The Accuracy of Measuring Perimeter Points: Use of GPS vs. Bearing and Distance

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When performing a survey, there are often questions about what method to use when measuring ground position. This article discusses the importance of comparing the error range of GPS readings to that of the bearing and distance (compass and tape) method. Both techniques will result in a measured ground position that includes a certain margin of error. When working over smaller distances, the bearing and distance technique on average will produce more accurate results than when a GPS is used. This is due to the fact that the maximum error using compass and tape is proportionate to the distance measured while the maximum error of a GPS is consistent, and additive, for every single point recorded.

However, there is a threshold above which the accumulated errors of the bearing and distance technique are equal to or greater than those of a GPS. The following examples show how to calculate these errors and how to choose an appropriate technique or a combination of techniques for the most accurate field surveys.

Calculating error at point locations along a perimeter

The error estimates of this example are based upon the following assumptions:

- For a handheld GPS (not differentially corrected):
  - The GPS positioning error is 12m (6m radius from displayed position).
- For compass and tape:
  - Magnetic declination and anomalies are known and corrected accordingly.
  - The error of a compass angle reading is 1 degree.
  - The error of a distance measurement is insignificant compared to the size of the distance measured. (If a distance of 10 or 1000m is measured, the error is 1mm).
- The coordinates of the reference point are of high accuracy. (Measured either by a differential GPS or an alternative technique with high precision.).

Of course, two caveats must be borne in mind when evaluating error using either method. First, actual GPS error (for a typical non-differentially corrected handheld GPS) varies from minute to minute (and actually from second to second), due to variations in the atmosphere, GPS satellite orbits, and other factors beyond the control of the user. Therefore, the amount of GPS error almost certainly will vary from point to point. Second, the amount of error in a compass reading will depend on the quality of the compass, conditions in the field, and the ability and experience of the person taking the compass reading. Therefore, errors in compass readings will also vary from point to point, but are more likely to be consistent if the readings are taken by a single individual.

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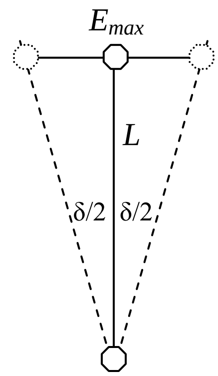
Maximum error when working with bearing and distance:

$$E_{\max} = 2 * \left| \sin\left(\frac{\delta}{2}\right) \right| * L$$



With a maximum GPS error of 12m ($E_{\max}$) and compass angle reading error of 1 degree (δ) the compass and tape based distance that produces the same error as a GPS reading is:

$$L_{\lim it} = E_{\max} \frac{1}{2 * \left| \sin\left(\frac{\delta}{2}\right) \right|} = 12[m] * \frac{1}{2 * \left| \sin(\frac{1}{2}) \right|} = 687.6m$$

According to this estimate, a GPS will produce a more accurate point location than the bearing and distances approach when working over a distance larger than 687.6m.

This same formula can be expanded to show that, when measuring a perimeter, the total error at each successive point is equal to the sum of the errors from all previous points:

$$E_{Tot} = 2 * \left| \sin\left(\frac{\delta_1}{2}\right) \right| * L_1 + 2 * \left| \sin\left(\frac{\delta_2}{2}\right) \right| * L_2 + \ldots + 2 * \left| \sin\left(\frac{\delta_n}{2}\right) \right| * L_n$$

With a constant compass angle error: $\delta_1 = \delta_2 = \ldots = \delta_n = \delta$

$$E_{Tot} = 2 * \left| \sin\left(\frac{\delta}{2}\right) \right| * L_1 + 2 * \left| \sin\left(\frac{\delta}{2}\right) \right| * L_2 + \ldots + 2 * \left| \sin\left(\frac{\delta}{2}\right) \right| * L_n = 2 * \left| \sin\left(\frac{\delta}{2}\right) \right| \left(L_1 + L \ \ldots + L_n\right)$$

Results in: $E_{Tot} = 2 * \left| \sin\left(\frac{\delta}{2}\right) \right| * \sum_1^n L_i$

This derivation suggests that the positioning error at the final or n-th perimeter point measured by bearing and distance is the sum of all intermediary errors. Thus the total length of the surveyed perimeter (along with the GPS and compass error) is all that is needed to determine whether the bearing and distance approach is more accurate than GPS readings (within the limits of the preceding assumptions).

Calculating an error threshold

This same concept was used to determine the maximum perimeter length for the bearing and distance technique given a variety of GPS and compass errors (Table 1). It should be noted that the most accurate readings can be acquired using a combination of both techniques: measuring all points using a compass and tape until the threshold perimeter length is reached (i.e., the length at which the cumulative error using GPS would equal the cumulative error using compass and tape), then collecting the remaining points using a GPS. This concept will be expanded on in the next example.

GPS	Bearing & Distance	
Position Error [m]	Compass angle error [Degrees]	Maximum perimeter length [m])
6	1	343.8
6	2	171.9
12	1	687.6
12	2	343.8
18	1	1031.3
18	2	515.7
24	1	1375.1
24	2	687.6

Table 1. Maximum perimeter length that can be measured using bearing and distance before GPS readings will be more accurate. Maximum (threshold) perimeter lengths are calculated for assumed GPS errors of 6, 12, 18, and 24 meters and compass bearing errors of 1 and 2 degrees.

Accumulated error with distance and bearing

The error estimates of this example are based upon the previously stated assumptions, as well as the following:

- Any errors in the background map image are insignificant.

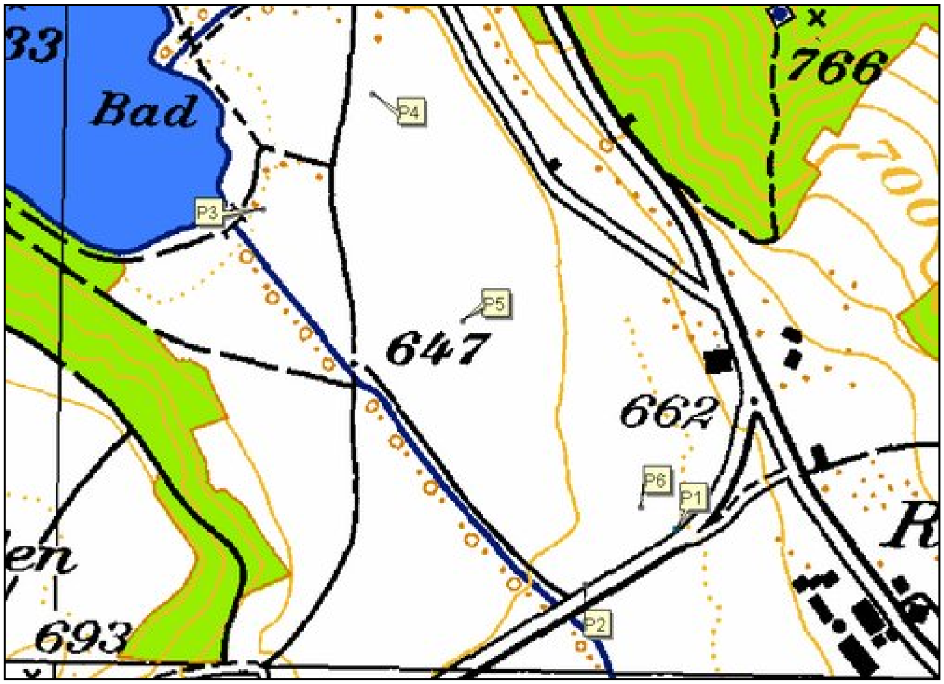


Figure 1. Position of perimeter points used in this example.

Working with a set of six perimeter points P1..P6 (Figure 1), it is possible to calculate the maximum positional error given the distance between points (Table 2). With a 1-degree compass angle error, this table provides the error between any two perimeter points, “Error per Segment,” and the accumulated error from P1 to Pn, “Error Accumulated”.

Start point	End point	Segment Length [m]	Accumulated Length [m]	Error per Segment [m]	Error Accumulated [m]
P1	**P2**	93	93	1.62	1.62
P2	**P3**	439	533	7.66	9.28
P3	**P4**	143	676	2.50	11.78
P4	**P5**	223	900	3.89	15.67
P5	**P6**	232	1132	4.05	19.72

Table 2. Accumulated positioning error of perimeter points using bearing and distance technique.

With the assumed GPS positioning error of 12m, points 5 and 6 would have been more accurate when measured with a GPS (Figures 3 and 4). Point 4, located at the accumulated perimeter length of 676m, represents the threshold where the accuracy of measurements taken by bearing and distance are comparable to those taken by a GPS.

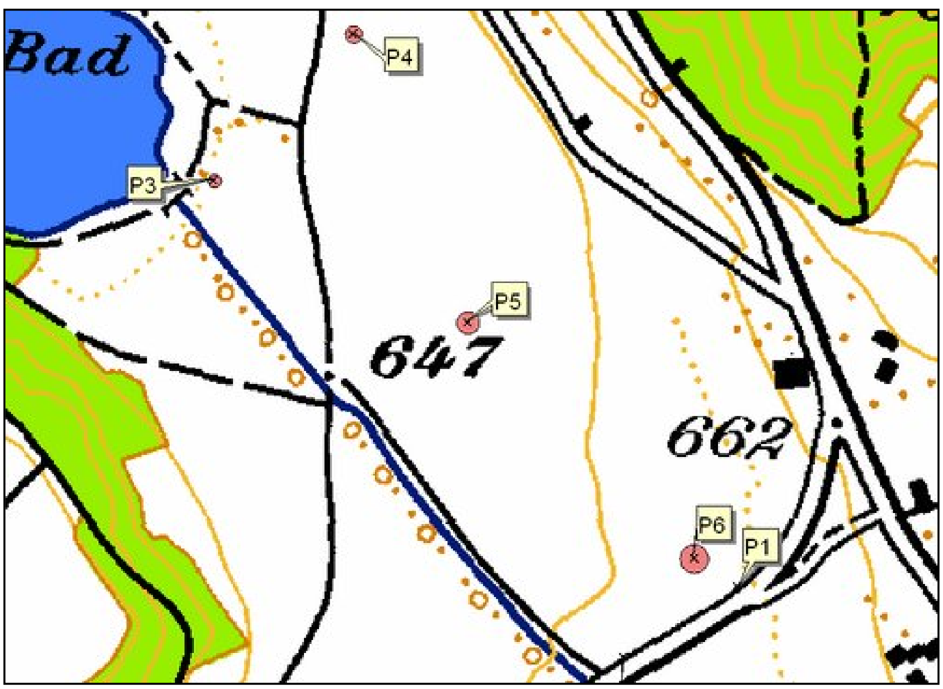


Figure 3. Red shaded circles show the accumulated error when points are measured by the bearing and distance technique.

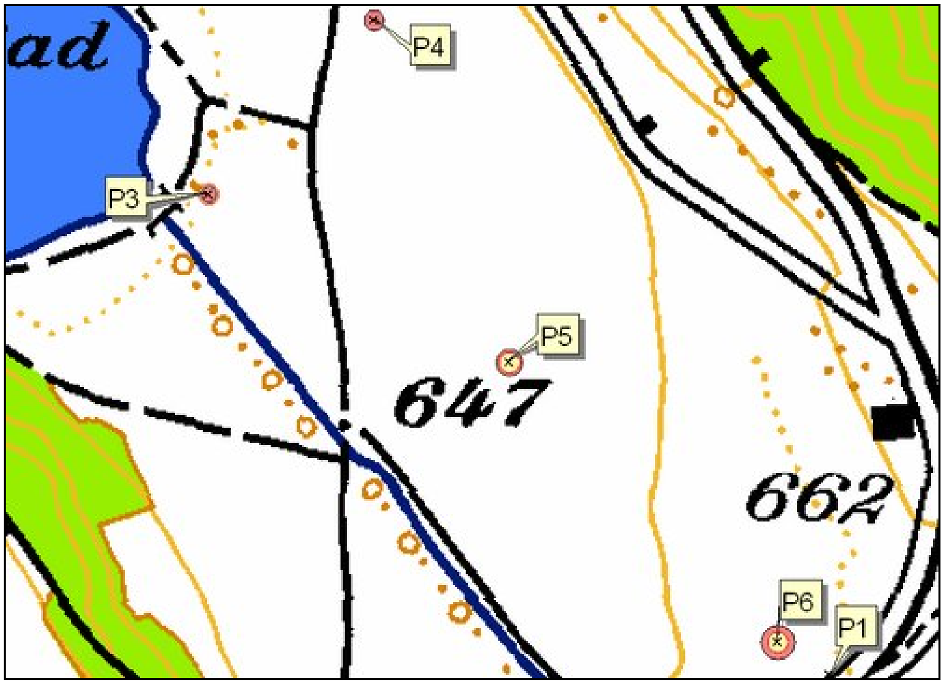


Figure 4. Comparing the accuracy of the two measurement techniques. The yellow circles indicate the GPS positioning error (P5 and P6), the red shaded circles the accumulated positioning error when measured by the bearing and distance technique.

An improvement to the accuracy of the bearing and distance technique could be achieved by measuring all perimeter points from the reference point, P1 (Table 3). Replacing the accumulated length with the distance from P1 to the individual points reduces the total length and thus the positioning error of the perimeter points (Figure 5).

Start point	Endpoint	Segment Length [m]	Positioning Error[m]
P1	P2	93	1.6
P1	P3	460	8.0
P1	P4	469	8.2
P1	P5	260	4.5

Table 3. Positioning error of points. Technique used – Bearing and Distance from reference point to each individual point.

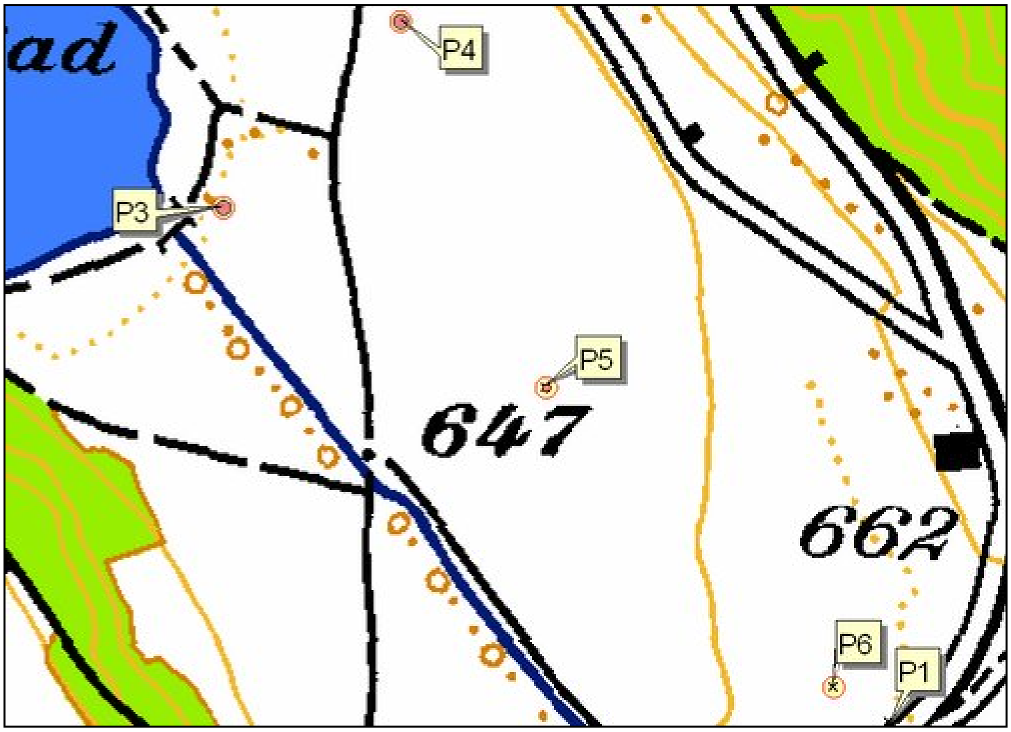


Figure 5. Comparing the accuracy of the two techniques when all distance measurements are made from the reference point (P1). The yellow circles indicate the GPS positioning error and the red circles show the error when measured by the bearing and distance technique.

While the final example shows that the GPS error is constantly larger than the bearing and distance technique using a single reference point, this approach could not be applied to an actual minefield survey as it would require the survey team to travel through the minefield to measure all points.

Errors in polygon areas

The previous examples demonstrated the positional errors that can occur when performing a perimeter survey with a GPS or the bearing and distance method. It is also important to investigate the accuracy of the polygon or area derived from these point locations. The final example explores this concept using points measured with a GPS.

The three concentric circles shown in Figure 6 represent potential GPS measurement errors of 10, 20, and 30m diameters. If the position to be measured is on one side of the street (small red dot), a measurement made with a GPS of 10 m accuracy might report a position in the middle of the street, while a GPS of 30 m accuracy could potentially report a position on the other side of the street.

As discussed previously, all point locations measured with the same GPS have a consistent maximum potential error. Figure 7 shows multiple positions measured using a GPS with 10m diameter maximum error. The blue circles represent perimeter points while the red circle indicates the location of a benchmark (the diameter of the circles is proportional to the amount of error)

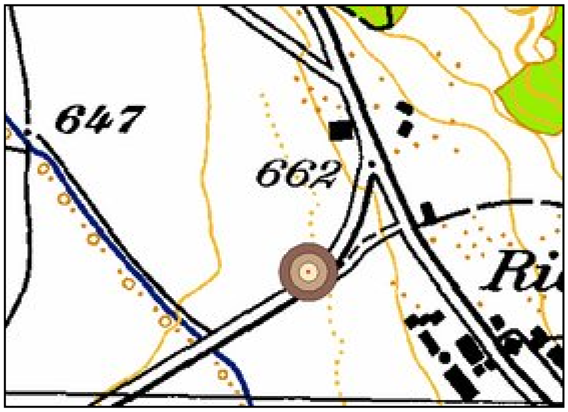


Figure 6. Review of positioning error of GPS.

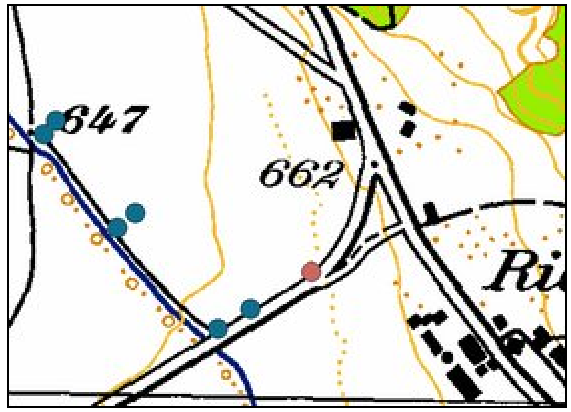


Figure 7. Measuring multiple positions with GPS.

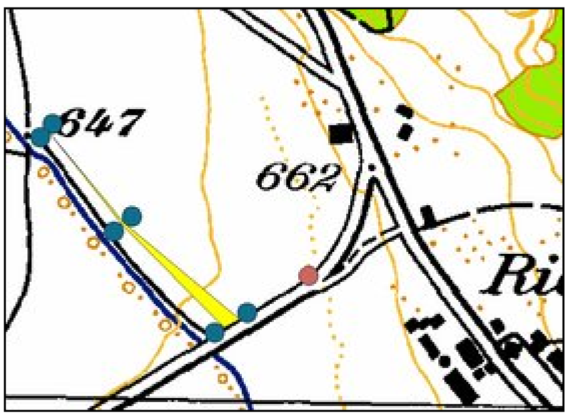


Figure 8. Minimum area polygon.

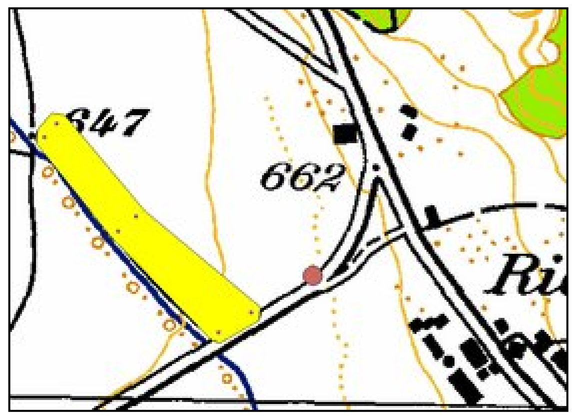


Figure 9. Maximum area polygon.

Figure 8 and Figure 9 demonstrate the two extreme cases of what might occur when generating a perimeter from these points measured with a GPS. Figure 8 assumes that all measured points are located at the position within the error circle that represents the *shortest* distance to the adjacent perimeter points – producing a polygon of minimum area. Figure 9 assumes that all measured points are located at the position within the error circle that represents the *maximum* distance to the adjacent perimeter points – producing a polygon of maximum area.

When the minimum and maximum area polygons are overlaid, the resulting buffer zone (shown in yellow) represents the full error-range of the polygon described by the GPS readings (Figure 10).

Figure 11 illustrates one additional case, where the benchmark point is measured by GPS and the perimeter points are measured using bearing and distance. For this example, errors of measuring bearing and distance are minimal. The resulting polygon maintains the correct shape, yet the position of the entire polygon will shift relative to the positioning error of the original GPS measured benchmark.

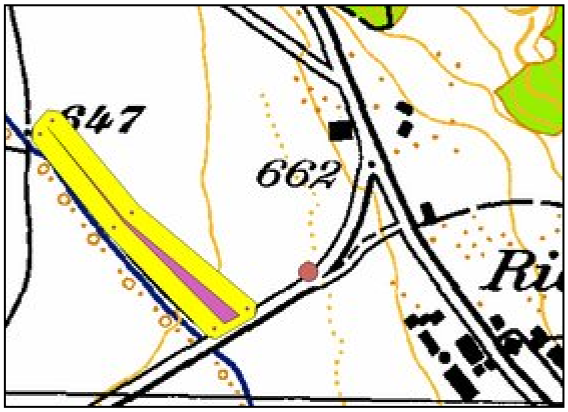


Figure 10. Overlay of minimum and maximum polygons.

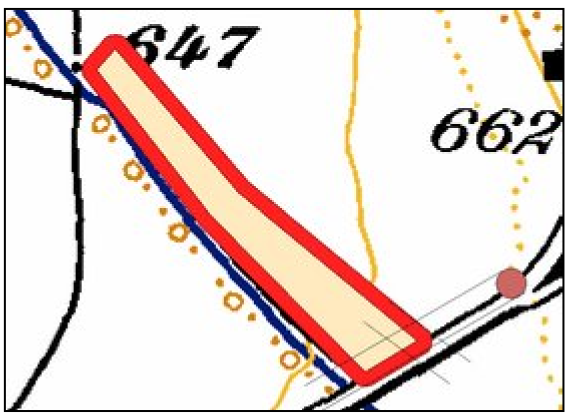


Figure 11. Polygon error based upon an initial GPS benchmark and perimeter points measured by bearing and distance.