## The TEG Version of the Model

I rewrite the labor equation (7) as

$$(1-u)\frac{2}{1+\tau} = \frac{1}{\lambda k} + \frac{Ix}{a_F}$$

I substitute for the wage from from (9) in the R&D equation (6):

$$\frac{x}{\lambda k a_F} = e^{-r/q}\frac{\beta 2\lambda k - \beta k(\tau+1)}{r+I-n}\left(\frac{1}{\lambda k} + \frac{Ix}{a_F}\right)$$

and then for $\frac{1}{\lambda k} + \frac{Ix}{a_F}$ from the labor equation above to obtain:

$$x = \lambda k a_F e^{-r/q}\beta k\frac{2\lambda - \tau - 1}{r+I-n}2\frac{1-u}{1+\tau}.$$

I use that expression for $x$ in the labor equation (7) to obtain:

$$\left(1 - \frac{n+I}{q^{1-\frac{1}{\gamma}}+n+I}\right)\left(\frac{1}{1+\tau} - \frac{I\lambda e^{-r/q}\beta k^2}{r+I-n}\frac{2\lambda-\tau-1}{1+\tau}\right) = \frac{1}{2\lambda k}, \qquad (11)$$

where I have substituted for unemployment from (8) and then for $p = q^{1-\frac{1}{\gamma}}$ to express

$$1-u = 1 - \frac{n+I}{q^{1-\frac{1}{\gamma}}+n+I}.$$

Clearly the left-hand side of (11) decreases as $q$ increases.

I am interested in how the left-hand side of (11) changes as a results of a lowering in $\tau$. Since only the expression in the second brackets contains $\tau$, let it be denoted by $B$ and for brevity let also $A \equiv \frac{I\lambda e^{-r/q}\beta k^2}{r+I-n}$. Then,

$$\begin{aligned}
\frac{\partial B}{\partial \tau} &= -\frac{1}{(1+\tau)^2} - A\left(\frac{-(1+\tau)-2\lambda+\tau+1}{(1+\tau)^2}\right) \\
&= \frac{A2\lambda - 1}{(1+\tau)^2}.
\end{aligned}$$

The denominator is positive. The numerator is positive if $A2\lambda > 1$ or after substituting for $A$:

$$\frac{I\lambda e^{-r/q}\beta k^2}{r+I-n} 2\lambda > 1.$$

If the above inequality holds, then $\frac{\partial B}{\partial \tau} > 0$.

The right-hand side of (11) remains unchanged as a result of a lower $\tau$, while the left-hand side decreases if $\frac{\partial B}{\partial \tau} > 0$. This can be offset only through a falling $q$ which increases the left-hand side. From this follows that trade liberalization, given that $A2\lambda > 1$ holds, leads to a lower $q$. This in turn means that $p$ increases.

If $A2\lambda < 1$, then $\frac{\partial B}{\partial \tau} < 0$, which would mean that the left-hand side of (11) increases when $\tau$ decreases. This can be offset by a rising $q$, which would decrease the left-hand side. This in turn would mean that $p$ would decrease.

## The PEG Version of the Model

In this section I show how I find the sign of $\frac{\partial I}{\partial \tau}$. In the labor equation (7) I substitute for unemployment and solve for $p$:

$$\begin{aligned}
\frac{p}{p+n+I} &= \frac{1+\tau}{2}\left(\frac{1}{\lambda k} + \frac{Im}{a_F}\right) \\
p &= \frac{\frac{1+\tau}{2}\left(\frac{1}{\lambda k} + \frac{Im}{a_F}\right)}{1 - \frac{1+\tau}{2}\left(\frac{1}{\lambda k} + \frac{Im}{a_F}\right)}(n+I) \\
p &= \frac{n+I}{\frac{2}{1+\tau}\left(\frac{1}{\lambda k} + \frac{Im}{a_F}\right)^{-1} - 1}.
\end{aligned}$$

Keeping in mind that $p = q^{1-\frac{1}{\gamma}}$, I can write $e^{-r/q} = z(I, \tau)$, where I substitute for $q$ with $p$, which equals the expression above. $e^{-r/q}$ is a function of $I$ and $\tau$. Keeping $I$ constant, a decreasing $\tau$, decreases $p$. Lower $p$ means a higher $q$, which in turn means higher $e^{-r/q}$. I can therefore write that $\frac{\partial z}{\partial \tau} < 0$. Similarly, keeping $\tau$ constant, a decrease in $I$ decreases $p$ which in turn increases $q$, which in turn means that $e^{-r/q}$ goes up. I can therefore conclude that $\frac{\partial z}{\partial I} < 0$.

From the R&D equation after substituting for $x = m$ I obtain

$$\begin{aligned} m &= e^{-r/q} \frac{\beta k \left(2\lambda - \tau - 1\right)}{r + I - n} \left(a_F + I m \lambda k\right) \\ I &= \frac{e^{-r/q} R \frac{a_F}{m} - (r - n)}{1 - e^{-r/q} R \lambda k}, \end{aligned}$$

where $R \equiv \beta k \left(2\lambda - \tau - 1\right)$. It is clear that $\frac{\partial R}{\partial \tau} < 0$. I can rewrite the above equation as

$$I - \frac{Z \frac{a_F}{m} - (r - n)}{1 - Z \lambda k} = 0,$$

where for convenience I abbreviate and write $Z \equiv z(I, \tau) R$. Since $\frac{\partial z}{\partial \tau} < 0$ and $\frac{\partial R}{\partial \tau} < 0$, it follows that $\frac{\partial Z}{\partial \tau} < 0$. Since $R$ does not change with the innovation rate $I$ and $\frac{\partial z}{\partial I} < 0$, it follows that $\frac{\partial Z}{\partial I} < 0$. From $1 - Z\lambda k > 0$ (which must hold for $I$ to be positive) and $\frac{\partial Z}{\partial \tau} < 0$, follows that the left-hand side of the above expression decreases with a decreasing $\tau$, or $\frac{\partial LHS}{\partial \tau} > 0$. Also from $1 - Z\lambda k > 0$ and $\frac{\partial Z}{\partial I} < 0$, follows that the left-hand side of the above expression decreases with a decreasing $I$, or $\frac{\partial LHS}{\partial I} > 0$. From $\frac{\partial LHS}{\partial I} > 0$ and $\frac{\partial LHS}{\partial \tau} > 0$ follows that

$$\frac{\partial I}{\partial \tau} < 0.$$