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Abstract

This paper examines the interactions between employment and training policies. Their effectiveness in stimulating income may be interdependent for various important reasons. For example, the more employment policies stimulate the employment rate, the greater the length of time over which workers use the human capital generated by training policies. Moreover, the greater the government expenditures on employment and training subsidies, the higher the taxes required to finance these expenditures and these higher taxes reduce aggregate income. On account of such effects, employment and training policies may be complementary or substitutable with respect to aggregate income. To analyze these interactions, we construct a simple, dynamic model of hiring decisions, derived from microfoundations. The model is calibrated with German data. Surprisingly, the simulation shows that, for reasonable parameter values, the complementarities are weak or absent. The analysis provides a methodology for examining policy interactions which may be useful well beyond the bounds of employment and training policies.

Keywords: complementarities; hiring subsidies; training subsidies; vocational training; employment; unemployment;

JEL classification: J21, J23, J24, J64, J68

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1 Introduction[1]

When analyzing the impact of labor market policies, it is important to take into account possible interactions between different policies. In this context, one important aspect are complementarities between two policies, i.e. the effect of each policy on e.g. unemployment is greater when it is implemented in conjunction with the other policy than in isolation. Ignoring the possibility of complementarities could distort the evaluation of the performance of policies. This paper offers a methodology which captures potential complementarities of labor market policies.

As an example we consider two important policies: training subsidies and hiring subsidies[2]. Most OECD countries have implemented such policies to encourage both employment and training. To varying degrees, both policies serve a similar purpose, namely, to improve the employment and income perspectives, particularly for low-skilled workers. In this context, there are two important transitions in the labor market: the transition from school into training and the transition from training into work. Whereas training policies are meant to ease the transition into training, hiring subsidies are meant to facilitate the transition from apprenticeship to work. This paper deals with two questions: (i) Are there complementarities between both policies? (ii) Given the existence of complementarities, on what institutional and policy features of the economy the size of complementarities depends?

One possible channel whereby both policies are complementary is the following: the more hiring subsidies stimulate the employment rate, the greater the length of time over which workers use the human capital generated by training policies. Hiring subsidies facilitate the transition from training to work as they increase the probability that an apprentice continues working after having finished training successfully. Thereby, hiring subsidies improve the effectiveness of training policies. As hiring subsidies also increase the expected profits being generated by a former apprentice, the training incentives of the firms raise. Thus, hiring subsidies indirectly increase the number of people being hired as apprentices. This broadens the target group for training subsidies. However, both subsidies might be substitutes because both subsidies increase the trained labor force and reduce the non-trained labor force. They only differ with respect to the transition they are targeted at. Whereas training subsidies aim at increasing the number of people being in training (first transition), hiring subsidies aim at increasing the hiring of successful apprentices as trained employees (second transition). Moreover, the greater the government expenditures on employment and training subsidies, the higher the taxes required to finance these expenditures and these higher taxes reduce the expected life time income. On account of these effects, employment and training policies may be complementary or substitutable with respect the expected life time income.

Our analysis tackles these issues by presenting a macro model of the labor market that is rich enough to capture the various groups of workers relevant to these alternative policy options, while at the same time being simple enough to generate straight-forward, intuitively transparent, policy guidelines. The model allows to identify and qualify each effect being associated with the subsidies.

[1]We would like to thank Alessio J.G. Brown, Wolfgang Lechthaler and Christian Merkl for valuable comments. Moreover we would like to thank the participants of the 2007 Annual Meeting of the Verein für Socialpolitik, of the 3rd Annual IZA Conference on the Evaluation of Labor Market Programs, in particular Jose Galdo, and the participants of the IAB workshop on "Evaluation of Passive and Active Labour Market Policies for the Long-Term Unemployed and Social Benefit Recipients" for the discussion. Besides, we are very grateful to Markus Hummel (IAB) and Jessica Erbe (BIBB) for providing us data and valuable information.

[2]Sometimes, "hiring subsidies" are also called "employment subsidies" or "employment vouchers" and are implemented through a wide variety of policy instruments, such as tax breaks or grants. As they all - given that they are awarded only for a limited period of time - have analogous effects on labor market activities and government budgetary outlays, this paper groups them together under the broad heading of "hiring subsidies".

The analysis is based on a model which takes some important labor market imperfections – such as wage bargaining, hiring and separation costs as well as imperfections related to the tax and transfer system – as given. Thus, we do not intend to derive policies as first-best responses to labor market failures. In fact, we suppose that the institutions being responsible for labor market failures can be modified only successively and not in the short term. Given this, we analyze the impact of the two subsidies in the presence of these institutions. This approach is supposed to be very relevant to the decisions that policy makers commonly have to make.

We calibrate the model for the German labor market, which is especially characterized by a high unemployment of the low-skilled people. Moreover, the transition to training and from training to work has become more difficult in recent years.

This paper is organized as follows. Section 2 provides the underlying ideas and the relation to the literature. Section 3 presents the theoretical model of the labor market. In section 4, we solve a simplified model analytically in order to give the underlying intuition. In section 5 we calibrate the model and discuss the numerical results, we start with the simplified version of the model and enlarge it gradually. Finally, section 6 concludes.

2 Underlying ideas and relation to the literature

Our study is related to a variety of previous studies analyzing the impact and optimal design of employment subsidies, both, theoretically and empirically. The initial work was done by Pigou (1933) and Kaldor (1936).[3] Often, the search and matching framework (see Mortensen and Pissarides (1994)) has been used to analyze connections between the labor market and the education sector (see e.g. Charlot et al. (2005)) or the effect of labor market policies (see e.g. Cardullo and van der Linden (2006), Danthine (2005), Boone and van Ours (2004), Mortensen and Pissarides (2003) and Bovenberg et al. (2000)). However, the matching technology[4] is assumed to be stable through time. This assumption is acceptable given that the matching technology can be considered independent of input and output of the matching process. However, some empirical studies estimating search and matching functions (see Blanchard and Diamond (1989) for the United States, and Fahr and Sunde (2001, 2004) for Germany) have not confirmed the stability through time but have found a negative time trend.[5]

Moreover, given that the matching process itself may not be invariant to a policy change, it is not admissible to use the matching function to analyze labor market policies. To prevent running afoul of the Lucas Critique, we do not base our analysis on a policy-invariant matching function. Instead, similar to Brown et al. (2006), we analyze explicitly how policies affect people's incentives given an intertemporal maximization of economic agents. In our analysis, the focus is on the firm side. This has two reasons: (i) labor demand, especially with respect to the low-skilled labor force, is the short side of the market in economies with high unemployment and (ii) the subsidies which are analyzed are paid to the firms. The household side gets involved through the wage bargaining.

[3] For a survey of the empirical literature, see for example Katz (1998). For US evidence, see Woodbury and Spiegelman (1987) and O'Leary et al. (2005). For British evidence, see Bell et al. (1999).

[4] Like a production function, the matching technology describes the relation between input – the number of unemployed (U) and the number of vacancies (V) – and output given by the number of matches (M): $M = f(U, V)$. Often, a Cobb Douglas function is used: $M = U^{\alpha}V^{\beta}$. However, if $\alpha + \beta$ do not sum up to 1, the results are input dependent.

[5] Besides, many empirical studies reject the hypothesis of constant returns to scale (e.g. Warren (1996) for the United States, Fahr and Sunde (2001) for Germany).

Many studies in this area are static and only account for the short-run impact of an employment policy.[6] There are, however, good theoretical and empirical reasons to believe that longer-run effects are important, often more important than the short-run effects.[7] In this context, our study differs from the literature, as we explicitly capture the dynamic effects of subsidies by specifying the transition rates between employment, unemployment and training as a function of the hiring incentives of the firm.

The existing dynamic frameworks for evaluating subsidies are mainly deterministic and not well suited to analyze the impact of the policy. Mortensen and Pissarides (2003) explore the effects of taxes and subsidies on job creation, job destruction, employment and wages in a search and matching equilibrium model. However, in their model, like in the models of Albrecht and Vroman (2002) as well as Cardullo and van der Linden (2006), migration between skill groups, which is an essential component in our model, does not take place. In this context, we contribute to the existing literature by explicitly allowing for migration from the low-skilled to the medium-skilled labor force.

This detailed grid allows us to analyze and contrast the effects of training and hiring subsidies in a common framework, explicitly taking the budgetary effects into account[8]. As several authors have stressed the importance of training programmes aimed at fighting unemployment, training subsidies to support human capital accumulation are supposed to play a crucial role. According to Kluve and Schmidt (2002) training subsidies can combat European unemployment: as "educational credentials matter considerably"; they come to the result, that programmes improve the labor market prospects of economically disadvantaged individuals by facilitating the transition out of unemployment.

The analysis is based on an ex ante policy evaluation. In this context, the paper contrasts with the ex post policy evaluation which is omnipresent in the literature (e.g. de Koning, 2007). However, according to Wolpin (2007), there is "little methodological or applied research explicitly concerned with ex ante policy evaluation using nonexperimental methods, ...".[9] The following analysis wants to contribute to fill the gap by using a macro approach with a special focus on complementarities.

The analysis of complementarities between labor market institutions and policies is prevalent in the literature (see e.g. Belot and van Ours (2001)). Theoretical analyses of complementarities can be found in Coe and Snower (1997), Orszag and Snower (1999a), L'Haridon (2002) as well as Burda and Weder (2002). However, they focus on complementarities between institutions or policies other than in this paper and do not deal with the question by how far the size of complementarities is affected by different features of the economy.

We now proceed to present the underlying model.

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[6] See, e.g., Layard et al. (1991: 490-492) and Snower (1994).

[7] Orszag and Snower (2000) show that the dynamic, long-run effects of employment subsidies differ from what may be expected in the short run, once the corresponding lagged adjustment processes have worked themselves out.

[8] Orszag and Snower (2003a and 2003b) stress that the literature has disregarded the total impact of employment subsidies on the government budget constraint. On the one hand, aggregate payroll taxes finance subsidies but on the other hand, payroll taxes can be reduced if the subsidies reduce unemployment and thereby unemployment benefit payments. In this paper we follow their line of reasoning.

[9] One example in this context is the analysis by Todd and Wolpin (2007). They assess the impact of a school subsidy program in Mexico by using a theoretical framework for an ex ante evaluation. However, there model is restricted to the micro level.

3 The model

The analysis is based on a Markov model of the labor market. The dynamics are governed by a Markov matrix that summarizes the transition probabilities among the different labor market states. The transition probabilities are the result of an optimization principle of the firms.

The model is meant to provide an analytical framework for analyzing the effectiveness of hiring and training subsidies. As noted, our model is meant to be both rich enough to capture the relevant heterogeneity of the labor market but it also aims to be simple enough to generate straight-forward, intuitively transparent, policy guidelines. Thus, the model involves some judicious compromises between analytical simplicity and the depiction of heterogeneous labor market behaviors. Specifically, the labor force is differentiated according to its different skill levels, which are defined by the level of educational attainment. We assume, that each skill level corresponds to a certain productivity level. Total population is divided into eight groups (see table 1): school leavers or newborn (B),[10] people joining vocational training (apprentices, T) and those being either employed (N) or unemployed (U). The employed and unemployed, respectively, are divided into three subgroups according to the skill level: low-skilled, medium-skilled and high-skilled. Vocational training takes p periods, so

state	variable
low-skilled employment	N_l
low-skilled unemployment	U_l
medium-skilled employment	N_m
medium-skilled unemployment	U_m
high-skilled employment	N_h
high-skilled unemployment	U_h
vocational training	T
school leavers	B

Table 1: The Labor Market States.

that there are p cohorts. In each period, a fraction $\rho + \theta$ leaves the vocational training where ρ is the mortality rate and θ is the break off rate of training. So, given the inflow into vocational training, A^1, the outflow p period later is given by $A^1(1-\rho-\theta)^p$. The stock of people being in vocational training is given by $T = A^1\sum_{c=1}^{p}(1-\rho-\theta)^{c-1}$.[11]

The individual person is either unemployed or employed (respectively engaged in vocational training). For simplicity, there is no capital. Moreover, we assume constant returns to labor. Let a_i be the productivity of an employee with a skill level $i = l, m, h$. (Here, as well as for other variables below, the subscript l stands for "low-skilled"; the subscript m for "medium-skilled", the subscript h for "high-skilled"). The firms face a random operation cost $\epsilon_{i,t}$ which is iid across workers and time within the skill group i. This cost may be interpreted as an operating cost or a productivity shock. With respect to all employees, its mean of future values is normalized to zero and its cumulative distribution $\Gamma_i(\epsilon_i)$ is time invariant.

In the model, training takes place within the "dual system of vocational training", the dominant form in Germany.[12] This is a combination of vocational training provided by a private employer and

[10] In the analysis, both interpretations of the variable B are possible: the essential point is that people leaving this state enter the labor market.

[11] In the initial steady state, A is exogenously given. In the presence of subsidies, A is determined endogenously.

[12] For a survey of the German dual system of vocational training, see Franz and Soskice (1995).

theoretical education in vocational schools. With respect to the latter, the associated costs (e.g. for school buildings, salaries of the teachers) are distributed among the population. We assume that these costs have a highly fix character, so that a change in the number of apprentices does not influence the level. For the sake of simplicity, they are ignored in the remainder. The costs which are important in the model are the costs for the employer which are caused by the provision of vocational training (e.g. wages of additional employees being in charge of the instruction of apprentices within the firm). With respect to the distribution of these costs, it is necessary to distinguish two types of training: general training and specific training.[13] With respect to the German system of vocational training, one can argue, that training has a mostly general character. Due to detailed plans determining the content of on-the-job training and central examinations, vocational training within the dual system is highly standardized on a national level.[14] According to the original theory by Becker (1964), in the presence of competitive markets, the employee receives all the returns from general training and thus also has to pay for training. However, German employers also pay a substantial amount of money for the training of apprentices [15] and thus - from the theoretical point of view - also pay for general training. It can be shown, that firms - in contrast to the original theory - have an incentive to invest in general vocational training, given that there are imperfections in the labor market.[16] In addition, the hold-up problem (see Williamson (1985) and Grout (1984)) does not occur if the investor (i.e. the firm paying for general education) receives the full marginal return on investment and therefore will not under-invest. However, a hold-up problem may arise if the costs of general education for the firms arise.[17] Without a shift of financial burden away from the firm sector, the fraction of firms providing vocational training would decrease. The firms would under-invest.

Agents in our model pursue the following sequence of decisions. First, the government sets the income tax rate to ensure that its tax receipts are equal to its expenditures. Second, wages are determined through bargaining. Third, the random operating costs are revealed and finally, employment decisions are made.

We now continue by presenting the dynamic structure.

3.1 The dynamic structure

The transitions among the labor market states are summarized in figure (1). In all states besides B, people face a probability ρ of dying. With respect to the school leavers, a fraction $\eta_{B,A}$ is hired as apprentice.[18] The residual part $(1-\eta_{B,A})B$ tries to get a job as a low-skilled employee; only a fraction η_l is hired. A low-skilled employee faces a probability φ_l of being fired and a probability $\eta_{N,A}$ of being hired as apprentice, a low-skilled unemployed faces a probability $\eta_{U,A}$ of being hired as apprentice and a probability $(1-\eta_{U,A})\eta_l$ of being hired as low-skilled employee. With a probability θ per period, an apprentice breaks off training, however, we assume that an apprentice is not fired.

[13] See Becker (1964). For a recent survey of the literature on private sector training see Leuven (2005).

[14] See for further details Harhoff and Kane (1993: 11) and Lindner (1998: 413).

[15] According to Beicht and Walden (2002), an employer being engaged in vocational training faces a net cost of 8,705 EUR per apprentice in 2000.

[16] See e.g. Harhoff and Kane (1993) and Acemoglu and Pischke (1999). Appendix A1 contains an analysis showing that firms have an incentive to invest in general vocational training.

[17] The costs of education could rise e.g. because of a shift of time from productive workplace activities to education. The additional financial burden (fall in output) could cause firms to reduce vocational training. See for an analysis of the changing environment and its consequences for vocational training e.g. Büchel (2002) and Woessmann (2004, 2006).

[18] In Appendix A4, we present in model, in which $\eta_{B,A}$ is supply-driven.

An apprentice who has finished training successfully and did not die, is hired as a medium-skilled employee with a probability η_{A,N_m}. A medium-skilled employee faces a probability φ_m of being fired. An unemployed medium-skilled is hired with a probability η_m. In the remainder, we call the transition rate $\eta_{B,A}$ describing the hiring of school leavers as apprentices "training rate" and not "hiring rate" in order to avoid confusion with the hiring of successful apprentices as medium-skilled employees.

In order to keep the model simple, we assume that the number of break offs and deaths is equal to the number of school leavers, so that the relevant population $(N_l + U_l + N_m + U_m + T + B)$ is constant. Moreover, we treat the high-skilled labor force $(N_h + U_h)$ as a quasi-fix factor, i.e. we assume, that these states are not affected by the introduction of subsidies. However, given the government budget constraint, it is necessary to take these states into account, because N_h also carries a part of the fiscal burden and U_h is responsible for a part of the fiscal burden. Ignoring it would bias the amount of the fiscal burden which has to be carried by the low-skilled and medium-skilled employees. Given this, the labor market system can be described as follows:

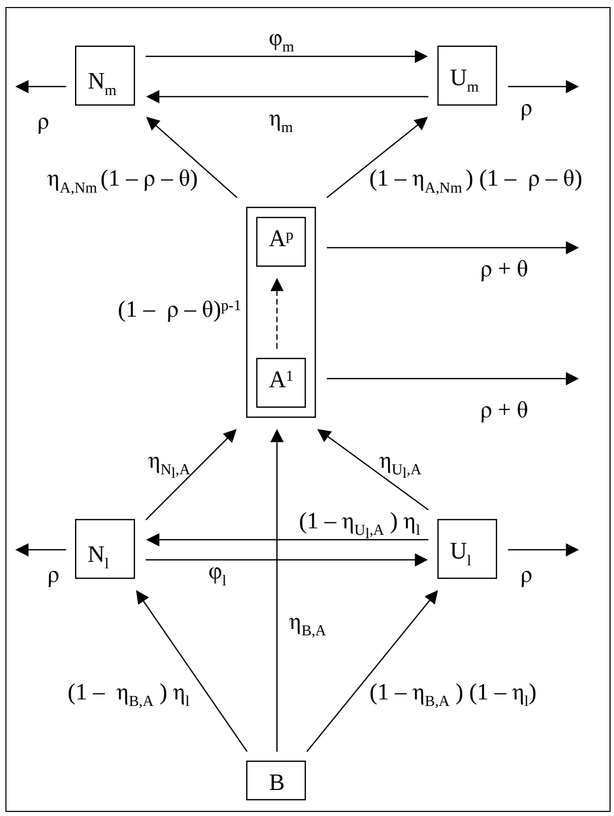

Figure 1: The dynamic structure of the model.

$$S_{t+1} = M_{t+1}\ S_t \tag{1}$$

where S_t is a vector of the different states:

$$S_t = (N_{m,t}, U_{m,t}, N_{l,t}, U_{l,t}, A^1_t, B_t) \tag{2}$$

and M_t is a Markov matrix of transition probabilities: (3)

$$M_t = \begin{pmatrix} (1-\varphi_m-\rho) & \eta_m & 0 & 0 & \eta_{A,N_m}(1-\rho-\theta)^p & 0 \\ \varphi_m & (1-\eta_m-\rho) & 0 & 0 & (1-\eta_{A,N_m})(1-\rho-\theta)^p & 0 \\ 0 & 0 & (1-\varphi_l-\eta_{N,A}-\rho) & (1-\eta_{U,A})\eta_l & 0 & (1-\eta_{B,A})\eta_l \\ 0 & 0 & \varphi_l & (1-(1-\eta_{U,A})\eta_l-\eta_{U,A}-\rho) & 0 & (1-\eta_{B,A})(1-\eta_l) \\ 0 & 0 & \eta_{N,A} & \eta_{U,A} & 0 & \eta_{B,A} \\ \rho & \rho & \rho & \rho & 1-(1-\rho-\theta)^p & 0 \end{pmatrix}$$

With respect to the transition rates $\eta_{N,A}$ and $\eta_{U,A}$, we assume that a transition is connected with a change of the firm. This is based on the assumption that there are two types of firms: (1.) firm which employ only low-skilled and (2.) firms which employ only medium-skilled and can be engaged in vocational training.[19]

3.2 The characteristics of the subsidies

In the following, we analyze two kinds of subsidies:

- Hiring subsidies ($\sigma^{\eta_{A,N_m}}$) are paid to firms which hire successful apprentices as medium-skilled employees. The subsidy is paid during the first period of the employment spell.
- Training subsidies (σ^{k_v}) are paid to firms which hire school leavers as apprentices. The training subsidies are paid per apprentice and per period over the whole phase of vocational training. They reduce the costs of the firm being associated with vocational training.

Whereas hiring subsidies aim at improving the employment situation of the successful apprentices by increasing the hiring incentive of the firm, training subsidies aim at improving the human capital in a first step and then in a second step, the long-term employment perspective, given that the employment rate of the medium-skilled labor force is higher than the employment rate of the low-skilled labor force.

3.3 Government Budget Constraint

When analyzing the government budget constraint, four policy instruments are taken into account: (i) a payroll tax with a tax rate τ_i, (ii) an unemployment benefit f_i, (iii) a hiring subsidy, $\sigma^{\eta_{A,N_m}}$ and (iv) a training subsidy, σ^{k_v}. Our model has three levels of payroll tax rates, in order to match the German progressive tax system ($\tau_h > \tau_m > \tau_l$). The ratios are assumed to be exogenous, whereas the levels are set so that the tax receipts are equal to the government's expenditures. We assume that people being engaged in vocational training do not pay taxes. Given the presence of the subsidies above, the government budget constraint is expressed as follows:

$$\begin{aligned} \textstyle\sum_{i=l,m,h}\tau_i w_i N_i = {} & \textstyle\sum_{i=l,m,h}\beta_i w_i(1-\tau_i)U_i \\ & + \sigma^{k_v}\ (\eta_{B,A}\ B)\textstyle\sum_{i=0}^{p-1}(1-\rho-\theta)^i \\ & + \sigma^{\eta_{A,N_m}}\ \eta_{A,N_m}(1-\rho-\theta)^p\ A^1 \end{aligned} \tag{4}$$

[19] This can be justified by the assumption that the firms are engaged in different sectors with different requirements with respect to human capital.

where the left-hand side stands for the tax receipts, to be paid by the employees of different skill groups. The term on the right-hand side represents the sum of the unemployment benefits, with the net replacement rate β_i, the training subsidies σ^{k_v} paid to firms for hiring school leavers as apprentices and finally the hiring subsidies, $\sigma^{\eta_{A,Nm}}$, paid to firms which hire successful apprentices as medium-skilled employees.[20]

3.4 Wage Determination

For simplicity, let the wage $w_{i,t}$ for each skill level i be the outcome of a Nash bargain between the median employee of that skill level and the firm. The median insider faces no risk of dismissal at the negotiated wage. The wage is renegotiated in each period. We start by calculating the relevant surplus for both, the employee and the firm.

3.4.1 Surplus of the employee

Each person has the following utility function:

$$u_{i,t}(C) = \frac{1}{1-\gamma}[C_{i,t}]^{1-\gamma} \text{ with } i = l, m, h \tag{5}$$

where γ is the coefficient of relative risk aversion (CRRA). Utility depends positively on consumption $C_{i,t}$.[21] Under bargaining agreement, the employee receives the net wage $w_i(1-\tau_i)$ in each period. The expected present value of the employee´s utility, $V_{i,t}^N$, is:

$$V_{i,t}^N = \frac{1}{1-\gamma}[w_{i,t}(1-\tau_{i,t})]^{1-\gamma} + V_{i,t}^F \tag{6}$$

where $V_{i,t}^F$ is the present value of utility resulting from the expected future life time income. (Here, as well as for other variables below, the superscript N stands for "employed"; the superscript U for "unemployed"). Under disagreement, the employee´s fallback position is $f_{i,t}$, assumed to be equal to the unemployment benefit. Assuming that disagreement in the current period does not affect the expected future life time income $V_{i,t}^F$, the corresponding expected present value of utility is:

$$V_{i,t}^{d,N} = \frac{1}{1-\gamma}[f_{i,t}]^{1-\gamma} + V_{i,t}^F \tag{7}$$

Given the expected present values in the cases of agreement and disagreement, we can calculate the bargaining surplus for the employee ($S_{i,t}^E = V_{i,t}^N - V_{i,t}^{d,N}$):

$$S_{i,t}^E = \frac{1}{1-\gamma}[w_{i,t}(1-\tau_{i,t})]^{1-\gamma} - \frac{1}{1-\gamma}[f_{i,t}]^{1-\gamma} \tag{8}$$

[20] Recall, that A^1 is the inflow into training and $(1-\rho-\theta)^p A^1$ is the outflow from training.

[21] In this model, for simplicity, workers consume all their income, i.e. either the net wage in the case of employment or the unemployment benefit in the case of unemployment.

3.4.2 Surplus of the firm

Under bargaining agreement, the firm receives the expected profit $(a_{i,t} - w_{i,t})$ in each period t. The expected present value of the profit π with respect to a medium-skilled employee (with $i = m$) is:

$$\pi_{m,t} = (a_{m,t} - w_{m,t}) + \delta((1 - \varphi_{m,t+1} - \rho)\pi_{m,t+1} + \varphi_{m,t+1}(-\varsigma_{m,t+1})) \tag{9}$$

where ς_m are the firing costs and δ is the discount factor. The firm's fallback profit is approximated by the firing costs $(-\varsigma_m)$, during disagreement the employee imposes the maximal cost on the firm (e.g. through strike, work-to-rule, sabotage) short of inducing dismissal. Again, we assume that disagreement in the current period does not affect future returns. Thus, the present value of expected profit under disagreement is:

$$\pi^d_{m,t} = -\varsigma_{m,t} + \delta((1 - \varphi_{m,t+1} - \rho)\pi_{m,t+1} + \varphi_{m,t+1}(-\varsigma_{m,t+1})) \tag{10}$$

With respect to a low-skilled worker $(i = l)$ and the future profit generated by the low-skilled worker, there is the additional probability $(\eta_{A,Nm})$ that the worker is hired as apprentice by another firm. However, this possibility affects the expected profit of the firm in the case of agreement as well as in the case of disagreement. Thus, when calculating the surplus, this component cancels out. Finally, we can calculate the bargaining surplus for the firm $(S^F_{i,t} = \pi_{i,t} - \pi^d_{i,t})$ independent of the skill level i:

$$S^F_{i,t} = (a_{i,t} - w_{i,t}) \ + \varsigma_{i,t} \tag{11}$$

3.4.3 Bargaining

To determine the wage, we use the Nash bargaining solution. The bargaining power of the employee is denoted by $\mu \, \epsilon \, \langle 0, 1\rangle$, and the bargaining power of the firm by $1 - \mu$. The wage $w_{i,t}$ solves:

$$\max_{w_{i,t}} \ (S^E_{i,t})^{\mu} \ (S^F_{i,t})^{1-\mu} \tag{12}$$

For the steady state, we assume that the firing costs are $\varsigma_i = c_\varphi \ w_i$ and that the unemployment benefit level $f_i = \beta_i \ w_i \ (1 - \tau_i)$ is defined on the economywide average net wage $w_i \ (1 - \tau_i)$ for each skill group. The negotiated wage is (with $\xi = 1 - \gamma$):

$$w_i = \frac{a_i \ \mu \ \xi}{1 - \beta_i^{\xi}(1 - \mu) - \mu(1 - (1 - c_\varphi)\xi)} \ \text{with } i = l, m, h \tag{13}$$

3.5 Training and Hiring Rate

With respect to the analysis of hiring and training subsidies, two hiring decisions are of importance as they are influenced by the subsidies: (i) the hiring of the school leavers[22] and (ii) the hiring of successful apprentices as medium-skilled employees. The training rate as well as the hiring rate are functions of the corresponding expected profits. Recall that, a worker of type i produces a_i of output per period, the firm has to pay a wage w_i.

[22] Recall that training subsidies are not paid for hiring low-skilled employees (N_l) or low-skilled unemployed (U_l) as apprentices. The corresponding transition rates, which would be affected $(\eta_{N_l,A})$ and $(\eta_{U_l,A})$ by the subsidy are very low, in particular in relation to $(\eta_{B,A})$. Thus, the quantitative impact of subsidies would be low. However, the complexity of the model would increase signficantly.

First, we derive the hiring rate η_{A,N_m}. The expected present value of profit generated by the employee, after the random cost $\epsilon_{\eta_{A,N_m}}$ is observed is:[23]

$$\begin{aligned}\pi_{A,N_m,t} &= -\epsilon_{\eta_{A,N_m,t}} + \sigma^{\eta_{A,N_m}} \\ &\quad + (a_{m,t} - w_{m,t}) \sum_{i=0}^{\infty} \delta^i (1 - \varphi_{m,t+i} - \rho)^i - \delta\ \varphi_{m,t+1}\ \varsigma_{m,t+1} \sum_{i=0}^{\infty} \delta^i (1 - \varphi_{m,t+i} - \rho)^i\end{aligned} \tag{14}$$

Under the assumption that the hiring decision is made by the firm which also conducted the vocational training, the hiring costs are zero because the successful apprentice to be hired as medium-skilled employee is already in the firm. Given this, the person is hired, if the expected present value of profit is positive: $\pi_{A,N_m,t} > 0$, thus:

$$\epsilon_{\eta_{A,N_m,t}} < \sigma^{\eta_{A,N_m}} + \frac{a_m - w_m - \delta\ \varphi_m\ \varsigma_m}{1 - \delta(1 - \varphi_m - \rho)} \tag{15}$$

We assume that $\epsilon_{\eta_{A,N_m,t}}$ is uniformly distributed between $\epsilon^-_{\eta_{A,N_m}}$ and $\epsilon^+_{\eta_{A,N_m}}$. The corresponding hiring rate is:[24]

$$\eta_{A,N_m} = \frac{(\sigma^{\eta_{A,N_m}} + \frac{a_m - w_m - \delta\ \varphi_m\ \varsigma_m}{1 - \delta(1 - \varphi_m - \rho_m)}) - \epsilon^-_{\eta_{A,N_m}}}{\epsilon^+_{\eta_{A,N_m}} - \epsilon^-_{\eta_{A,N_m}}} \tag{16}$$

Second, we consider the hiring of school leavers (B) as apprentices. If the corresponding training rate would only depend on the profit in the phase of vocational training, there would be no hiring, because the profit in the training phase is negative. The output a_v generated by the apprentice is smaller than the sum of the wage, w_v, and the additional costs of vocational training, k_v, the firm faces. However, the firm regards the costs of vocational training as an investment, which causes a profit (π_{A,N_m}, see eq. (14)), once the successful apprentice continues working in the firm as a medium-skilled employee with a certain probability $\eta_{A,N_m}(1 - \rho - \theta)$.[25] Therefore, also the latter phase has to be taken into account, when determining the training rate. The expected present value of profit generated by an apprentice is:[26]

$$\begin{aligned}\pi_{B,A,t} &= -\epsilon_{\eta_{B,A},t} + (a_{v,t} - w_{v,t} - k_{v,t} + \sigma^{k_v}) \sum_{i=0}^{p-1} \delta^i (1 - \rho - \theta)^i \\ &\quad + \delta^p (1 - \rho - \theta)^p\ \eta_{A,N_{m,t+p}}\ E(\pi_{A,N_m})\end{aligned} \tag{17}$$

Given the firing costs $\chi_{K,A}$, a school leaver is hired as apprentice if $\pi_{B,A} > \chi_{B,A}$, thus:

$$\begin{aligned}\epsilon_{\eta_{B,A},t} &< -\chi_{B,A} + (a_{v,t} - w_{v,t} - k_{v,t} + \sigma^{k_v}) \sum_{i=0}^{p-1} \delta^i (1 - \rho - \theta)^i \\ &\quad + \delta^p (1 - \rho - \theta)^p\ \eta_{A,N_{m,t+p}}\ E(\pi_{A,N_m})\end{aligned} \tag{18}$$

[23] Recall that eq. (14) is equal to eq. (9) for $-\epsilon_{\eta_{A,N_m,t}} = 0$ and $\sigma^{\eta_{A,N_m}} = 0$.

[24] For a derivation of the hiring rates, see Appendix A2.

[25] The assumption that a hired successful apprentice stays in the firms implies that the hiring rate η_{A,N_m} is equivalent to the take over rate.

[26] Note that $E(\pi_{A,N_m,t})$ is given by equation (14) with $-\ \epsilon_{\eta_{A,N_m,t}} = 0$.

We assume that $\epsilon_{\eta_{B,A},t}$ is uniformly distributed between $\epsilon^-_{\eta_{B,A}}$ and $\epsilon^+_{\eta_{B,A}}$. The corresponding hiring rate is:

$$\eta_{B,A,t} = \tag{19}$$
$$\frac{[-\chi_{B,A} + (a_v - w_v - k_v + \sigma^{k_v}) \sum_{i=0}^{p-1} \delta^i (1-\rho-\theta)^i + \delta^p (1-\rho-\theta)^p \; \eta_{A,N_m} \; E(\pi_{A,N_m})] - \epsilon^-_{\eta_{B,A}}}{\epsilon^+_{\eta_{B,A}} - \epsilon^-_{\eta_{B,A}}}$$

Whereas training subsidies only have a direct effect - they increase the training rate $\eta_{B,A}$ -, hiring subsidies also have an indirect effect. Primarily, they aim at easing the transition from training to work, i.e. they increase the hiring rate $\eta_{N,A}$ (eq. (16)). Moreover they also have an indirect effect as they increase the training rate (eq. (19)).[27] The decision to hire an apprentice also depends on the probability, that the person continues working in the firm as medium-skilled. The higher the hiring rate, $\eta_{N,A}$, the higher is the probability and the higher is the incentive to hire a person as apprentice. Thus, hiring subsidies not only increase the fraction of apprentices who are hired as medium-skilled employees, they also increase the number of apprentices.

All other transition rates, especially the hiring and separation rates of the low-skilled and the medium skilled, respectively, (η_i and ϕ_i with $i = l, m$) are not affected by the implementation of subsidies. They are treated as constant factors, so that a microfoundation is not necessary.

The critical reader might argue, that not all successful apprentices are taken over by the firm which conduct vocational training. Some of the successful apprentices will get a job in a firm which is not engaged in vocational training. So far, we have assumed that the hiring rate η_{A,N_m} has been identical to the take over rate. Now, we assume, that there are two hiring rates: (1.) η^1_{A,N_m} being associated to the firm which is engaged in vocational training, and (2.) η^2_{A,N_m} being associated to the firm which is not engaged in vocational training. Now, only η^1_{A,N_m} can be interpreted as take over rate and is given by eq. (16). With respect to η^2_{A,N_m} and in contrast to eq. (16), hiring costs (χ_m) have to be taken into account, as the apprentice is hired by another firm for which the successful apprentice is an outsider. Therefore, the hiring rate η^2_{A,N_m} is calculated as follows:

$$\eta^2_{A,N_m} = \frac{[-\chi_m + \sigma^{\eta_{A,N_m}} + \frac{a_m - w_m - \delta\ \phi_m\ \varsigma_m}{1-\delta(1-\phi_m-\rho_m)})] - \epsilon^-_{\eta^2_{A,N_m}}}{\epsilon^+_{\eta^2_{A,N_m}} - \epsilon^-_{\eta^2_{A,N_m}}} \tag{20}$$

The crucial difference with respect to the previous analysis is the substitution of η_{A,N_m} in eq. (19) by η^1_{A,N_m}, i.e. hiring subsidies paid to firms which are not engaged in vocational training but hire successful apprentices (with the hiring rate η^2_{A,N_m}), do not have an impact on the training rate $\eta_{B,A}$.

3.6 Labor Market Equilibrium

The following system of equations constitutes the labor market equilibrium:

- the six labor market dynamic equations given by the transition matrix (3)
- the government budget constraint, equation (4),[28] and finally

[27] The hiring subsidy increases $\eta_{N,A}$ and $E(\pi_{A,N_m})$ and thereby according to eq. (19) $\eta_{B,A}$.

[28] As the equation describing the government budget constraint contains three tax rates, it is necessary to introduce two additional equations in order to close the model. They describe the ratios between the tax rates, t_h, t_m and t_l.

- the equations determining the hiring rates, equations (16), (19) and (20).

Now, we continue by presenting an analytical solution of a simplified version of the model. After having illustrated the solution of the benchmark model, we make some robustness checks with respect to the parameters whose values are highly uncertain.

4 A simple analytical solution

In the following analysis, aggregate income is calculated as the income aggregated over all labor market states:[29]

$$\Phi = \sum_{i=m,l,h} [N_i \ w_i(1-t_i) + U_i \ \beta_i \ w_i(1-t_i)] + T \ w_v \tag{21}$$

4.1 The benchmark model

Before starting to solve the model numerically, we present a simple analytical solution, which helps to understand the underlying intuition. In order to get an analytically traceable solution, we make the following simplifications: (1.) there are no taxes:[30] $t_i = 0$ with $i = l, m, h$, (2.) training takes one period: $p = 1$ and (3.) training is not broken off: $\theta = 0$. With respect to the high-skilled labor force, all variables are treated as constant. With respect to the medium- and low-skilled labor force, the labor market states can be expressed as functions of the transition rates, i.e. in particular as functions of the hiring rates (eq. (16), eq. (20) and eq. (19)) and thereby as a function of the hiring and training subsidies. The corresponding steady state expressions can be derived on the base of the Markov Matrix.

According to Coe and Snower (1997) "policies are complementary in the sense that the effect of each policy is greater when implemented in conjunction with the other policy than in isolation". To check the presence of complementarities, we calculate the cross derivative of Φ with respect to the two subsidies, σ^{k_v} and $\sigma^{\eta_{A,N_m}}$, which is

$$\frac{\partial^2 \Phi}{\partial \sigma^{k_v} \ \partial \sigma^{\eta_{A,N_m}}} = [\underbrace{w_m(1-\beta_m) \ (\Sigma_\epsilon) \ (1-\rho)\rho^2 \ (\eta_l + \eta_{N,A}(1-\eta_l) + \rho + \phi_l)}_{<0}] \ / \tag{22}$$

$$[\underbrace{(\Pi\epsilon) \ (1+\rho)}_{<0} \underbrace{(\eta_l \ (1-\eta_{U,A})(\eta_{N,A}+\rho) + (\eta_{U,A}+\rho)(\eta_{N,A}+\rho+\phi_l))}_{>0} \underbrace{(\eta_m + \rho + \phi_m)}_{>0}]$$

with:

$\Sigma_\epsilon = (\epsilon^-_{\eta^1_{A,N_m}} - \epsilon^+_{\eta^1_{A,N_m}} + \epsilon^-_{\eta^2_{A,N_m}} - \epsilon^+_{\eta^2_{A,N_m}}) < 0$ and

$\Pi_\epsilon = (\epsilon^-_{\eta^1_{A,N_m}} - \epsilon^+_{\eta^1_{A,N_m}})(\epsilon^-_{\eta^2_{A,N_m}} - \epsilon^+_{\eta^2_{A,N_m}})(\epsilon^-_{\eta_{B,A}} - \epsilon^+_{\eta_{B,A}}) < 0.$

The cross derivative is unambiguously positive. Hence, the result reveals - at least for a very simple version of the model - the existence of complementarities with respect to aggregate income in the sense that the impact of one subsidy one aggregate income is bigger in the presence of the other subsidy.

The intuition behind this is as follows: hiring subsidies facilitate the transition from vocational training to work. They increase the probability that an apprentice continues working in the firm as

[29] The income in the labor market state B is 0.

[30] In the presence of taxes, a purely analytically examination with the given model is not possible. Therefore we have done the analysis with a different, 2-period-model, which is presented in appendix A3.

a medium-skilled employee after having finished vocational training successfully. Thereby, hiring subsidies improve the effectiveness of training policies, since the higher probability will amplify the positive impact of training subsidies. Moreover, hiring subsidies indirectly increase the number of people being hired as apprentices. This broadens the target group for training subsidies.

4.2 Robustness Checks

This section discusses the impact of different parameter values on the size of complementarity. In this context, we calculate the derivative of the cross derivative $\frac{\partial^2 \Phi}{\partial \sigma^{k_v} \partial \sigma^{\eta_{A,N_m}}}$ with respect to the parameters of interest. We focus on three parameters which face a relatively high uncertainty (compared to the calibration of other parameters, see section below): the bargaining power of the medium-skilled (μ_m), the degree of risk aversion ($\gamma = 1 - \xi$) and finally, the limits of the ϵ-distributions. We start by analyzing the impact of the lower limit of the $\epsilon_{\eta^q_{A,N_m}}$-distribution.

4.2.1 Distribution parameter 1: $\epsilon^-_{\eta^q_{A,N_m}}$

When calculating the derivative with respect to $\epsilon^-_{\eta^q_{A,N_m}}$ (with $q = 1, 2$), we get the following expression:

$$\partial(\frac{\partial^2 \Phi}{\partial \sigma^{k_v} \ \partial \sigma^{\eta_{A,N_m}}})/\partial \epsilon^-_{\eta^q_{A,N_m}} = \frac{\partial^2 \Phi}{\partial \sigma^{k_v} \ \partial \sigma^{\eta_{A,N_m}}} \frac{-1}{(\Sigma_\epsilon)} \frac{\epsilon^-_{\eta^{2/q}_{A,N_m}} - \epsilon^+_{\eta^{2/q}_{A,N_m}}}{\epsilon^-_{\eta^q_{A,N_m}} - \epsilon^+_{\eta^q_{A,N_m}}} \tag{23}$$

The impact of an increasing lower limit of the distribution ($\epsilon^-_{\eta^q_{A,N_m}}$) on the size of complementarity is unambiguously positive. An increase of the lower limit $\epsilon^-_{\eta^q_{A,N_m}}$ (with $q = 1, 2$) raises the wage elasticity of the hiring rate which amplifies the impact of a given subsidy.[31]

4.2.2 Distribution parameter 2: $\epsilon^-_{\eta_{B,A}}$

Now, we calculate the impact the distribution parameter $\epsilon^-_{\eta_{B,A}}$ on the size of complementarity. We obtain:

$$\partial(\frac{\partial^2 \Phi}{\partial \sigma^{k_v} \ \partial \sigma^{\eta_{A,N_m}}})/\partial \epsilon^-_{\eta_{B,A}} = \frac{\partial^2 \Phi}{\partial \sigma^{k_v} \ \partial \sigma^{\eta_{A,N_m}}} \frac{-1}{\epsilon^-_{\eta_{B,A}} - \epsilon^+_{\eta_{B,A}}} \tag{24}$$

Again, the impact of an increasing lower limit of the distribution ($\epsilon^-_{\eta_{B,A}}$) on the size of complementarity is unambiguously positive. The increase of the lower limit $\epsilon^-_{\eta_{B,A}}$ raises the wage elasticity of the training rate which amplifies the impact of a given subsidy.[32]

4.2.3 Parameters of the wage equation

When analyzing the impact of the bargaining power (μ_m) and the degree of risk aversion ($\gamma = 1-\xi$), it has to be noticed, that the only transmission channel is the wage. According to eq. (22), both parameters do not affect the size of the complementarity directly, but only indirectly via the wage

[31] An increase of the upper limit $\epsilon^+_{\eta^q_{A,N_m}}$ has the opposite impact.

[32] An increase of the upper limit $\epsilon^+_{\eta_{B,A}}$ has the opposite impact.

(see eq. (13) for $i = m$). Therefore, we first we analyze the impact of the wage (w_m) on the size of the complementarity by calculating:

$$\partial\left(\frac{\partial^2 \Phi}{\partial \sigma^{k_v}\ \partial \sigma^{\eta_{A,N_m}}}\right)/\partial w_m = \frac{\partial^2 \Phi}{\partial \sigma^{k_v}\ \partial \sigma^{\eta_{A,N_m}}} \frac{1}{w_m} \tag{25}$$

A higher wage (w_m) raises the size of the complementarity. Given this result, it then can be shown, that a higher degree of risk aversion ($\gamma = 1 - \xi$) and a higher bargaining power (μ) both raise the size of the complementarity as they both raise the wage (w_m). The higher the bargaining power, the higher is the fraction of the surplus and finally the wage, the employee gets. The higher the degree of risk aversion, the higher is the weight of the fallback position in the Nash bargaining relative to income in the case of agreement, which implies a higher wage.[33]

5 Numerical Evaluation

5.1 Calibration

The impact of the policies under consideration and thereby the credibility of the ex ante policy evaluation heavily depends on the validity of the chosen parameter values. To check the validity, a possible "diagnostic is to test the restrictions the model places on the data" (Wolpin, 2007). Therefore in the remainder, we not only illustrate the calibration, we also make some cross checks in order to test, whether parameter values which can be derived endogenously correspond to the values given by the literature.

We calibrate the model for Germany with a year as unit of time. As far as possible, the values of the parameters are based on observed data for the period 1997-2003. The annual interest rate, i, is set at 3.5 percent,[34] which leads to a discount rate of $\delta = 0.966$. The coefficient of relative risk aversion (CRRA, γ) is set at 1.5.[35] The value is relatively low, but taking into account the whole calibration, it can be justified. The period of analysis and thus the minimum duration of unemployment in the model is one year. Thus, the risk of unemployment is much bigger than in the real world, there agents could leave unemployment before the end of a year. In reality they therefore have a higher possibility to smooth income. Taking this into account, calibrating the utility function with a relatively low degree of risk aversion is justified as it compensates the higher risk in the model.

The number of periods, p, a person is engaged in vocational training is set at 3.[36] The break off rate θ, is set at 0.035 which implies (taking the death rate into account) that about 16% of the apprentices do not finish vocational training successfully, which corresponds to the empirical data.[37] Moreover, some transition rates are taken as exogenously given. According to Wilke´s

[33] In the Nash bargaining, the wage increases with the value of the fallback position. Here, the impact is similar. The fallback position does not change but it gets a higher relative weight.

[34] This is the average real interest rate over the whole period, calculated as the difference between the nominal interest rate of long term government bonds and the inflation rate. Nominal values are transformed to real values by using the consumption deflator.

[35] A value of 1.5 is located within the reasonable scope. According to Rodepeter (1999) and Dohmen et al. (2006), the limit values for CRRA are 1 and 5.

[36] This corresponds to the typical length of vocational training within the dual system in Germany.

[37] Using data for the number of new apprentices and the number of successful apprentices which are delivered by the Statistisches Bundesamt (2006a), this value is confirmed.

(2005) Kaplan-Meier functions for Germany, the hiring rates for the high-skilled, medium-skilled and the low-skilled are set at $\eta_h = 0.55$, $\eta_m = 0.59$ and $\eta_l = 0.49$.

The labor market states are defined and quantified as follows. The low-skilled labor force ($N_l + U_l$) includes those with an educational attainment corresponding to less than upper-secondary education.[38] The medium-skilled labor force ($N_m + U_m$) contains all people with vocational upper secondary education. People with post-secondary and tertiary education are considered as being high-skilled ($N_h + U_h$). The corresponding values can be calculated on the basis of OECD data.[39] However, according to these data, the group of the low-skilled employed, N_l, also contains the apprentices. Indeed, for the purpose of the following analysis, we have to distinguish explicitly between those who are only working and the apprentices. Based on data from the Statistisches Bundesamt (2006a, 2006b), we calculate the fraction of apprentices in the total labor force (4%). Given this, we can quantify the level of apprentices and adjust the level of low-skilled employment, correspondingly. Given all these data, now, we can calculate simultaneously, the number of the school leavers, B, the separation rate of the medium-skilled employees, ϕ_m (= 0.05), the number of people in the first period of training A^1 and the death rate, ρ (= 0.023).[40] The value of the death rate corresponds to an average working life time of about 44 years, which is in line with the empirical data.[41] After this, we normalize all relevant values (N_m, U_m, N_l, U_l, B, A), so that the sum of the relevant population[42] is 1. Table (2) shows the values for the relevant states.

	low-skilled labor	vocational training	medium-skilled labor
employment	0.149789	0.0576113	0.666851
unemployment	0.0345668	-	0.066253
school leavers	0.0249283		

Table 2: Values of the relevant labor market states in the initial steady state.

Next, we calibrate the productivities and the wages. Based on data from the German national accounts, the aggregate productivity, a, is set to $52,575$ EUR and the aggregate wage, w, which is calculated as average gross wage per employee plus social security payments, is set equal to $31,290$ EUR.[43] In order to get the wages for different skill groups, OECD indices for the relative earnings of the population with income from employment for different skill groups are used, they yield the following ratios: $w_h/w = 1.27$, $w_m/w = 0.92$ and $w_l/w = 0.72$.[44] According to Beicht and Walden (2002), the wage of a person being engaged in vocational training is set to $w_v = 8269$ EUR, the productivity of a person being engaged in vocational training is $a_v = 7730$ EUR and the additional costs k_v are set at 5716 EUR. [45]

[38] This corresponds to the conventional definition which classifies people with an educational attainent corresponding to at most level 2 of the International Standard Classification of Education (ISCED) as low-skilled.

[39] See OECD (1999-2005). The underlying labor force contains people between 15 and 64.

[40] This is done by using the equations $1, 2$ and 6 of the matrix M and the fact, that the number of people being in vocational training is given by $A^1 \sum_{i=0}^{p-1} (1 - \rho - \theta)^i$.

[41] Working life begins at the age of 17 and according to Brussig and Wojtkowski (2006), retirement takes places between 62 and 63.

[42] As mentioned at the beginning, the high-skilled labor force has to be taken into account due to the budget constraint. However, as it is treated as a quasi fix factor, this part of the labor force is not relevant when analysing the transition between labor market states.

[43] Average of annual data from the Statistisches Bundesamt (2006b).

[44] See OECD (1999-2005). These values imply a ratio $w_m/w_l = 1.26$ which is in line with the corresponding data reported by Wienert (2006).

[45] According to Beicht and Walden (2002) additional, non-wage costs are about 8166 EUR. However, according to

In order to calculate the tax rates, we first determine the ratios of the tax rates. This is done by using the income tax scale of the year 2002 described in Boss and Elendner (2003) which illustrates the progressive character of the German tax system.[46] We obtain the following ratios: $\tau_h/\tau_l = 1.437$ and $\tau_m/\tau_l = 1.178$. In a next step, we define some wage-related parameter values. The net replacement rates β_i are set to 78.25 percent for the low-skilled unemployed ($i = l$), to 68.25 percent for the medium-skilled unemployed ($i = m$), and to 64.67 percent of high-skilled unemployed ($i = h$).[47] According to Chen and Funke (2005), we set the hiring costs to 10 percent of the wage and the firing costs are set to 60 percent of the wage, thus the corresponding parameters are $c_\eta = 0.1$ and $c_\varphi = 0.6$. Based on data for transition rates between the education and training system on the one hand and the labor force on the other hand,[48] we calculate (i) the fraction of low-skilled people who enter vocational training: $\eta_{B,A} = 0.70$ and (ii) the ratio: $\eta_{U,A}/\eta_{N,A} = 3.6$. Moreover, according to the Bundesministerium für Bildung und Forschung (2004) about 25 % of the people who have successfully finished vocational training become unemployed, thus the corresponding hiring rate, η_{A,N_m} is set at 0.75. However, about 52 % of all the successful apprentices stay in the firm in which they have been trained ($\eta^1_{A,N_m} = 0.52$),[49] this part of the hiring rate can be regarded as take over rate. The residual is hired by another firm ($\eta^2_{A,N_m} = 0.23$). Based on these values and the given equations, the missing values characterizing the initial steady state can be calculated unambiguously. The derived values are summarized in table (3).[50]

parameter	variable	basis of calculation	value
bargaining power	μ	eq.(13) for $i = l, m, h$	0.245
productivity (high-skilled)	a_h	and $a =$	75107
productivity (medium-skilled)	a_m	$\frac{a_l N_l + a_m N_m + a_h N_h + a_v T}{N_l + N_m + N_h + T}$	48597
productivity (low-skilled)	a_l		27248
tax rate (high skilled)	τ_h	ratios of tax rates	0.0697
tax rate (medium-skilled)	τ_m	and eq.(4)	0.0572
tax rate (low-skilled)	τ_l	with: $\sigma^{\eta_{A,N_m}} = 0$, $\sigma^{k_v} = 0$	0.0485
firing rate (low-skilled)	ϕ_l		0.096
training rate (low-skilled unempl.)	$\eta_{U,A}$	equations (3) and (4) of	0.041
training rate rate (low-skilled empl.)	$\eta_{N,A}$	Matrix M and ratio of hiring rates	0.011

Table 3: Derived parameter values for the initial steady state.

The value of the bargaining power (0.25) is significantly lower than traditionally assumed in the literature (0.5). However, there is a lot of evidence arguing for a decline in the bargaining power of the employees (e.g. OECD 2007). Given this, the value is reasonable.

Finally, we have to determine the remaining parameter for each hiring rate, i.e. the lower and

Dohmen and Hoi (2004) the additional non-wage costs are about 30 % lower, because the costs can be deducted from the tax.

[46] Given the ratios of the tax rates, the wages of each skill group, the replacement rates as well as the values for the different labor market states, we can calculate the tax rates for the initial steady state (i.e. $\sigma^{k_v} = 0$ and $\sigma^{\eta_{A,N_m}} = 0$) by using eq. (4).

[47] The values are net replacement rates (unweighted average across six family types) of workers with 67, 100 and 150 percent of average productivity. See OECD (2006b).

[48] See Reinberg and Hummel (2006).

[49] Bundesministerium für Bildung und Forschung (2007).

[50] Given all these values, it can be checked easily, that the firms have an incentive to engage in vocational training as it was assumed in section 3.2 (see Appendix A1 for a detailed calculation).

upper limits of the distribution of ϵ: $\epsilon^{-}_{\eta_{B,A}}$ and $\epsilon^{+}_{\eta_{B,A}}$ as well as $\epsilon^{-}_{\eta^q_{A,N_m}}$ and $\epsilon^{+}_{\eta^q_{A,N_m}}$ (with $q = 1, 2$). For the sake of simplicity we assume that the lower limit is 0, only $\epsilon^{-}_{\eta_{B,A}}$ is set to 5000 in order to fulfill the restriction below-mentioned. As all the other variables and parameters of the equations determining the hiring rates as well as the hiring rates themselves are given for the initial steady state, one can easily solve the equations determining the hiring rates, eq. (19) for $\epsilon^{+}_{\eta_{B,A}}$, eq. (16) for $\epsilon^{+}_{\eta^1_{A,N_m}}$ and eq. (20) for $\epsilon^{+}_{\eta^2_{A,N_m}}$ and get the values shown in table (4).[51] The choice of the uniform distribution also has implications for the wage elasticities of the hiring rates. We now examine, whether the implicit elasticities are in line with the empirical literature. Given the functional form of the hiring rates and the parameter values for $\epsilon^{+}_{\eta_{B,A}}$ and $\epsilon^{+}_{\eta^q_{A,N_m}}$ (with $q = 1, 2$), we can calculate the hiring rates for the initial values of the corresponding wage and for a wage which is equal to 90 percent of the corresponding initial wage. Then, we calculate the elasticity. Given the empirical estimates, as summarized in Orszag and Snower (1999b) hiring elasticities range from -0.5 to -4.0. Thus, the elasticities are in line with the literature.

parameter	$\epsilon^{+}_{\eta^1_{A,N_m}}$	$\frac{\partial \eta^1_{A,N}}{\eta^1_{A,N}} / \frac{\partial w_m}{w_m}$	$\epsilon^{+}_{\eta^2_{A,N_m}}$	$\frac{\partial \eta^2_{A,N}}{\eta^2_{A,N}} / \frac{\partial w_m}{w_m}$	$\epsilon^{+}_{\eta_{B,A}}$	$\frac{\partial \eta_{B,A}}{\eta_{B,A}} / \frac{\partial w_v}{w_v}$
value	335044	-1.56	745010	-1.60	69784.1	-0.52

Table 4: Derived parameter values for the uniform distribution.

5.2 Simulation Results

In the following calculations, we assume that the training subsidy, σ^{k_v}, and the hiring subsidy, $\sigma^{\eta_{A,N_m}}$, respectively can take two values: 0 and 5000 EUR. The subsidies are paid per person and period. We then calculate the impact on aggregate income for three alternatives: either only one subsidy is implemented or both subsidies are implemented simultaneously. The size of complementarity is calculated as follows: First, we calculate the percental increase in aggregate income for the two cases, in which only one subsidy is implemented, and then calculate the sum. Second, we compute the percental increase in aggregate income given that both subsidies are implemented simultaneously. Third, we compare both results, i.e. we answer the question by how much (in percent) the increase in aggregate income given that both subsidies are implemented simultaneously is higher than the sum of the separate effects. Moreover, we make the same calculation with respect to employment.

5.2.1 Benchmark solution

Before analyzing the model by taking into account all parameters, we first calculate the numerical solution of the simple model given above, i.e. we assume, that (1.) there are no taxes, (2.) training is not broken off ($\theta = 0$), and (3.) the duration of training is one period ($p = 1$). For this case, the impact of the subsidies on aggregate income and employment is shown in table (5). The analytical solution is confirmed by the numerical exercise. With respect to aggregate income, there are complementarities between the two subsidies, however, they are quite weak. With respect to employment, the size of complementarity is significantly higher. An alternative way to examine

[51] The values are the result of the full calibration of the model in the absence of subsidies. In the case of the simplified version, the values are marginally different.

policy increase of ... in %	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 0$	$\sigma^{k_v} = 0$ $\sigma^{\eta_{A,N_m}} = 5000$	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 5000$	size of complementarity (%)
aggregate income	0.505544	0.531254	1.03951	0.261515
employment	0.515151	0.75335	1.28258	1.10965

Table 5: The size of complementarity in the benchmark case.

complementarities with respect to aggregate income is to calculate the value of the cross derivative according to equation (22). For the benchmark model we get $\frac{\partial^2 \Phi}{\partial \sigma^{k_v} \, \partial \sigma^{\eta_{A,N_m}}} = 4.29265 * 10^{-8} > 0$.

5.2.2 Realistic calibration of the training phase

Now, we still assume the absence of taxes. However, with respect to vocational training, we adjust the values to realistic ones ($\theta = 0.035$ and $p = 3$). The impact of the subsidies on aggregate income and employment is shown in table (6). As in the benchmark model, there are complementarities with

policy increase of ... in %	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 0$	$\sigma^{k_v} = 0$ $\sigma^{\eta_{A,N_m}} = 5000$	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 5000$	size of complementarity (%)
aggregate income	1.43988	0.426991	1.87968	0.685955
employment	1.45472	0.484117	1.95999	1.0913

Table 6: The size of complementarity for a realistic calibration of the training phase.

respect to employment and aggregate income, which are still weak. However, the complementarity with respect to aggregate income is - in relative terms -, significantly higher than in the benchmark case,[52] whereas the complementarity with respect to employment is roughly the same. The result is caused by another remarkable result: Now, as the training takes $p = 3$ periods, the training subsidies which are paid per person and per period have a higher impact than in the benchmark case.

The critical reader might argue that the fraction of apprentices which breaks off vocational training, θ, is not exogenous but has to be treated as a function of the hiring subsidy, $\sigma^{\eta_{A,N_m}}$, i.e. hiring subsidies do not only influence the demand for successful apprentices but via θ they also influence the supply of apprentices partially. The reasoning is the following: a subsidy rate increases the hiring rate η_{A,N_m} and thus the probability of the apprentice to get a medium-skilled job. The higher probability of getting a high, medium-skilled income could be expected to reduce the incentive to break off training. Therefore θ should be treated as an endogenous variable with $\frac{\partial \theta}{\partial \sigma_{\eta_{A,N_m}}} < 0$. However, studies trying to identify reason for breaking off training do not give any argument to think that financial incentives would reduce θ. One main reason to break off training is a problem in the relationship between apprentice and the instructor in the firm. Many apprentices breaking off training do not intend to finally stop training but they try to get a training position somewhere else. Thus, for the aggregate, the financial incentive cannot be supposed to play a role.

[52] Also, the value of the cross derivative is higher, it lies between $2.04506 * 10^{-7}$ and $2.19894 * 10^{-7}$. Here, in contrast to the benchmark solution, the cross derivative is also a function of $\sigma^{\eta_{A,N_m}}$ and σ^{k_v}, so that we get a range of values.

5.2.3 Impact of taxes

In contrast to the analysis so far, we now implement taxes. For the moment, we assume, that only the expenditures for unemployment benefits have to be financed via taxes. According to eq. (4), the tax rates are set in a way which ensures that the tax receipts of the government are equal to its expenditures for unemployment benefits. However, with respect to the government budget constraint, we ignore the presence of subsidies, they are assumed to be 0 in eq. (4). The impact of the subsidies on aggregate income and employment is shown in table (7). Again, there are complementarities. With respect to employment, the result is absolutely identical to the previous result. As the introduction of taxes does not affect the producer wages and thereby the hiring rates, there is also no impact on employment. With respect to aggregate income, the impact of both subsidies and thus the size of complementarity is higher than in the previous simulation.[53] Now, the implementation of subsidies affects aggregate income over two channels. The first channel is the same as in the previous simulation: subsidies increase employment and reduce unemployment. Additionally, there is a second channel: a higher level of subsidies also reduces unemployment benefits compared to the initial steady state. By leading to a fall in the number of people requiring unemployment benefits and an increase in the number of employed people paying taxes, subsidies generate subsidy-induced revenue for the government. Consequently, the tax rates can be reduced and the net wages increase.[54] Due to the second channel, aggregate income increases more strongly than in the previous simulation.

policy increase of ... in %	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 0$	$\sigma^{k_v} = 0$ $\sigma^{\eta_{A,N_m}} = 5000$	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 5000$	size of complementarity (%)
aggregate income	2.07699	0.640015	2.73923	0.818134
employment	1.45472	0.484117	1.95999	1.0913

Table 7: The size of complementarity in the presence of taxes.

In the case of lump sum taxation, the result does not change. The tax volume is determined by the level of unemployment and thus by the level of the subsidies. However, the subsidies are determined exogenously. Therefore also the tax volume is determined exogenously. In the case of a lower unemployment level, people in all labor market states benefit from the lower required tax volume, independent of the tax system. In the case of lump sum taxation this is evident, as the amount of taxes a person has to pay is reduced for all states, i.e. also for the unemployed. In the case of a payroll tax, the unemployed also realize a benefit, as the net wage and therefore the unemployment benefit ($f_i = \beta_i \ w_i \ (1 - \tau_i)$ with $i = l, m, h$) increases. Only the distribution of the tax burden over the labor markets states depends on the tax system.[55] However, as aggregate income is calculated as the simple sum of income over all states, the distribution is not relevant for the final result.

In section 4.2 it was shown, that a rising lower limit of the ϵ-distribution increases the size of complementarity. Now, the extent of the increase is worth knowing. Table (8) shows the impact of

[53] Also the value of the cross derivative is higher than in the previous simulation. It lies between $3.34347 * 10^{-7}$ and $3.57591 * 10^{-7}$.

[54] Recall that a rise of the net wage implies a higher unemployment benefit.

[55] Even if there is no progression with respect to the payroll tax, the distributions of the tax burden are different between the two systems.

a gradual increase of the lower limit $\epsilon^{-}_{\eta^{q}_{A,N_m}}$ with $q = 1,2$ and the lower limit $\epsilon^{-}_{\eta_{B,A}}$, respectively. A higher lower limit causes an increase of the corresponding elasticity. Given this, a higher subsidy has a higher impact. Finally, the size of complementarity increases.[56]

$\epsilon^{-}_{\eta^{q}_{A,N_m}}$ with $q = 1,2$	0	0	80000
$\epsilon^{-}_{\eta_{B,A}}$	5000	9000	5000
$\frac{\partial\eta^{1}_{A,N_m}}{\eta^{1}_{A,N_m}} / \frac{\partial w_m}{w_m}$	-1.56057	-1.56057	-2.88556
$\frac{\partial\eta^{2}_{A,N_m}}{\eta^{2}_{A,N_m}} / \frac{\partial w_m}{w_m}$	-1.60346	-1.60346	-3.00766
$\frac{\partial\eta_{B,A}}{\eta_{B,A}} / \frac{\partial w_v}{w_v}$	-0.517585	-0.567656	-0.517585
complementarity (aggregate income)	0.818134	0.876066	1.15171
complementarity (employment)	1.0913	1.1506	1.59393

Table 8: Robustness checks concerning the size of complementarity in the presence of taxes.

Given that $\epsilon^{-}_{\eta^{q}_{A,N_m}} = 0$ (with $q = 1,2$) and $\epsilon^{-}_{\eta_{B,A}} = 5000$, we conduct some further robustness checks, with respect to the bargaining power (μ_m) and the degree of risk aversion ($\gamma = 1 - \xi$). In contrast to the benchmark case, we do no longer assume that the bargaining power of the medium-skilled (μ_m) is equal to the bargaining power of the low-skilled (μ_l), but we assume that the medium-skilled have a higher bargaining power: $\mu_m = 1.4\ \mu_l$. This implies (1.) for given wages, a lower difference $a_m - a_l$, which is however still positive and (2.) a training rate $\eta_{B,A} < 1$. Moreover, we assume a higher degree of risk aversion (γ). According to Dohmen et al. (2006), a realistic upper value is $\gamma = 5$. Here, $\gamma = 4$ still guarantees that $\eta_{B,A} < 1$. In table (9), we show the size of the complementarity for different combinations of μ_m and γ. The result confirms the result

γ	1.5	1.5	4
μ_m	$\mu_l =$ 0.245	1.4 $\mu_l =$ 0.251	$\mu_l =$ 0.363
complementarity (aggregate income)	0.818134	0.884566	0.889588
complementarity (employment)	1.0913	1.17389	1.18011

Table 9: Robustness checks concerning the size of complementarity in the presence of taxes.

of the analytical solution. A higher bargaining power as well as a higher degree of risk aversion raise the size of the complementarity. However, a higher value for γ also implies a higher μ_m. In order to eliminate the second effect, one could adjust the ratio (μ_m/μ_l) in a way, which guarantees a stable $\mu_m (= 0.245)$. However, for $\gamma = 4$, a constant value $\mu_m = 0.245$ requires that $\mu_l = -6.7\ \mu_m$, which is not possible.

The critical reader might argue that the aggregate income given by eq. (21) is not an appropriate measure of welfare as it is not based on utility. Therefore, we also calculate the impact of the subsidies on welfare (Ψ). In this context, we use the concept of "consumption equivalents".[57] We quantify the welfare change of a given policy reform for each labor market state by asking by how

[56] A value of $\epsilon^{-}_{\eta_{B,A}} > 9000$ or a value of $\epsilon^{-}_{\eta^{i}_{A,N_m}} > 80000$ imply a value of the training rate $\eta_{B,A} > 1$ and finally a value of $N_l < 0$ which is not reasonable.

[57] This is a common concept in the literature. See e.g. Conesa and Krueger (1999).

much an individuals ´s consumption has to be increased in the old steady state so that her expected present value of utility equals that under a specific policy reform. Thus, for the benchmark case, the utility function becomes:

$$u_{i,t}(C) = \frac{1}{1-\gamma}[C_{i,t}(1+\lambda)]^{1-\gamma} \text{ with } i = l, m, h \tag{5a}$$

Given this, eq. (21) changes as follows in order to calculate welfare:

$$\Psi = \sum_{i=m,l,h}[N_i \ \frac{1}{1-\gamma}(w_i(1-t_i)(1+\lambda))^{1-\gamma} + U_i \ \frac{1}{1-\gamma}(\beta_i \ w_i(1-t_i)(1+\lambda))^{1-\gamma}] \tag{21a}$$
$$+ T \ \frac{1}{1-\gamma}(w_v(1+\lambda))^{1-\gamma}$$

For example, an λ of 0.1 implies that if a certain combination of subsidies is implemented, a person in the population considered will experience an increase in welfare due to the subsidies equivalent to receiving 10% higher consumption in the initial steady state (in all future models of her event tree).

The impact of the subsidies on the consumption equivalent and finally on welfare is shown in table (10).

policy increase of λ in % for	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 0$	$\sigma^{k_v} = 0$ $\sigma^{\eta_{A,N_m}} = 5000$	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 5000$	size of complementarity (%)
$\gamma = 0$	2.00352	0.617804	2.64208	0.791692
$\gamma = 1.5$	1.95228	0.603361	2.58402	1.1104
$\gamma = 2$	1.60979	0.506088	2.13585	0.943901
$\gamma = 2.5$	1.06539	0.35083	1.42061	0.309602

Table 10: The size of complementarity with respect to welfare in the presence of taxes.

The higher the degree of risk aversion, the lower is the impact of the subsidies. Subsidies increase the proportion of the medium-skilled workforce being connected with a higher wage. However, the higher the degree of risk aversion, the lower is the weight of this state when calculating welfare. Thus, a higher degree of risk aversion reduces the impact of the subsidies, independent of whether they are implemented separately or simultaneously. However, the size of complementarity does not decrease monotonously.

5.2.4 Impact of the government budget constraint

In the next step, we analyze the impact of the government budget constraint, i.e. in contrast to the previous analysis, also subsidies have to be financed by taxes. According to eq. (4), the tax rates are now set in a way which ensures that the tax receipts of the government are equal to its total expenditures, i.e. the sum of unemployment benefits and subsidies. The impact of the subsidies on aggregate income and employment is shown in table (11). With respect to employment, the result is absolutely identical to the previous result for the same reason as before. But there are no longer complementarities with respect to aggregate income.[58] In contrast to the previous simulation, in

[58] Also the value of the cross derivative is higher than in the previous simulation. It lies between $-9.08308 * 10^{-7}$ and $-7.91423 * 10^{-7}$.

policy increase of ... in %	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 0$	$\sigma^{k_v} = 0$ $\sigma^{\eta_{A,N_m}} = 5000$	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 5000$	size of complementarity (%)
aggregate income	1.25663	0.464608	1.65755	-3.70008
employment	1.45472	0.484117	1.95999	1.0913

Table 11: The size of complementarity in the presence of the government budget constraint.

the presence of the government budget constraint, the implementation of subsidies does not cause a reduction but an increase of the tax rates, as the subsidies are not self-financing. As before, subsidies create additional employment. By leading to a fall in the number of people requiring unemployment benefits and an increase in the number of employees paying taxes, subsidies generate subsidy-induced revenue for the government. However, the revenue is not sufficient to compensate the additional expenditures caused by the subsidies. Consequently, the tax rates have to be increased in order to ensure that the receipts are equal to the expenditures. In the given framework, hiring subsidies are almost self-financing whereas training subsidies are not self-financing. Therefore, in the case of the government budget constraint, the impact of the hiring subsidy with respect to aggregate income is still 72.6% (= 0.464608/0.640015) of the impact in the previous section, whereas the impact of the training subsidy is only 60% (= 1.25663/2.07699) of the impact in the previous section.

The analysis of the necessary increases of the tax rates reveals a new type of complementarity. The necessary increase of the tax rates given that both subsidies are implemented simultaneously is higher than the sum of the necessary increases given that both subsidies are implemented separately. With respect to aggregate income this type of complementarity overcompensates the positive type of complementarity illustrated in the previous simulation, so that there is no complementarity with respect to aggregate income.

Now, we do some robustness checks, first with respect to the lower limit of the $\varepsilon-$distribution and second with respect to the parameters of the wage equation. For the sake of comparison, we make the same modifications as before. We start by increasing the lower limit of the ε-distribution. The results are shown in table (12). For the reason already mentioned, the size of the complementarity with respect to employment does not changed compared to the result in table (9). With respect to aggregate income, there are still no complementarities. Second, we do some robustness checks

$\epsilon^-_{\eta^q_{A,N_m}}$ with $q{=}1{,}2$	0	0	80000
$\epsilon^-_{\eta_{B,A}}$	5000	9000	5000
$\frac{\partial\eta^1_{A,N}}{\eta^1_{A,N}} / \frac{\partial w_m}{w_m}$	-1.56057	-1.56057	-2.88556
$\frac{\partial\eta^2_{A,N}}{\eta^2_{A,N}} / \frac{\partial w_m}{w_m}$	-1.60346	-1.60346	-3.00766
$\frac{\partial\eta_{B,A}}{\eta_{B,A}} / \frac{\partial w_v}{w_v}$	-0.517585	-0.567656	-0.517585
complementarity (aggregate income)	-3.70008	-3.48305	-3.84615
complementarity (employment)	1.0913	1.1506	1.59393

Table 12: Robustness checks concerning the size of complementarity in the presence of the government budget constraint.

with respect to the parameters of the wage equation. Assuming that $\epsilon^{-}_{\eta^{q}_{A,N_m}} = 0$ (with $q = 1, 2$) and $\epsilon^{-}_{\eta_{B,A}} = 5000$, we conduct robustness checks with respect to the bargaining power (μ_m) and the degree of risk aversion ($\gamma = 1 - \xi$). In table (13), we show the size of the complementarity for different combinations of μ_m and γ. For reasons already mentioned, the results with respect to

γ	1.5	1.5	4
μ_m	$\mu_l =$ 0.245	1.4 $\mu_l =$ 0.251	$\mu_l =$ 0.363
complementarity (aggregate income)	-3.70008	-3.47877	-3.46345
complementarity (employment)	1.0913	1.17389	1.18011

Table 13: Robustness checks concerning the size of complementarity in the presence of the government budget constraint.

employment are equal to the results in the previous section. With respect to aggregate income, the values increase for a higher bargaining power and for a higher degree of risk aversion. However, they remain negative.

Finally, we calculate the impact of the subsidies on the consumption equivalent and finally on welfare. The results are shown in table (14). In contrast to the previous section, now the impact of

policy increase of λ in %	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 0$	$\sigma^{k_v} = 0$ $\sigma^{\eta_{A,N_m}} = 5000$	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 5000$	size of complementarity (%)
$\gamma = 0$	1.19018	0.442789	1.57092	-3.79969
$\gamma = 1.5$	1.21185	0.444637	1.60868	-2.88622
$\gamma = 2$	0.924236	0.358269	1.23565	-3.65373
$\gamma = 2.5$	0.458971	0.218369	0.629417	-7.07512

Table 14: The size of complementarity with respect to welfare in the presence of the government budget constraint.

the subsidies on welfare is not a monotonous function of the degree of risk aversion. For $\gamma = 1.5$, the impact of the subsidies is the biggest. Moreover, there are no complementarities.

6 Conclusion

This paper has examined in particular the interactions between employment policies and training policies. We have shown that there are good theoretical reasons for these policies to be either complementary or substitutable. Our definition of complementarity is straightforward: two policies are complementary, when the effect of each policy on, say, aggregate income is greater when it is implemented in conjunction with the other policy than in isolation. We define substitutability as a negative complementarity, i.e. the effect of one policy is smaller when implemented in conjunction with the other policy than in isolation. Ignoring the possibility of complementarities could distort the evaluation of the performance of policies.

As an example we consider hiring subsidies and training subsidies, which are supposed to play a crucial role in reducing unemployment, especially in Europe. In many European countries, high unemployment is one of the biggest economic problems, especially, for the low-skilled people. In particular in Germany, the unemployment rate of this group is relatively high. However, in Germany, the unemployment rate of the youth relative to those of the unemployment of the prime-age people (25 - 54 years) is the lowest in the OECD, a fact which is often explained by the German dual system of vocational training. However, with respect to the vocational training, the situation has decreased in recent years. There has been an increasing gap between the declining demand for apprentices by firms and the supply, i.e. the number of school leavers wishing to enter the apprenticeship. Therefore it is often argued that the financial burden of firms providing vocational training should be reduced. A second problem for young people is the transition from vocational training to work. A significant fraction of apprentices who have successfully finished vocational training becomes unemployed.

In our model, we have analyzed the effects of subsidies which are expected to reduce the problems. Training subsidies are paid to employers in order to increase their incentive to provide vocational training. Hiring subsidies are meant to increase the transition from apprenticeship to work. They are provided for a limited period of time, in which they drive a wedge between the income, the worker receives, and the labor costs the employer is confronted with. For a certain labor income, the producer wage is reduced in the presence of a hiring subsidy and thus the hiring incentives are increased.

Our analysis has tackled the assessment of complementarities by presenting a macro model of the labor market that allows us to identify and qualify each effect being associated with the two subsidies when implemented in isolation and when implemented in conjunction. Taking into account the possibility of complementarities helps to avoid distortions in the evaluation of labor market policies, policy makers are increasingly interested in. Moreover, to make our analysis expressly relevant to the decisions that policy makers commonly face in practice, we do not follow the mainstream practice of deriving policies as first-best responses to labor market failures. Instead, the model takes a variety of common labor market imperfections as given.

The simulation results reveal that complementarities between the particular policies are quite weak in absolute terms. When focusing on the relative size of complementarity of different model structures and alternative calibrations, there are significant differences. However, in the presence of the government budget constraint, hiring subsidies and training subsidies are substitutes. Our analysis provides a methodology for examining policy interactions which may be useful well beyond the bounds of the hiring and training subsidies.

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Appendix

A1 Incentive of the Firm to engage in general vocational Training

When answering the question why firms have - in contrast to the initial theory - an interest to pay for general training, two aspect are noteworthy with respect to the German labor market. (1.) High firing costs can create an incentive to invest in general training: Since an employer can decide not to take over an apprentice at the end of the training phase, but faces considerable costs when firing a regular employee, the firm may be willing to subsidize vocational training. Such a training may serve as an expensive employment test for which employers are willing to pay (Harhoff and Kane, 1993), i.e. during the training phase the employer can screen potential employees. (2.) Another reason for the willingness of the firm to pay for general training can lie in the wage structure. A compressed wage structure, caused by labor market frictions can create an incentive for the firm to invest in general training. This aspect is analyzed in detail by Acemoglu and Pischke (1999). As there is a reason to believe that this is an essential aspect in Germany, the theoretical background is shown in the following: Assume that the amount of training is a continuous variable, Θ. The product of the worker $a(\Theta)$ as well as the wage $w(\Theta)$ is a function of the amount of training. The worker gets a wage which corresponds to the outside option $o(\Theta)$. Given that there are no frictions, $w(\Theta) = o(\Theta) = a(\Theta)$. The profit of the employer is $\pi = a(\Theta) - w(\Theta) = 0$. The employer has no incentive to invest in general human capital, a higher amount of human capital would cause an increase of the productivity and an increase of the wage in the same extent, thus the profit is not affected. Now, assume that there are labor market frictions (e.g. mobility costs). The outside option of the worker is $o(\Theta) = a(\Theta) - \Delta(\Theta)$ with $\Delta'(\Theta) > 0$. Again, the worker gets a wage corresponding to his outside option: $w(\Theta) = o(\Theta) = a(\Theta) - \Delta(\Theta)$. But now, due to $\Delta(\Theta)$, the wage structure is compressed, which is illustrated by the fact that $\frac{\partial w(\Theta)}{\partial \Theta} < \frac{\partial a(\Theta)}{\partial \Theta}$. Now, the profit of the firm is given by $\pi(\Theta) = a(\Theta) - w(\Theta) = \Delta(\Theta)$. As the profit increases with Θ, the firm has an interest to invest in general training.

So far, there was the implicit assumption that the wage can be set by the firm, thus the bargaining power, μ, of the employee is 0. Now, we assume $\mu > 0$. For the moment we assume, that there are no firing costs, i.e. $c_\phi = 0$. The wage is calculated as follows:

$$w(\Theta) = \frac{\mu}{1 - \beta + \beta\mu}\, a(\Theta)$$

where β is the replacement rate. Thus, the profit of the firm, π, can be calculated as follows:

$$\pi = a(\Theta) - w(\Theta) = a(\Theta)(1 - \frac{\mu}{1-\beta+\beta\mu})$$

Thus, as $\frac{\partial a(\Theta)}{\partial \Theta} > 0$, an increase in the amount of training, Θ, has a positive impact on the profit of the firm. This implies that (1.) $(1 - \frac{\mu}{1-\beta+\beta\mu}) > 0 \Leftrightarrow \mu < 1$ and (2.) $(1 - \frac{\mu}{1-\beta+\beta\mu}) > 0 \Leftrightarrow \beta < 1$. Both conditions have to be satisfied: $\mu < 1$ implies that the firm must be able to capture a fraction of the profit in the bargaining process. This is also one central conclusion of the analysis by Kessler and Lülfesmann (2006). They focus on the incentive complementarity between employer sponsored general and specific training. They argue that the possibility to provide specific training leads the employer to invest in general human capital. The second condition again stresses what has already been mentioned. Here, β is the replacement rate, but in a more general interpretation it can also be seen as the ratio of the outside option relative to the wage. $\beta < 1$ implies that the skilled workers face a relatively worse outside option. This causes a compression of the wage structure, which is a necessary condition for the firm to invest in general training. Taking also firing costs into account (i.e. $c_\phi > 0$), the conditions to be fulfilled is $\frac{\mu}{1-\beta+\beta\mu-\mu c_\phi} < 1$. Given that $\mu = 0.245$, $c_\phi = 0.6$ and $\beta = 0.6825$, this condition is satisfied!

Now, we deliver a short numerical exercise showing, that firms have an incentive to invest in general training, i.e. they have an financial interest to conduct vocational training. During the training phase, the firm realize a loss: $(7730 - 8269 - 5716)\sum_{t=0}^{3-1}\delta^t(1-\rho-\theta)^t = -17130.1$ EUR. Given that the apprentice will be employed (with a probability: 0.52 $(1-\rho-\theta) = 0.490$) and generate an expected profit of 174223 EUR (according to eq. (9) which has to be discounted (by the factor $\delta^p = 0.90$), the overall surplus is 59844 EUR. Thus, the firm (on average) has an incentive to engage in vocational training.

However, one might argue, that a positive overall surplus is not enough because the opportunity costs have to be taken into account. In our model, the opportunity costs are 0 as we assume two exogenously given types of firms: firms which are engaged in vocational training and firms which are not. In contrast to our model one might argue, that firm have the decision between the employment of a low-skilled employee and the training of the person. In this case, the opportunity costs are given by the expected profit generated by a low-skilled person. In our model, the corresponding profit is 22122.8 EUR, which is lower than the expected overall surplus in the case of training.

A2 Hiring and Training Rate

Hiring rate

The decision whether to hire a successful apprentice as a medium-skilled is a function of the associated expected profit, which is calculated as follows:

$$\begin{aligned}
\pi_{A,N_m,t} = & -\epsilon_{\eta_{A,N_m},t} + (a_m - w_m + \sigma^{\eta_{A,N_m}}) \\
& + \delta^1(1-\varphi_m-\rho)^1(a_m - w_m) - \delta\varphi_m\,\varsigma_m \\
& + \delta^2(1-\varphi_m-\rho)^2(a_m - w_m) - \delta\varphi_m\,\varsigma_m\delta^1(1-\varphi_m-\rho)^1 \\
& + \delta^3(1-\varphi_m-\rho)^3(a_m - w_m) - \delta\varphi_m\,\varsigma_m\delta^2(1-\varphi_m-\rho)^2 \\
& + \ \ldots
\end{aligned} \tag{A2.1}$$

This can be rewritten as:

$$\pi_{A,N_m,t} = -\epsilon_{\eta_{A,N_m},t} + \sigma^{\eta_{A,N_m}} \; + (a_m - w_m) \sum_{t=0}^{\infty} \delta^t (1-\varphi_m-\rho)^t - \delta \; \varphi_m \; \varsigma_m \sum_{t=0}^{\infty} \delta^t (1-\varphi_m-\rho)^t \tag{A2.2}$$

The term on the right hand side can be simplified, so that the equation becomes:

$$\pi_{A,N_m,,t} = -\epsilon_{\eta_{A,N_m},t} + \sigma^{\eta,A,N_m} + \frac{a_m - w_m \; - \; \delta\varphi_m \; \varsigma_m}{1 - \delta(1 - \varphi_m - \rho)} \tag{A2.3}$$

A successful apprentice is hired whenever: $\pi_{A,N_m,,t} > 0$. As the person is already in the firm, there are no hiring costs. Solving for the random component $-\epsilon_{\eta_{A,N_m},t}$, we obtain the following equation:

$$-\epsilon_{\eta_{A,N_m},t} < \sigma^{\eta,A,N_m} + \frac{a_m - w_m \; - \; \delta\varphi_m \; \varsigma_m}{1 - \delta(1 - \varphi_m - \rho)} \tag{A2.4}$$

With respect to $\epsilon_{\eta_{A,N_m}}$, we assume a uniform distribution, i.e. is uniformly distributed between $\epsilon^-_{\eta_{A,N_m}}$ and $\epsilon^+_{\eta_{A,N_m}}$. Given all this, the corresponding hiring rate can be expressed as follows:

$$\eta_{A,N_m} = \frac{(\sigma^{\eta_{A,N_m}} + \frac{a_m - w_m - \delta \; \varphi_m \; \varsigma_m}{1-\delta(1-\varphi_m-\rho_m)}) - \epsilon^-_{\eta_{A,N_m}}}{\epsilon^+_{\eta_{A,N_m}} - \epsilon^-_{\eta_{A,N_m}}} \tag{A2.5}$$

Training rate

The decision to hire a school leaver as apprentice is a function of the associated profit, which is calculated as follows:

$$\begin{aligned} \pi_{B,A,t} = & -\epsilon_{\eta_{B,A},t} + (a_v - w_v - k_v + \sigma^{k_v}) \\ & + \delta^1 (1-\rho-\theta)^1 (a_v - w_v - k_v + \sigma^{k_v}) \\ & + ... \\ & + \delta^{p-1} (1-\rho-\theta)^{p-1} (a_v - w_v - k_v + \sigma^{k_v}) \\ & + \delta^p (1-\rho-\theta)^p \eta_{A,N_m} E(\pi_{A,N_m,t}) \end{aligned} \tag{A2.6}$$

with: $E(\pi_{A,N_m,t}) = \sigma^{\eta,A,N_m} + \frac{a_m - w_m \; - \; \delta\varphi_m \; \varsigma_m}{1-\delta(1-\varphi_m-\rho)}$.

$$\begin{aligned} \pi_{B,A,t} = -\epsilon_{\eta_{B,A},t} + (a_v - w_v - k_v + \sigma^{k_v}) \sum_{i=0}^{p-1} \delta^i (1-\rho-\theta)^i \\ + \delta^p (1-\rho-\theta)^p \eta_{A,N_m} \; (\sigma^{\eta,A,N_m} + \frac{a_m - w_m \; - \; \delta\varphi_m \; \varsigma_m}{1 - \delta(1 - \varphi_m - \rho)}) \end{aligned} \tag{A2.7}$$

A person is hired as apprentice whenever: $\pi_{B,A} > \chi_{B,A,t}$. Solving for the random component $-\epsilon_{\eta_{B,A}}$, we obtain the following equation:

$$\begin{aligned} \epsilon_{\eta_{B,A},t} < [-\chi_{B,A,t} + (a_v - w_v - k_v + \sigma^{k_v}) \sum_{i=0}^{p-1} \delta^i (1-\rho-\theta)^i \\ + \delta^p (1-\rho-\theta)^p \eta_{A,N_m} \; (\sigma^{\eta,A,N_m} + \frac{a_m - w_m \; - \; \delta\varphi_m \; \varsigma_m}{1 - \delta(1 - \varphi_m - \rho)})] \end{aligned} \tag{A2.8}$$

Again, we assume a uniform distribution, i.e. $-\epsilon_{\eta_{B,A}}$ is uniformly distributed between $\epsilon^-_{\eta_{B,A}}$ and $\epsilon^+_{\eta_{B,A}}$. The corresponding hiring rate can be expressed as follows:

$$\eta_{B,A} = \frac{[-\chi_{B,A,t} + (a_v - w_v - k_v + \sigma^{k_v}) \sum_{i=0}^{p-1} \delta^i (1-\rho-\theta)^i + \delta^p (1-\rho-\theta)^p \ \eta_{A,N_m} \ E[\pi_{A,N_m,t}]] - \epsilon^-_{\eta_{B,A}}}{\epsilon^+_{\eta_{B,A}} - \epsilon^-_{\eta_{B,A}}} \tag{A2.9}$$

with $E[\pi_{A,N_m,t}] = \sigma^{\eta,A,N_m} + \frac{a_m - w_m - \delta\varphi_m \varsigma_m}{1-\delta(1-\varphi_m-\rho)}$.

A3 A simple model with government budget constraint

In contrast to the main model, the following one is a 2-period model. This and some other properties allow to get an analytical solution even in the presence of the government budget constraint. The worker´s possible labor market states are illustrated in Figure (2): training T, medium-skilled employment N_m, medium-skilled unemployment U_m, low-skilled employment in period 1 and 2, N_l^1 and N_l^2, and finally low-skilled unemployment in period 1 and 2, U_l^1 and U_l^2. At the end of the

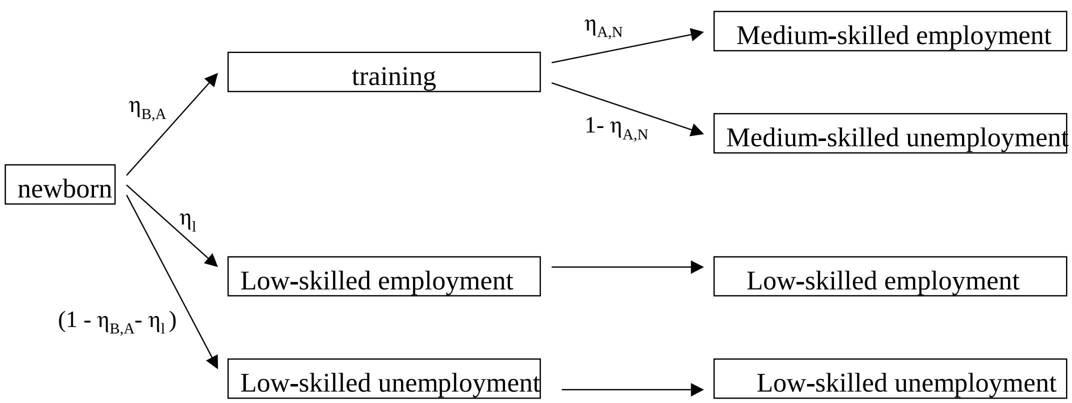

Figure 2: The dynamic structure of the simple model.

second period, all people die. We assume, that the number of deaths is equal to the number of newborn, so the labor force is constant. A newborn is hired with a probability $\eta_{B,A}$ as apprentice and with a probability η_l as low-skilled employee. The rest remains unemployed. A trained person becomes a medium-skilled employee in the 2^{nd} period with a probability $\eta_{A,N}$. For the sake of simplicity, we assume that low-skilled people do not change their labor market state in the second period. Given all this, we get the following expressions for the steady state:

$$N_l^1 = \eta_l \ (\ N_m + U_m + N_l^2 + U_l^2) \tag{A3.1}$$

$$U_l^1 = (1 - \eta_{B,A} - \eta_l) \ (\ N_m + U_m + N_l^2 + U_l^2) \tag{A3.2}$$

$$N_m = \eta_{A,N} \ T \tag{A3.3}$$

$$U_m = (1 - \eta_{A,N})\ T \tag{A3.4}$$

$$N_l^2 = N_l^1 \tag{A3.5}$$

$$U_l^2 = \ U_l^1 \tag{A3.6}$$

Moreover, we normalize the total labor force to 1

$$1 = T + N_m + U_m + N_l^1 + U_l^1 + N_l^2 + U_l^2 \tag{A3.7}$$

Given these equations, we can calculate the steady state expressions for each labor market state as a function of the transition rates: $N_m = 0.5\ \eta_{A,N}\ \eta_{B,A}$, $U_m = 0.5(1 - \eta_{A,N})\eta_{B,A}$, $T = \frac{\eta_{B,A}}{2}$, $N_l^{1,2} = \frac{\eta_l}{2}$ and finally $U_l^{1,2} = 0.5(1 - \eta_{B,A} - \eta_l)$. Assuming a uniform distribution of the random component, the hiring rates $\eta_{A,N}$ and $\eta_{B,A}$ which are affected by the subsidies are expressed as:

$$\eta_{A,N} = \frac{a_m - w_m + \sigma^{\eta,A,N_m}}{\epsilon^+_{\eta_{A,N_m}}} \tag{A3.8}$$

and:

$$\eta_{B,A} = \frac{a_v - w_v + \ \sigma^{k_v} + \eta_{A,N}\ (a_m - w_m + \sigma^{\eta,A,N_m})}{\epsilon^+_{\eta_{B,A}}} \tag{A3.9}$$

where $w_m = (a_m + \sigma^{\eta,A,N_m})\mu$ which is given by eq. (13) for $\xi = 1$, $\beta = 0$ and $c_\phi = 0$. Given these expression, the labor markets states can be written as function of the subsidies. In order to determine aggregate income, we have to connect each labor market state with a value. In the case of training the value corresponds to the wage w_v and in the case of employment, the value corresponds to the net wage $w_i(1 - \tau)$ with $i = N_m, N_l^1, N_l^2$. With respect to unemployment, we assume the absence of unemployment benefits, so that the value associated with unemployment is 0.

For $\tau = 0$, aggregate income is calculated as follows:

$$W = T\ w_v + N_m\ w_m + (N_l^1 + N_l^2)\ w_l \tag{A3.10}$$

Substituting the variables presenting the labor market stated in eq. (A3.10) by their steady state expressions gives an expression of aggregate income as a function of the two subsidies. Then, we can calculate the cross derivative of aggregate income w.r.t. to the two subsidies:

$$\frac{\partial^2 W}{\partial \sigma^{\eta,A,N_m} \partial \sigma^{k_v}} = \frac{(1-\mu)\mu(a_m + \sigma^{\eta,A,N_m})}{2\ \epsilon^+_{\eta_{A,N_m}} \epsilon^+_{\eta_{B,A}}} \tag{A3.11}$$

The two subsidies are complementary w.r.t. aggregate income. The cross derivative is unambiguously positive.

In the presence of the government budget constraint, $\tau > 0$, so that aggregate income is calculated as follows:

$$W = Tw_v + N_m w_m(1 - \tau) + (N_l^1 + N_l^2)\ w_l\ (1 - \tau) \tag{A3.12}$$

The government budget constraint is given by:

$$\tau\ N_m w_m + \tau\ (N_l^1 + N_l^2)\ w_l = T\ \sigma^{k_v} + N_m \sigma^{\eta,A,N_m} \tag{A3.13}$$

Solving for τ yields:

$$\tau = (T\ \sigma^{k_v} + N_m \sigma^{\eta,A,N_m})/(N_m w_m + \ (N_l^1 + N_l^2)\ w_l) \tag{A3.13a}$$

Substituting the variables presenting the labor market stated in eq. (A3.12) and in eq. (A3.13a) by their steady state expressions and then substituting τ in eq. (A3.12) by the expression in eq. (A3.13a) gives an expression of aggregate income as a function of the two subsidies. Then, we can calculate the cross derivative of aggregate income w.r.t. to the two subsidies:

$$\frac{\partial^2 W}{\partial \sigma^{\eta,A,N_m} \partial \sigma^{k_v}} = \frac{(1-\mu)\mu(a_m + \sigma^{\eta,A,N_m}) - (1-\mu)(3(a_m + \sigma^{\eta,A,N_m})(1-\mu) + \sigma^{\eta,A,N_m})}{2\ \epsilon^{+}_{\eta_{A,N_m}} \epsilon^{+}_{\eta_{B,A}}} \tag{A3.14}$$

This equation shows the contrast to the cross derivative in the absence of the government budget constraint, which is given by the last term on the right-hand side. The additional term is unambiguously negative. In order to get to know the sign of the whole expression, we make some transformations which yield:

$$\frac{\partial^2 W}{\partial \sigma^{\eta,A,N_m} \partial \sigma^{k_v}} = \frac{(1-\mu)\ ((4\mu - 3)a_m - (1-\mu)\ 4\ \sigma^{\eta,A,N_m})}{2\ \epsilon^{+}_{\eta_{A,N_m}} \epsilon^{+}_{\eta_{B,A}}} \tag{A3.14a}$$

Now, in the presence of the government budget constraint, there are no complementarities. The term on the right-hand side in brackets is for a plausible parameter value ($\mu < 0.75$) negative.

A4 The impact of the risk aversion on the size of complementarity

In the benchmark model it is assumed, that the transition from state B to state A is demand driven, so that the transition probability $\eta_{B,A}$ can be considered as a hiring rate. However, one might argue, that the transition is supply-driven. In this case, the school leavers decide whether they start an apprenticeship or try to find a job as low-skilled. In the following, we model the transition as supply-driven. School leavers compare the difference of the expected return of entering into training and the expected return of entering the low-skilled labor market with their education costs. We assume that they are heterogenous with respect to their education costs. For a fraction of the school leaver the costs (costs induced by a skill adverse environments or mobility costs) are so high that an apprenticeship is not worthwhile. This may explain why about 15 000 vocational training vacancies have not been filled in 2006 (Bundesministerium für Bildung und Forschung, (2007)). In the remainder, the training rate $\eta_{B,A}$ (i.e. the proportions of school leavers who enter vocational training) is modelled as function of the differences in the expected utilities being associated with the two alternative labor market states. For the marginal school leaver the sum of the expected life-time utility in the case of joining training, V^{T_1}, and the education costs (db) is equal to the expected life time utility in the case of joining the low-skilled labor force.

$$\eta_l V^{N_l} + (1-\eta_l) V^{U_l} = V^{T_1} - db^* \tag{A4.1}$$

with

$$V^{N_l} = \frac{1}{\xi}(w_l)^{\xi} + \delta((1 - \rho - \varphi_l - \eta_{N,A}) V^{N_l} + \varphi_l V^{U_l} + \eta_{N,A} V^{T_1}) \tag{A4.2}$$

$$V^{U_l} = \frac{1}{\xi}(f_l)^{\xi} + \delta((1 - \rho - (1-\eta_{U,A})\eta_l - \eta_{U,A}) V^{U_l} + (1-\eta_{U,A})\eta_l V^{N_l} + \eta_{U,A} V^{T_1}) \tag{A4.3}$$

and

$$V^{T_1} = \frac{1}{\xi}(w_v + \sigma^{k_v})^{\xi} + \delta(1-\rho)V^{T_2} \tag{A4.4}$$

with

$$V^{T_2} = \frac{1}{\xi}(w_v + \sigma^{k_v})^{\xi} + \delta(1-\rho)V^{T_3} \tag{A4.5}$$

and

$$V^{T_3} = \frac{1}{\xi}(w_v + \sigma^{k_v})^{\xi} + \delta(1-\rho)(\eta^1_{A,N} + \eta^2_{A,N})V^{N_m} + (1 - \eta^1_{A,N} + \eta^2_{A,N})V^{U_m} \tag{A4.6}$$

Given also the equations for V^{N_m} and V^{U_m} we have a system of 7 equation and 7 unknown variables which can be solved so that we get the steady state expression for the value being associated with each labor market state. Substituting V^{T_1}, V^{N_l} and V^{U_l} in eq. (A4.1) by their corresponding steady state expressions, we obtain an equation for $d^B = d^{B*}$, the education cost of the marginal school leaver. A school leaver enters vocational training if and only if $d^B \leq d^{B*}$.

However, we are interested in the proportion of the school leavers, who enter training ($\eta_{B,A}$). Therefore, we have to illustrate the relationship between d^B and $\eta_{B,A}$. The value of d^{B*} represents the disutility of the marginal worker, i.e. the worker who is indifferent between acquiring human capital via training and remaining low-skilled. Only people with $d^B \preceq d^{B*}$ decide to enter vocational training. Ordering the workers in terms of their individual costs, from the lowest to the highest, we let the cumulative distribution of costs be approximated by a continuum given by the function $d^B = f(\eta_{B,A})$, $(\partial d^B / \partial \eta_{B,A}) > 0$.

As we are interested in $\eta_{B,A}$, we use the inverse function in the remainder: $\eta_{B,A}(d^B)$, with $(\partial \eta_{B,A} / \partial d^{B*}) > 0$. The intuition behind this is as follows: the higher is d^{B*}, the higher is the fraction of school leavers for which $d^B \preceq d^{B*}$ and therefore the higher is the fraction of newborn who decide to join training, $\eta_{B,A}$. For simplicity, we assume: $\eta_{B,A} = x_\eta \, d^{B*}$, with the education cost parameter $x_\eta > 0$. Using the expression for d^{B*}, calculated as described above, we obtain an equation for the proportion of school leavers entering vocational training $\eta_{B,A}$. This equation is then used in the model instead of equation (19).

Doing the same policy simulations, the qualitative results do not change.