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1361

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June 2007

This paper is part of the Kiel Working Paper Collection No. 2

"The Phillips Curve and the Natural Rate of Unemployment"
June 2007

http://www.ifw-kiel.de/pub/kap/kapcoll/kapcoll_02.htm

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Kiel Working Paper No. 1361

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Preliminary

May 23, 2007

This paper is being prepared for the Symposium "The Phillips Curve and the Natural Rate of Unemployment," Kiel Institute for the World Economy, June 3-4, 2007. I am indebted to Marvin Goodfriend and Edward Nelson for many helpful discussions.

ABSTRACT

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It is clear that at present various versions of the Calvo (1983) model of price adjustment are dominant in monetary policy analysis—see, e.g., Woodford (2003). This is true despite well-known criticisms including Mankiw (2001) or Mankiw and Reis (2002) and the well-documented need for the addition of ad-hoc features if actual inflation and output data are to be matched. Accordingly, there is ample reason, to give consideration to alternative models. In this paper, a new look is given to the P-bar model utilized by McCallum and Nelson (1999a, 1999b), based on previous work by Mussa (1981) and others. Relative to the Calvo model, the P-bar specification has three significant advantages: it satisfies the strict version of the natural rate hypothesis; it relies on costs of adjusting output, which are more tangible than menu costs of changing prices; and its basic version produces more realistic autocorrelation patterns than does the basic Calvo specification. The present paper develops these comparisons more completely and systematically than in previous work.

1. Introduction

It is generally agreed that the "Phillips Curve" relationship—i.e., the component of a macroeconomic model that describes the way in which price adjustments are made—is crucial for understanding the link between monetary policy actions and the behavior of real macroeconomic aggregates such as output and employment. Indeed, this is in effect agreed to even by real-business-cycle (RBC) proponents, when they specify that price adjustments are virtually complete within each period, for they are thereby implicitly adopting a limiting case of the Phillips curve—one that implies that the effects of monetary policy on the cyclical behavior of these real aggregates are almost non-existent.

In addition, it is currently the case that a discrete-time version of the Calvo (1983) model of staggered price-setting is by far the leading—indeed, dominant—specification of the Phillips curve relationship.[1] There are reasons, however, to be somewhat dissatisfied with this state of affairs. First, it has been persuasively argued by critics including Mankiw (2001), Mankiw and Reis (2002), Estrella and Fuhrer (2002), and Rudd and Whelan (2007) that the basic form of the Calvo model is drastically inconsistent with crucial properties of the basic time-series data on inflation, output, and employment. Arguably effective counter-arguments have been developed by Woodford (2003), Sbordone (2006), Gali, Gertler, and Lopez-Salido (2005) and others, but it remains troublesome that sophisticated analysis is required to avoid serious discrepancy with the most basic facts. In addition, there are a priori reasons for objection to the Calvo specification, including the highly stylized timing structure, the emphasis on costs of price adjustments per se, the absence of any implied autoregressive components, and the

[1] This claim probably needs little justification, but is supported by the extensive use of the Calvo model in Woodford's (2003) seminal treatise and in Walsh's (2005) excellent graduate-level textbook.

model's failure to satisfy the natural-rate hypothesis. There is ample reason, accordingly, to give consideration to alternative models of price adjustment.

In what follows, such consideration is given to one particular alternative, namely, the "P-bar" model utilized by McCallum and Nelson (1999a, 1999b), which is based on previous work by McCallum (1980, 1994), Mussa (1981), and Barro and Grossman (1976). This specification lacks certain features favored by proponents of the Calvo model,[2] but has three significant advantages. First, the P-bar model satisfies the strict version of the natural rate hypothesis, whereas the Calvo model does not satisfy even the weaker "accelerationist" version.[3] Second, the P-bar model relies on costs of adjusting output or employment, which are more tangible and better documented than menu costs of changing prices. Third, the unadulterated version of the P-bar model implies the existence of autoregressive components in the implied time-series processes for output and inflation. Consequently, in many cases it produces more realistic autocorrelation patterns for crucial variables (including output and inflation) than does the basic Calvo specification in a standard three-equation macro model consisting of the price adjustment equation, an optimizing IS-type demand relationship, and a Taylor-style monetary policy rule for the nominal interest rate.

One non-standard feature of the investigation of dynamics conducted below is its attention to the possibility that U.S. monetary policy practice has changed significantly over the last 25 years, with the Volcker disinflation of 1979-1984 leading to a subsequent improvement that can be approximated as the elimination of a random-walk component in the Fed's implicit inflation target. This type of change could be responsible for the

[2] The main such feature is the hypothesized distribution of different prices by sellers of differentiated goods, which gives rise to resource misallocation resulting from inflation.
[3] See McCallum and Nelson (1999a) or Mankiw and Reis (2002), as well as Section 6 below.

significant reduction in inflation persistence that is found in the post-1987 data, which the P-bar model matches somewhat better than data pertaining to the previous era—for example, 1954-1986.

The paper's organization is as follows. In Section 2 the basic features of the P-bar model are presented in an informal manner. Sections 3 and 4 then develop a more formal analysis of a flexible-price economy and a general approach to the introduction of sticky prices with demand-determined output into the foregoing framework. For comparative purposes, the more familiar Calvo model is discussed in a parallel manner in Section 5 after which Section 6 contrasts the status of the Calvo and P-bar models with respect to the fundamental natural-rate hypothesis of Lucas (1972). Section 7 develops a calibration to be used in the dynamic investigation of these two models that is conducted in Section 8, and Section 9 provides a short conclusion.

2. Basic Features of the P-Bar Model

The simplest and most basic way of introducing price stickiness into a macro model is to specify that prices for each period t are set at the start of that period, on the basis of information available from previous periods, at their expected market-clearing or "natural-rate" levels.[4] Then quantities demanded at those prices are supplied by sellers even when shocks result in conditions different from those expected. Letting p_t denote the log of a typical seller's price P_t, this specification would be that $p_t = E_{t-1}\overline{p}_t$, where $E_{t-1}\overline{p}_t \equiv E(\overline{p}_t | \Omega_{t-1})$ with $\overline{p}_t$ denoting the market-clearing price and Ω_t including observations on all variables in periods t, t−1, t−2, ... Thus the basic adjustment friction is simply that current-period observations are not available to agents when setting prices

[4] These levels may reflect the influence of market power, as mentioned by Friedman (1968).

for that period. The dynamics resulting from the foregoing specification are excessively limited, however, so some additional structure is needed to have any hope of matching actual price-output data. In that regards, the Rotemberg (1982) model posits quadratic costs of changing nominal prices from one period to the next, and leads to a reduced-form expression that is almost exactly the same as that pertaining to the aggregate price level in the Calvo model. It would seem, however, that the costs of adjusting employment (or output) between periods are more tangible and more significant that those of changing prices. Accordingly, the P-bar model assumes instead that there are (quadratic) costs of changing output. More precisely, there is a quadratic cost of changing the output gap, $\tilde{y}_t = y_t - \bar{y}_t$, where y_t is the log of output and $\bar{y}_t$ is the log of the natural rate of output, i.e., the flexible-price rate of output.[5] Let us now consider the implications for price adjustment.

We have in mind, as is standard, use of the Dixit-Stiglitz consumption index, based on the assumption that the typical household has CES preferences for individual goods with a common elasticity of substitution of θ (it is assumed that θ > 1). This setup gives rise to aggregate demands for each good with price elasticities of θ. Therefore, for each firm one can define the (log) price $\bar{p}_t$ such that $\bar{p}_t - p_t = (1/\theta)(y_t - \bar{y}_t)$. Thus, $\bar{p}_t$ is the price that would make the demand for a firm's product equal to its "natural-rate" quantity. In a symmetric equilibrium, then, $\bar{p}_t$ becomes the price that would prevail if prices adjusted promptly to current conditions, in other words, the flexible-price price level (associated with the flexible-price level of output, $\bar{y}_t$).

[5] It is the change in the gap that is relevant because it is costly to change employment levels while changes is labor productivity, brought about by technology shocks, do not require changes in the work force.

In a given period t, a seller will incur adjustment costs if $y_t - \overline{y}_t$ differs from $y_{t-1} - \overline{y}_{t-1}$ and will also incur basic allocational costs whenever $y_t - \overline{y}_t$ differs from zero. Treating these costs as quadratic, it follows that the optimal choice of $y_t - \overline{y}_t$ is a weighted average of zero and $(y_t - \overline{y})_t - (y_{t-1} - \overline{y}_{t-1})$.[6] Thus we assume that sellers set prices so as to make the expected value of $\tilde{y}_t \equiv y_t - \overline{y}_t$ equal to a weighted average of zero and $\tilde{y}_t - \tilde{y}_{t-1}$, and then produce quantities that will satisfy the demand that is forthcoming in t at that price. In short, sellers set prices so as to make $E_{t-1}\tilde{y}_t = \phi\tilde{y}_{t-1}$, where ϕ is a positive fraction whose value depends upon the relative costs of (i) adjusting output and (ii) having output differ from its natural-rate value.

This behavior can equivalently be specified in terms of p_t and $\overline{p}_t$ by inserting $p_t - E_{t-1}\overline{p}_t$ in place of $E_{t-1}\tilde{y}_t$ in the foregoing equation and $p_{t-1} - \overline{p}_{t-1}$ in place of $\tilde{y}_{t-1}$, the θs and −1's cancelling out.[7] Thus we obtain $p_t - E_{t-1}\overline{p}_t = \phi(p_{t-1} - \overline{p}_{t-1})$ and from this, rearrangement yields

$$(1) \qquad p_t - p_{t-1} = (1-\phi)(\overline{p}_{t-1} - p_{t-1}) + E_{t-1}(\overline{p}_t - \overline{p}_{t-1}),$$

which is the relationship that was termed the "P-bar model" by McCallum (1994).[8] Intuitively, the two right-hand side terms represent the extent of price "disequilibrium" in period t−1 and the change in the "equilibrium" value between t−1 and t. A more formal derivation is provided below, in Appendix A.

[6] This is true even if the price setting decision is based on dynamic optimizing behavior that takes account of implications for all future periods; see McCallum and Nelson (1999a). The value of ϕ is different than it would be if the forward-looking nature of behavior were not taken into account.
[7] Note that $p_t = E_{t-1}p_t$, since the former is predetermined.
[8] The model was developed and utilized by Herschel Grossman, Robert Barro, Michael Mussa, and McCallum in the 1970s and early 1980s; for references see McCallum (1994, pp. 251-252).

3. Flexible-Price Relations

The foregoing discussion concerns departures of current prices and quantities from their flexible-price (natural-rate) values that would prevail in the absence of any price stickiness. To develop the picture more analytically, it is therefore useful to examine the nature of the flexible-price values that serve as points of reference. We do so with a rather simple model of optimizing behavior on the part of households that consume Dixit-Stiglitz bundles of consumption goods and specialize in production of a single good, in the sale of which they have some market power. Accordingly, consider a typical household with utility function

$$(2) \qquad \frac{c_t^{1-\sigma}}{1-\sigma} - \Psi \frac{n_t^{1+\gamma}}{1+\gamma}$$

where $\sigma > 0$ is the inverse of its intertemporal elasticity of substitution and $\gamma > 0$ is the labor supply elasticity. The household can sell its labor on the market for a real wage of w_t and can buy or sell bonds at a real interest rate of r_t. Also, the household operates a production facility and sells its output $Y_t = A_t k^a nd_t^{1-a}$ at the nominal price P_t according to the demand function $Y_t^A (P_t / P_t^A)^{-\theta}$, where Y_t^A and P_t^A are aggregate demand and the aggregate price level, respectively, with nd_t being labor used in production . Then the household's budget constraint for t is

$$(3) \qquad w_t(n_t - nd_t) + Y_t^A (P_t / P_t^A)^{1-\theta} - \frac{b_{t+1}}{1+r_t} + b_t - tx_t - c_t = 0$$

and there is a second constraint to require that sales be equal to production, viz.,

$$(4) \qquad A_t k^a nd_t^{1-a} - Y_t^A (P_t / P_t^A)^{-\theta} = 0.$$

From these we write a Lagrangian function that is the discounted present value of

(2) over the infinite future, plus the discounted present values of the left-hand sides of (3) and (4), multiplied by the Lagrange multipliers λ_t and ξ_t, respectively. Then taking derivatives with respect to c_t, n_t, nd_t, b_{t+1}, and P_t we obtain—abstracting from the difference between expected values and realizations—the first-order conditions:

(5) $$c_t^{-\sigma} - \lambda_t = 0$$

(6) $$-\Psi n_t^{\gamma} + \lambda_t w_t = 0$$

(7) $$-\lambda_t w_t + \xi_t (1-\alpha) A_t k^{\alpha} nd_t^{1-\alpha} = 0$$

(8) $$\frac{\lambda_t}{1+r_t} + \beta E_t \lambda_{t+1} = 0$$

(9) $$\lambda_t (1-\theta) Y_t^A \frac{P_t^{-\theta}}{(P_t^A)^{1-\theta}} - \xi_t Y_t^A (-\theta) \frac{P_t^{-\theta-1}}{(P_t^A)^{-\theta}} = 0 \;\Rightarrow\; \lambda_t P_t = \xi_t P_t^A \frac{\theta}{\theta-1} \; .$$

These seven equations (3)-(9) determine the household's choices for the variables c_t, n_t, nd_t, b_{t+1}, P_t, λ_t, and ξ_t in response to the exogenous (to the household) variables A_t, w_t, r_t, tx_t, Y_t^A, and P_t^A. Actually, of course, these are seven difference equations that govern the paths of the seven variables, given an initial condition on b_t and relevant transversality conditions.

Now we consider general market equilibrium, resulting when a large number of households, similar to the one just described but each producing and selling its own differentiated good, interact competitively. The symmetric monopolistic competition equilibrium will have the same values for each household for the various variables and will also satisfy $P_t = P_t^A$ and $n_t = nd_t$. In addition, the government determines values for g_t and tx_t and is subject to the government budget constraint

(10) $g_t - tx_t = \dfrac{b_{t+1}}{1+r_t} - b_t.$

Thus we can add an additional equation, the Fisher identity

(11) $R_t = r_t + E_t\pi_{t+1}$,

where

(12) $\pi_{t+1} = p_{t+1} - p_t$,

and then specify that R_t is controlled by the central bank according to a monetary policy rule such as

(13) $R_t = \mu_0 + \mu_1(\pi_t)$

with $\mu_1 > 1$. Then the system (3)-(13) can be considered as governing the behavior of c_t, n_t, b_{t+1}, P_t, λ_t, ξ_t, w_t, Y_t, r_t, R_t, and π_t, given exogenous determination of g_t, tx_t, and A_t, the first two by policy and the latter by technology.[9]

4. Sticky Prices

We now wish to modify the system at hand to reflect the phenomenon of price level stickiness. In order to do this, let us use $\bar{Y}_t$ to represent the values of Y_t that are determined by the system (3)-(13). Notice that these values can be regarded as determined by a time-invariant function of the system's exogenous variables A_t and g_t, the tax/transfer variable tr_t being irrelevant because the specified system has the property of Ricardian equivalence.[10] With the path for $\bar{Y}_t$ and therefore $\bar{y}_t$ given, we can return to our system (3)-(13) but with (9) replaced by a specification of price adjustment

[9] Here we write Y_t in place of Y_t^A.

[10] In the counting exercise above, we can replace (3) with the overall resource constraint in which b_t and tr_t do not appear, in which case the only role for (10) is to determine b for given values of tr.

behavior that reflects the presumed nominal stickiness. In the case at hand, that equation is (1). It adds the endogenous variable $\overline{p}_t$ but we also have the demand relationship

$$(14) \qquad \overline{p}_t - p_t = \theta(y_t - \overline{y}_t)$$

and in addition the linearized overall resource constraint[11]

$$(15) \qquad y_t = \omega_c c_t + \omega_g g_t$$

to complete the sticky-price system. That is, equations (1), (5), (8), and (11)-(15) determine, with P-bar price behavior, the eight endogenous variables c_t, p_t, $\overline{p}_t$ λ_t, y_t, r_t, R_t, and π_t.

But what about the labor market variables, one might ask: are they determined by including (3), (4), (6), and (7) to explain n_t, nd_t, w_t, and ξ_t? Clearly not, for with $n_t = nd_t$ inclusion of these relations would overdetermine the system. That is because we have specified the model to include (14), which makes output strongly demand determined. Thus our assumptions imply that one or more of the relations (3), (4), (6), (7), and $n_t = nd_t$ must be violated.

Our preferred specification in that regard is as follows. With y_t determined as outlined above, labor employed, nd_t, is dictated by the production function in (4). We then suppose that the labor supply and demand relations (6) and (7) are irrelevant for the determination of employment <u>within</u> each period, although they play essential roles in determining $\overline{y}_t$. Instead, we visualize workers and producers agreeing in advance (i) that employment in period t will be whatever it needs to be to satisfy the employment (nd_t) magnitude just specified and (ii) bargaining between producers and workers so as to determine, in advance of t, expected values of output and the real wage rate w_t. Then the

[11] Obviously, ω_c and ω_g are the steady-state shares of consumption and government in national income.

nominal wage for t is set at the value that equals this real wage times the expected price level. Under this arrangement, it is presumed that neither (6) nor (7) will prevail in each period; they are replaced by the bargaining process. Thus, for example, we explain w_t, the nominal wage W_t, n_t, and nd_t by the four equations (4), $n_t = nd_t$, $\log(W_t) = E_{t-1}[\log(w_t) + \log(P_t)]$, and $nd_t = (Y_t / A_t k^a)^{1/(1-a)}$.

In passing, it might be noted that this last method of demand-determination of output could be applied to a Calvo-type form of price adjustment. In that case, we would have $\Delta p_t = \beta \Delta p_{t+1} + \kappa[\overline{p}_t - p_t]$, with $\overline{p}_t$ being the price under full price flexibility. And if one followed the usual analysis, $\overline{p}_t - p_t$ would move proportionately with real marginal cost. But one can assume instead that (14) holds, with output being demand determined and the labor market functioning as specified in the previous paragraph. It would seem, accordingly, that this latter version of the Calvo equation should perhaps be termed the "New Keynesian" model of the Phillips curve, with the usual version then designated as the "New Neoclassical Synthesis" model, as in Goodfriend and King (1997).

5. Comparison with Calvo Model

The development of the P-bar model just given treats the labor market quite differently from typical discussions in the recent literature. Accordingly, it may be helpful to demonstrate that, as asserted in the previous paragraph, an analogous specification can be applied as well to the Calvo model. Let p_t^* denote the (log of the) nominal price that would be optimal in period t if prices were fully flexible. Our present objective, then, is to derive the relationship between these p_t^* values and the prices p_t that are chosen when the Calvo frictions are operative. We do this first for a typical seller, assuming that sellers are alike but behave independently.

Let $1-\alpha$ denote the fraction of sellers permitted to change their prices within any period and also the probability that a given seller can change his price in any specific period. Now p_t^* is the price that a seller would choose in period t if it could be changed again in t +1; but with probability α the seller will not be able to do so. If he knew that it would in this case be able to change his price in t+2, then in t he would choose a weighted average of p_t^* and $E_t p_{t+1}^*$ with weights 1 and $\alpha\beta$, namely, $E_t \dfrac{p_t^* + \alpha\beta p_{t+1}^*}{1+\alpha\beta}$.

(With probability $1-\alpha$, p_{t+1}^* is irrelevant in t.) Here the β terms are discount factors pertaining to events in the future.[12] Similarly, if instead the seller could with certainty change his price in t+3, he would choose $E_t \dfrac{p_t^* + \alpha\beta p_{t+1}^* + \alpha^2\beta^2 p_{t+2}^*}{1+\alpha\beta+\alpha^2\beta^2}$. But, continuing, since there is no future period in which he will for certain be able to change his price, in period t the seller charges

$$(16) \qquad E_t \frac{p_t^* + \alpha\beta p_{t+1}^* + \alpha^2\beta^2 p_{t+2}^* + \cdots}{1+\alpha\beta+\alpha^2\beta^2+\cdots}.$$

Consequently, since $1 + \alpha\beta + \alpha^2\beta^2 + \cdots = (1 - \alpha\beta)^{-1}$, the optimal <u>reset price</u> x_t is a probabilistically discounted present value of expected current and future values of p^*_{t+j}:

$$(17) \qquad x_t = (1 - \alpha\beta)\, E_t[p^*_t + \alpha\beta\, p^*_{t+1} + (\alpha\beta)^2\, p^*_{t+2} + \ldots].$$

Here x_t is the same for all sellers, but since some do not get to change their price in t, we need to determine the average price p_t (i.e., the price level) in t. Since the

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[12] This result (and those below) depends upon the presumption that for $p_t \neq p_t^*$, the seller's loss is quadratic in $p_t - p_t^*$, which makes "certainty equivalence" results applicable. That is, the value of x_t that minimizes $k(x_t - p_t^*)^2 + \alpha\beta k(x_t - p_{t+1}^*)^2 + \alpha^2\beta^2 k(x_t - p_{t+2}^*)^2 + \cdots$ (where $k > 0$) is $x_t = \dfrac{p_t^* + \alpha\beta p_{t+1}^* + \alpha^2\beta^2 p_{t+2}^* + \cdots}{1+\alpha\beta+\alpha^2\beta^2 \cdots}$.

fraction of sellers charging x_t will be $(1 - \alpha)$, and the fraction charging x_{t-j} will be $(1 - \alpha)\alpha^j$, we see that

(18) $p_t = (1 - \alpha)[x_t + \alpha x_{t-1} + \alpha^2 x_{t-2} + \ldots]$.

Now, by using (17) to eliminate the x_{t+j} terms in (18) we can find the relationship between price levels p_t and those p_t^* values that would be optimal in the absence of the Calvo price adjustment frictions. This result can be expressed in the familiar form

(19) $\Delta p_t = \beta E_t \Delta p_{t+1} + \kappa(p^*_t - p_t)$,

where $\kappa > 0$. The object now is to demonstrate that together (17) and (18) imply (19).

First we write (17) and (18), using the lag operator L, defined by $z_{t-j} = L^j z_t$, as[13]

(17') $x_t = (1-\alpha\beta)[1 + \alpha\beta L^{-1} + \alpha^2\beta^2 L^{-2} + \ldots]\, p_t^* = (1-\alpha\beta)\dfrac{1}{1-\alpha\beta L^{-1}}\, p_t^*$

(18') $p_t = (1-\alpha)[1 + \alpha L + \alpha^2 L^2 + \ldots]x_t = (1-\alpha)\dfrac{1}{1-\alpha L}\, x_t$.

Substituting (17') into (18') we obtain

(20) $(1-\alpha L)(1-\alpha\beta L^{-1})p_t = (1-\alpha)(1-\alpha\beta)p_t^*$.

Now multiply the terms in parentheses on the left-hand side and subtract $(1-\alpha)(1-\alpha\beta)p_t$ from each side. Then four terms can be cancelled out on the left-hand side, yielding

(21) $(\alpha - \alpha L - \alpha\beta L^{-1} + \alpha\beta)p_t = (1-\alpha)(1-\alpha\beta)[p_t^* - p_t]$.

Next factor out α on the left-hand side and obtain

(22) $\alpha[(1-L) - \beta(L^{-1} - 1)]p_t = (1-\alpha)(1-\alpha\beta)[p_t^* - p_t]$.

Finally, the latter can be expressed as

[13] Here I am ignoring the expectation operator; this matter will be discussed below.

$$(23) \qquad \Delta p_t = \beta \Delta p_{t+1} + \kappa_1 [p_t^* - p_t] \quad , \qquad\qquad \kappa_1 = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}.$$

This is much like the familiar Calvo formula, except for the absence of the E_t operator on Δp_{t+1}. But we could just as well have applied E_t to both (17) and (18) before beginning, which would eliminate that difference and give the desired relationship between p_t and p_t^* values.

But how does the term $[p_t^* - p_t]$ relate to the term that usually appears in presentations of the Calvo model, the price markup times marginal cost? To determine this, let us refer to the flexible-price model presented in (3)-(13) above. There the price being chosen is labeled P_t and the seller relates it optimally, under price flexibility, to the economy-wide average price, denoted P_t^A, according to equation (9), which we write as $P_t / P_t^A = (\xi_t / \lambda_t)[\theta/(\theta-1)]$. But equation (7) shows unambiguously that the ratio (ξ_t / λ_t) equals real marginal cost for the producer, so (9) is the exact counterpart of $[p_t^* - p_t] = \log(\xi_t / \lambda_t) + \log[\theta/(\theta-1)]$, log of the markup times real marginal cost. Thus the approach developed in this section yields exactly the same price equation, when the Calvo-type friction is present, as the version utilized by Woodford (2003), Gali and Gertler (1999), Sbordone (2002), Walsh (2003), and many others.

Note, however, that the specification of $p_t^* - p_t$ in (23) should depend upon the labor market structure in the economy being modeled. If workers are hired on a spot market, it is appropriate to specify that $p_t^* - p_t$ is given by the departure of marginal cost from its steady-state (and flexible price) value, as we have just done. But if workers are employed via contracting arrangements of the type specified above in Section 4 for the

P-bar model, then the relation $[p_t^* - p_t] = \overline{p}_t - p_t = \theta(y_t - \overline{y}_t)$ is again appropriate, and the value of the slope coefficient attached to $[p_t^* - p_t]$ in the price adjustment equation $\Delta p_t = \beta \Delta p_{t+1} + \kappa_1 \theta [y_t - \overline{y}_t]$ is determined mainly by demand, not supply, conditions.

It is interesting to note that the formulation developed in this section corresponds quite closely in several respects to the original presentation in Calvo (1983), although here in discrete time. Specifically, Calvo's equations (2) and (3) are continuous-time analogs of (17) and (18) above, but with symbols V_t, P_t, and $P_t + \beta E_t$ in place of our x_t, p_t, and p_t*.[14] The meaning of these symbols will therefore be exactly the same if Calvo's $P_t + \beta E_t$ is taken to be the same as our p_t*, the price in the absence of the nominal friction. His interpretation is that P_t reflects "the average price set by competitors" and E_t represents "excess demand." Since the latter will be zero with full price flexibility, his expression reduces to ours when $E_t = 0$ and serves the same function under other conditions.

6. The Natural Rate Hypothesis

As suggested above, a fundamental concept in monetary macroeconomics is the "natural-rate hypothesis," introduced by Friedman (1966, 1968) and refined by Lucas (1972). Friedman's version of this hypothesis is that differing steady-state inflation rates will not keep output (or employment) permanently high or low relative to the "natural-rate" levels that would prevail in the absence of nominal price stickiness in the relevant economies. Lucas's version is stronger; it asserts that there is <u>no</u> monetary policy that can permanently keep output (or employment) above its natural-rate value, not even with an ever-increasing (or ever-decreasing) inflation rate. It should be noted that both of these concepts are distinct

[14] Calvo's parameter β is positive but is <u>not</u> the discount rate.

from monetary superneutrality: an economy can be one in which superneutrality does not obtain, in the sense that different permanent inflation rates lead to different steady-state levels of capital and thus natural rates of output, without any implied failure of the natural-rate hypothesis (NRH), which concerns the difference between actual and natural-rate levels of output (and other real variables).

The validity of the NRH—or of Friedman's weaker version, the "accelerationist" hypothesis—was a matter of much analysis and debate in the late 1970s and early 1980s. The earliest empirical tests were not supportive of the NRH, but the arguments of Lucas (1972) and Sargent (1971), emphasizing that the utilized test procedures would be inappropriate under rational expectations, led to a reversal of typical findings and by 1980 even self-styled Keynesian economists were agreeing to the proposition that the NRH was basically valid. In recent years, however, this agreement has seemingly been implicitly overturned, not by explicit argument but mainly by practice, via the widespread adoption of the Calvo (1983) adjustment mechanism. As stated above, the Calvo model posits that price adjustments can be made during any period by only a fraction of all sellers, with all others holding their nominal prices fixed at their previous-period values. Then, with a Cobb-Douglas production function, the adjustment equation (23) above can be written as follows, where y_t is the log of output and $\bar{y}_t$ is its natural-rate value (Walsh, 2005, pp. 238-9):

(24) $\pi_t = \beta E_t \pi_{t+1} + \kappa(y_t - \bar{y}_t)$ $\qquad \kappa \neq \kappa_1$

Here β is a discount factor satisfying $0 < \beta < 1$ so, in a steady state, we have an implied relationship between inflation and the (constant) output gap, i.e., the constant value of $y_t - \bar{y}_t$. Therefore the Calvo model does not satisfy even the accelerationist hypothesis, much less the stronger NRH. It is surprising to me that relationships similar to (24) would be used so frequently in today's analysis.[15] I would think that analysts would, at a minimum, replace (24) with something like the following:

(24') $\pi_t - \pi = \beta(E_t\pi_{t+1} - \pi) + \kappa(y_t - \bar{y}_t)$.

Here π represents the steady-state inflation rate under an existing policy rule, assumed to be one that permits a steady-state inflation rate.[16] Such a relationship would result if it is assumed that those sellers that do not have an opportunity (in a given period) to reset their prices optimally, have their prices change automatically at the ongoing inflation rate (rather than being held constant). From a steady-state perspective, (24') would imply $y_t - \bar{y}_t = 0$, thereby satisfying the accelerationist hypothesis, Friedman's weaker version of the NRH. (Even so, specification (24') does not satisfy the stronger Lucas version, which pertains to inflation paths more general than steady states.)

In what way would this change affect current reasoning regarding monetary policy? Basically, it would imply that different steady-state inflation rates would not induce different steady-state output gaps. In the influential analysis of Woodford (2003, Ch. 6), the optimal steady-state inflation rate is zero, in the absence of traditional shoe-leather costs of inflation (due to transaction frictions which give money its medium-of-exchange role).[17] Thus with these frictions included, as in Friedman (1969), the optimal rate will lie between zero and the negative value implied by Friedman's analysis. But with our suggested change to price adjustment specification (24'), different inflation rates will not have any permanent effect on the (zero) output gap, and the Friedman rate (which reduces the opportunity cost of holding money to zero) would seem to be implied from the steady-state perspective.

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[15] I have used them several times myself, but mainly for illustrative purposes (as below).
[16] Other reference values for inflation yield similar results.
[17] Also see King and Wolman (1999).

To complete this section, let us note that the P-bar model does satisfy even the strict version of the NRH, arguing as follows. In the model, the output gap conforms to the relation $E_{t-1}\tilde{y}_t = \phi\tilde{y}_{t-1}$, as we have seen above. But ϕ is a positive fraction, so expectationally $\tilde{y}_t$ behaves as a stable autoregressive process, with the gap approaching zero asymptotically. Thus the only possible steady-state value for $\tilde{y}_t$ is zero. In this sense, accordingly, the model satisfies the NRH. Alternatively, one can see from (1) that on average we have $E[\overline{p}_t - p_t] = 0$, implying that $E[y_t - \overline{y}_t] = 0$.[18]

Are there other price adjustment specifications that satisfy the NRH? Yes, the Fischer-Gray-Lucas relation $y_t - \overline{y}_t = \phi_1(p_t - E_{t-1}p_t) + \phi_2(y_{t-1} - \overline{y}_{t-1})$ of the decade 1975-1985 does, as well as the more recent Mankiw-Reis (2002) "sticky information" formulation.[19] Unfortunately, some of the attractive features of the latter—delayed and hump-shaped impulse responses to monetary shocks—do not obtain when policy is modeled realistically as being conducted via an interest rate instrument, rather than the AR(1) nominal income growth relation used by Mankiw and Reis in place of a policy rule.

7. Calibration of the Basic Models

Let us now calibrate a benchmark version of the P-bar model for use in quantitative exercises designed to explore its dynamic properties, both alone and in relation to an analogous model that replaces the P-bar price-adjustment equation with the Calvo equation. The relevant equations, sufficient for determination of the key variables π_t, $\tilde{y}_t$, and R_t, are as follows:

[18] Given its justifiably great influence, it is ironic that Lucas's model in (1972b) features a Phillips Curve specification, $y_t = \phi_3(p_t - E_t p_{t+1})$, that does not obey the NRH.
[19] Of course, the real-business-cycle version of the Phillips curve, $y_t = \overline{y}_t$, satisfies the NRH trivially.

(25) $\tilde{y}_t = E_t\tilde{y}_{t+1} + b(R_t - E_t\pi_{t+1}) + v_t$

(26) $E_{t-1}\tilde{y}_t = \phi\tilde{y}_{t-1}$

(27) $R_t = (1-\mu_3)[(1+\mu_1)\pi_t + \frac{\mu_2}{4}\tilde{y}_t] + \mu_3 R_{t-1} + e_t$

Here (25) is the expectational IS equation that arises from the combination of (5), (8), and (15) above, in the manner familiar in the literature (e.g., McCallum and Nelson, 1999a). For its slope parameter we set $\sigma = 1.0$ and $\omega_c = 1.00$, obtaining $b = -1.0$.[20] In the P-bar equation (26), we set ϕ at 0.89, the value estimated by McCallum and Nelson (1999a). The remainder of the basic calibration pertains to the disturbance processes. We take the monetary policy shock e_t to be white noise with a standard deviation SD(e) = 0.002, close to the value estimated in McCallum and Nelson (1999a).[21] The shock process in the IS function, v_t, is more complex as it incorporates both preference shocks $v1_t$—not explicitly mentioned in the discussion above—and unexpected changes in both the level of government spending and natural-rate output $\bar{y}_t$. In the exercises below, we specify the preference shock to be white noise with SD(v1) = 0.01, as in McCallum and Nelson (1999a), and the $\bar{y}_t$ process to be AR(1) with AR coefficient equal to 0.95 and innovation SD of 0.007—close to values reported in the RBC literature. We neglect government spending, thereby treating all output as if it were consumption.

For the second model, with Calvo price adjustments, the only change is to replace the P-bar equation with the Calvo equation as in (24), with slope coefficient $\kappa = 0.03$.

[20] This value is several times as large as in McCallum and Nelson (1999a), but is only about 1/5 as large as the values used by Rotemberg and Woodford (1997, 1999) and Woodford (2003). It represents an intermediate position more generally, I believe, in terms of the literature circa 1999-2006 on quantitative monetary policy analysis.

[21] In this calibration section, SDs are given in quarterly fractional units, not annualized percentage units as in Tables 1 and 2 below.

This value is slightly higher than typically used by Woodford (2003), but seems quite representative of the recent literature.

With both models we will consider various parameter values for the monetary policy rule (27). The original Taylor rule, put forth in Taylor (1993), has $\mu_1 = 0.5$, $\mu_2 = 0.5$, and $\mu_3 = 0.0$ but results with other values are explored as well. In particular, the inclusion of interest rate smoothing is represented by a setting of $\mu_3 = 0.8$. Furthermore, we shall—for reasons mentioned above—consider cases in which a randomly changing target inflation rate is included. It is modeled as a random walk with innovation SD of 0.001, and appears inside the square brackets in (27). The inclusion of this last term is reported in Tables 1 and 2 below by the indication $\mu_4 = 1$.

In these exercises the object will be to see how well—or how poorly—the calibrated P-bar and Calvo models produce simulated time series that match the most basic dynamic facts of the U.S. time series data. The facts that we have in mind are the ones given in the following Table 1, which pertain to the standard deviations and (first) autocorrelation coefficients of quarterly observations on the three basic variables inflation, output gap, and nominal interest rate. The measures used for these variables are the change in the log of the consumer price index, the Hodrick-Prescott cycle component of the log of real GDP, and the federal funds rate.[22] The historical period considered is 1954.1-2005.4, but we are interested in the two subperiods 1954.1-1987.3 and 1987.4-2005.4, whose break date corresponds to the date at which Alan Greenspan became Chairman of the Board of Governors of the Federal Reserve System. This is a convenient and fairly standard date to use to separate the recent period of superior monetary policy

[22] I am highly aware of various weaknesses of the H-P cycle measure of the output gap, but have adopted it to keep from departing too much from standard practice.

Table 1: U.S Statistics

Cell entries are standard deviations (per cent p.a.) and AR(1) coeffs for $\Delta p_t, \tilde{y}_t,$ and R_t

	inflation rate SD/autocorr	output gap SD/autocorr	interest rate SD/autocorr
1954.1-2005.4	3.00 0.841	1.58 0.848	3.36 0.955
1954.1-1987.3	3.51 0.863	1.82 0.842	3.74 0.947
1987.4-2005.4	1.37 0.463	1.00 0.890	2.26 0.970

Table 2: Properties of Basic Model with P-Bar Price Adjustment

Cell entries are standard deviations (per cent p.a.) and AR(1) coeffs for $\Delta p_t, \tilde{y}_t,$ and R_t

	$\mu_3 = 0, \mu_4 = 0$	$\mu_3 = 0.8, \mu_4 = 0$	$\mu_3 = 0, \mu_4 = 1$	$\mu_3 = 0.8, \mu_4 = 1$
$\mu_1 = 0.5$ $\mu_2 = 0.0$	5.08 8.00 7.64 0.871 0.871 0.874	3.35 4.69 6.82 0.129 0.875 0.875	5.56 8.05 8.00 0.886 0.870 0.878	4.04 4.90 3.51 0.360 0.871 0.913
$\mu_1 = 0.5$ $\mu_2 = 0.5$	5.24 3.84 6.24 0.875 0.875 0.791	3.78 2.56 2.49 0.240 0.875 0.859	5.60 3.76 6.56 0.889 0.870 0.805	4.28 2.65 3.20 0.396 0.872 0.904
$\mu_1 = 2.0$ $\mu_2 = 0.5$	2.56 6.46 5.08 0.874 0.874 0.670	3.04 5.03 3.19 0.254 0.870 0.843	3.35 6.79 5.60 0.915 0.873 0.709	3.76 5.34 3.87 0.468 0.875 0.871
$\mu_1 = 2.0$ $\mu_2 = 1.0$	2.58 4.25 4.48 0.876 0.875 0.512	3.29 3.80 3.01 0.323 0.874 0.813	3.31 4.38 4.92 0.913 0.871 0.595	3.92 3.95 3.65 0.475 0.873 0.861

Calibration: $b = -1$, $\phi = 0.89$, SD(v) = 0.01, SD(a) = 0.007, SD(e) = 0.002, SD(ζ) = 0.001

Table 3: Properties of Basic Model with Calvo Price Adjustment

Cell entries are standard deviations (per cent p.a.) and AR(1) coeffs for $\Delta p_t, \tilde{y}_t,$ and R_t

	$\mu_3 = 0, \mu_4 = 0$	$\mu_3 = 0.8, \mu_4 = 0$	$\mu_3 = 0, \mu_4 = 1$	$\mu_3 = 0.8, \mu_4 = 1$
$\mu_1 = 0.5$ $\mu_2 = 0.0$	0.64 1.28 1.24 0.894 0.082 0.539	0.64 1.54 1.14 0.758 0.409 0.761	2.45 1.05 2.63 0.965 0.113 0.836	2.51 1.63 2.58 0.937 0.439 0.931
$\mu_1 = 0.5$ $\mu_2 = 0.5$	0.46 0.90 1.20 0.875 0.052 0.382	0.46 1.26 1.06 0.748 0.292 0.738	2.23 0.92 2.49 0.969 0.096 0.791	2.23 1.32 2.38 0.945 0.329 0.918
$\mu_1 = 2.0$ $\mu_2 = 0.5$	0.20 0.84 1.11 0.662 0.015 0.216	0.30 1.14 0.965 0.618 0.196 0.698	2.26 0.86 2.51 0.968 0.049 0.761	2.30 1.19 2.41 0.957 0.249 0.925
$\mu_1 = 2.0$ $\mu_2 = 1.0$	0.18 0.76 1.30 0.676 0.004 0.160	0.24 1.05 0.93 0.593 0.133 0.679	2.22 0.79 2.56 0.969 0.064 0.688	2.28 1.10 2.39 0.961 0.180 0.925

Calibration: $b = -1$, $\kappa = 0.03$, SD(v) = 0.01, SD(a) = 0.007, SD(e) = 0.002, SD(ζ) = 0.001

performance from the post-World War II experience more generally.[23] To reflect the change, a shock to the inflation target will be introduced into (27) by changing π^* to $\pi^* + \mu_4\zeta_t$, with ζ_t a random walk and $\mu_4 = 1$ for 1987.4-2005.4 and 0 otherwise.

8. Dynamic Properties of the Models

Dynamic properties of the foregoing models were obtained, approximately, by means of stochastic simulations. In each cell in Tables 2 and 3, the reported magnitudes are mean values over 200 replications with each simulation pertaining to a sample of 200 periods (quarters), with 50 start-up periods discarded in each case. The magnitudes in each cell are, for the case at hand, the standard deviations of inflation, the output gap, and the interest rate (respectively) plus the first univariate autocorrelation coefficient for each of these same three variables. The standard deviations, which may be thought of as reflecting root-mean-square targeting errors for inflation and the output gap, are expressed as annualized percentages.[24]

Table 2 reports results for several variants of the policy rule. In the first column both μ_3 and μ_4 are kept equal to zero, so the rule does not feature interest rate smoothing or a time-varying inflation target. In the first row, μ_1 equals 0.5 to represent mild policy response to inflation deviations, well above the 0.0 value needed to reflect adherence to the Taylor principle, while $\mu_2 = 0$ represents an absence of response to the output gap. In this case, variability of all three variables—inflation, gap, and interest rate—is considerably greater than is found in the U.S. data. All three of these variables have (first) autocorrelation coefficients of approximately 0.87, representing strong—and

[23] This dating does not imply any disagreement with the notion that a (perhaps "the") crucial step in the move toward responsible monetary policy took place over 1979-1984 as the Volcker disinflation.
[24] For the output gap the figure is of course not annualized, since it has no time-unit dimension.

reasonably realistic—persistence in their time series properties.

Increasing μ_2 to 0.5 in the second row reduces the variability of $\tilde{y}_t$ but leaves the SDs high for inflation and interest. Adoption of $\mu_1 = 2.0$ and $\mu_2 = 1.0$ in the fourth row reduces the variability of inflation to a realistic level. It leaves SD($\tilde{y}_t$) and SD(R_t) higher than is realistic, but not by an excessive margin, and produces serial correlation coefficients for inflation and gap that are quite realistic, and for R_t that is low but not terribly so.

In the second column, interest rate smoothing is introduced in all cases. This has the effect of reducing the variability of all three reported variables. Persistence remains high for $\tilde{y}_t$ and R_t measures, but falls to rather low values for inflation. For that reason, columns 3 and 4 are included to investigate the possibility that ρ_π would be increased to realistic levels by inclusion of the random-walk inflation target (by setting $\mu_4 = 1$). As will be seen, these values are increased but not to the strong-persistence magnitudes observed in actual data for the years prior to 1987.

In Table 3, we have similar exercises for the model but with the Calvo price adjustment equation used instead of the P-bar relation. Here the difficulties are even more serious for the calibration at hand. In the first two columns the SD values for all three variables are well below realistic values; for inflation the SD is less than 1.0 in all cases (falling as low as 0.18). Furthermore, the autocorrelation coefficients are very low for the output gap in both columns and for the interest rate in column 1. When we turn to columns 3 and 4, with the variable inflation target, inflation rate and interest rate SD values rise to realistic levels, but SD($\tilde{y}_t$) remains rather low. Most significantly, however, the autocorrelation coefficient for the output gap is quite low in all cases, reaching a moderate level only in row 1, column 4, where it still remains below 0.5. Overall, the impression conveyed by these results is that the basic version of the Calvo model performs even more poorly than the basic version of the P-bar model.

9. Conclusion

Let us now conclude with an overview of the arguments developed in the preceding sections. Basically, we have compared the P-bar model of price adjustment with the currently dominant Calvo specification. From a theoretical perspective, we have argued that the P-bar model is more attractive, since it depends upon adjustment costs for physical quantities rather than nominal prices, while incorporating a one-period information lag. Furthermore, the resulting adjustment relation is more completely free of "money illusion," in terms of dynamic relationships, and therefore satisfies the natural rate hypothesis of Lucas (1972a), which is arguably a property that any neoclassical model should possess—but which is not satisfied by the Calvo model in any of its variants. Along the way, we have shown that both the P-bar and Calvo models can be formulated in versions in which current real wages are, or are not, allocative—i.e., in which the labor market clears each period or depends upon some form of prior contracts according to which sellers choose quantities to satisfy demand at predetermined prices with those prices set to maximize revenue to be divided between the seller and his contracted workers according to some rule set by bargaining.

Quantitatively, we have examined crucial dynamic properties of a calibrated model in which the P-bar and Calvo equations are alternatively included. For a given calibration of the demand parameters, the implied time series properties of the inflation rate, output gap, and nominal interest rate are determined for various policy parameters, and are compared with quarterly data for the U.S. economy. Neither model dominates; the P-bar model tends to imply more variability than actually observed for both inflation and output, whereas the Calvo model implies less variability than actual, especially for inflation. In terms of serial correlation, the P-bar model implies a realistically high degree of first-order autocorrelation of the output gap but somewhat less than actual autocorrelation of inflation in those cases in which interest-rate smoothing is included in the policy rule. The Calvo specification, on the other hand, does a better job of matching inflation persistence (i.e., serial correlation) but a much poorer job with respect to the output gap.

Overall, our comparison seems somewhat more favorable to the P-bar model and, in any case, certainly does not provide support for the dominant position held by the Calvo model in current monetary policy analysis.

Appendix A

Here the purpose is to derive the P-bar equation (1) from the cost-of-adjustment model of Section 2. The seller's objective is to minimize, at time t,

(A1) $$E_{t-1}\sum_{j=0}^{\infty}\beta^j[(p_{t+j}-\overline{p}_{t+j})^2+c_2(\tilde{y}_{t+j}-\tilde{y}_{t+j-1})^2]$$

where the first term for period t+j represents the cost of having a selling price different from the value that would obtain in the absence of price stickiness, with the second term representing the cost of changing output.[25] Letting $\tilde{p}_t = p_t - \overline{p}_t$ and using the Dixit-Stiglitz demand relationship $\tilde{y}_t = -\theta\tilde{p}_t$, (A1) becomes

(A2) $$E_{t-1}\sum_{j=0}^{\infty}\beta^j[\tilde{p}_{t+j}{}^2+c_2\theta^2(\tilde{p}_{t+j}-\tilde{p}_{t+j-1})^2].$$

The first-order condition for minimization is then

(A3) $$E_{t-1}[\tilde{p}_t + c(\tilde{p}_t-\tilde{p}_{t-1})-c\beta(\tilde{p}_{t+1}-\tilde{p}_t)]=0,$$

where $c = c_2\theta^2$, or, equivalently,

(A4) $$\tilde{p}_t = \alpha\tilde{p}_{t-1}+\alpha\beta p_{t+1}$$

where $\alpha = c/(1+c+c\beta)$—<u>not</u> the Calvo probability of Section 5—and the expectation operator is implicit. We can see that the relevant solution is of the form $\tilde{p}_t = \phi\tilde{p}_{t-1}$, implying that $\tilde{p}_{t+1} = \phi^2\tilde{p}_{t-1}$, so substitution into (A4) shows that ϕ must satisfy $\alpha\beta\phi^2 - \phi + \alpha = 0$, and the relevant solution is

(A5) $$\phi = \frac{1-\sqrt{1-4\alpha^2\beta}}{2\alpha\beta}.$$

Thus we have, recognizing again the expectation operator,

[25] The constant c_2 reflects the importance of the second cost relative to the first.

(A6) $\quad p_t - E_{t-1}\overline{p}_t = \phi(p_{t-1} - \overline{p}_{t-1}),$

as was argued informally in Section 2. It is shown in McCallum and Nelson (1999a, p. 25) that ϕ is real and satisfies $0 < \phi < 1$.

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