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Working Paper

A Simple Model for Calculating Ballistic Missile Defense Effectiveness

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Abstract

This paper develops a probabilistic model that can be used to determine the number of ballistic missile defense interceptors required to meet a specific defense objective, given the technical performance of the defense. The defense objective is stated as some probability that no warheads leak through the defense. The defense technical performance is captured by the interceptor single-shot probability of kill and the warhead detection, tracking, and classification probability. Attacks are characterized by the number of warheads and decoys that cannot be discriminated by the defense. Barrage and shoot-look-shoot firing modes are examined, with the optimal interceptor allocation derived for the shoot-look-shoot mode. Applications of this model to national and theater missile ballistic missile defense are discussed.

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A Simple Model for Calculating Ballistic Missile Defense Effectiveness

Introduction

In the current ballistic missile defense (BMD) debate, as with so many public policy debates, one must understand the connection between the resources devoted to a particular task and the objectives one wishes to accomplish. Thus, the perennial question: "How much is enough?" or more specifically, "How much national or theater ballistic missile defense is required to accomplish specific political/military objectives, assuming a given level of technical performance for the defense?" This paper develops a simple BMD model to answer these questions.

The paper begins with a short discussion of defense objectives. It then treats interceptor-based defenses as a Bernoulli trial problem. Next it examines the target kill probability—the main parameter in the Bernoulli trial model. The paper goes on to derive the size of the defense required to meet the defense performance criterion for barrage and shoot-look-shoot defense firing doctrines. The optimal interceptor allocation for shoot-look-shoot defenses is also presented. Finally, the paper gives two examples of how this model can be applied: first to national missile defense and then to theater missile defense. Cost has not been included in this model, although it could be added if one wants to examine the extent to which different defense objectives can be met within specific financial constraints.

One can image several plausible objectives for ballistic missile defense, e.g., completely blocking an attack of a given size, attenuating an attack by a certain percentage, or protecting some target set so that a specified fraction of the defended targets survive a given attack. Since the current ballistic missile defense debate is driven largely out of a concern for the proliferation of weapons of mass destruction and the possibility, however remote, that an accidental or unauthorized ballistic missile attack might be launched from Russia (or China), the criterion chosen for this model is to destroy all attacking warheads with a specified probability. This is a very stringent criterion. The model also can be adapted to a criterion that allows some leakage.

For example, one might specify that a national missile defense completely block, with a probability of 0.80, relatively small accidental and unauthorized attacks, or small intentional threats from Third World countries to which long-range ballistic missiles might proliferate in the future. According to the Ballistic Missile Defense Organization, U.S. national missile defense (NMD) systems should be effective against attacks ranging in size from 4 to 20 reentry vehicles, accompanied by varying levels of penetration aids.[1] Moreover, NMD systems are supposed to meet the following criterion: 95 percent confidence of destroying 95 percent of the incoming attack, assuming four interceptors are fired at each incoming target.[2] Interpreting this to mean a probability of 0.95 that no more than one out of 20 warheads leaks through the defense, this criterion is equivalent to a probability of 0.67 that no warheads leak through the defense for an attack containing 20 warheads, assuming the probability with which the defense destroys an attacking warhead is the same for all warheads in the attack.

Theater missile defenses should defend U.S. and allied military forces and allied cities, the latter requiring the more stringent defense criterion.[3] To the extent allied leaders rely on defenses to protect cities, instead of deterrence, they will demand very good defense performance. For example, theater missile defenses may be required to achieve a probability of 0.50 that no warheads land on allied cities. This appears to be poor performance at first glance. However, it is a demanding criterion because it applies to the entire course of a hypothetical war, i.e., to the opponent's entire theater-range ballistic missile arsenal, and not just a single attack as in the national missile defense example given above. Moreover, theater ballistic missile arsenals may contain between 200 and 400 missiles.[4] Therefore, this criterion states that there should be no more than a 50-50 chance that any warhead penetrates the defense even if the opponent's entire arsenal is launched at an ally's capital.

Defense As a Bernoulli Trial Problem

It is difficult to calculate the effectiveness of BMD systems and, hence, the size of the defense required to meet specific performance criteria without a detailed understanding of the sensors and interceptors that make up the defense, as well as a detailed characterization of the targets the defense is attempting to shoot down. Nevertheless, one can develop a simple parametric model that will give approximate results using simple assumptions regarding the technical performance of the defense. This model, in turn, can be used to

[1] See the *1995 Report to the Congress on Ballistic Missile Defense*, the Ballistic Missile Defense Organization, U.S. Government Printing Office, Washington, D.C., September 1995, p. 3-3.

[2] See Michael Dornheim, "Missile Defense Design Juggles Complex Factors," *Aviation Week and Space Technology*, February 24, 1997, p. 54.

[3] Military forces can probably continue functioning if a few ballistic missiles armed with weapons of mass destruction land in their vicinity. Biological and chemical weapons, in particular, would largely be ineffective if troops wear protective gear. However, even one weapon of mass destruction landing on an allied city would be a threat greater than most allied leaders would tolerate.

[4] For example, North Korea may have around 100 Scud B and 100 Scud C missiles that can threaten South Korea, but it is unlikely to have 200 No Dong missiles with which to threaten Japan in the next decade. On the other hand, Iraq reportedly had around 400 Al Hussayn missiles before the 1991 Gulf War. See *Jane's Strategic Weapons Systems*, September 1995, Issue 19.

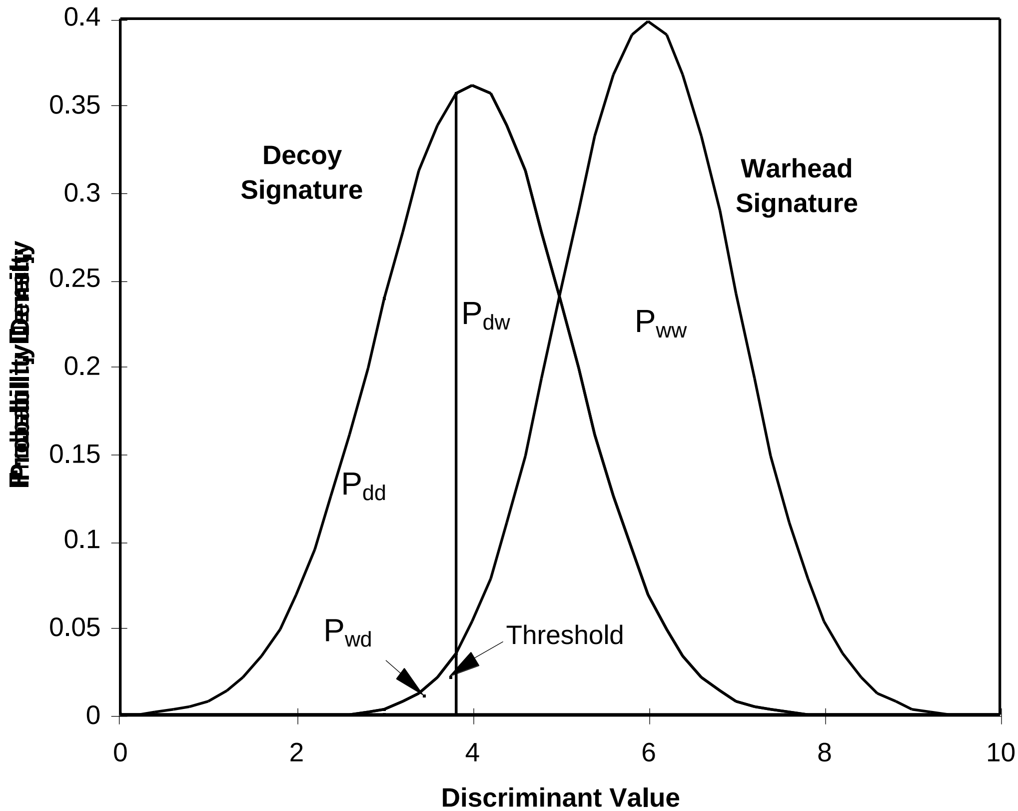

Fig. 1 - Decoy Discrimination

provide general observations about the level of technical performance that must be achieved if ballistic missile defenses are to provide militarily, if not politically, significant protection against nuclear, biological, and chemical threats.[5]

Ballistic missile attacks may contain warheads and decoys. Decoys can be partially discriminated from warheads depending on the discriminant signatures (e.g., infrared signature, radar cross section, etc.). Fig. 1 illustrates this situation for some threshold value of the discriminant for a case where decoy discrimination is relatively poor. One obtains the following four probabilities: the probability that a warhead is actually classified as a warhead, P_{ww}, which equals the integral of the warhead signature to the right of the threshold; the probability that a warhead is classified as a decoy, P_{wd}, which equals the integral of the warhead signature to the left of the threshold; the probability

[5] For a review of similar weapon-target allocation models, some of which include the number of weapons leaking through a defense, see Samuel Matlin, "A Review of the Literature On The Missile-Allocation Problem," *Operations Research*, Vol 18, 1970, pp. 334–373. In particular, optimal weapon-target allocation models that yield expected values for the number of warheads that penetrate the defense, assuming no decoys in the attack, are given in J. S. Przemieniecki, *Mathematical Methods in Defense Analysis*, 2nd Edition, American Institute of Aeronautics and Astronautics, Washington, D.C., 1994, pp. 154–159; and N.K. Jaiswal, *Military Operations Research: Quantitative Decision Making*, Kluwer Academic Publishers, Boston, MA, 1997, pp.169–172.

that a decoy is classified as a decoy, P_{dd}, which equals the integral of the decoy signature to the left of the threshold; and the probability that a decoy is classified as a warhead, P_{dw}, which equals the integral of the decoy signature to the right of the threshold. P_{wd} represents Type I errors and P_{dw} represents Type II errors in the decoy discrimination process. Note that P_{ww}=1-P_{wd} and P_{dd}=1-P_{dw}. If there are W warheads and D decoys in the attack, the apparent number of warheads in the attack, W^*, is given by

Eq. 1 $$W^* = P_{ww}W + P_{dw}D ,$$

i.e., this is the number of targets with which the defense must contend.

There are two ways by which warheads can leak through the defense: (1) the warhead is classified as a warhead and is not shot down by the defense and (2) the warhead is classified as a decoy, in which case it gets a free ride. Obviously, the discriminant threshold should be set so as to make P_{wd} as small as possible, recognizing that as the threshold decreases more decoys will appear as warheads in the attack (i.e., P_{dw} increases). The probability, q, that a warhead leaks through the defense is given by

$$q = P_{wd} + P_{ww}(1 - K_w^*) = 1 - P_{ww}K_w^* ,$$

where K_w^* is the conditional probability that the defense shoots down a warhead given that it has been classified as a warhead. We now define

$$K_w = P_{ww}K_w^* ,$$

where K_w equals the overall probability that a warhead is detected and destroyed by the defense.

Defenses based on interceptors can be modeled as a Bernoulli trial problem where the probability $P(x)$ that x attacking warheads (or missiles) will penetrate the defense is given by the binomial distribution,

Eq. 2 $$P(x) = \binom{W}{x} q^x (1-q)^{W-x} = \binom{W}{x} (1-K_w)^x (K_w)^{W-x} = \frac{W!}{x!(W-x)!}(1-K_w)^x K_w^{W-x} ,$$

where K_w is assumed to be the same for all warheads and W is the number of warheads in the attack. For the case where the defense shoots down all warheads, i.e., x=0, this reduces to

Eq. 3 $$P(0) = (K_w)^W ,$$

as one might expect. Figure 2 shows a plot of this probability as a function of the number of warheads in the attack for different values of K_w. If the criterion for defense performance states that the number of warheads that leak through the defense must be less than or equal to L, the probability with which this occurs is given by

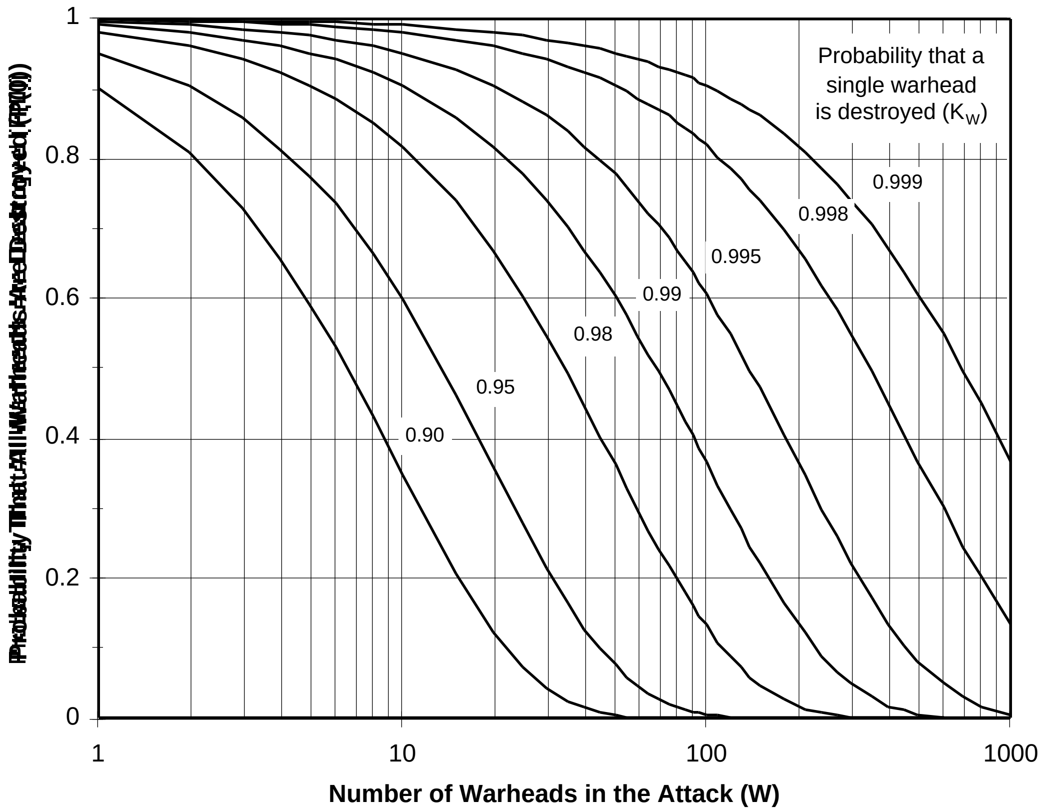

Fig. 2 - Probability That All Warheads Are Destroyed vs. Attack Size

$$P(\le L) = \sum_{x=0}^{L} P(x) \; .$$

Target Kill Probability

The probability with which a single attacking warhead or target can be destroyed is given quite generally by the equation:

$$K_j = [1-P_j(\text{common mode failure})]P_j(\text{kill}|\text{no common mode failure}),$$

where K_j is the probability that a target of type j (i.e., warhead or decoy) is destroyed, P_j*(common mode failure)* is the probability that some common mode failure affects all shots taken at the target, and P_j*(kill|no common mode failure)* is the probability that the defense can shoot down target type j if no common mode failures occur. Examples of common mode failures are a failure to detect and track the target, misclassifying a warhead as a decoy, and the defense system reliability (e.g., a failure of the BMD command and control system to transmit the target coordinates to the defense interceptors).[6] The target kill probability can be rewritten as

Eq. 4 $$K_j = P_j(track)P_j(kill|track),$$

where all sources of common mode failure have been included in $P_j(track)$, i.e.,

$$P_j(track) = P_{\text{det}\&track} ?P_{classify} ?P_{rel} ,$$

where $P_{det\&track}$ is the probability of detecting and successfully tracking a target with sufficient precision to commit a defense interceptor, $P_{classify}$ is the probability that the warhead or decoy is classified as a warhead (i.e., P_{ww} for warheads and P_{dw} for decoys), and P_{rel} is that part of the defense system reliability that affects all shots taken by the defense.

The conditional probability that target type *j* is destroyed if successfully tracked, $P_j(kill|track)$, is given by

$$P_j(kill|track) = 1-(1-k_{j1})(1-k_{j2})..(1-k_{jn}),$$

where k_{ji} is the conditional single-shot probability of kill (SSPK) for target type *j* associated with the i[th] shot, given that all prior shots have failed to destroy the target. Note that this equation is valid if the shots are statistically dependent. Statistical dependence may occur for several reasons. For example, if the i[th] shot does not destroy the target but knocks it off course or changes its signature, then subsequent shots may have a greater or lesser chance of destroying the target. Or, if multiple shots are taken at incoming targets, the SSPK of subsequent shots may degrade because the fire control radar cannot track and communicate with a large number of interceptors simultaneously during their flyout. If multiple targets enter the defense radar coverage simultaneously, this problem gets worse.[7] Finally, this equation assumes the interceptor SSPK is uniform throughout the defended area or defense "footprint," which may not always be the case because the SSPK may degrade the farther the interceptor must fly to make its engagement.

If one assumes the shots are statistically independent with identical SSPKs, the above equation reduces to

$$P_j(kill|track) = 1-(1-k_j)^n = 1-l_j^{\,n},$$

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[6] Launch platform reliability would also be a common mode error if a single launch platform launches all the interceptors fired at the incoming targets. Otherwise, the launch reliability of the individual interceptors is included in the kill probability $P_j(kill|no\ common\ mode\ failure)$. Intermediate cases, where multiple launch platforms participate in the defense but several interceptors in a single engagement come from a single launch platform, are more complex and are not treated here.

[7] If a prior shot breaks a target into multiple pieces without actually destroying the warhead, for example, a Scud missile intercept where the empty fuel tank is cut in two but the warhead remains viable, then these equations must be modified to account for the extra target fragments (assuming the resulting fragments cannot be discriminated from the warhead after the failed intercept). This added complexity has been left out of this model.

where k_j is the SSPK associated with each shot at target type j, l_j=1-k_j is the leakage rate associated with each shot, and n is the number of shots taken at each target of type j. The same formula can be applied as an approximation for statistically dependent shots if l_j is defined to be the geometric mean of the leakage rate for the statistically dependent shots. Therefore, the defense kill probability, K_j, can be written as

Eq. 5 $$K_j = P_j(track)\left[1 - (1 - k_j)^n\right] = P_j(track)(1 - l_j^n) \ .$$

The existence of common mode failures implies that unlimited improvements in defense effectiveness cannot be achieved simply by adding more shots. Fig. 3 illustrates this for the case where *P(track)* equals 0.98.

Current BMD designers apparently expect interceptor SSPKs around 0.80 to 0.85.[8] While this may not be unrealistic for current threats, i.e., unitary warheads without countermeasures, such high SSPKs may be difficult to achieve in the presence of countermeasures, although it is also difficult to assess how low the SSPK might drop. The SSPK against decoys that cannot be discriminated from warheads is likely to be smaller than that for warheads, at least for hit-to-kill interceptors, because decoys usually are smaller than warheads. Similarly, if one is optimistic about emerging sensor technologies, sensor architectures should be possible against simple ballistic missile threats that have values for *P(track)* above 0.95, and perhaps above 0.99. Again, the important question is how high *P(track)* might be in the presence of enemy countermeasures.

Countermeasures

The most significant recent technical advance with respect to ballistic missile defense is the advent of hit-to-kill interceptors. This is the critical technology upon which all current generation BMD interceptors rely. To date, 20 hit-to-kill intercept tests have been performed against ballistic missile targets six of which have been successful. Other critical BMD technologies are radar and infrared sensors for missile and warhead detection, tracking, and decoy discrimination. Over the past decade, advances in radar (e.g., imaging radar) and optical sensors (e.g., long-wave infrared sensors) probably make it possible to discriminate crude decoys and booster fragments from warheads, especially when data from different sensors are combined. However, a sophisticated opponent may be able to build decoys that cannot be readily discriminated. Hence, decoy discrimination remains the greatest technical challenge for effective ballistic missile defense.

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[8] For national missile defense, designers apparently hope for SSPKs around 0.85. See Michael Dornheim, “Missile Defense Design Juggles Complex Factors,” *Aviation Week and Space Technology*, February 24, 1977, p. 54. For theater missile defense, the SSPK of the Theater High-Altitude Area Defense has been given as 0.80. See “THAAD,” *World Missiles Briefing*, Teal Group Corporation, Fairfax VA, 1996, p. 6. While optimistic, these numbers are not unreasonable for simple threats since other systems have achieved comparable performance, e.g., U.S. air-to-air missiles have SSPKs on the order of 0.5 to 0.8, even in the face of enemy countermeasures.

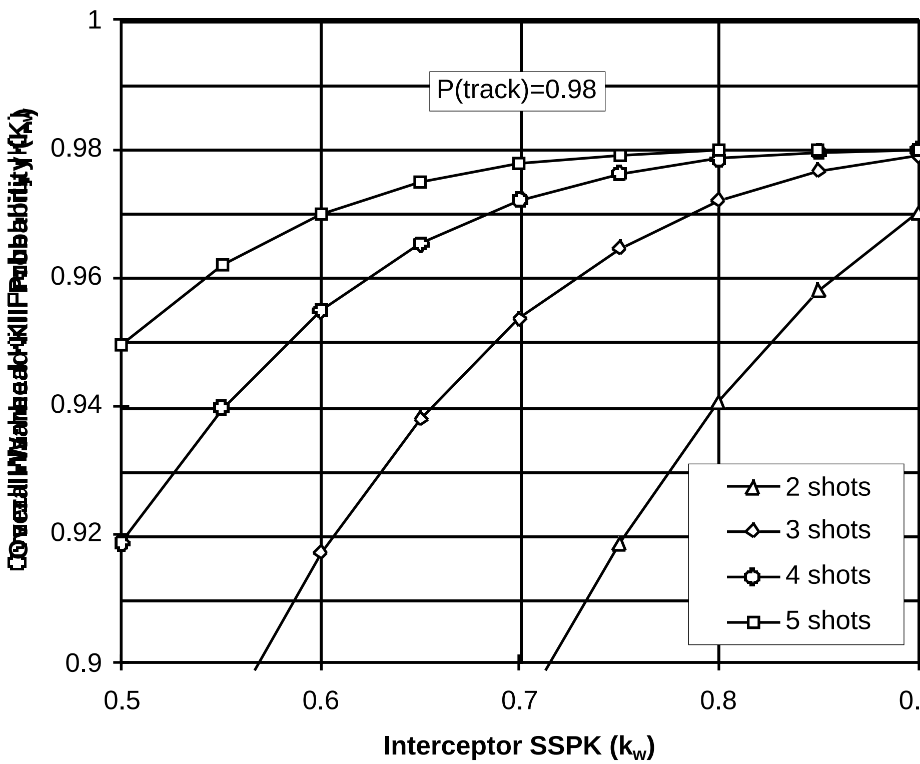

Fig. 3 - Warhead Kill Probability with Multiple Shots

For the purpose of this discussion countermeasures can be grouped into three categories: those that reduce the probability of successful target detection and tracking (thereby delaying the launch of the interceptor and causing the defended footprint to shrink, or causing it not to be launched at all), those that reduce the interceptor SSPK after it is launched, and those that simply overwhelm the defense with too many targets. Examples of countermeasures that reduce *P(track)* would be stealth techniques that reduce the target radar cross section, radar or infrared (IR) jammers that increase sensor noise levels, and large attacks that converge on the defense simultaneously, thereby saturating the tracking and fire control capabilities of the defense (i.e., saturating the defense “battle space”).

Countermeasures that reduce exoatmospheric interceptor SSPKs involve cooling the warheads so the hit-to-kill IR seekers cannot lock onto their targets (cool warheads heat up upon reentry) and encapsulating warheads in large balloons, thereby obscuring the precise warhead location so the hit-to-kill interceptor misses its target. Maneuvering warheads may reduce endoatmospheric interceptor SSPKs.[9]

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[9] Exoatmospheric intercepts occur above 100 km. Endoatmospheric intercepts occur at altitudes below 80–100 km. This distinction is important principally because simple lightweight decoys (balloons, chaff, etc.) are removed from the “threat cloud” at altitudes between 80 and 100 km, thus reducing the decoy discrimination problem for ground-based BMD systems that conduct endoatmospheric intercepts. One should note that simple lightweight decoys are not effective on missiles with ranges less than about 300 km

Finally, the defense may be saturated with a large number of decoys that cannot be discriminated from warheads (i.e., for large values of $P_{dw}D$), a large number of multiple independently targeted reentry vehicles (MIRVs) for nuclear-tipped long-range missiles, or a large number of chemical or biological bomblets contained on fractionated chemical or biological warheads. Fractionated chemical or biological warheads pose one of the most difficult responsive threats to theater missile defenses because tens or hundreds of bomblets may be deployed on each missile. Moreover, bomblets may be difficult to detect and track because of their small radar cross section and, if tracked, may be difficult to intercept with hit-to-kill interceptors because of their small size. Explosive fragmentation warheads would work better against bomblets; however, the existing exoatmospheric theater missile defenses (e.g., the Theater High-Altitude Area Defense and the Navy Theater-Wide defense) only have hit-to-kill warheads. Booster fragments may also appear as exoatmospheric decoys although, as mentioned above, current radar techniques may be able to discriminate these from warheads.

The 1991 Gulf War provided an example of BMD countermeasures when booster fragments and unintentional maneuvering on the part of poorly designed Al Hussayn missiles undermined Patriot (PAC-2) defense effectiveness. The PAC-2 SSPK apparently was much lower than originally estimated, i.e., in the range 0.09–0.25 and possibly lower, demonstrating that battlefield surprises may lower the SSPK below what one expects based on engineering estimates and test data alone.[10]

Again, it is important to note that the current theater ballistic missile threat does not include these countermeasures (except for unintentional maneuvering of reentry vehicles and booster fragments acting as decoys). Nor is it clear when they might appear, although it seems reasonable to assume they will because these countermeasures, while not trivial, are probably within the reach of most countries that have the ability to produce ballistic missiles indigenously. However, the outcome of the measure-countermeasure competition is very difficult to determine in advance. On the one hand, the offense frequently has the last move, suggesting that modestly sophisticated adversaries might be able to develop simple countermeasures after defenses are deployed. On the other hand, the United States has superior technical and financial resources relative to most regional opponents. Therefore, it is not inconceivable that the United States could engage less developed countries in an offense-defense competition and maintain the effectiveness of

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because these missiles never leave the atmosphere. For short-range missiles the opponent must design more sophisticated, heavier decoys or other penetration aids.

[10] Initial claims shortly after the Gulf War placed PAC-2 intercept performance at 96 percent against Iraqi Scud missiles. However, this estimate was lowered after critics challenged this assessment. Today, the Army claims that 40 percent of the Scuds aimed at Israel were successfully intercepted by PAC-2, where "successful intercept" is defined as an intercept that either destroys, damages, or knocks the warhead off course sufficiently to avoid damaging the intended target (with a unitary high-explosive warhead), and that 70 percent of the Scuds fired at Saudi Arabia were successfully intercepted. However, the Army also claims that only 25 percent of the intercepts destroyed the Scud warhead with "high confidence." On the other hand, a General Accounting Office investigation of these claims concluded that the available evidence only supports a 9 percent intercept rate, with the possibility that fewer than 9 percent of the Scud warheads were actually destroyed. See John Conyers, Jr., "The Patriot Myth: Caveat Emptor," *Arms Control Today*, November 1992, p. 9; David Bond, "Army Scales Back Assessments Of Patriot's Success in Gulf War," *Aviation Week and Space Technology*, April 13, 1992, p. 64; and Jeremiah Sullivan, Dan Fenstermacher, Daniel Fisher, Ruth Howes, O'Dean Judd, and Roger Speed, *Technical Debate over Patriot Performance in the Gulf War*, Arms Control, Disarmament, and International Security Research Report, University of Illinois at Urbana-Champaign, 4 April 1998.

its BMD systems. This is likely to be the case against North Korea and may be the case against states such as Iraq, Iran, Syria, and Libya, but it is unlikely to be the case against Russia or China.

A definitive technical assessment of these countermeasures and BMD counter-countermeasures is beyond the scope of this paper. Suffice it to say that the effectiveness of BMD systems against simple threats (i.e., unitary warheads with only crude penetration aids) may be fairly good, assuming adequate testing can solve the engineering problems that plague current-generation hit-to-kill interceptors. However, BMD effectiveness against future threats is less certain and will depend largely on the ability to defeat offensive countermeasures of the sort mentioned above. The impact of countermeasures is captured in the following analysis by analyzing the variation in defense effectiveness as a function of parametric variations in the warhead detection and tracking probability, the interceptor SSPK, and the number of decoys that cannot be discriminated from warheads.

Sample Calculations of Target Kill Probability

Since there is some uncertainty as to the values one should use for *P(track)* and the interceptor SSPK, it is useful to plot the warhead kill probability, K_w, as a function of these variables, as illustrated in Figs. 4–8 for different numbers of shots. Alternately, these figures can be used to determine the level of *P(track)* and interceptor SSPK that must be achieved if the overall warhead kill probability for the defense is to attain a specified value. Once the warhead kill probability is estimated, Fig. 2 can be used to determine the likelihood that no warheads leak through the defense for a given attack size. For example, if 3 shots are taken at all incoming warheads and the SSPK of each is 0.70 and *P(track)* equals 0.98, then the warhead kill probability is a little over 0.95 (see Fig. 6). With this performance there is at least a 60 percent chance that no warheads will leak through the defense if the attack contains fewer than 10 warheads.

Defense Required to Meet Performance Criterion

From Eqs. 3 and 5 one can derive the number of shots that must be fired at each incoming warhead to achieve a given level of defense effectiveness, *P(0)*, if *W* warheads are in the attack. The parameters k_w and $P_w(track)$ capture the technical performance of the defense. The result is

Eq. 6
$$n = \frac{\ln\left(1 - \frac{P(0)^{1/W}}{P_w(track)}\right)}{\ln\left(1 - k_w\right)} .$$

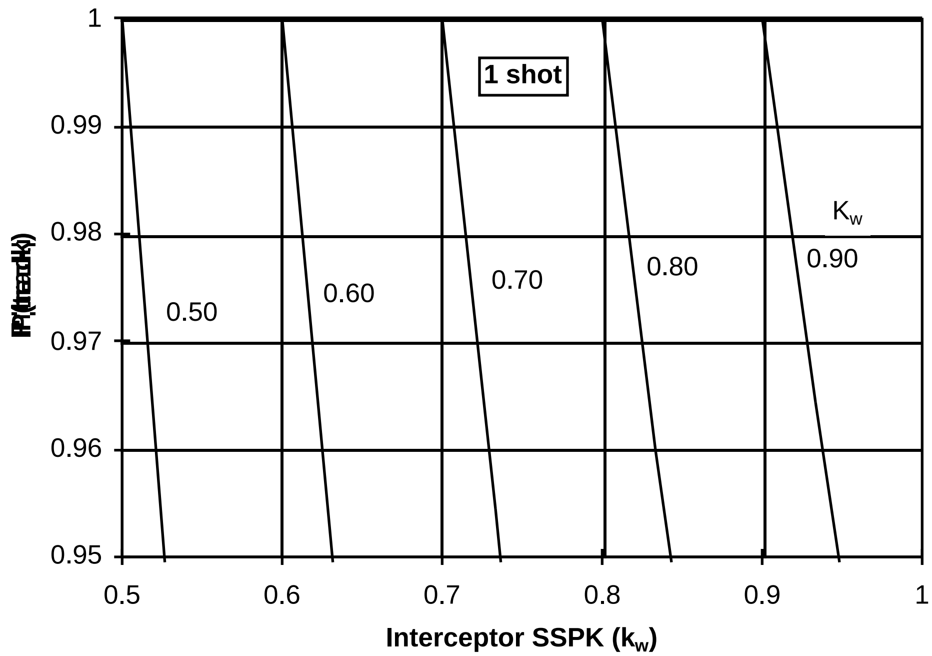

Fig. 4 - Overall Warhead Kill Probability (1 shot)

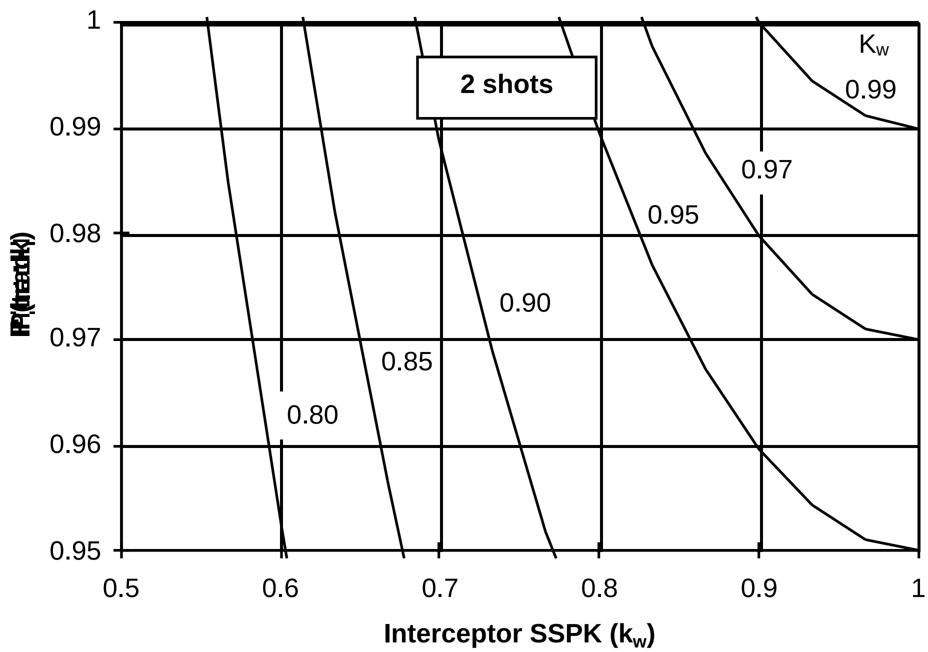

Fig. 5 - Overall Warhead Kill Probability (2 shots)

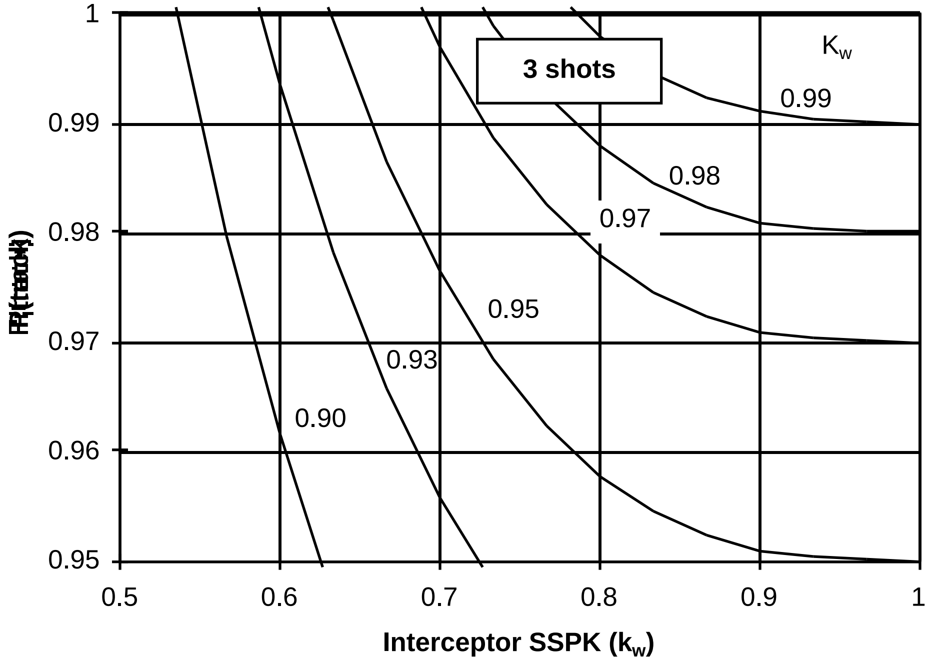

Fig. 6 - Overall Warhead Kill Probability (3 shots)

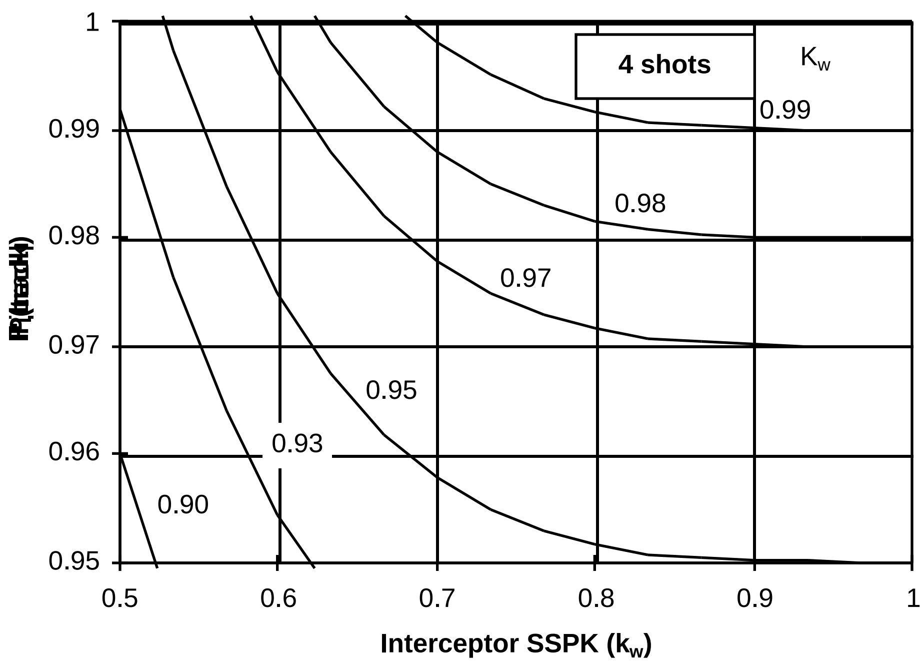

Fig. 7 - Overall Warhead Kill Probability (4 shots)

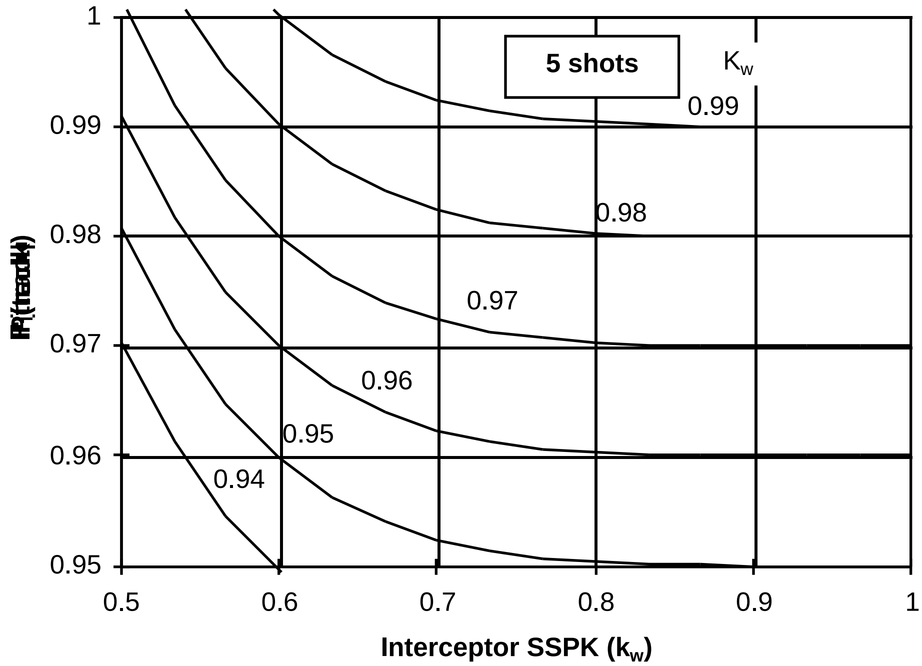

Fig. 8 - Overall Warhead Kill Probability (5 shots)

This equation is valid whether decoys are present or not. In general, this equation yields non-integral values for the number of interceptors allocated to each incoming warhead, *n*. The integral value of *n* that meets the defense effectiveness criterion is simply

$$n^* = Int(n) + 1 \,,$$

where *Int(n)* is the integer part of *n*.

However, a reasonable interpretation can be given for non-integral values of *n*, namely, some fraction of the incoming targets have *Int(n)* interceptors allocated against them and the remainder have *Int(n)*+1. The fraction with *Int(n)*+1 interceptors equals the fractional part of *n* (i.e., *n-Int(n)*). This interpretation for non-integral values of n is approximate as can be seen by recalculating Eq. 3 with mixed interceptor allocations. The correct equation under these circumstances would be

$$P(0) = K_{w_i}^{(1-f)W} K_{w_{i+1}}^{fW} \,,$$

where K_{wi} is the warhead kill probability if *i* interceptors are fired at each of *(*1-*f)W* targets, where *f* is the fractional part of *n*, and K_{wi+1} is the warhead kill probability if *i*+1 shots are fired at the remainder. This can be reduced to

$$P(0) = P(track)^W \left[(1-l^i)^{1-f}(1-l^{i+1})^f\right]^W ,$$

from which one identifies

Eq. 7 $$\langle K_w \rangle = P_w(track)(1-l_w^i)^{1-f}(1-l_w^{i+1})^f$$

as the average warhead kill probability associated with a split interceptor allocation. Equation 5 is a reasonable approximation to Eq. 7 for values of *n* above 1, as can be seen in Fig. 9 (for *P(track)*=1 and *k*=0.7). Note that for n<1 Eq. 7 is zero because some of the warheads have no interceptors allocated against them. Figure 10 illustrates the probability that no warheads leak through the defense for both cases, assuming *W*=5 or *W*=50, and using the same values for $P_w(track)$ and k_w as in Fig. 9.

Barrage Firing Doctrine[11]

The number of BMD interceptors required to meet the defense performance criterion depends on the apparent size of the attack, i.e., *W** in Eq. 1, and the firing doctrine one assumes for the defense. The simplest doctrine is barrage fire, where n interceptors are fired near simultaneously at each incoming target. The number of interceptors required for barrage fire, I_B, is simply

Eq. 8 $$I_B = W^* n = W^* \left(\frac{\ln\left(1 - \frac{P(0)^{1/W}}{P_w(track)}\right)}{\ln(1-k_w)} \right) .$$

Again, *P(0)* is the desired probability that no warheads leak through the defense, *W** is the apparent number of warheads in the attack, *W* is the number of real warheads in the attack, $P_w(track)$ is the warhead detection, tracking, and discrimination probability, and k_w is the interceptor SSPK against warheads. For barrage fire, the number of interceptors required increases linearly with the apparent attack size, provided the number of real warheads in the attack is fixed.

Equation 8 assumes that all interceptors can engage the incoming attack. This is true for a single BMD site with a footprint large enough to cover the territory of interest. It is also true if multiple BMD sites are required to cover the entire territory, provided the interceptors and warheads are spread uniformly across all defended footprints. However, if the attack is launched against targets located within a single BMD footprint of a multi-site

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11 For a discussion of uniform (barrage), random, and shoot-look-shoot firing doctrines see J.S. Przemieniecki, *Mathematical Methods in Defense Analysis*, 2nd Edition, American Institute of Aeronautics and Astronautics, Washington, D.C., 1994, pp. 154–159.

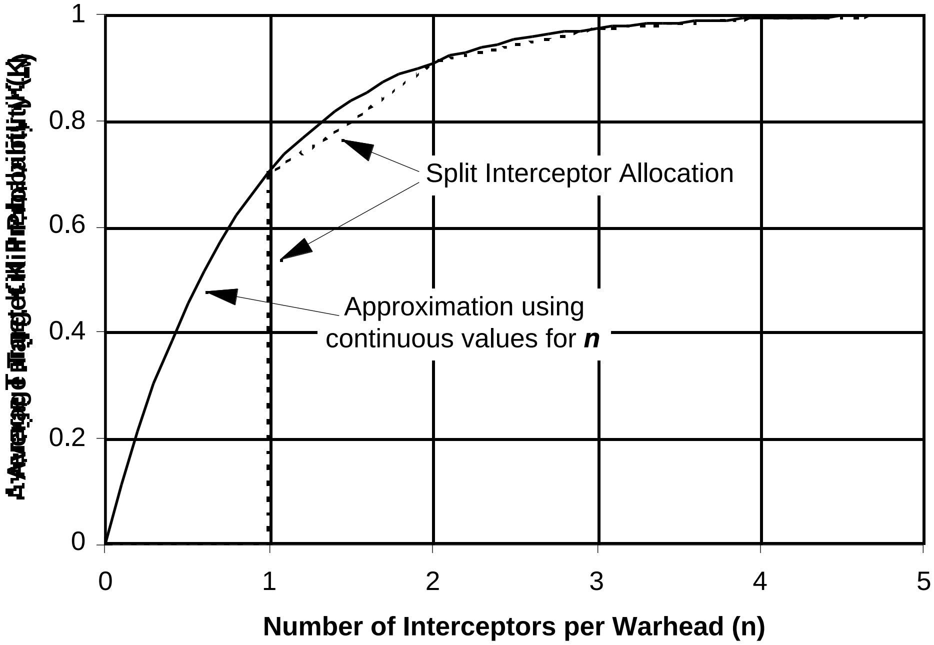

Fig. 9 - Comparison of Target Kill Probabilities

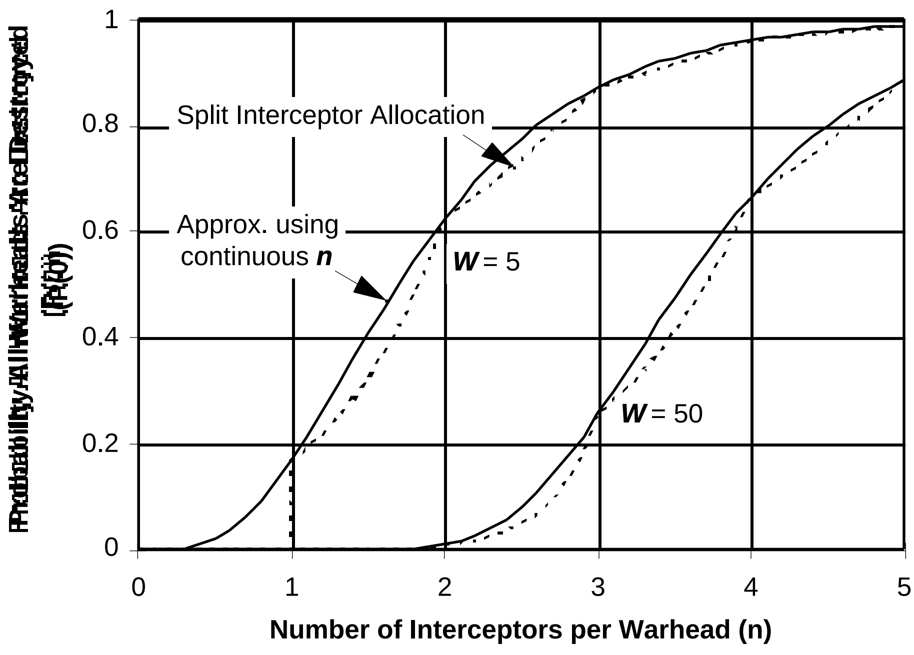

Fig. 10 - Probability That All Warheads Are Destroyed

system in an attempt to saturate the defense, then each BMD footprint must have I_B interceptors. Therefore, to defend against concentrated attacks with a multi-site defense, a total of mI_B interceptors must be deployed, where m is the number of BMD sites required for complete coverage.[12]

Optimal Shoot-Look-Shoot Firing Doctrine

If shoot-look-shoot tactics are possible, fewer interceptors are required for equivalent defense effectiveness because the second shot(s) are directed only at those warheads and decoys that are not destroyed in the first shot opportunity. However, shoot-look-shoot is more demanding technically because it requires accurate kill assessment after the first shot opportunity and, typically, requires high-speed interceptors for reasonable defended footprints because less time is available for the second intercept attempt.

To simplify this calculation, the expected value for the number of apparent warheads leaking through the first shot opportunity is used. A more accurate calculation would use the probability distributions for the number of warheads and decoys classified as warheads that leak though the first shot opportunity. Using the expected value approach, the number of interceptors required assuming shoot-look-shoot tactics, I_{SLS}, is given by

Eq. 9 $$I_{SLS}(s) = sW^* + \left[(1-k_w)^s P_{ww}W + (1-k_d)^s P_{dw}D\right](n-s) \ ,$$

where s is an integer representing the number of shots taken at each apparent warhead in the first shot opportunity, n is the number of shots that must be taken at each warhead to achieve the defense performance criterion (given by Eq. 6), and the other variables are defined earlier. This can be rewritten as the number of interceptors per apparent warhead as follows,

Eq. 10 $$\frac{I_{SLS}}{W^*} = n^* = s + \left[\alpha(1-k_w)^s + (1-\alpha)(1-k_d)^s\right](n-s) \ ,$$

where

$$\alpha = \frac{P_{ww}W}{\left(P_{ww}W + P_{dw}D\right)}$$

is the fraction of apparent attacking objects that are warheads and 1-α is the fraction of apparent attacking objects that are decoys. Frequently s=1, although minimizing I_{SLS} may require s to be greater than 1.

The optimal number of shots, s_o, in the first shot opportunity that minimizes I_{SLS} can be determined from the following definition of the minimum value,

$$n^*(s_o+1) \ ? \ n^*(s_o) \le n^*(s_o-1) \ .$$

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[12] More complicated preferential offense and defense strategies can be considered. However, such modeling complexities do not alter appreciably the defense size one calculates using uniform attacks against uniform defenses. See N.K. Jaiswal, *Military Operations Research: Quantitative Decision Making*, Kluwer Academic Publishers, Boston, MA, 1997, pp.169–172.

These inequalities can be reduced, after a bit of algebra, to the following inequalities,

Eq. 11 $$F(n,s_o) \le 1 \le F(n,s_o - 1) ,$$

where

Eq. 12 $$F(n,s) = \left[\alpha k_w (1-k_w)^s + (1-\alpha) k_d (1-k_d)^s\right](n-s) + \left[\alpha (1-k_w)^{s+1} + (1-\alpha)(1-k_d)^{s+1}\right].$$

For a given value of *n*, s_o can be found implicitly from the equation,

$$F(n,s_o - 1) = 1 .$$

The integral value for the optimal number of shots taken at each apparent warhead in the first shot opportunity is simply the integral part of s_o, or *Int*(s_o).

If the interceptor SSPK is the same for decoys and warheads (i.e., $k_w = k_d$), or if decoys are not included in the attack (i.e., α=1), the above equation reduces to,

Eq. 13 $$\left[k_w\left(n - s_o\right) + 1\right]\left(1 - k_w\right)^{s_o - 1} = 1 .$$

Note that in this case the optimal interceptor allocation is independent of the apparent attack size, *W**. Hence, the number of interceptors required for the optimal shoot-look-shoot defense, I_{SLS}, is proportional to the apparent attack size for a fixed number of real warheads in the attack, assuming k_w equals k_d as the apparent attack size increases, as was the case for barrage firing doctrines.

Alternately, by picking an integral value for *s*, one can determine when the number of shots taken at each attacking object becomes large enough so that the optimal number of shots taken in the first shot opportunity changes from *s* to *s*+1.[13] A series of "transition" numbers, n_s, are thus obtained such that the optimal number of shots in the first shot opportunity is *s* provided that

$$n_s \le n < n_{s+1} ,$$

where n_s is the solution to the equation

$$F(n_s, s-1) = 1 ,$$

Table 1
Transition Numbers vs. Interceptor SSPK

	Interceptor SSPK				
	0.5	0.6	0.7	0.8	0.9

[13] This approach was originally suggested by David Vaughan. For a similar treatment, not including decoys, see Eric Larson and Glenn Kent, *A New Methodology for Assessing Multilayer Missile Defense Options*, MR-390-AF, RAND, Santa Monica, 1994, pp. 12–16.

n_1	1	1	1	1	1
n_2	4	4.5	5.33	7	12
n_3	9	11.75	17.44	33	113
n_4	18	28.38	55.48	159	1114

for integral values of *s*. This approach has the advantage that Eq. 12 can be solved explicitly for n_s to give,

Eq. 14 $$n_s = (s-1) + \frac{1-\left[\alpha(1-k_w)^s + (1-\alpha)(1-k_d)^s\right]}{\left[\alpha k_w(1-k_w)^{s-1} + (1-\alpha)k_d(1-k_d)^{s-1}\right]}.$$

Again, if the interceptor SSPK is the same for warheads and decoys, or if no decoys are contained in the attack, this reduces to

Eq. 15 $$n_s = s - \frac{1}{k_w} + \frac{1}{k_w(1-k_w)^{s-1}}$$

Table 1 gives the first four values for n_s for different values of the interceptor SSPK, assuming decoys and warheads have the same SSPK. Practical defense systems rarely will fire more than 10 interceptors at incoming targets. However, one can see that the optimal shoot-look-shoot allocation rarely requires more than 2 shots in the first shot attempt unless the interceptor SSPK is less than 0.5. Non-integral values of n_s are interpreted the same way as before, i.e., they provide the transition numbers that minimize Eq. 10 given that *s* shots are taken in the first shot opportunity and the second shot opportunity has a split interceptor allocation with some targets having *Int(n-s)* interceptors and the remainder having *Int(n-s+1)* interceptors allocated against them. If the interceptor SSPK against warheads and decoys is different, as is likely to be the case for hit-to-kill interceptors, then Eq. 14 must be used to find the transition numbers.

Figure 11 illustrates the minimum number of interceptors required per apparent warhead, i.e., n^*, as a function of the number of attacking warheads for different fractions α. The number of interceptors per warhead for barrage fire is also shown for comparison ($n^*=n$ in this case). The interceptor SSPK in Fig. 11 is assumed to be 0.7 for warheads and one-third this value for decoys, *P(track)* equals 0.99, and the defense criterion is a 0.80 probability that no warheads leak through the defense. The graphs stop for attacks equal to 22 warheads because the criterion that all warheads must be shot down with a probability of 0.80 cannot be met for larger attacks. Figure 12 shows the same set of curves for near-perfect detection, tracking, and warhead classification (i.e., *P(track)*=0.999). Note that shoot-look-shoot firing

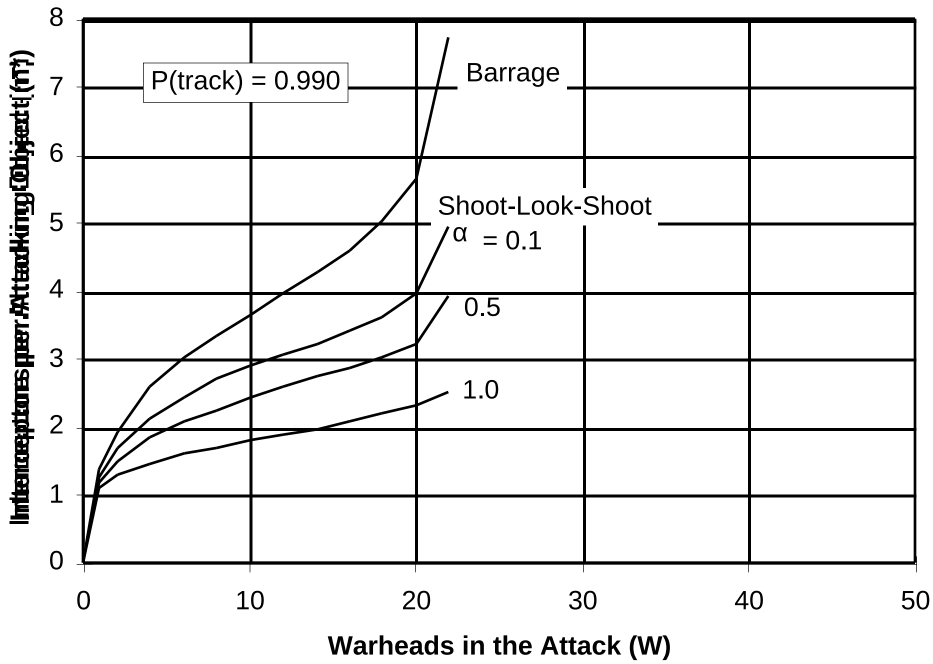

Fig. 11 - Interceptors Allocation vs. Attack Size (*P(track)*=0.99)

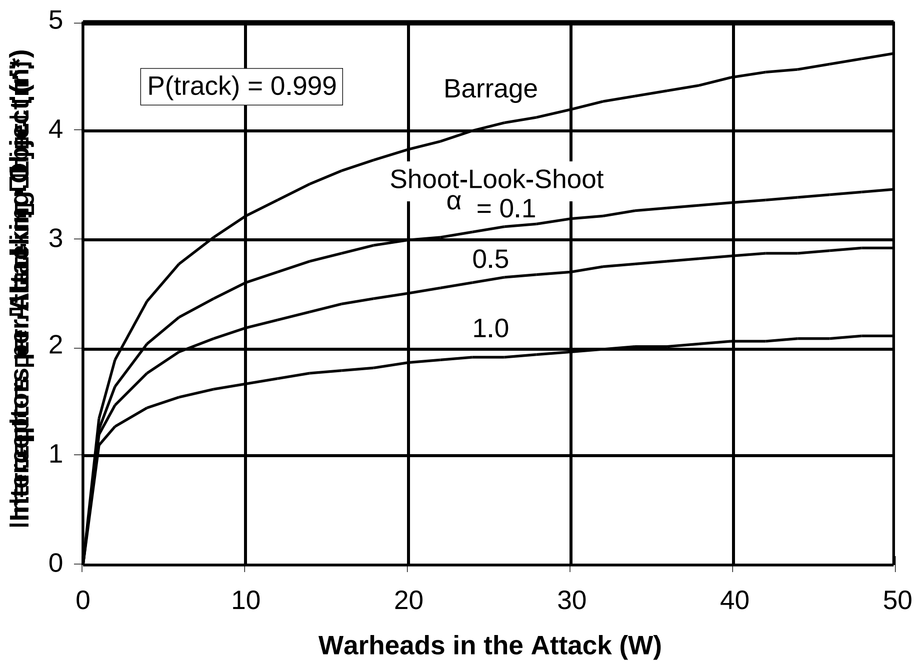

Fig. 12 - Interceptors Allocation vs. Attack Size (*P(track)*=0.999)

doctrines become less effective as more undiscriminated decoys are included in the attack. In fact, in the limit where $k_d \to 0$ and $D \to \infty$, the optimal shoot-look-shoot case is equivalent to the barrage case, as one might expect.

Layered Defenses

Layered defenses with separate defensive systems and independent detection and tracking sensors in each layer are used to achieve very high defense effectiveness. If one assumes common mode failures between layers are negligible (there are few ways to test this under realistic wartime conditions), then the effectiveness of a defense with m layers is

Eq. 16 $$K_w = 1\text{-}L_1 L_2 ... L_m$$

where L_i is the probability that a warhead leaks through the i^{th} layer of the defense.[14] For example, if one assumes the target kill probability is 0.97 in each of two layers, then the overall probability that a target can be destroyed is around 0.999, implying that such a defense could block 100 out of 100 warheads with a probability of 0.9 (see Fig. 2).

Applications

To answer the question "How much ballistic missile defense is enough?" one must address two questions: how many BMD sites does it take to adequately cover the territory of interest and how many interceptors should be deployed at each site to meet the defense performance criterion? The number of BMD sites required depends on the BMD footprint, which is a function of the interceptor flyout speed and the time available for interceptor flyout, the latter of which is a function of the speed (i.e., range) of the incoming target and the radar detection range. The radar detection range, in turn, is a function of the radar power-aperture product, the radar cross section of the target, and the number of targets that must be engaged simultaneously. Shoot-look-shoot footprints are smaller that barrage footprints because less flyout time is available for the second shot attempt. In general, the probability with which a warhead can be destroyed is not uniform throughout the defended footprint. Nevertheless, for purposes of this analysis, the interceptor SSPK is assumed to be uniform throughout the defended footprint and the footprint is assumed not to shrink appreciably with the attack sizes considered here.

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[14] Approximate methods for finding the optimal interceptor allocation between different layers of a multi-layer defense are given in Michael V. Finn and Glenn A. Kent, *Simple Analytic Solutions To Complex Military Problems*, N-2111-AF, RAND Corporation, Santa Monica, CA, August 1985, pp. 33–38.

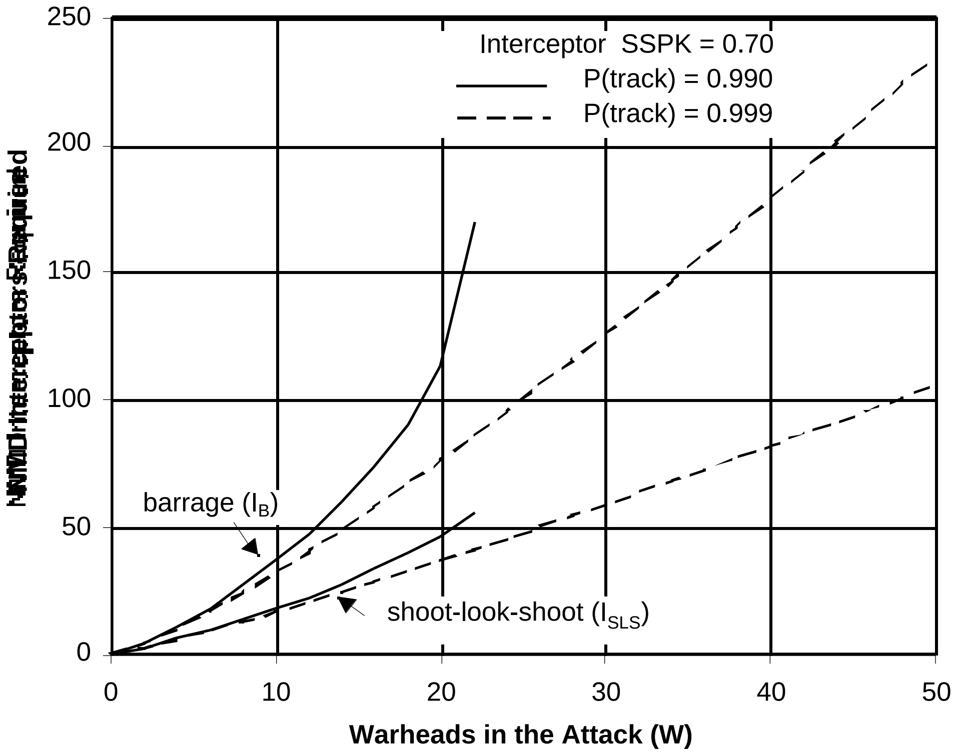

Fig. 13 - NMD Interceptors vs. Attack Size

National Missile Defense

Applying this model to national missile defense, Fig. 13 illustrates the number of NMD interceptors required at a single site for barrage and shoot-look-shoot firing doctrines to achieve a probability of 0.80 that all warheads in the attack are destroyed, as a function of the attack size. No decoys are included in this calculation and a high level of technical performance has been assumed, i.e., each NMD interceptor has an SSPK of 0.70 against warheads and a probability of successfully detecting and tracking incoming targets, *P(track)*, equal to 0.990 (solid line) or 0.999 (dashed line). In this case, approximately 113 NMD interceptors are required to destroy 20 out of 20 warheads with a probability of 0.80 if the defense operates in a barrage mode (for *P(track)* = 0.99), but only 47 interceptors are required if the defense operates in shoot-look-shoot mode. If the defense is ABM Treaty compliant, i.e., contains no more than 100 interceptors at a single site, then such a defense operating in barrage mode could completely block up to 19 warheads that enter the defended footprint with a probability of 0.80 (assuming *P(track)*=0.99).

Figure 14 illustrates a similar plot assuming one undiscriminated decoy accompanies each warhead (i.e., $P_{dw}D$=1), *P(track)* equals 0.99, and that the interceptor SSPK against

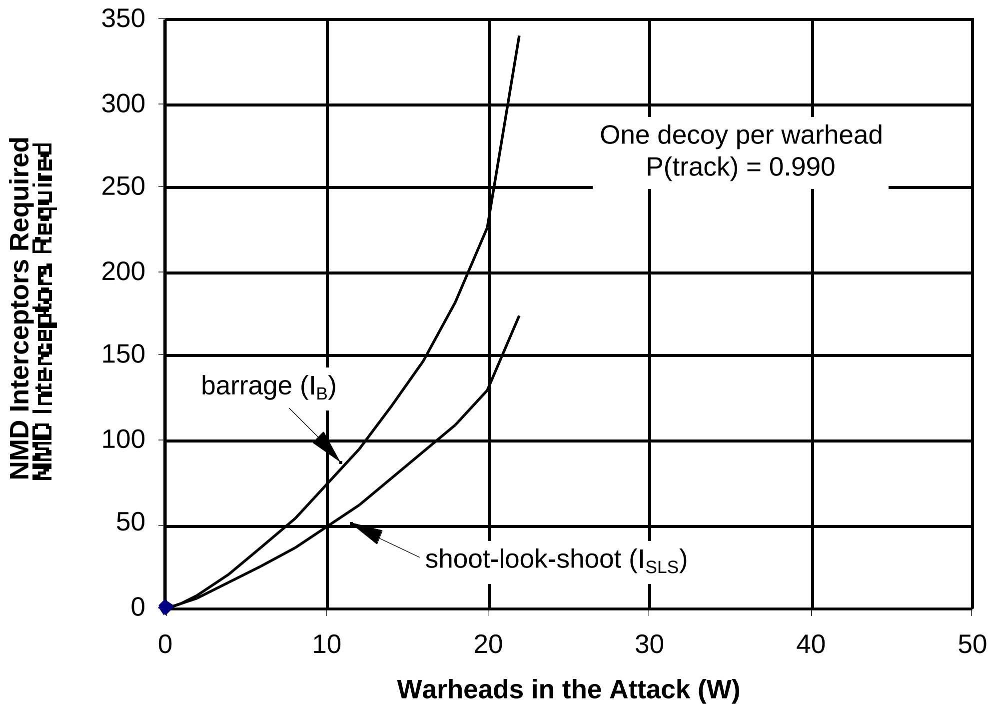

Fig. 14 - NMD Interceptors vs. Attack Size (One Decoy per Warhead)

warheads is 0.7 and against decoys is 0.23. As one can see, 73 interceptors fired in a barrage would be required to block (with probability 0.80) 10 out of 10 warheads accompanied by 10 sophisticated decoys, whereas 48 interceptors would be required to achieve the same effectiveness if they are fired in shoot-look-shoot mode. An ABM Treaty-compliant defense should be able to completely block attacks containing up to 12, or 17, warheads if the defense operates in barrage or shoot-look-shoot mode, respectively, again assuming *P(track)* equals 0.99.

The curves in Figs. 13 and 14 are derived by multiplying the appropriate curves in Figs. 11 and 12 by the apparent attack size. The number of shots taken at each incoming warhead to meet the defense criterion varies along the length of each curve in Figs. 13 and 14, with the maximum never exceeding five for *P(track)*=0.999, and rarely exceeding six for *P(track)*=0.99.[15] Note also that the barrage curve in Fig. 14 is equal to the barrage curve in Fig. 13 with the ordinate multiplied by 2 (i.e., W^*/W), but that this is only approximately true for the shoot-look-shoot curves. Again, the curves with *P(track)* equal

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[15]Taking more than 6 shots at each incoming target may stress the NMD command and control system since each interceptor must be tracked as it flies toward the intercept point. In addition, the desire not to waste too many interceptors on early arriving targets in case subsequent attacks occur tends to limit the number of interceptors the defense will allocate to each target, assuming the attack is believed to be intentional. While these arguments are suggestive, they do not set rigid limits on the number of shots an NMD system can take. In principle, a large number of NMD interceptors might be fired in a barrage against an incoming target, although numbers beyond 10 seem implausible.

to 0.990 stop for attacks equal to 22 warheads because the criterion that all warheads must be shot down with a probability of 0.80 cannot be met for attacks above this size.

One concept under discussion for an initial national missile defense is to deploy 20 interceptors at Grand Forks, North Dakota—the site of the ABM Treaty-compliant defense the United States abandoned in 1974.[16] With sufficiently high-speed interceptors this defense may be able to cover the entire continental United States, but only with a barrage firing doctrine. Therefore, such a defense could defend against threats containing up to 6 warheads, or 4 warheads accompanied by 4 decoys that cannot be discriminated by the defense, assuming NMD interceptors can achieve SSPKs above 0.7 and NMD detection and tracking systems can achieve values for *P(track)* above 0.99. If multiple sites are envisioned for a more robust national missile defense, one must multiply the number of interceptors at each site by the number of sites required for adequate coverage to obtain the total number of interceptors—with the understanding that the attacker may concentrate his attack against targets located within a single defended area to saturate the defense.

Frequently one is interested more in the variation in the required number of interceptors as a function of *P(track)* and the interceptor SSPK, either because these parameters are not well known or because countermeasures may reduce the values for *P(track)* and/or the interceptor SSPK. Figure 15 illustrates this variation for a barrage firing doctrine for an attack containing 10 warheads and no decoys. The defense performance is held constant (i.e., a probability of 0.80 that 10 out of 10 warheads are destroyed). Note that the defense cannot meet the performance criterion if *P(track)* is below 0.978, regardless of the interceptor SSPK or how many shots are fired. For a barrage firing doctrine the number of shots taken at each warhead is constant along each of the NMD interceptor contour lines. For example, along the 40 interceptor contour, 4 shots are taken at each warhead. If *P(track)* equals 0.99, the SSPK for each interceptor must be above 0.67 to meet the defense performance criterion if no more than 4 shots are fired at each incoming object. If the SSPK drops to 0.50, the required number of interceptors begins to grow rapidly, leading to a situation where 6 or more shots, depending on *P(track)*, must be fired at each incoming warhead, thereby potentially creating problems with battle space saturation.

Figure 16 illustrates the same variations for a shoot-look-shoot firing doctrine, again for an attack containing 10 warheads and no decoys. In this figure the number of shots taken at each incoming warhead is not constant along the interceptor contour lines. Moreover, the optimal number of shots taken in the first shot opportunity varies across the domain, with one shot typically taken in the first shot opportunity between the 10 and 24 interceptor contour lines, two shots between the 24 and 40 interceptor contour lines, and three shots taken in the first shot opportunity to the left of the 40 interceptor contour line.

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[16] See Stanley Kandebo, “U.S. Pursues NMD System To Prepare for ‘Rogue’ Threat,” *Aviation Week and Space Technology*, March 3, 1997, pp. 44–45.

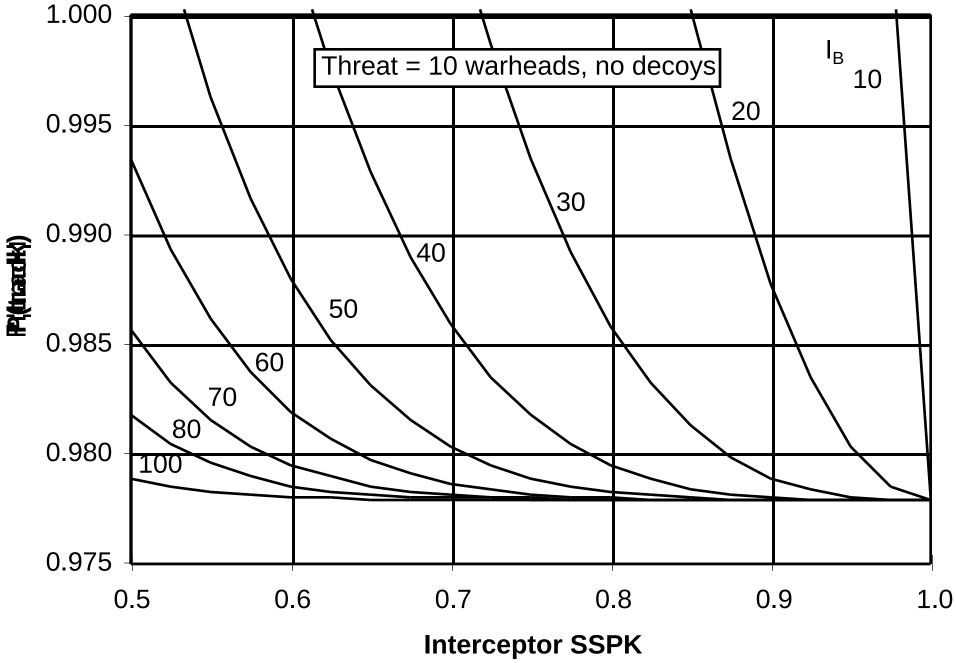

Fig. 15 - NMD Interceptors (Barrage) vs. *P(track)* and SSPK

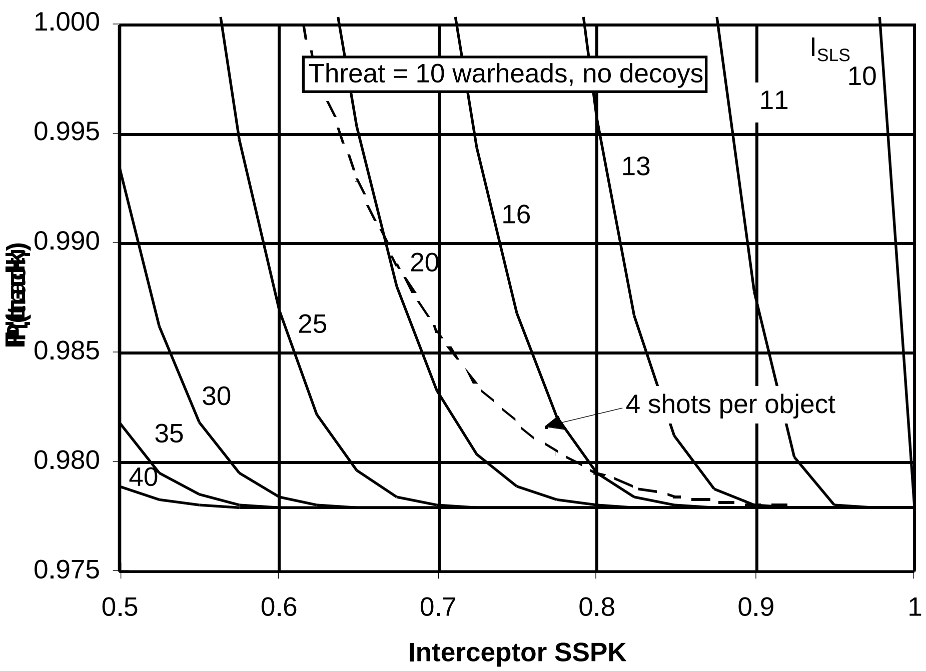

Fig. 16 - NMD Interceptors (Shoot-Look-Shoot) vs. *P(track)* and SSPK

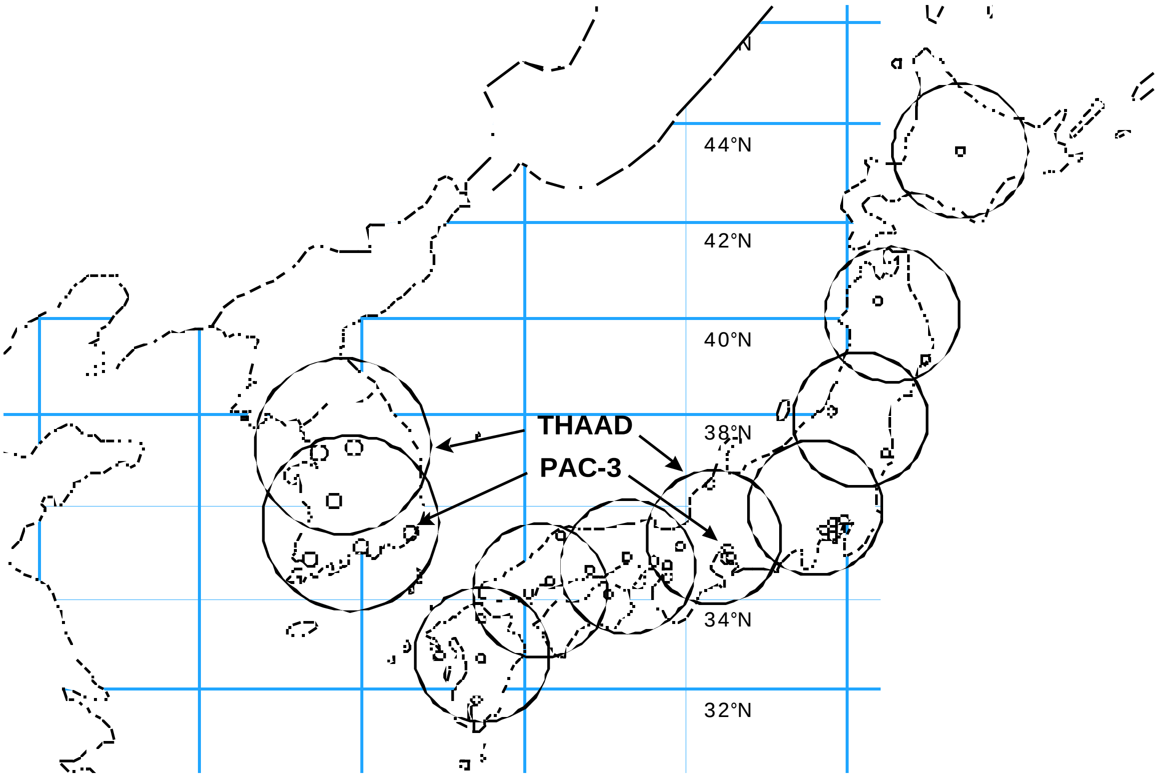

Fig. 17 - THAAD Coverage of Northeast Asia

Theater Missile Defense

TMD systems generally have smaller defended footprints due to the shorter flight times for theater-range ballistic missiles and the slower interceptor speeds associated with TMD interceptors. Hence, several sites typically are required to achieve adequate coverage. For example, Fig. 17 illustrates that approximately 10 Theater High-Altitude Area Defense (THAAD) sites are required to defend South Korea and Japan. The footprints covering Japan are smaller than those defending South Korea because they must engage longer-range (i.e., higher-speed) missiles (i.e., the No Dong), whereas South Korea may be attacked with shorter-range missiles (i.e., the Scud B and Scud C).[17] A

[17] The THAAD footprints covering South Korea are approximately 450 km in diameter and those covering Japan are approximately 350 km in diameter. These are based on estimates of THAAD performance made by the Congressional Budget Office. See *The Future of Theater Missile Defense*, Congressional Budget Office, Washington, D.C., June 1994. They represent an upper bound on THAAD coverage because they are based only on the kinematics of a single THAAD intercept, i.e., the distance out to which a THAAD interceptor can fly to meet a single incoming warhead, given an estimate of the detection range for the THAAD ground-based radar. If multiple warheads attack targets located within a single defended area simultaneously, the defended footprint shrinks, and if the THAAD radar is queued by other sensors, the footprint increases¾especially for long-range theater ballistic missiles. In addition, the probability with which a warhead will be destroyed is not uniform throughout this footprint, although for purposes of this analysis a uniform SSPK has been assumed.

similar number of THAAD sites would be required to cover important areas within the Persian Gulf.

The number of THAAD upper-tier interceptors required at each site can be calculated once the architecture is specified. For illustrative purposes, a layered defense with THAAD operating as an upper-tier and PAC-3 as a lower-tier defense is assumed, where both layers are assumed to be equally effective, i.e., THAAD and PAC-3 interceptors are assumed to have the same SSPK and both systems are assumed to have the same value for *P(track)*.[18] In other words, the leakage rate for each layer is assumed to be the same. While this generally will not be the case, this assumption allows one to observe how the required number of upper-tier interceptors varies with the overall technical performance of the defense. In addition, both layers are assumed to operate only in barrage mode (to achieve wide area coverage for THAAD and because insufficient time exists for PAC-3 to operate in anything but barrage mode). Therefore, this layered architecture provides a thin area defense over the entire theater with selected high-value targets protected by both upper and lower-tier defenses.

The required number of THAAD upper-tier interceptors, operating in a barrage mode, to ensure that no warheads leak through the upper and lower tiers with a probability of 0.5 is shown in Fig. 18 as a function of the number of warheads in the attack, assuming no decoys (or alternately very good decoy discrimination). Both the upper and lower tiers are assumed to have a detection, tracking, and warhead classification probability of 0.98 and upper and lower-tier interceptor SSPKs that vary simultaneously between 0.5 and 0.9. The number of shots taken at incoming warheads is assumed to be the same for the upper and lower tier—a number which varies along each of the curves.

From Fig. 18 one observes that between 280 and 930 upper-tier interceptors must engage the attack to defeat 200 warheads with a probability of 0.5, assuming the upper and lower-tier interceptor SSPKs are between 0.5 and 0.9. A nominal upper and lower-tier interceptor SSPK of 0.7 implies that approximately 540 upper-tier interceptors are required to defeat a 200-warhead attack. If the attack is directed uniformly at all defended areas (i.e., 10 THAAD defended footprints), then these 540 upper-tier interceptors must be spread evenly among the sites to defeat the attack.

However, an opponent will likely concentrate the attack in an effort to saturate one defended site, with the constraint that not all ballistic missiles can be launched in a single day. The maximum daily launch rate is limited by the number of available launchers. Although Scud-type mobile missiles can be reloaded within several hours, few launchers will be able to launch more than one missile per day because of the need to move and hide immediately after launch to avoid U.S. air attacks. Historically, countries have deployed between 4 to 15 missiles per mobile launcher (Iraq had about 13 missiles per launcher), suggesting that at most 7–25 percent of the arsenal can be launched in a single day. For example, during the 1991 Gulf War no more than 14 Al Hussayn missiles were fired in a single day (80 percent of which were launched at night), which corresponds to 3.5 percent of

[18] Patriot Advanced Capability-3 (PAC-3) is a more advanced version of the PAC-2 system used in the 1991 Gulf War.

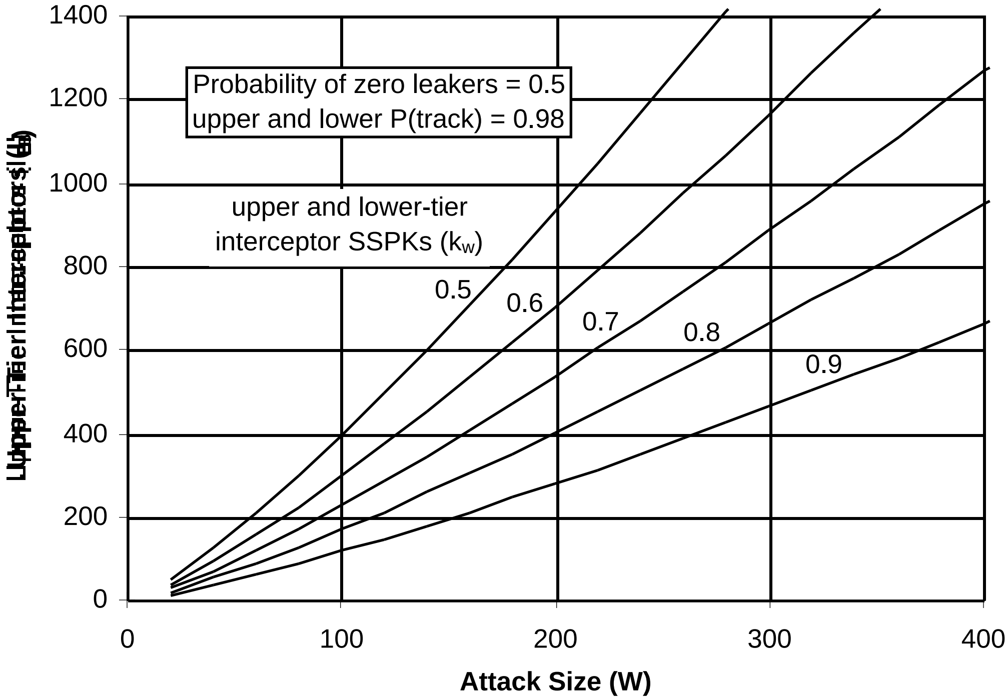

Fig. 18 - Upper-Tier Interceptors vs. Attack Size

Iraq's arsenal prior to the war.[19] Therefore, each defended site must have enough interceptors to handle this fraction of the total arsenal, under the assumption that subsequent attacks can be handled by replenishing the interceptors at the site(s) being exhausted within less than 24 hours.

If 10 THAAD sites are required to cover South Korea and Japan and the number of upper-tier interceptors found from Fig. 18 are spread evenly across all 10 sites, then each site can handle attacks of up to 10 percent of the total ballistic missile arsenal in a single day. In this case, the total number of interceptors that must be deployed to the theater to guarantee that no single site can be saturated is equal to 1.9 times the number of interceptors given in Fig. 18. If North Korea can launch more than 10 percent of its arsenal against a single defended area in one day, then the number of interceptors that must be deployed at each site increases.[20] If the threat contains decoys, then the number of upper-tier interceptors shown

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[19] This is around half the maximum daily launch rate Iraq could have attained. One should also note that the average daily Scud launch rate dropped threefold (from 4.7 to 1.5 launches per day) after U.S. Scud-hunting operations began. See The *Gulf War Air Power Survey, Summary Report*, Vol. I, Part II (Washington, D.C.: U.S. Government Printing Office, 1993), pp. 84–87.

[20] The general equation for the total number of interceptors that must be deployed to the theater is $I(S+M-1)/M$ where I is the total number of interceptors that must engage the attack from Fig. 18, S is the number of TMD sites required for complete coverage, and M is the number of theater ballistic missiles per launcher in the arsenal.

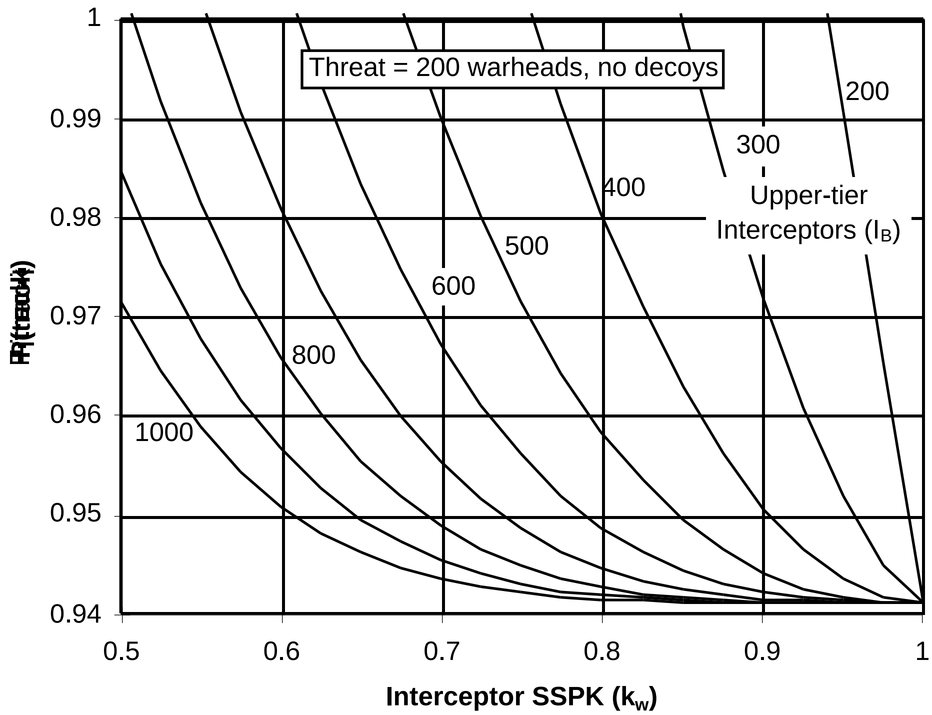

Fig. 19 - THAAD Interceptors (Barrage) vs. *P(track)* and SSPK

on the ordinate in Fig. 18 should be multiplied by W^*/W to get the number of interceptors required to meet the defense performance criterion in the presence of decoys.

The impact of countermeasures that reduce warhead detection and tracking probabilities or interceptor SSPKs can be assessed with Fig. 19, which shows the number of THAAD interceptors required to defeat a 200-warhead attack with probability 0.5, assuming both THAAD and PAC-3 systems operate in barrage mode and that they have the same effectiveness. Obviously, larger interceptor inventories can compensate for reduced technical performance, with the caveat that *P(track)* for the THAAD and PAC-3 systems must be above 0.94 in order to meet the defense performance criterion. Moreover, if a large number of interceptors must be fired at each warhead to meet the performance criterion, the defense performance at some point will degrade because the upper-tier battle space will become saturated. A critical assumption underlying Figs. 18 and 19 is that no common mode errors occur between the upper and lower tiers. If common mode errors exist between these layers, they must be below 0.1 percent for the plots in these figures to remain valid.

Concluding Observations

The above discussion suggests that a single-layer defense, e.g., a thin national missile defense, may be able to achieve high confidence that all incoming warheads are destroyed, but only if the defense can attain high levels of technical performance (i.e., multiple shots can be taken with interceptors having SSPKs above 0.6 and detection, tracking, and warhead classification probabilities above 0.98) and the attack contains fewer than 10–20 apparent warheads. Theater missile defenses will face larger threats. Hence, they must have multiple layers each with its own sensors and interceptors, especially for protecting high-value targets such as allied cities. Again, the TMD interceptor SSPKs must be above 0.6 and warhead detection and tracking probabilities must be above 0.94 for TMD systems to effectively protect cities against nuclear, biological, or chemical attacks. Common mode failures between layers must be negligible.

In general, the anticipated deployments for NMD and TMD systems currently under consideration by the Ballistic Missile Defense Office appear to be of the right size to handle near-term threats (i.e., unitary warheads without sophisticated decoys). However, if sophisticated decoys cannot be discriminated from warheads, the defense required to meet reasonable performance criteria grows nearly linearly with the size of the threat. Finally, defense performance degrades smoothly if other countermeasures are introduced that reduce the interceptor SSPK or interfere with warhead detection, tracking, and discrimination.