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**THE GLOBAL MACROECONOMIC  
CONSEQUENCES OF A DEMOGRAPHIC  
TRANSITION**

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# **The Global Macroeconomic Consequences of a Demographic Transition**

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# **The Global Macroeconomic Consequences of a Demographic Transition**

## **Abstract**

The world is in the midst of a significant demographic transition with important implications for the macroeconomic performance of the global economy. This paper summarizes the key features of the current and projected future demographic change that are likely to have macroeconomic effects. It then applies a new ten region global model (an extended version of the MSG-Cubed model) incorporating demographic dynamics, to examine the consequences of projected global demographic change on the world economy from 2005 to 2050. A distinction is made between the effects on each country of its own demographic transition and the effects on each country of the equally large demographic changes occurring in the rest of the world.

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# 1 Introduction

The world is undergoing a demographic transition from high to low population growth rates at different rates in different regions. What will be the impacts of this global demographic change on macroeconomic outcomes in major regions? How much of our current macroeconomic experience is caused by demographic change already under way? How important are demographic transitions within a country relative to the spillovers from the demographic transition occurring simultaneously in the rest of the world? This paper attempts to provide some preliminary quantitative insights into these questions.

Figure 1 shows the growth of population by major region from 1950 to 2050 as projected by the United Nations World Population Prospects report (the 2004 Revision) <sup>1</sup>. The broad patterns in Figure 1 are by now well known. After a period of strong population growth up to the 1970s, population growth in industrialized economies began to decline due to falling fertility rates. This phenomenon, although delayed in the developing world, also began to emerge in developing countries in the 1980s<sup>2</sup>. UN Projections from 2005 clearly indicate that not only is the global population growth rate projected to fall over the coming half century but also that the phenomenon is spread across all the regions shown. It is not only the trend in growth rates in Figure 1 that are interesting. Also important are the levels of growth rates and their differential across regions and across time. Note that for the former Soviet Union (FSU) the growth rate is negative on average beginning during 1990-95 and this is also projected to occur in Japan by 2010-15. Thus in some regions although the population growth rates are falling, the population levels are projected to continue rising for some time but at a slowing rate. In other regions with negative growth rates, the absolute population levels will decline. Both the size of populations and their growth rates should have impacts on overall macroeconomic performance and its composition throughout the global economy.

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1 It should be stressed that there is a great deal of uncertainty about these projections. See Lee (2003) for a discussion.

2 See the papers in Birdsall et al (2001) for an overview of the impacts of demographic change on developing countries.

Another important aspect of global demographic change are the changes in dependency ratios - i.e. the ratio of dependent age groups to the working population. It is well known that old age dependency ratios are increasing both as a result of falling fertility rates as well as increasing life expectancy. Figure 2 shows the elderly dependency ratio across the main regions, defined as the ratio of adults aged 65 and above to the working age population of adults aged 15 to 65. These projections from the UN mid case scenario out to 2050 are dramatic. Most obvious in Figure 2 is the rise in the Japanese dependency ratio from one elderly person to ten working age adult in 1970 to seven to ten working age adults by 2050 (i.e. the ratio rises from 0.1 to 0.7). Similar, but not quite as dramatic trends, are projected throughout the regions indicated.

At the other end of the demographic transition is a change in the child dependency ratios. This is the ratio of children below 15 to the working age population. Figure 3 shows that this ratio is projected to fall significantly from now until 2050. While the number of elderly that need to be supported by working adults is projected to increase over time, the number of dependent children is projected to decline.

An analysis of the impact of the global and regional differences in demographic change needs to take into account the effects of changing growth rates as well as the numbers of adults and children. This paper incorporates these projections into a general equilibrium model that allows for the changing composition of the population and captures its affect on labor supply, investment, growth potential, saving, asset markets, international trade and financial flows. Although extremely difficult to model, with a general equilibrium approach it is possible given recent analytical development, to get conditional insight into the overall impact of the observed and projected global demographic adjustment.

There is already a large literature on the impact of demographic change<sup>3</sup> particularly in Japan<sup>4</sup>. However most of this literature has focused only on what is happening in individual

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3 See IMF World Economic Outlook September 2004, Bloom and Williamson (1997), Borsch Supan et al (2003), Bosworth and Burtless (1998), Brooks (1998), Faruquee (2002a, 2002b, 2003a, 2003b), Higgins and Williamson (1997); Higgins (1998), Cutler et al (1990) and recent papers by Bryant (2004b) and Helliwell (2004).

countries without taking into account the global demographic picture. Exceptions are the recent work by Faruquee (2000a, 2003b), Bryant et al (1998,2001,2002,2004,2005), McKibbin and Nguyen (2004) and Batini et al (2005).

A number of alternative approaches have been followed in measuring the impact of demographic change on macroeconomic variables. Single equation econometric studies tend to focus on the aggregate impact of demographic variables such as dependency rates on aggregate savings. Some studies include investment and explore the implications for the current account either directly or via the impact on savings and investment. Times series studies on panels of countries such as Masson et al (1998) and Higgins (1998) find a strong negative impact of dependency rates on savings. Higgins (1998) also finds a strong negative impact of dependency rates on investment and the current account. This result for the current account implies the negative saving effect dominate the investment effect. The IMF WEO (2004) using a panel of 115 countries also find a strong negative impact of the elderly dependency ratio on savings and the current account balance.

Other studies focus directly on the current account. Chinn and Prasad (2003) find strong negative effects of dependency rates on the current account in a panel of 89 countries from 1971 to 1995. Similarly Luhrmann (2003) finds that higher youth and adult dependency ratios tend to worsen current accounts. In both cases the youth dependency rates were more powerful than the adult dependency rates. This is supported by Helliwell (2004).

Recently the econometric approach has been extended using vector autoregression (VAR) techniques (Kim and Lee (2005)) in which both economic variables and dependency rates are assumed to interact in a dynamic way. Although preliminary, Kim and Lee (2005) find a strong negative effect of dependency rates on aggregate saving rates. Most of this effect is on public

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4 See for example Endo and Katayama (1998), Horioka (1991), Meredith (1995), Ogawa and Retherford (1993), Takayama and Kitamura (1999), Takayama (1998), Takayama, Kitamura and Yoshida (1998), Yashiro and Oishi (1997).

savings rather than personal savings. They also find a strong negative effect of dependency rates on current accounts suggesting the savings effect dominates the investment effect.

An alternative to the direct econometric approach is to simulate economy wide or global models (either estimated or calibrated models). A variety of General equilibrium approaches are also emerging ranging from the multi-country OLG model of Auerbach and Kotlikoff (1997)<sup>5</sup> to an extension of the GTAP model in Chan et al (2005) and broad macroeconomic models<sup>6</sup>, however none of these approaches deal adequately with both the macroeconomic and financial issues that are the focus of this paper. This paper follows the approaches of Bryant et al (2001, 2002, and 2004) and McKibbin and Nguyen (2004) using an intertemporal general equilibrium model.

Section 2 summarizes the theoretical methodology for capturing key aspects of the macroeconomics of demographic change set out in McKibbin and Nguyen (2004) in an intertemporal general equilibrium model. Appendix A sets this out in more detail. The approach focuses on the impacts of changing demographics on labor supply, consumption and saving responses and how in general equilibrium these responses impact on investment, trade and capital flows and asset markets. The basic approach to modeling consumption and saving extends the methodology of Blanchard (1985), Weil (1989), Faruquee, Laxton, and Symansky (1997) and Faruquee (2000a, 2000b, 2003a, 2003b). The extension to allow for children follows Bryant et al (2001, 2002, and 2004) and McKibbin and Nguyen (2002). Section 3 summarizes how the theoretical approach is incorporated in a new ten region model of the global economy consisting of: USA; Japan; Europe; Rest of OECD; Eastern Europe and Former Soviet Union; China; India; Other Asia; Latin America; and other Developing Countries. In section 4 the methodology for calculating the impact of demographic change from 1985 to 2050 is outlined. Section 5 presents results for the contribution of current and projected demographic change to the macroeconomic outcomes, distinguishing between the effects of own country demographic

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5 Examples of this approach include the INGENUE (2001) model.

6 See Borsch-Supan et al (2003) and Guest and McDonald (2004)



change from the impacts of demographic change in the rest of the world on each country. A summary, conclusion and future research directions are set out in section 6.

## 2 A Theoretical Framework for Incorporating Demographic Change in a Multi-Country Model

The theoretical framework used in this paper is based on that of Bryant and McKibbin (2001) as extended in McKibbin and Nguyen (2004). For the purposes of this paper, the MSG-Cubed model (McKibbin and Wilcoxon (1998)) has been extended to include demographic considerations. Important changes include incorporating finite lives of individuals and allowing individual incomes to vary with age. Economic agents progress from being financially dependent children to eventually being adults who are financially responsible for their own children. The approach draws heavily on Faruqee (2000a, 2000b), who extended the Blanchard (1985) model of finitely-lived agents to include aging considerations. It is very similar to Bryant and Velculescu (2002) and Bryant (2004) in the way in which children are modeled. A key difference however it that in this paper we assume that all adults are assumed to bear the cost of providing support for children rather than having this support depend on the adult's age<sup>7</sup>.

We assume that children are born each year to the adult population. Children who are 17 in the current year become adults. The increment to the adult population is referred to as the adult maturity rate. It is this adult maturity rate rather than the child birth rate that affects the growth rate of the economy. The adult maturity rate is equivalent in concept to the birth rate that usually underlies models of economic growth. As a cohort of adults emerges they are assumed to inherit the level of productivity of the economy in the period they emerge. Over time as the cohort ages it is assumed that the productivity rate of that cohort follows a profile which is

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<sup>7</sup> Bryant and Velculescu (2002) show the sensitivity of the results to this assumption.

directly derived from an estimated age earnings profile of that economy. In other words, it is assumed that the age earning profile is directly related to the marginal productivity of a cohort over its working life. An example of the age earning profile for Japan is shown in figure 4. We assume that cohorts do not retire but just that their marginal productivity eventually declines to zero.

The potential labor force adjusted by productivity is calculated outside the MSG-Cubed model by aggregating up each cohort over time. This way it is possible to capture the changing composition of the labor force as well as the overall productivity of the labor force. We are also able to use the same technique to keep track of human wealth by cohort. Thus the impact of changes in population growth and child dependency rates on labor supply and consumption decisions can be captured relatively well. The remainder of the model captures the macroeconomic impact on investment through changes in the marginal productivity of capital induced by the demographic shock. There will also be general equilibrium impacts on fiscal positions, asset prices, trade and capital flows which are captured by the rest of the model. We abstract from the problems of financing social security systems although they can be introduced in this framework following the approach of Bryant (2004a).

In a model where countries are each experiencing differential changes in demographic variables it is very complex to untangle the underlying story of adjustment. In McKibbin and Nguyen (2004) we consider a number of simple experiments in a model which is a very much simplified version of the model developed in this paper. The reader is referred to that paper for a greater understanding of the economic adjustment story to a generic demographic shock. In the current paper we attempt to quantify the entire demographic effects and provide some intuition of the relative contribution of effects across countries.

Consider the dynamics due to a fall in the child birth rate in this model. The simple theoretical implications of this approach are dealt within detail in McKibbin and Nguyen (2004). Initially the disposable incomes of households (after deduction for supporting children) effectively rise in the first 15 years as there are fewer children to support. The real economic impacts on labor supply occurs when there are less children maturing into adults and entering the work force 16 years after the initial shock. Effective labour inputs are calculated using age earnings profiles so that as the cohort of lower birth rate adults move through the workforce, the effective loss of workers is magnified by the loss in workers when they move through their more productive years. The decline in labor supply has the biggest per unit impact at around age 40, or 40 years after the demographic shock began because this is the most productive stage of the “missing workers”. The demographic transition lasts well past 100 years. With a significant fall in the number of workers, the aggregate macroeconomic variables will show a sharp decline by about 40 years after the initial shock because there are less effective workers in the economy over time. Individual households will attempt to smooth their consumption over their lifetime knowing that there will be less economic activity in the future and more effective income in the short run. However it is ambiguous how large this effect will be because there will also be a fall in real interest rates through a lower expected future marginal product of capital and this will impact on the discounting of future income by households. Investment in the longer term will be lower because there will be less workers available and the marginal product of capital will be driven down. However there will also be a desire to substitute capital for workers to attempt to maintain consumption per capita in future years. The net effect in this model implies that initially investment rises as households raise savings to provide for future consumption but eventually aggregate investment falls even though the capital labour ratio rises over time.

### 3 The modified 10 region MSG-Cubed Multi-Country Model

The theoretical approach to demographics outlined in Section 2 is embedded into a 10 region version of the MSG-Cubed model. The country and region aggregation is summarized in Table 1.

The MSG-Cubed multi-country model is based on the theoretical structure of the G-Cubed model outlined in McKibbin and Wilcoxon (1998)<sup>8</sup>. This particular version has been aggregated to two sectors and is very similar to the original McKibbin Sachs Global Model (McKibbin and Sachs (1991), hence the use of the name MSG-Cubed. A number of studies—summarized in McKibbin and Vines (2000)—show that the G-cubed modeling approach has been useful in assessing a range of issues across a number of countries since the mid-1980s.<sup>9</sup> Some of the principal features of the model are as follows:

- The model is based on explicit *intertemporal* optimization by the agents (consumers and firms) in each economy<sup>10</sup>. In contrast to static CGE models, time and dynamics are of fundamental importance in the G-Cubed model. The MSG-Cubed model is known as a DSGE (Dynamic Stochastic General Equilibrium) model in the macroeconomics literature and a Dynamic Intertemporal General Equilibrium (DIGE) model in the computable general equilibrium literature.
- In order to track the macro time series, the behavior of agents is modified to allow for short run deviations from optimal behavior either due to myopia or to restrictions on the ability of households and firms to borrow at the risk free bond rate on government debt. For both

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8 Full details of the model including a list of equations and parameters can be found online at: [www.gcubed.com](http://www.gcubed.com)

9 These issues include: Reaganomics in the 1980s; German Unification in the early 1990s; fiscal consolidation in Europe in the mid-1990s; the formation of NAFTA; the Asian crisis; and the productivity boom in the US.

10 See Blanchard and Fischer (1989) and Obstfeld and Rogoff (1996).

Table 1: The MSG-Cubed 10 Region Demographic Model (version 58J)

**Countries:**

United States

Japan

Europe

Rest of OECD

Eastern Europe and Former Soviet Union

China

India

Other Asia

Latin America

Other Developing Countries

**Sectors:**

Energy

Non-Energy

Capital goods producing sector.

households and firms, deviations from intertemporal optimizing behavior take the form of rules of thumb, which are consistent with an optimizing agent that does not update predictions based on new information about future events. These rules of thumb are chosen to generate the same steady state behavior as optimizing agents so that in the long run there is only a single intertemporal optimizing equilibrium of the model. In the short run, actual behavior is assumed to be a weighted average of the optimizing and the rule of thumb assumptions. Thus aggregate consumption is a weighted average of consumption based on wealth (current asset valuation and expected future after tax labor income) and consumption based on current disposable income (following Campbell and Mankiw (1987)). Similarly, aggregate investment is a weighted average of investment based on Tobin's  $q$  (a market valuation of the expected future change in the marginal product of capital relative to the cost) and investment based on a backward looking version of  $Q$  (following Hayashi (1979, 1982)).

- There is an explicit treatment of the holding of financial assets, including money. Money is introduced into the model through a restriction that households require money to purchase goods.
- The model also allows for short run nominal wage rigidity (by different degrees in different countries) and therefore allows for significant periods of unemployment depending on the labor market institutions in each country. This assumption, when taken together with the explicit role for money, is what gives the model its “macroeconomic” characteristics. (Here again the model's assumptions differ from the standard market clearing assumption in most CGE models.)
- The model distinguishes between the stickiness of physical capital within sectors and within countries and the flexibility of financial capital, which immediately flows to where expected returns are highest. This important distinction leads to a critical difference between the quantity of physical capital that is available at any time to produce goods and services, and the valuation of that capital as a result of decisions about the allocation of financial capital.

As a result of this structure, the MSG-Cubed model contains rich dynamic behavior, driven on the one hand by asset accumulation and, on the other by wage adjustment to a neoclassical steady state. It embodies a wide range of assumptions about individual behavior and empirical regularities in a general equilibrium framework. The interdependencies are solved out using a computer algorithm that solves for the rational expectations equilibrium of the global economy. It is important to stress that the term ‘general equilibrium’ is used to signify that as many interactions as possible are captured, not that all economies are in a full market clearing equilibrium at each point in time. Although it is assumed that market forces eventually drive the world economy to a neoclassical steady state growth equilibrium, unemployment does emerge for long periods due to wage stickiness, to an extent that differs between countries due to differences in labor market institutions.

The theoretical approach to modeling demographics outlined in Section 2 is embedded into the large scale MSG-Cubed model in the key areas of labor supply and consumption and saving decisions. The rest of the model endogenizes the response of investment, asset prices and international trade and capital flows adjustment to changes in demographic inputs.



## 4 Calculating the Impacts of Demographic Change

The demographic changes projected over coming decades are large, but are they likely to have an important effect on the economies of advanced and developing countries? This section uses simulations from the MSG-Cubed model to investigate this issue. This is a difficult issue to untangle from the historical data as well as from future projections. It necessarily requires the use of a model. An earlier paper (Batini, Callen and McKibbin (2005)) explored this issue in a four region model. The same technique is used in this paper.

The approach is to first project the world economy from 1985 to 2100 assuming the UN (2002) mid range demographic projection. The projections from the UN data have to be modified to fit into our simplified analytical framework. In particular we have to modify the birth rates of children and maturity rates of adults to adjust for the fact that we assume a constant probability of death among adults, which is not consistent with the UN projections. We try as much as possible to match the population numbers from the UN projections. In addition we make assumptions about productivity growth by sector and country using the approach outlined in Bagnoli et al (1996) extended in McKibbin et al (2004). We build a baseline projection containing the actual and expected demographic transition.

We then want to ask the question “what is the impact of the demographic projections in this baseline projection”? There are a number of ways this could be addressed. One is to modify the demographic assumptions as in Bryant et al (2004) and Bryant and McKibbin (2004) to test the sensitivity of projections to different demographic assumptions. In this paper we want to produce an estimate of the entire impact of demographic change. In other words we want to see what would be happening over the next fifty years if there had not been any demographic change (defined as changes in birth rates relative to steady state). We explore this question by effectively removing the demographic change from our projections. How this is done is stylized in Figure 5. A stylized baseline projection for population is illustrated in Figure 5. Population growth rates are higher for developing economies and falling quickly over time towards the same steady state as for industrialized economies by 2100. In order to calculate the contribution of demographic

change to the projection, the future is recalculated assuming that the demographic variables in the model such as child birth rate and adult maturity rates for each country are set equal to the long run steady state rates. These are assumed to be the rates, equal in all countries where the population growth is zero in all countries, given a constant probability of death of adults of 1 percent. It should be stressed that in the intertemporal modeling approach in this paper there needs to be a well defined long run steady state with all countries growing at the same rate. This is forced by having productivity growth by country eventually converging and the birth rates of children and maturity rates of adults eventually converging. It is assumed that all countries will eventually have the same population growth rates in the steady state. In the counterfactual experiment we assume these demographic rates apply from 1985 onwards. The difference between the baseline with demographics built in and the counterfactual experiment with demographic growth rates at steady state values gives one measure of the contribution of the current demographic transition in the world economy. The steady state we assume will affect the results presented below. The results should be interpreted as indicative of the current contribution of demographic change to each economy and more usefully, how these changes projected to occur over time will lead to changes in economies over time. Even if the reader is skeptical about the steady state assumptions imposed in this study (these clearly affect the calculation throughout the period), the change in variables over time are less dependent on the steady state assumptions in the near term and primarily reflect the demographic consequences of the UN projections over time.

The process of removing a large shock of this nature is a conceptually difficult exercise because the model assumes rational expectations in a variety of markets. Thus the initial conditions for 1985 (i.e. the actual data) in the baseline have expectations about the future demographic transition already embodied in stock variables such as physical capital stocks, net asset positions (both domestic and foreign) and human capital. We thus have a problem in the counterfactual exercise, beginning in 1985, when we remove the demographic shock because in 1985 we will be capturing both the impact of the underlying demographic change as well as the impact of the change in expectations about future demographic change. For a period after the new information is announced there will be a large adjustment in asset stocks which reflects the revision in expectations. In an admittedly imperfect attempt to separate out the revision of

expectations from the underlying demographic change, we solve the model for 20 years to 2005 (and for many years beyond) so that much of this initial asset adjustment is completed. By 2005 we believe we are capturing more of the pure demographic effect and less of the revision to expectations. Thus in the following analysis all results are presented from 2005 to 2100, after the asset stock adjustment and asset price volatility to the change in information in 1985 have washed through the economy.

In order to undertake the analysis in our theoretical framework but as consistently as possible with actual data, we convert the 2002 UN population projections (the mid case scenario) into the parameters of the model, given the assumption in the model of a constant probability of death for adults and a different but constant probability of death for children. This conversion is done in a way which gives us as close as possible the aggregate adult and child populations over time for each country as projected by the United Nations. It is not an exact representation of the UN projections because the probability of death is changing over time in the UN projections and at this stage of the research we are unable to incorporate this feature into the model. Thus the results should be interpreted as illustrative (and with great care) rather than as precise predictions about the future.

Figures 6 and 7 present the deviation of the child birth rate and adult maturity rates from the long run steady state rates in each year commencing from 1985. These are the basic shocks that are removed from the baseline in the counterfactual simulation and which are interpreted as the demographic shocks in the baseline projection. Several important points can be seen in Figures 6 and 7. In each region from 1985 to 2020, child birth rates and adult maturity rates are falling. For Japan the fall in the growth rates are actually smaller than the fall in the growth rates for developing countries but this is a starting from a rate below the steady state rate. Thus the population is falling in Japan but still rising in developing countries despite a large projected fall in growth rates in developing countries. The difference between growth rates of the population and the level of the population will be shown to be important in the results. In an important sense the prospective demographic transition in developing countries is larger than the prospective demographic transition in many industrial economies including Japan. Much of Japan's demographic transition has already occurred in the second part of the twentieth century.



## 5 The Impact of Global Demographic Change

Results for the contribution of demographic change to each of the ten economies (or regions) in the model from 2005 to 2050 are contained in figures 8 through 13. There is one figure for each variable to be examined. Within each figure there are ten graphs – one for each country or region in the model. Within each graph there are two lines. The lines labeled “own” shows the consequences of demographic change that only occurs within that country. The line labeled “global” shows the impact of global demographic change on that country, including the demographic change within its borders. The second line captures global demographic change. The difference between the two lines shows the impacts of the demographic change in the rest of the world on each economy. Thus for example in Figure 8, US GDP growth in 2005 is estimated to be 0.4 percentage points higher as a result of US demographic change, considered in isolation, than it would have been if the steady state population growth rates applied in 2005. Furthermore, when we estimate the contribution of global demographic change, we find that US GDP growth is just under 0.6 percent higher (than otherwise) in 2005 because of demographic change occurring in both the US and in the rest of the world.

The spill-over of demographic change occurs both through changes in trade flows and capital flows. For example, a rise in the labour force outside the US would lead to a fall in the relative price of products from the rest of the world as well as a rise in the demand for US products. The US growth rate is persistently affected because higher foreign demand raises real wages and the real rental price of capital in the United States. In addition the lower imported input costs would expand the production function in the United States and raise the US growth rate during the transition. As can be seen from many of the results in figure 8 through 13 the global demographic shock is important for understanding domestic developments in each economy.

Consider the results in more detail. By 2005 there are more workers in the United States than there would have been without the demographic bulge even though the growth rate of labor is falling through the demographic adjustment. With more workers in the US, the marginal

product of capital is higher than it otherwise would be and capital accumulation would still be higher than without the demographic bulge. However, eventually the rate of capital accumulation declines over time consistent with fall in the growth rate of the labour supply. In each graph global demographic change (currently being experienced relative to the long run) implies higher current and future growth relative to what would have happened without the demographic transition. Other features of figure 8 that are worth noting include the result that the US demographic change is detracting from growth by 2040 in the United States but by 2010 in Japan, and 2018 in Europe and ROECD. In developing countries the timing is 2017 for China, 2020 for India, and after 2030 for other developing countries.

Figure 9 shows results for the level of GDP relative to baseline. Note that while the growth rate of GDP is above zero, the level difference in GDP will still be rising. Even though Japanese GDP growth is below the “no demographic change” case by 2010, the level of GDP is still above what it would have been without the demographic transition by 2020. It is interesting that by 2050 the Japanese economy is projected to have 28% less GDP than it would have had without the demographic transition in Japan<sup>11</sup>. Thus the relative size of countries due to changing demographics can change significantly. For example taking account of global demographic trends these results suggest that Indian GDP will be 90% bigger than it would have been without the demographic changes since 1985 compared to the fall in the size of Japan.

Figure 10 show the impact of demographic change on savings relative to GDP for each economy. One feature which stands out in these results is that global demographic change tends to lower savings relative to own demographic change. The primary reason for this is that with higher incomes expected in future years when global populations are rising, consumers will be better off globally and need to save less in order to smooth consumption relative to the no demographic case. Countries at a more advanced stage of aging such as Japan and Europe tend to have higher savings as a result of current and recent past demographic factors which eventually

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<sup>11</sup> This means says nothing in welfare or “wellbeing” terms since it is per capita variables that matter and per capita GDP is likely to be higher. This is discussed at length in McKibbin and Nguyen (2004).

decline as the population ages. Whereas countries still experiencing positive population growth and higher birth rates will only gradually increase savings over time in anticipation of the demographic transition. Countries with relatively closed capital accounts (such as most developing countries) are dominated by their own demographic adjustment whereas the more open capital accounts (the industrial economies) experience a larger difference between savings from domestic demographic change and savings driven by global demographic change. It is important to keep in mind that as well as being driven by household behavior, the outcomes for savings are also driven by adjustment to investment to be discussed below. The MSG-Cubed model has the property that saving and investment within countries tend to move together with some offset through international capital flows but to the extent that risk premia partly prevent capital from flowing across borders the domestic real interest rate acts to partially equilibrate domestic saving and investment. This is consistent with the empirical evidence in Obstfeld and Rogoff (2000) and Helliwell (2004).

Figure 11 shows the results for private investment. The investment paths closely follow savings in each economy although driven by different factors. As mentioned in the previous section there are different effects on investment partly driven by the changing marginal product of capital due to changing effective labor forces as well as shifts in spending patterns and reallocation of investment from countries with low capital productivity to countries with high capital productivity. In the model it is assumed that capital is highly mobile adjusted by a risk premium which is calculated on 2002 data. This risk premium is assumed to remain constant over the entire simulation period. Thus countries that restrict capital flows in 2002 will also relatively restrict these flows through the entire period<sup>12</sup>. Over time the countries experiencing rising labor forces are tending to attract investment and those experiencing falling labor forces are tending to lose investment. However this general pattern from own demographic change is changed in the global context. Japan for example experiences a far smaller fall in investment over the period in the global context because of the much stronger demand for Japan products

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<sup>12</sup> Batini, Callen and McKibbin (2005) explore the impacts of changing this assumption and allowing more capital to flow to developing countries.

from a rapidly growing world economy. A similar story applied for Europe where own demographic change tends to lower investment over time yet in the global context demographic change tends to raise European investment after 2015. This is both a US effect as well as an effect from developing countries in which a vast proportion of the population growth and economic growth is being sourced.

The net outcome of savings and investment is the current account balance. This is shown for all economies in Figure 12. As expected the demographic change in Europe and Japan contributes a sizeable amount to the current account surpluses of these economies. What is interesting is that the global demographic change in both cases adds further to the current account surpluses of both regions. In the domestic case resources are shifted from the aging economies into developing countries because of optimal saving decisions. The demographic change in developing countries and the United States tends to draw capital from the aging economies attracted by a higher return to capital. Thus there is both a “push” element and a “pull” element in the demographic impacts on the current account. The United States in particular but also China attracts significant amounts of savings from other industrialized economies. Overall in 2005 the model estimates that current demographic change on balance has contributed to current account surpluses in Japan, Europe, ROECD, China, Russia and Eastern Europe and deficits in the United States, rest of Asia, India, Latin America and the rest of the developing countries.

The price which adjusts to equilibrate saving and investment outcomes given the international flows of capital is the real interest rate. The real interest rate in each country is partly determined by savings and investment but also through uncovered interest rate parity adjusted by constant risk premia. The difference between real interest rates across countries reflects risk premia and expected future changes in real exchange rates. Own demographic change contributes to higher real interest rates in the United States and ROECD as well as developing countries. In Japan and Europe the declining marginal product of capital is reflected in lower real interest rates. Once the global demographic change is accounted for, higher real interest rates are experienced everywhere. This is not surprising as countries with high marginal products of capital because of rising effective labor forces will attract capital from aging



countries and hence drive up real interest rates globally even adjusting for capital restrictions. Over time as the demographic shocks work their way through the global economy real interest rates decline everywhere. These are quite dramatic changes. For example real interest rate fell by 200 basis points over 45 years in the United States. In most developing countries this change is closer to 400 basis points whereas China is an intermediate 300 basis points.

## **6 Summary and Policy Implications**

This paper has used a ten region global macroeconomic model with demographic dynamics to explore the impact of current and projected future global demographic change on the global economy. The paper illustrates that it is possible to use new theoretical developments in modeling overlapping generations in large scale quantitative models with significant country coverage to form a global picture of the consequences of demographic change. Clearly there are a large number of critical assumptions that drive the results and it is planned to undertake a wide range of sensitivity tests of these assumptions in future research. It is assumed that social security systems function reasonably well in each economy in order to capture the pure macroeconomic effects of demographic change. Although challenging to incorporate the features of social security systems in each of the ten regions, Bryant and his team have shown how important this can be in a small scale theoretical model<sup>13</sup>. In addition, more work is required in the modeling of developing countries which make up a majority of the countries in this new model and where important demographic changes are projected to occur. A number of key issues such as the impact of surplus labor, infrastructure needs, the role of the informal sector etc are being added to the G-Cubed models. These more representative features of developing countries may change the spill-over of demographic change in developing countries to the rest of the world.

There are at least two most important policy implications from this new research. The first is that the projected demographic transition in the global economy will likely have important

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13 See Bryant (2004)

macroeconomic impacts on growth, trade flows, asset prices (real interest rate and real exchange rates) and investment rates. The second result is that policymakers should not ignore the global demographic transition when focusing on domestic issues related to demographics. The fact that the demographic transition is at different stages across countries, particularly in industrialized countries relative to developing countries, implies that the global nature of demographic change cannot be ignored. This paper shows that the developing world has important impacts on the industrial economies.

As well as creating a framework for exploring a range of possible policy responses directly related to demographics, future work will use this new model to explore how other policies, apparently unrelated to demographics, might impact on the macro economy to offset any negative consequences or reinforce any positive consequences of global demographic change. A first attempt at this is contained in recent paper using a four region version of the model in this project by Batini, Callen and McKibbin (2005). That paper explored the impact of productivity improvements induced by economic reform and lowering barriers to international capital flows in developing countries. Other issues to which this new framework can be applied include the ability to use trade reform as a way of allowing labor intensive goods to flow into rapidly aging societies such as Japan, and enable domestic labor to be reallocated into activities that more directly support an ageing society. There is no reason why policies in other parts of the economy might not have a more substantial positive contribution to dealing with demographic change than the more direct policies that are usually proposed, such as increased migration, subsidies to child birth or changes in retirement ages. The ability offered by the model developed in this paper to assess a wide range of alternative policies aimed at dealing with the global demographic transition suggests an exciting new area of policy evaluation.

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## Appendix A: The analytical Approach

This appendix summarizes the analytical approach followed in incorporating demographics into the model. A summary of the key features of the MSG-Cubed model was outlined in section 3. The reader is referred to more detailed documentation in McKibbin and Wilcoxon (1998) and McKibbin and Nguyen (2004) as well as online at [www.gcubed.com](http://www.gcubed.com) .

### *a. Adult Population*

In each period, a cohort of children matures and joins the adult population. The size of the newly matured cohort, at time  $s$ , with respect to the existing adult population,  $N(s)$  is referred to as the maturity rate,  $b(s)$ . The maturity rate and its relationship to the population of children will be addressed in another section, below. Following Blanchard, we make the simplifying assumption that at any time  $s$ , all adults in the economy face the same mortality rate<sup>14</sup>,  $p$ , defined here as the probability of any given agent dying before the next period. The number of adults who matured at a previous time  $s$ , who are still alive at a subsequent time  $t$  is given by:

$$(1) \quad n(s,t) = b(s)N(s)e^{-p(t-s)}$$

The adult population size can then be determined for any time  $t$  by summing the number of living adults from all of the cohorts that have ever matured:

$$(2) \quad \begin{aligned} N(t) &= \int_{-\infty}^t n(s,t) ds \\ &= \int_{-\infty}^t b(s)N(s)e^{-p(t-s)} ds \end{aligned}$$

where  $N(t)$  represents the adult population size, at time  $t$ .

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<sup>14</sup> Blanchard notes that the assumption of a common mortality rate is a reasonable approximation for adults within the ages of 20 to 40. The fact that children and retirees, whose behaviour is of interest in studies of population aging, fall outside of this age bracket certainly indicates that the issue requires further attention.

Taking the derivative with respect to time yields an equation governing the evolution of the adult population size over time:

$$(3) \quad \frac{\dot{N}(t)}{N(t)} = b(t) - p$$

The above equation has a simple interpretation: the adult population grows at a rate determined by the maturity rate less the mortality rate.

### *b. Child Population*

In every period, a cohort of children is born. If we think of the adult population as representing the set of potential parents, then it follows that the size of a newly born cohort will depend upon the current adult population size and the birth rate,  $b_m$ . The expression for the number of children born at time  $s$  who are still alive at a later time  $t$ , is thus given by:

$$(4) \quad m(s, t) = b_m(s)N(s)e^{-p(t-s)}$$

The aggregate number of children,  $M(t)$ , can be calculated by summing the number of surviving children, who were born recently enough that they have not yet reached adulthood. If we let  $\Delta$  represent the fixed number of years from when a child is born to when it reaches adulthood, i.e. the period of childhood<sup>15</sup>, then:

$$(5) \quad M(t) = \int_{t-\Delta}^t m(s, t) ds$$

$$(6) \quad M(t) = \int_{t-\Delta}^t b_m(s)N(s)e^{-(t-s)p} ds$$

Differentiating with respect to time:

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<sup>15</sup> In the simulations that follow, the period of childhood is defined as the first 16 years of an agent's life.

$$(7) \quad \dot{M}(t) = -pM(t) + b_m(t)N(t) - b_m(t-\Delta)N(t-\Delta)e^{-p\Delta}$$

(Note that in the final exponential,  $p\Delta$  refers to the period of childhood multiplied by the mortality rate, it does not represent a change in  $p$ ).

### *c. Relationship Between the Birth Rate and the Maturity Rate*

Of the children who were born at time  $t-\Delta$ , those who survive will mature at time  $t$ , at which time they are added to the adult population. Thus, the maturity rate at time  $t$  is dependent on the birth-rate, and adult population size, of  $\Delta$  years past; as well as the mortality rate.

$$(8) \quad b(t)N(t) = b_m(t-\Delta)N(t-\Delta)e^{-p\Delta}$$

Now, we know that:

$$(9) \quad \begin{aligned} N(t-\Delta) &= N(t)e^{-\int_{t-\Delta}^t b(s)-p \, ds} \\ &= N(t)e^{p\Delta - \int_{t-\Delta}^t b(s) \, ds} \end{aligned}$$

so given the birth rate of  $\Delta$  years ago, and the maturity rates over the last  $\Delta$  years, the current maturity rate can be determined:

$$(10) \quad b(t) = b_m(t-\Delta)e^{-\int_{t-\Delta}^t b(s) \, ds}$$

Since the maturity rates over the last  $\Delta$  years will be dependent on previous values of the birth rate, it is clear that the rate of maturity is predetermined by any given series of birth rates.

### *d. Adult Consumption*

Adults attempt to maximize the expected utility derived from their lifetime consumption. Adults must take into account the uncertainty of their life-spans and thus they discount their

planned future consumption by the probability that they may not survive through to future periods. Assuming a logarithmic utility function, each agent will maximize the following:

$$(11) \quad \max \int_t^{\infty} \ln c(s, v) e^{-(\theta+p)v} dv$$

subject to the budget constraint:

$$(12) \quad \dot{w}(s, t) = [r(t) + p]w(s, t) + y(s, t) - c(s, t)$$

where  $c(s, t)$  is the consumption, at time  $t$ , of an adult who matured at time  $s$ ,  $\theta$  is the rate of time preference,  $w(s, t)$  is the financial wealth that an adult who matured at time  $s$  holds at time  $t$ ; and  $r(t)$  is the interest rate earned on financial wealth. Financial wealth included domestic bonds, equity, and foreign assets from all countries in the models. In addition to interest payments, adults also earn a rate of  $p$  on their holdings of financial wealth, due to the assumption of a life insurance market, as in Blanchard. Children do not play a part in the life insurance market, nor do they earn interest, as they are assumed to hold no financial wealth. In the full model the  $c(s, v)$  function also includes a complete set of all goods from all countries in the models. Within each period, once aggregate consumption for that period is determined the allocation across goods from different countries is determined based on relative prices of goods distinguished by place of production.

The optimal consumption path for an adult can be shown to be:

$$(13) \quad c(s, t) = (\theta + p)[w(s, t) + h(s, t)]$$

where  $c(s, t)$  is the consumption, at time  $t$ , of an adult who matured at time  $s$ , and  $h(s, t)$  represents the human wealth of the adult. An adult's human wealth is defined as the present value of the adult's expected income over the remainder of his or her lifetime:

$$(14) \quad h(s, t) = \int_t^{\infty} e^{-\int_t^v r(i)+p di} y(s, v) dv$$

At any time  $t$ , the sum of financial wealth and human wealth— $w(s,t)$  and  $h(s,t)$ —represents an adult's total wealth: the means by which the adult can pay for his or her future consumption. Adults consume a proportion of their total wealth each period, the proportion being determined by their rate of time preference, and their likelihood of perishing before the next period.

Aggregate adult consumption, aggregate financial wealth and aggregate human wealth are simply the sums of the consumption, financial wealth and human wealth for all adults in the economy.

$$(15) \quad C_N(t) = \int_{-\infty}^t c(s,t)n(s,t) ds$$

$$(16) \quad W(t) = \int_{-\infty}^t w(s,t)n(s,t) ds$$

$$(17) \quad H(t) = \int_{-\infty}^t h(s,t)n(s,t) ds$$

where  $C_N(t)$  represents aggregate adult consumption,  $W(t)$  is aggregate financial wealth, and  $H(t)$  is aggregate human wealth.

The adult aggregate consumption function can be shown to be given by:

$$(18) \quad C_N(t) = (\theta + p(t))[W(t) + H(t)]$$

#### *e. Labor Supply, and Demographic Considerations*

Empirically, one of the key economic characteristics that changes with age is the income that a person receives. This is usually summarized in an age-earning profile of a country. An example of the age-earnings profile for Japan is given in Figure 4. This shows the income of a cohort as they move through time relative to their initial earning as the 15-24 cohort. This is remarkably stable over 30 years and is similar (although not identical across countries). It shows that as new workers enter the workforce their income gradually rises over time presumably reflecting productivity improvements over time. From the 50-55 cohort, income begins to fall over time but there is still positive income after age 65. We introduce this age-earnings profile

into the model, such that an agent's income is determined by his or her age. We then aggregate over cohorts over time. Further, we assume that only adults earn labor income, and that children are completely dependent upon adults. Faruqee (2000a) utilizes hump-shaped age-earnings profiles for adults, fitted to Japanese data using non-linear least squares (NLS). Intuitively, the hump-shaped profile of age-earnings reflects the fact that young adults generally have incomes that are increasing as the young individuals age and gain more experience. After a certain age, however, earnings decline, reflecting first the decreasing productivity associated with aging, and then eventually reflecting retirement behavior.

Individual income is not specified as suddenly dropping to zero, at a given retirement age, for two reasons. Firstly, in practice, people typically retire at various ages, and some retirees continue to earn alternative forms of income even after retirement. Secondly, a discontinuous age-earnings profile introduces complications with respect to implementation in the MSG-Cubed model.

We model the evolution of income over the lifecycle by beginning with the assumption that individuals are paid a wage for each unit of effective labor that they supply. We also assume that effective labor supply is a function of an individual's age and of the current state of technology. Aside from aging considerations, note that as time passes, the technological progress in the economy has a positive effect on the value of effective labor supplied by all agents.

The effective labor supply, at time  $t$ , of an agent who has been an adult since time  $s$ , is given by:

$$(19) \quad l(s,t) = e^{\mu t} [a_1 e^{-\alpha_1(t-s)} + a_2 e^{-\alpha_2(t-s)} + (1 - a_1 - a_2) e^{-\alpha_3(t-s)}]; \quad (a_i > 0, \alpha_i > 0 \text{ for } i=1 \text{ to } 3)$$

The  $e^{\mu t}$  component (where  $\mu$  is the rate of technological progress) captures productivity increases due to advancements in technology. The remaining terms represent the non-linear functional form used to estimate the hump-shaped profile. The  $a_i$  and  $\alpha_i$  parameters are estimated, based on

empirical data, using NLS<sup>16</sup>. The hump-shaped effective labor supply specification will in turn lead to a hump-shaped age-earnings profile.

Individual labor supply can be re-written as:

$$(20) \quad l(s, t) = \sum_{i=1}^3 l_i(s, t)$$

where:

$$(21) \quad l_i(s, t) = e^{\mu t} a_i e^{-\alpha_i(t-s)}; \quad (a_i > 0, \alpha_i > 0)$$

and:

$$(22) \quad a_3 = (1 - a_1 - a_2)$$

Thus, the evolution of an adult's labor supply over time is given by:

$$(23) \quad \dot{l}(s, t) = \sum_{i=1}^3 (\mu - \alpha_i) l_i(s, t)$$

Aggregate effective labor supply in the economy for any time  $t$ ,  $L(t)$ , is the sum of the effective labor supplied by all adults in the economy:

$$(24) \quad \begin{aligned} L(t) &= \int_{-\infty}^t n(s, t) l(s, t) ds \\ &= \sum_{i=1}^3 L_i(t) \end{aligned}$$

where:

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<sup>16</sup> Values used in this paper for Japan are as estimated by Faruqee for Japan:  $\alpha_1 = 0.073$ ,  $\alpha_2 = 0.096$ ,  $\alpha_3 = 0.085$  and  $a_1 = a_2 = 200$ . In the theoretical model, to be consistent with Bryant (2004) we use the US parameters for both countries:  $\alpha_1 = 0.08152$ ,  $\alpha_2 = 0.12083$ ,  $\alpha_3 = 0.10076$  and  $a_1 = a_2 = 200$ .

$$(25) \quad L_i(t) = \int_{-\infty}^t n(s,t) l_i(s,t) ds$$

It can then be shown that:

$$(26) \quad \begin{aligned} \dot{L}(t) &= L_1(t) + L_2(t) + L_3(t) \\ &= (\mu - \alpha_1 - p)L_1(t) + (\mu - \alpha_2 - p)L_2(t) + (\mu - \alpha_{3i} - p)L_3(t) + e^{\mu t} b(t) N(t) \end{aligned}$$

The intuition behind the equation above is that the aggregate labor supply of the economy changes as the entire population ages, and also as new agents mature into the labor force.

In the application in this paper we use the estimate age earnings profile for Japan and the estimate US age earnings profile for all other regions. This is a crude approximation in lieu of getting sufficient data to estimate a more extensive set of age earnings profiles.

### *f. Intergenerational Transfers*

In our stylized model, children differ from adults, in that they do not provide labor supply (and thus do not receive payment for labor) and they do not hold financial wealth. Children are dependent upon their parents; each child receives an intergenerational transfer every period,  $c(t)$ , which is completely consumed by the child. As they do not make any consumption decision, but rather just entirely consume their transfer, we do not need to account for their human wealth.

We assume that  $c(t)$  grows at the rate of productivity growth,  $\mu$ —as the economy becomes more efficient in production, children benefit.

$$(27) \quad c(t) = c_0 e^{\mu t}$$



The simplest specification<sup>17</sup> for adult transfer payments is to assume that adults share the burden of supporting children equally, i.e.

$$(28) \quad j(s, t) = j(t)$$

where  $j(s, t)$  is the payment that an individual adult, who became an adult at time  $s$ , is liable for at time  $t$ . Note that transfer payments are bound by the following budget constraint, which constrains aggregate child receipts to equal aggregate adult payments:

$$(29) \quad c(t)M(t) = \int_{-\infty}^t j(t)n(s, t) ds$$

Thus:

$$(30) \quad j(t) = \frac{c(t)M(t)}{\int_{-\infty}^t n(s, t) ds}$$

$$(31) \quad j(t) = c(t)\delta(t)$$

Aggregate consumption for the whole economy, then, is the sum of aggregate adult consumption and aggregate child consumption:

$$C(t) = (\theta + p)[A(t) + H(t)] + c(t)M(t) \quad (1)$$

### *g. Income and Human Wealth*

In the presentation above, individual human wealth was defined as the expected present-value of future income over an adult's remaining lifetime. Having defined the profile of labor supply over the lifecycle, we can now be more explicit with respect to income. An adult's

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<sup>17</sup> Bryant and Velculescu (2001) for example make most expenses for children fall on younger adults whereas we assume that adults of all ages contribute equally.

income is after-tax labor income, plus government transfers, less lump sum taxes and intergenerational transfers:

$$(32) \quad y(s,t) = [1 - \tau(t)]w(t)l(s,t) + tr(t) - tx(t) - j(t)$$

where  $y(s,t)$  denotes the income, at time  $t$ , of an adult who matured at time  $s$ ;  $l(s,t)$  is the individual effective labor supply;  $\tau(t)$  is the marginal tax rate; and  $w(t)$  is the wage paid per unit of effective labor. We assume that the distribution of lump sum taxes,  $tx$ , and government transfers,  $tr$ , is uniform across the population, thus the year of an individual's coming of age is not a determinant of either of these two variables.

We define aggregate adult income as:

$$(33) \quad Y(t) = \int_{-\infty}^t y(s,t)n(s,t) ds$$

Taking the time derivative of  $h(s,t)$ , after substituting in the expression for individual income, we obtain:

$$(34) \quad \dot{h}(s,t) = [r(t) + p]h(s,t) - [1 - \tau(t)]w(t)l(s,t) - [tr(t) - tx(t) - j(t)]$$

The intuition for the equation above is that as time passes, future earnings are no longer as distant in time, and should therefore be discounted by a lesser magnitude—this explains the  $(r + p)$  growth—while at the same time, some income has just been received, and thus can no longer be considered part of human wealth—this explains why the current period's income is subtracted.

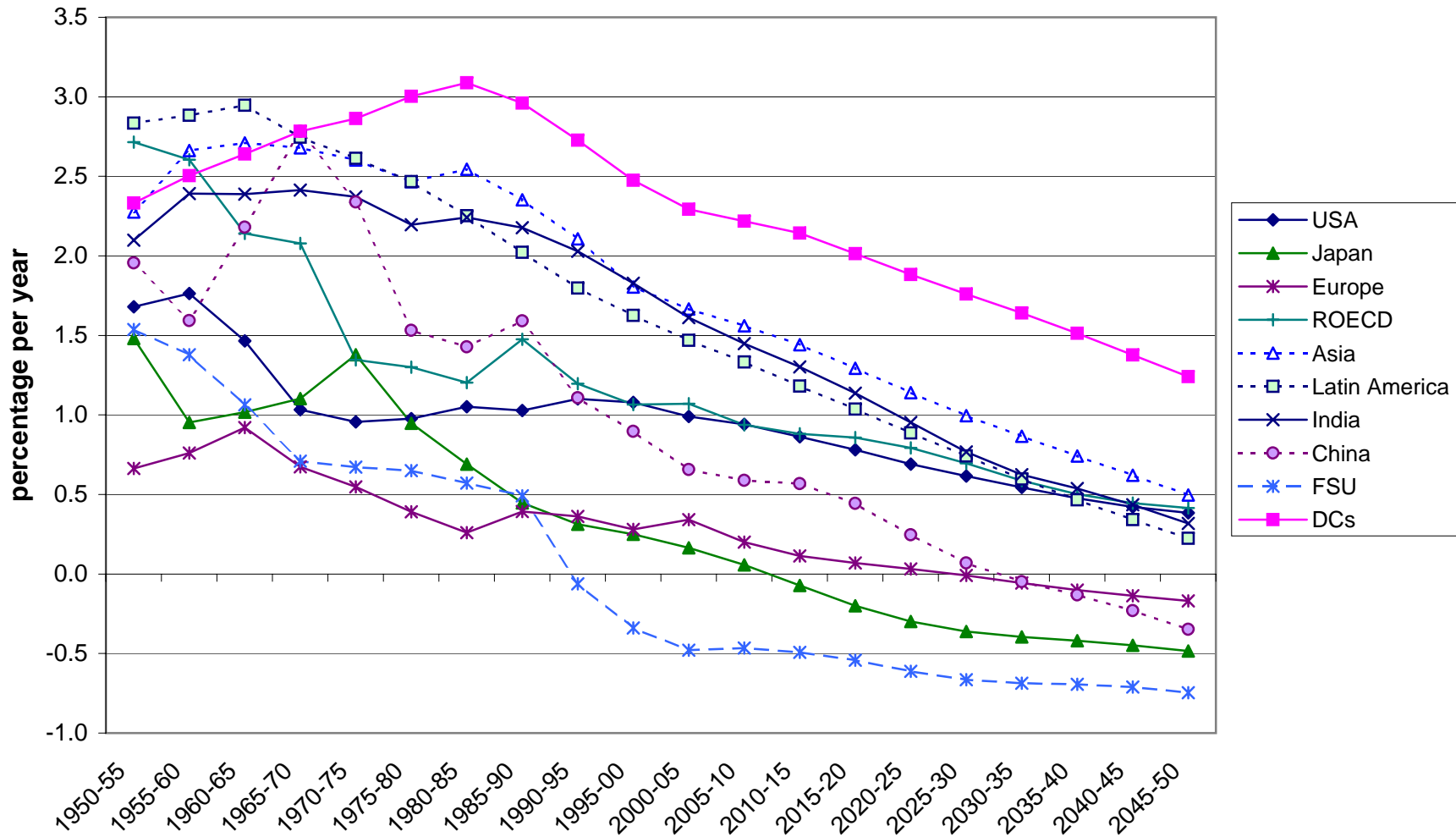
We can show that the evolution of aggregate human wealth is governed by the following relationship:

$$\dot{H}(t) = r(t)H(t) - Y(t) + h(t,t)n(t,t)$$

The intuition behind the equation above is that aggregate human wealth changes over time as future income draws nearer, thus  $H$  grows at the rate of  $r$ ; the presence of death, and hence  $p$ , does not affect aggregate human wealth, because insurance companies redistribute the wealth of the dead. Further, in each period, people receive income, and having been received, it can no longer be considered human wealth. The last term on the right hand side represents the new human wealth that the newly-matured cohort brings to the economy, each period.

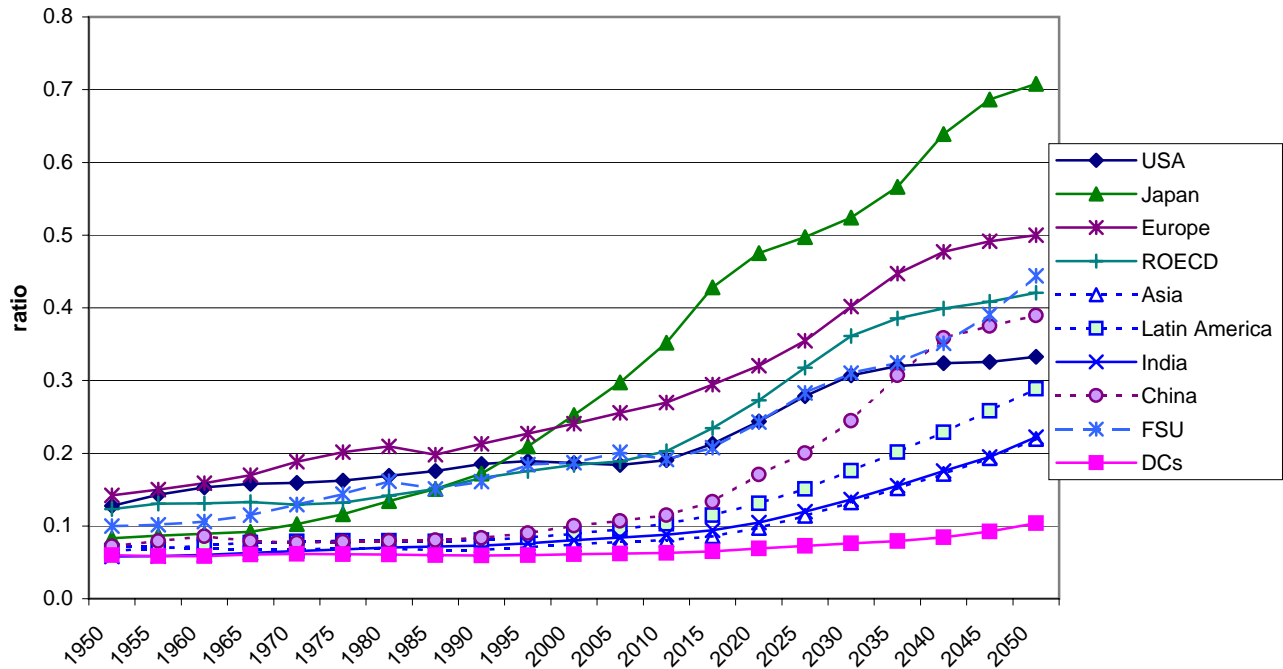
Further details on the approach followed in this paper and the implications of various simplifying assumptions can be found in Bryant (2004), Bryant and McKibbin (2004), Bryant and Velculescu (2001,2002), Bryant et al (2004) and McKibbin and Nguyen (2004).

# Figure 1: Population Growth Rate 1950-2050



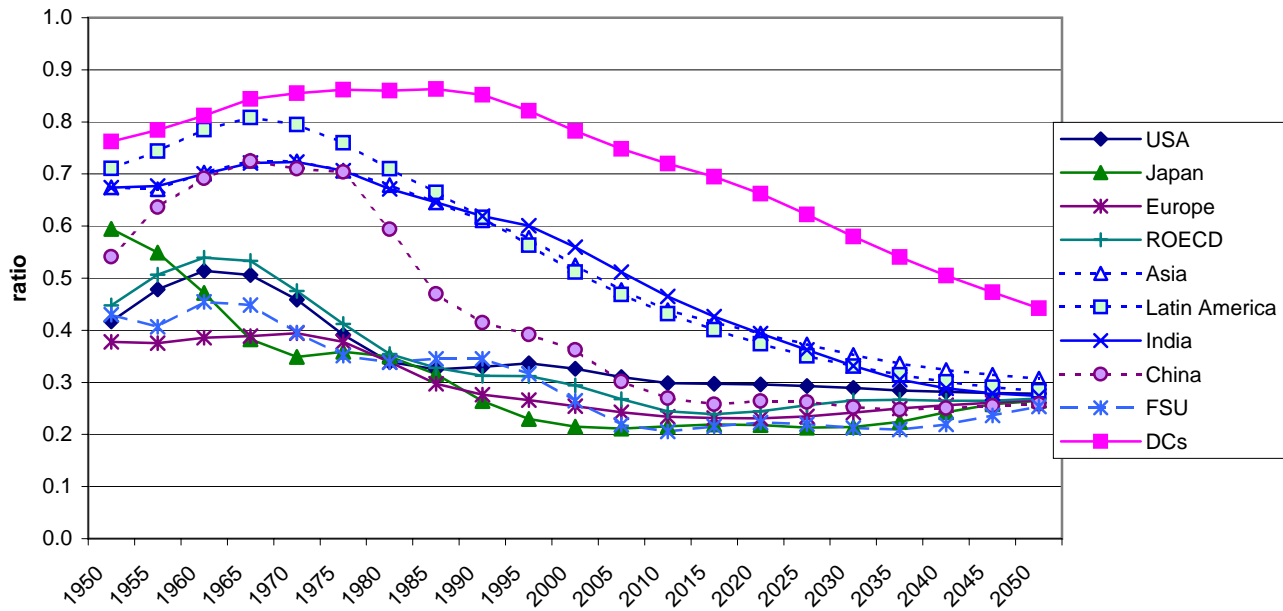
Source: UN, World Population Prospects: The 2004 Revision

**Figure 2: Elderly Dependency Ratio 1950-2050**  
(ratio of adults 65+ to adults 15-65)



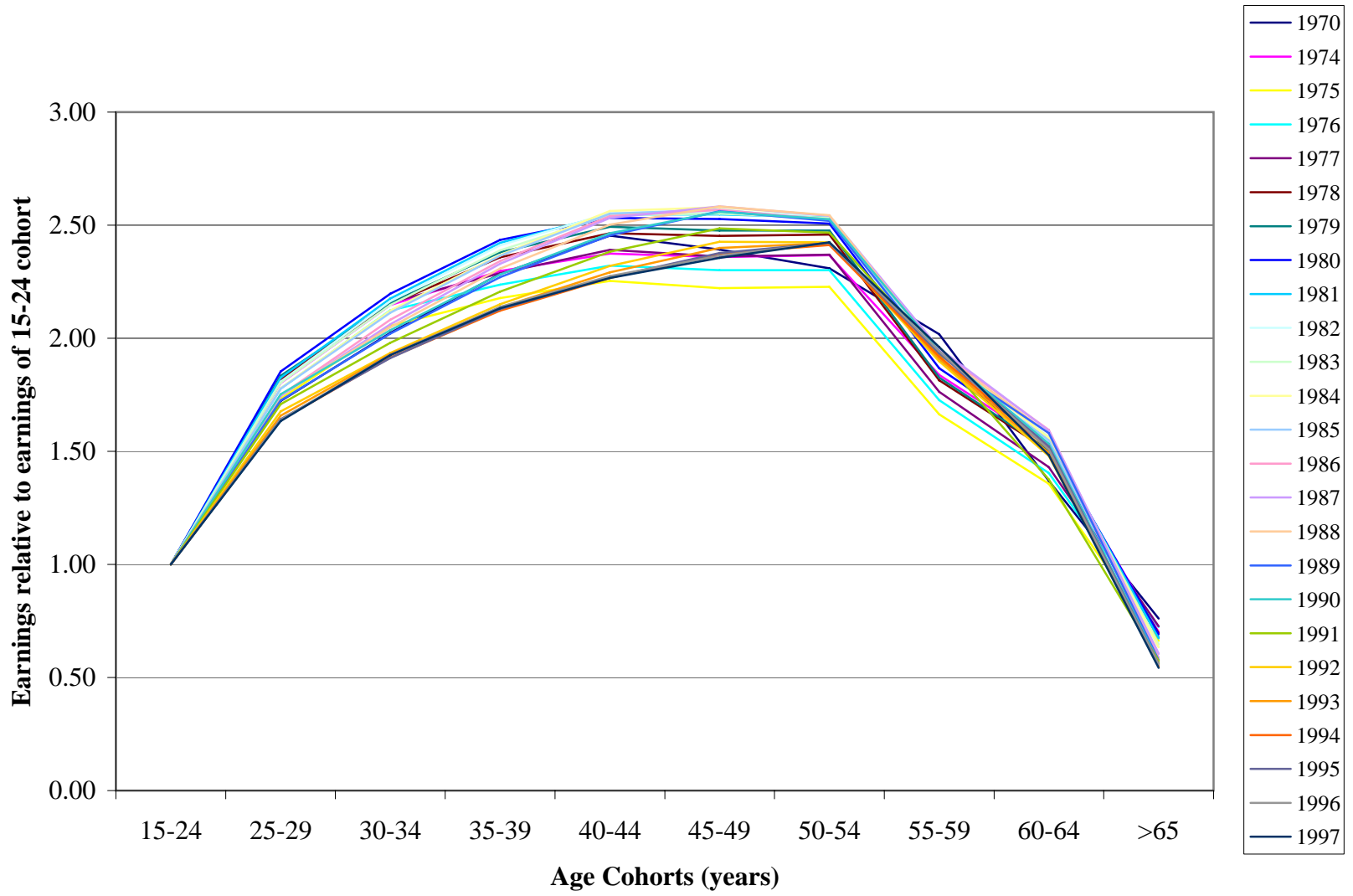
Source: UN, World Population Prospects: The 2004 Revision

**Figure 3: Child Dependency Ratio 1950-2050**  
(ratio of children 0-14 to adults 15-65)



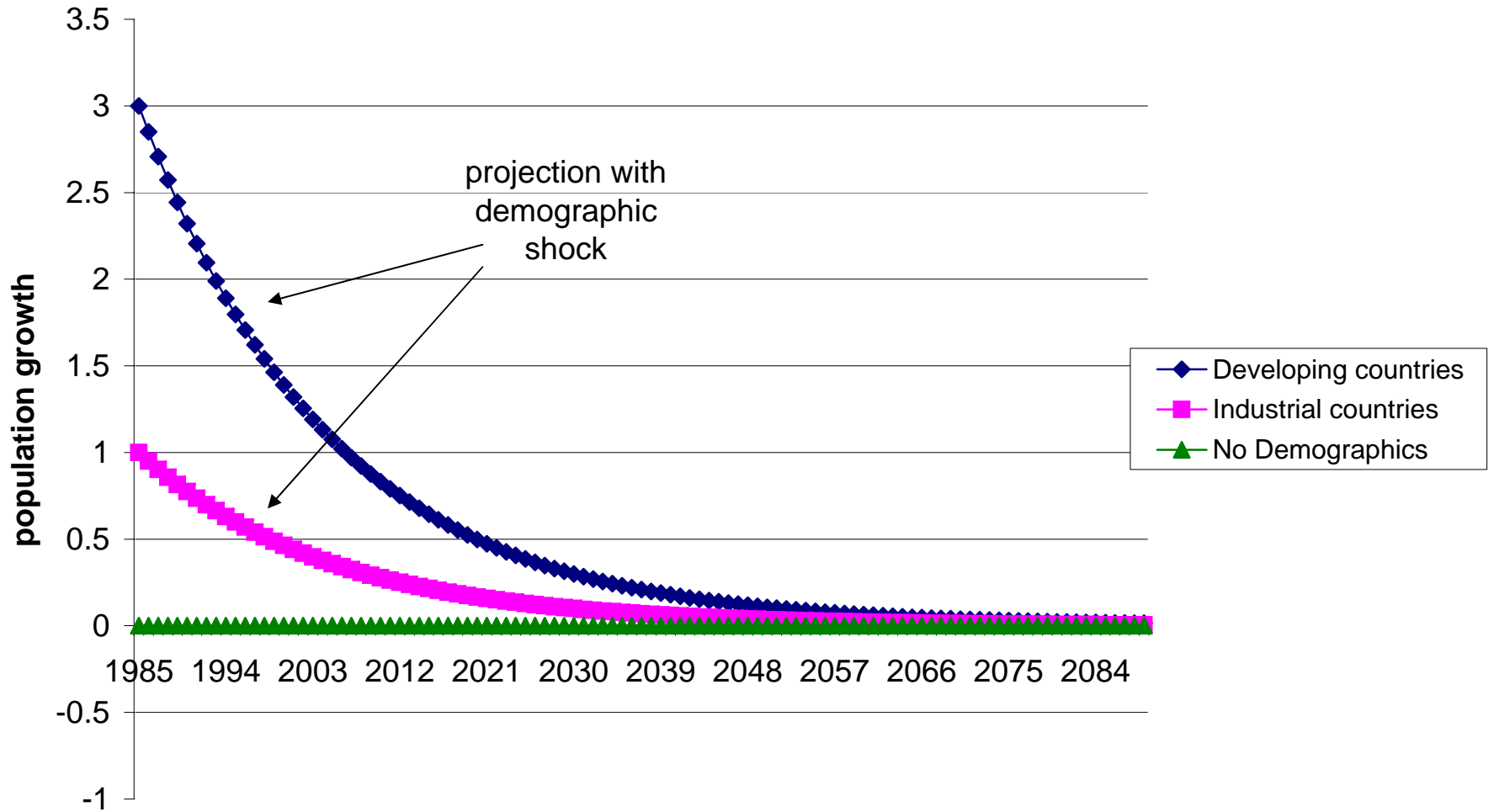
Source: UN, World Population Prospects: The 2004 Revision

**Figure 4. Age Earnings Profiles, Japanese Data 1970-1997**



source: Bryant and McKibbin (2004)

**Figure 5: Removing the Demographics  
(A stylized Representation)**







**Figure 7: Change in Adult Maturity Rate (relative to steady state)**

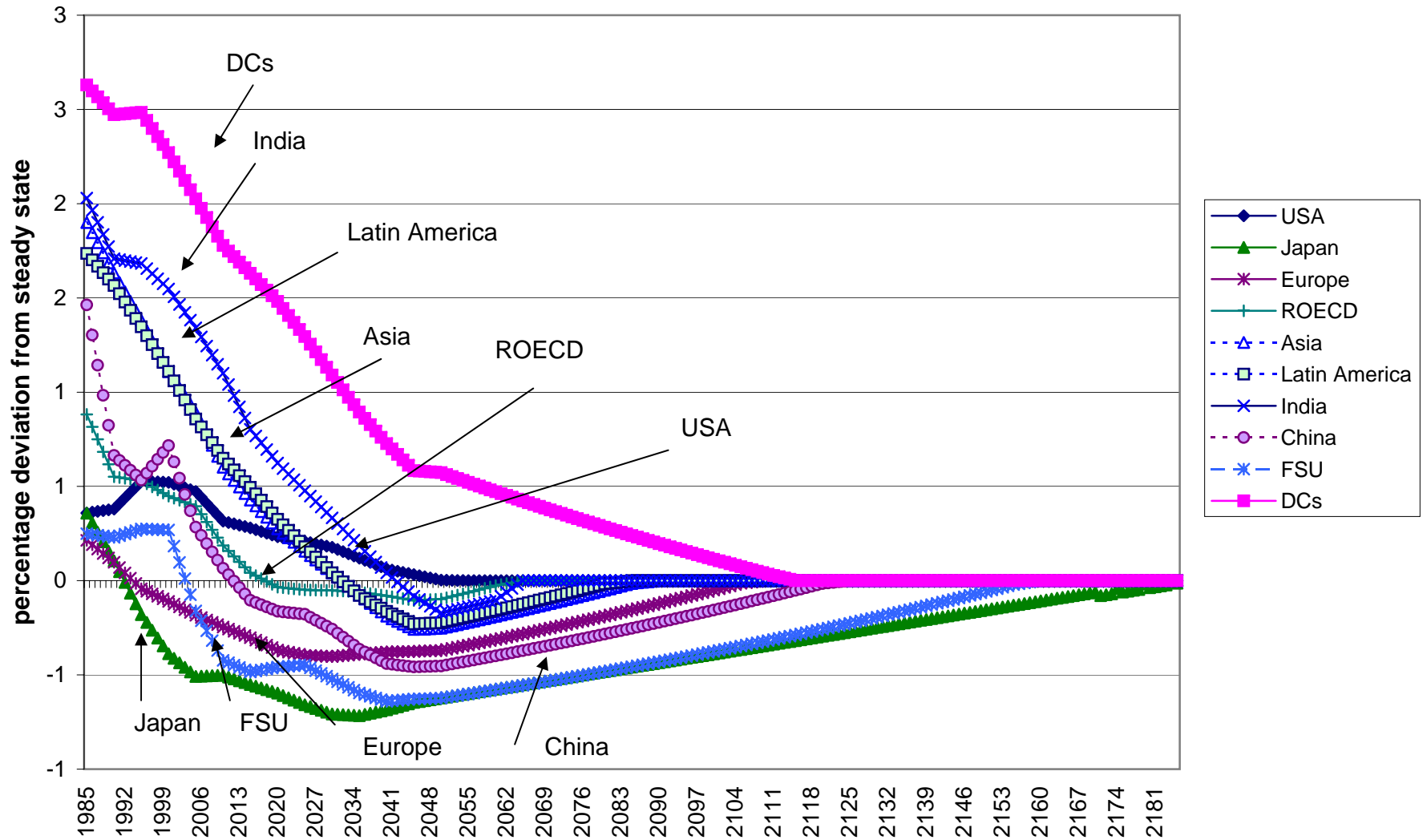
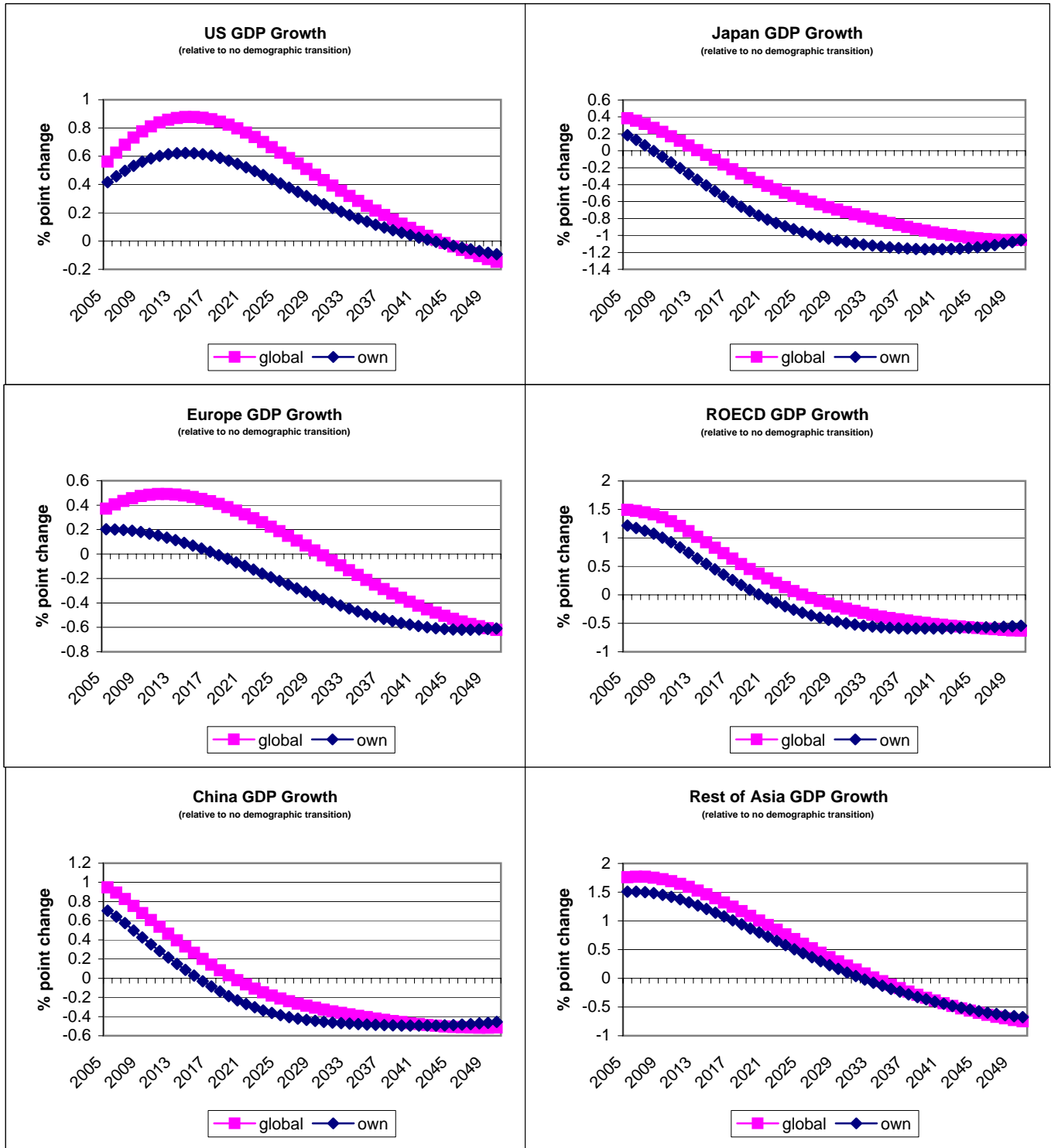
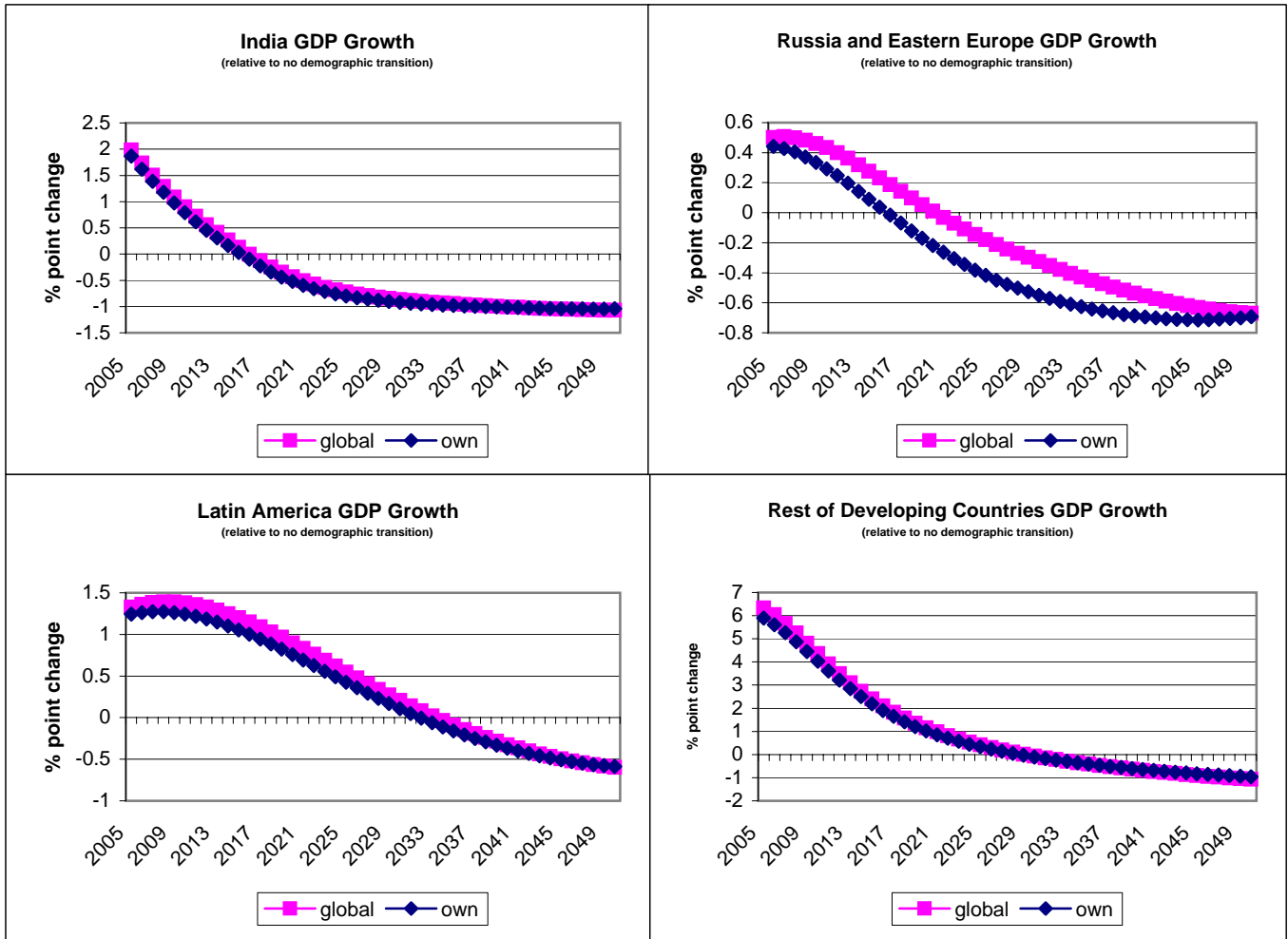


Figure 8: Contribution to GDP growth of Own versus Global Demographic Change



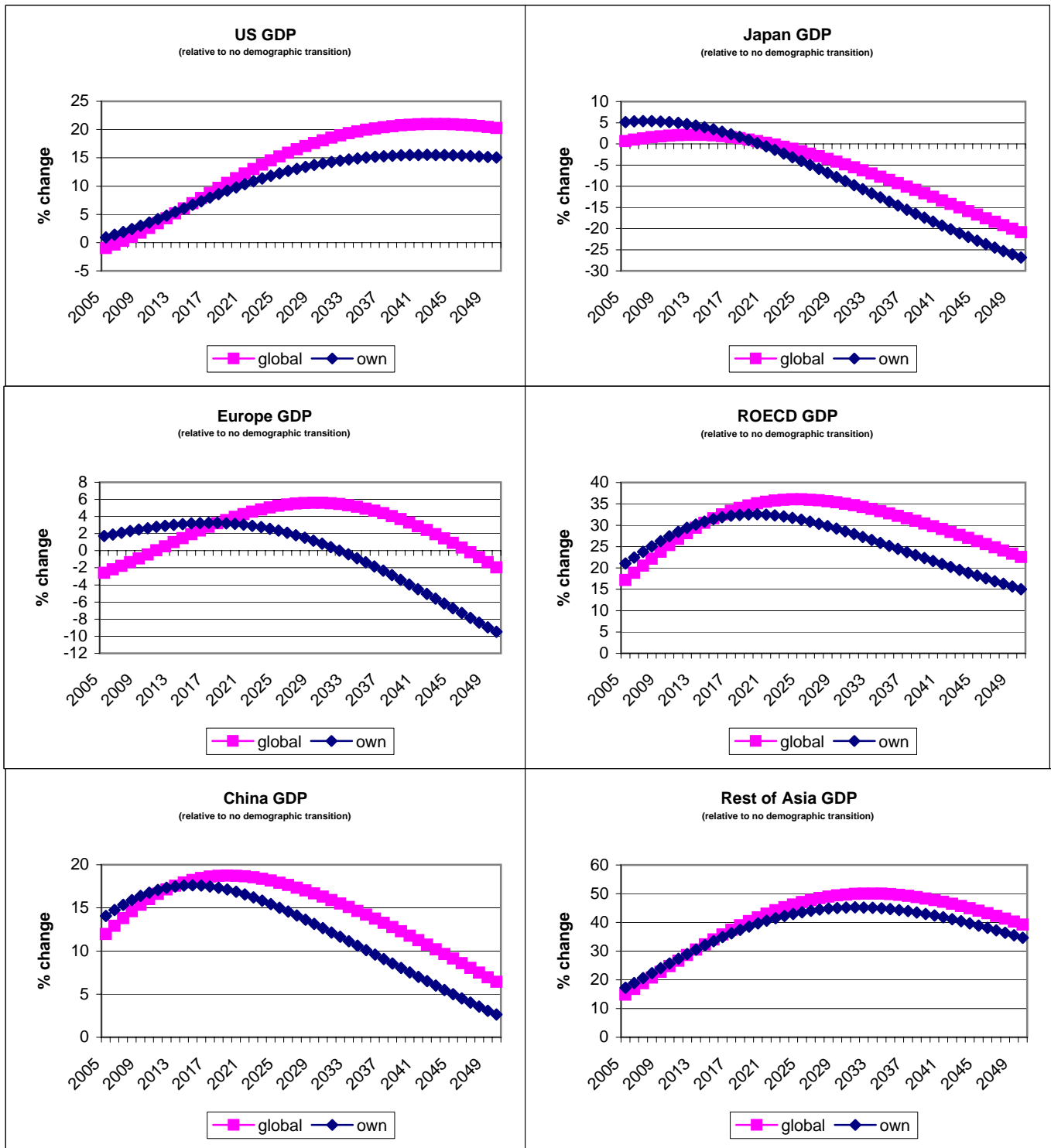
Source: MSG-Cubed (demography) version 58J

Figure 8 (cont): Contribution to GDP Growth of Own versus Global Demographic Change



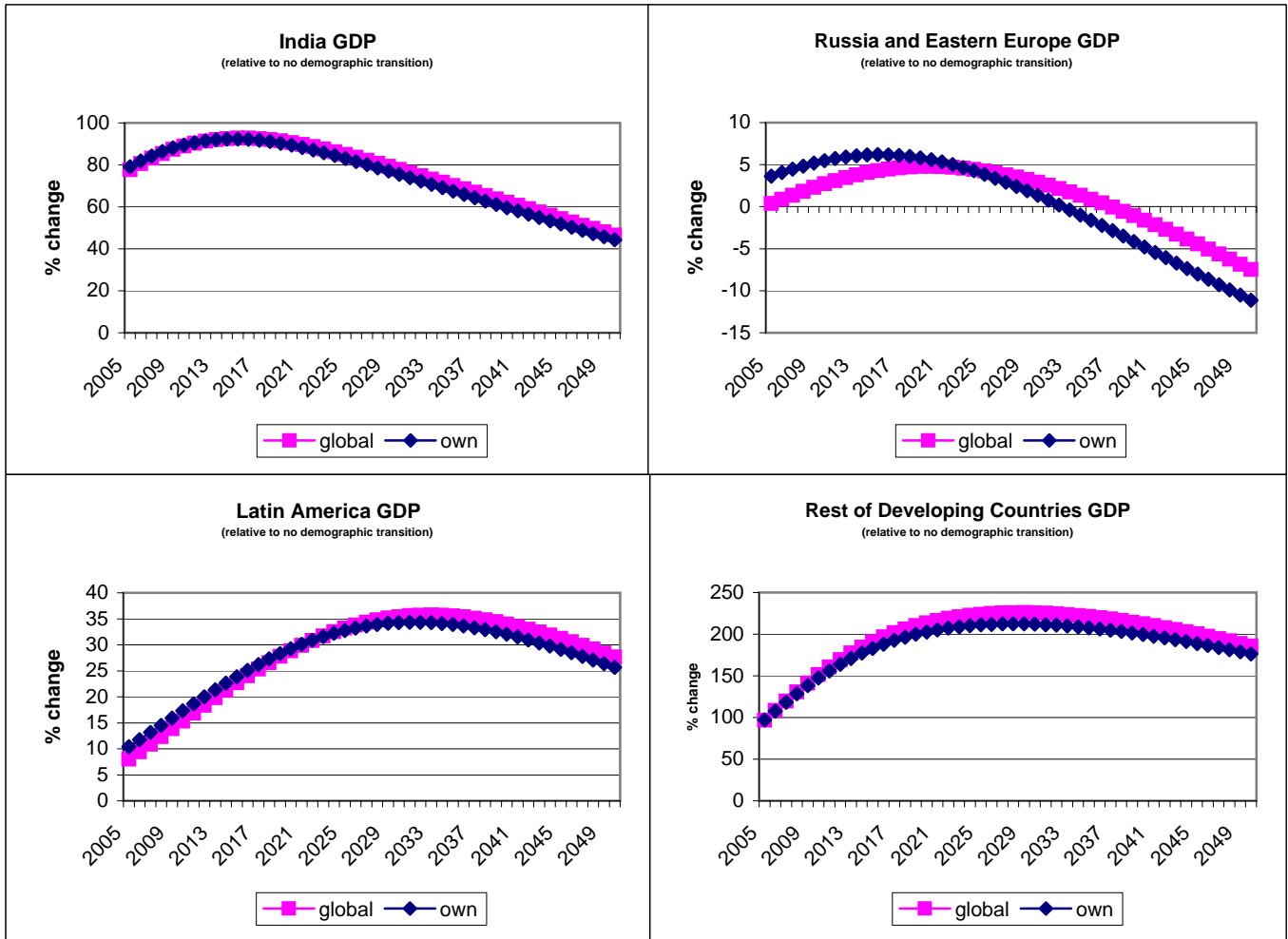
Source: MSG-Cubed (demography) version 58J

Figure 9 : Contribution to GDP level of Own versus Global Demographic Change



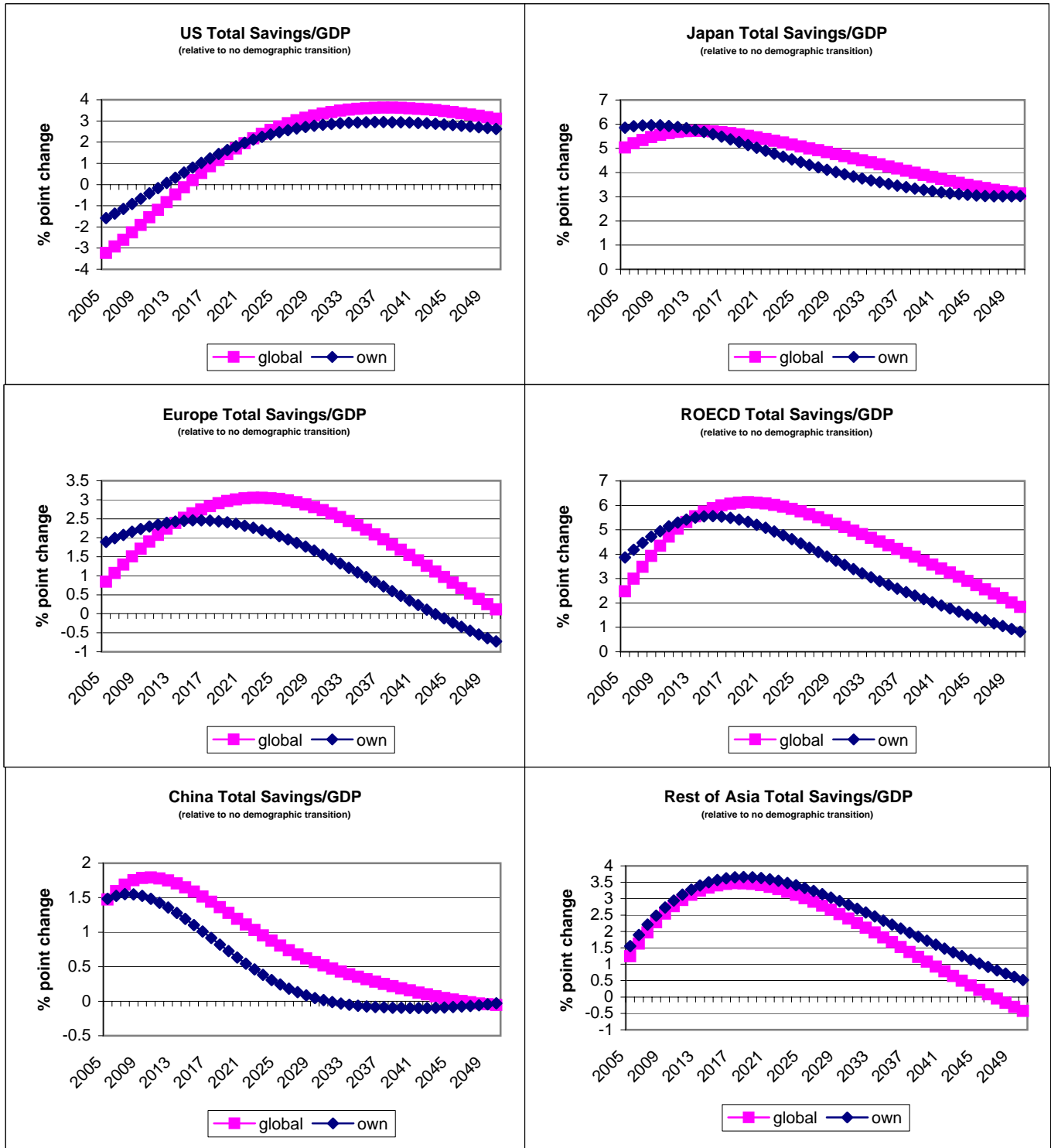
Source: MSG-Cubed (demography) version 58J

Figure 9 (cont) : Contribution to GDP level of Own versus Global Demographic Change



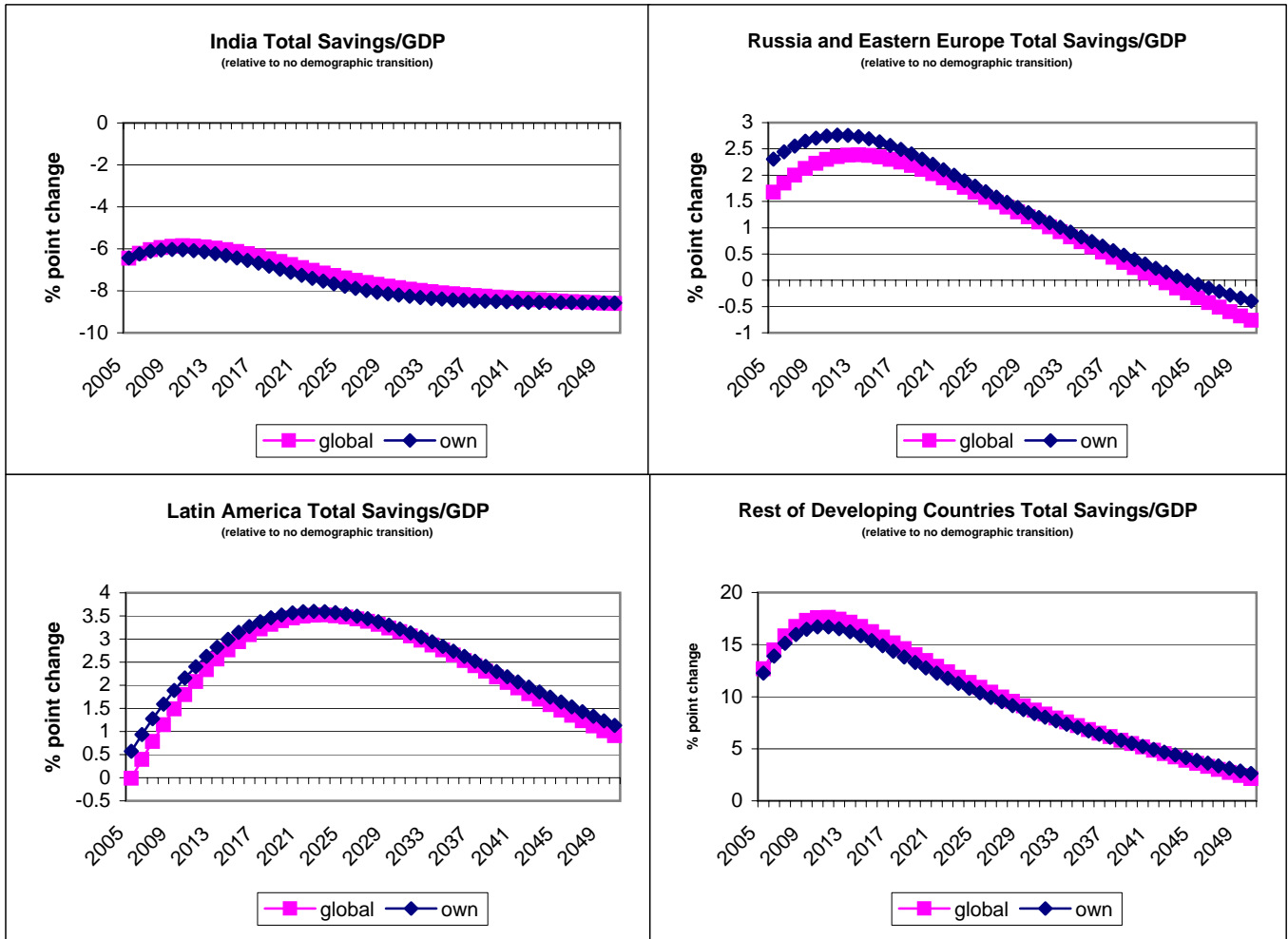
Source: MSG-Cubed (demography) version 58J

Figure 10 : Contribution to National Savings of Own versus Global Demographic Change



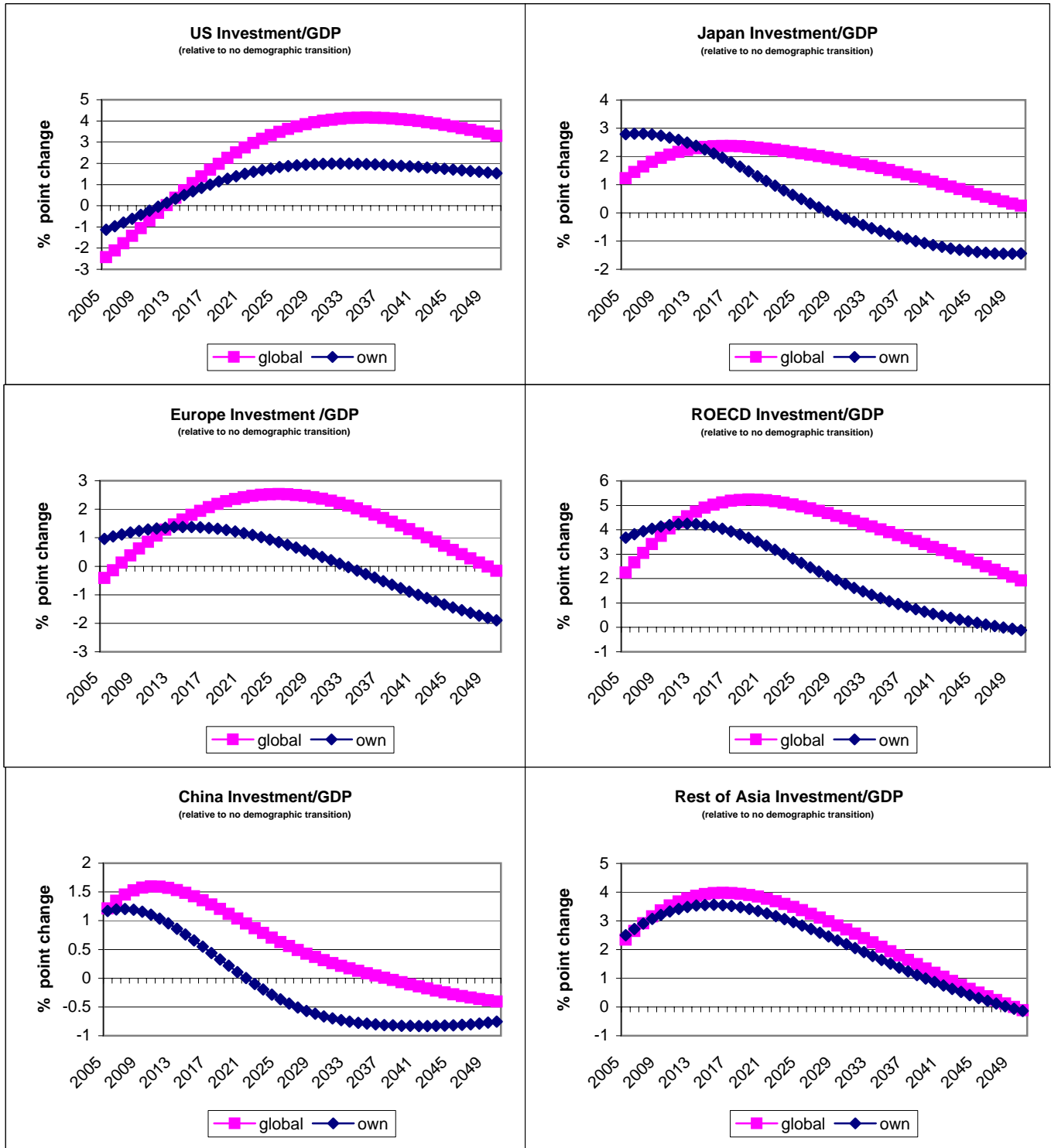
Source: MSG-Cubed (demography) version 58J

Figure 10(cont) : Contribution to National Savings of Own versus Global Demographic Change



Source: MSG-Cubed (demography) version 58J

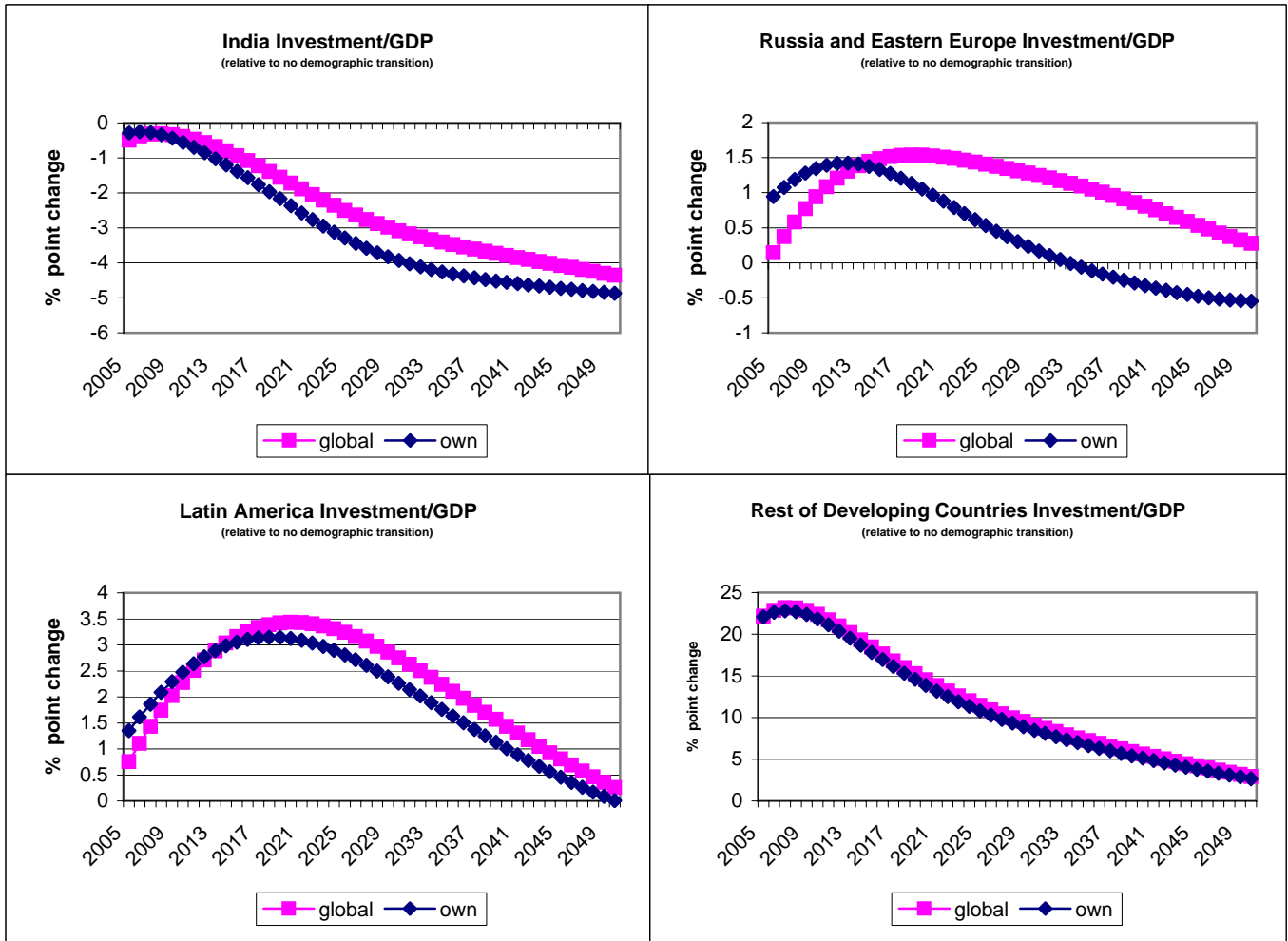
Figure 11 : Contribution to Private Investment of Own versus Global Demographic Change



Source: MSG-Cubed (demography) version 58J

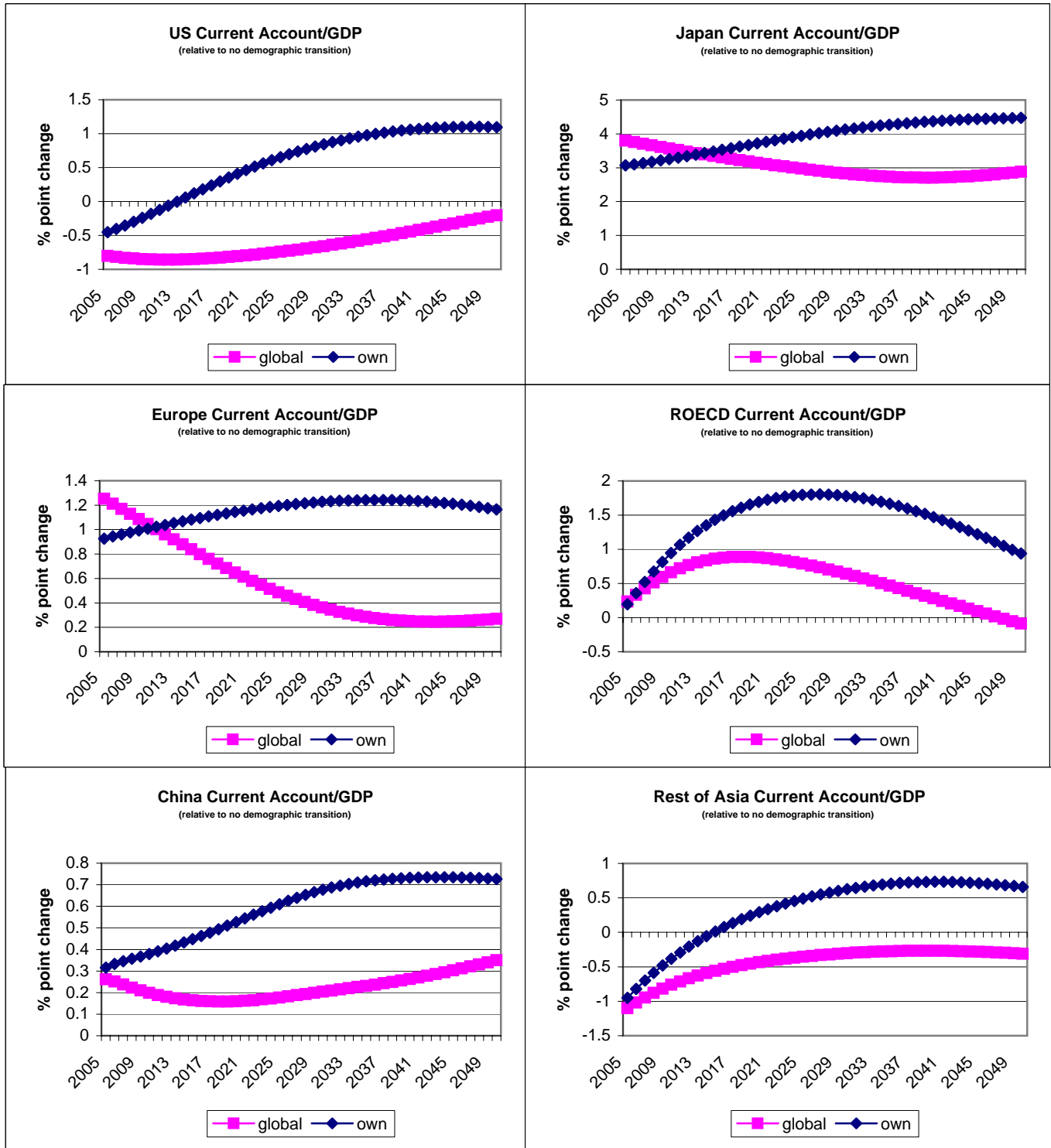


Figure 11 (cont) : Contribution to Private Investment of Own versus Global Demographic Change



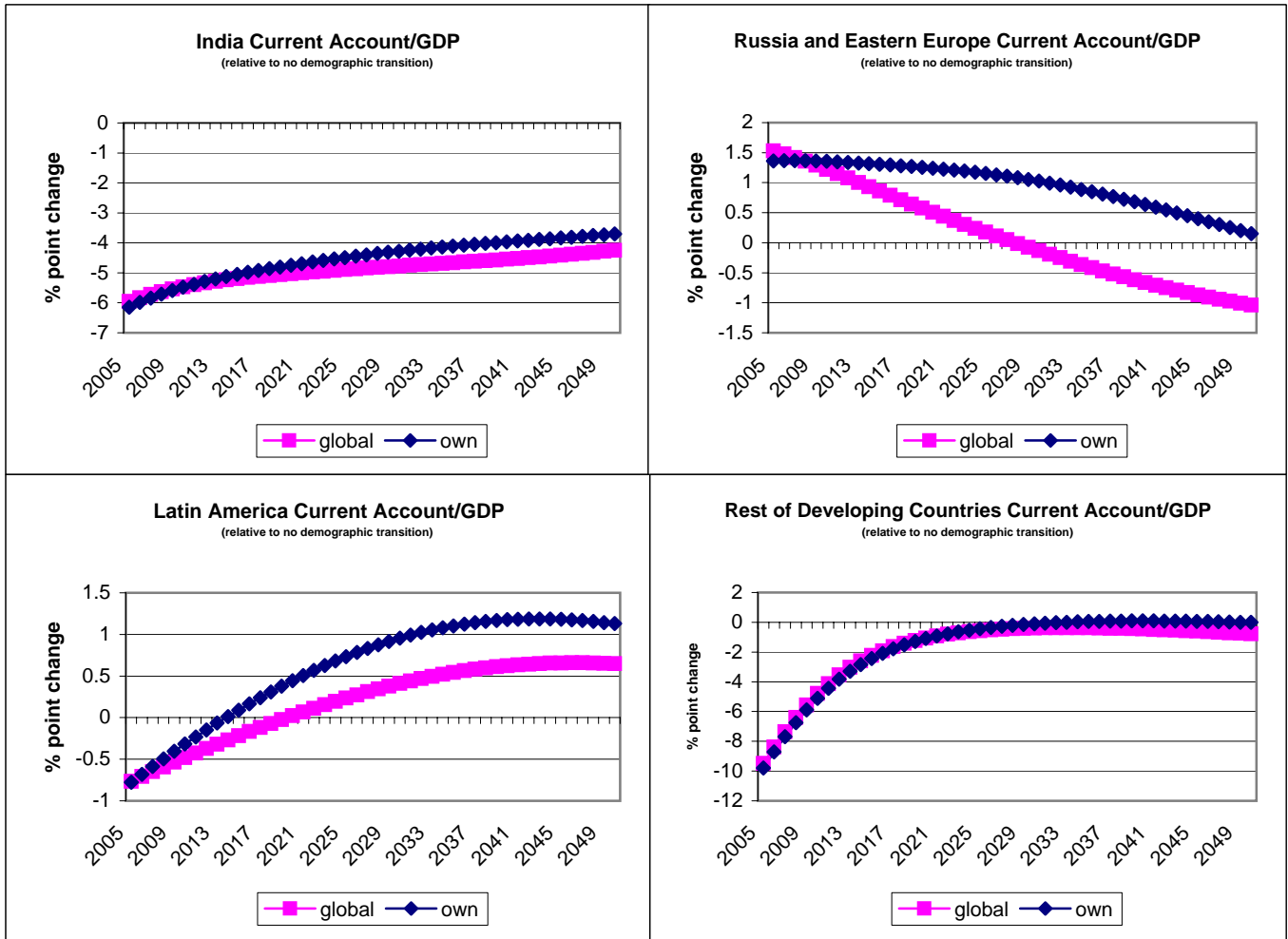
Source: MSG-Cubed (demography) version 58J

Figure 12 : Contribution to Current Accounts of Own versus Global Demographic Change



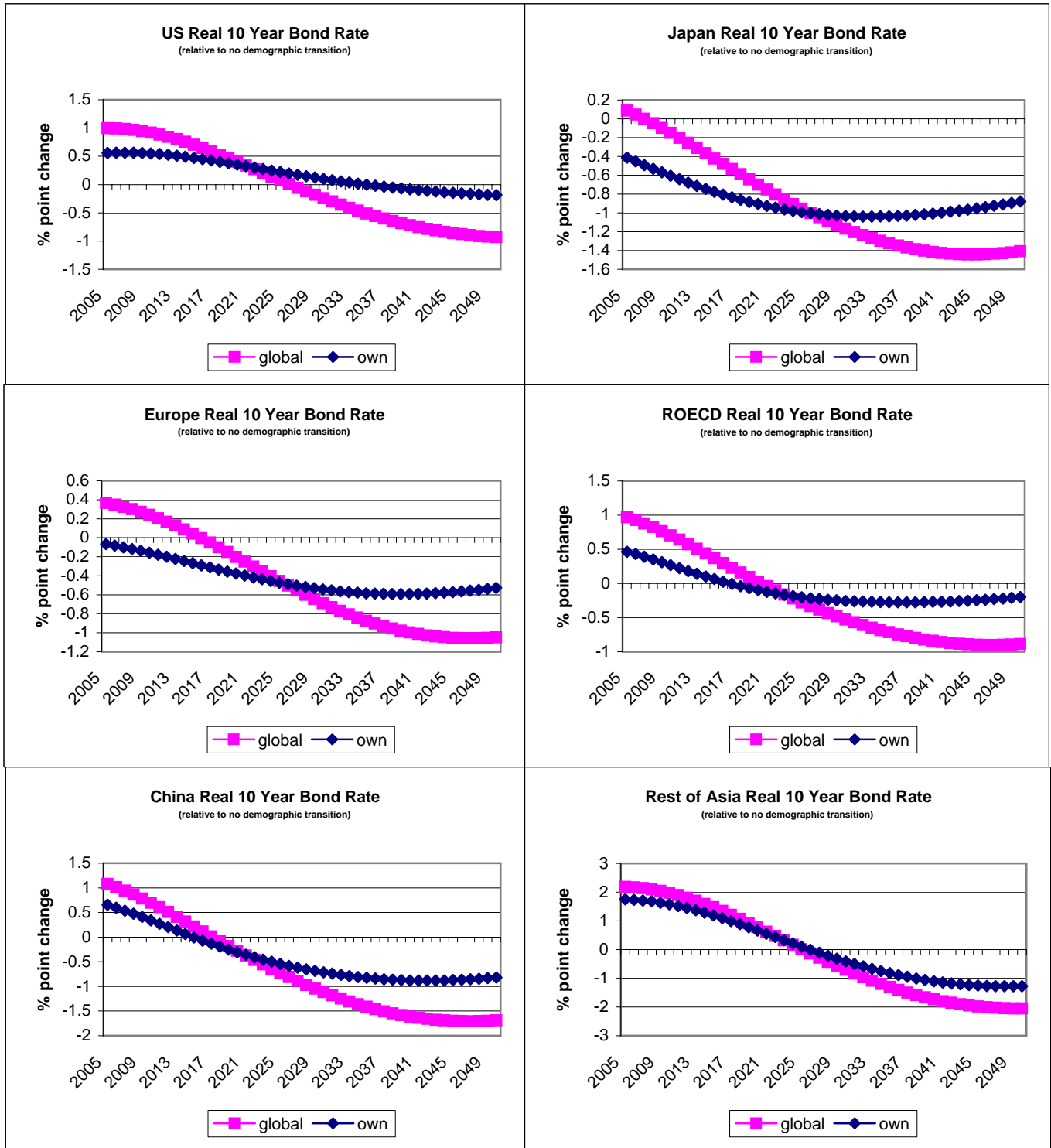
Source: MSG-Cubed (demography) version 58J

Figure 12 (cont): Contribution to Current Accounts of Own versus Global Demographic Change



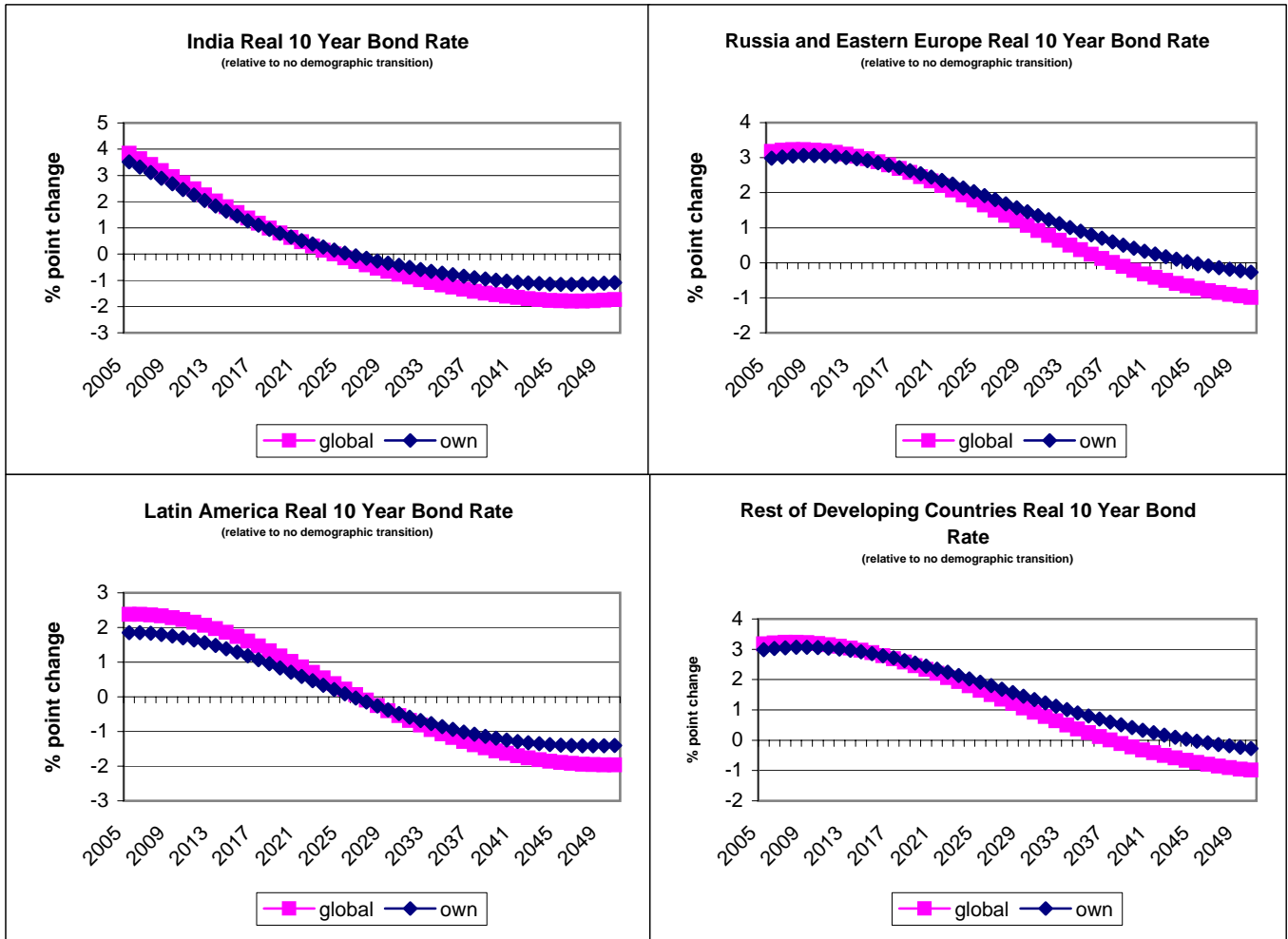
Source: MSG-Cubed (demography) version 58J

Figure 13 : Contribution to Long Term Real Interest Rates of Own versus Global Demographic Change



Source: MSG-Cubed (demography) version 58J

Figure 13 (cont): Contribution to Long Term Real Interest Rates of Own versus Global Demographic Change



Source: MSG-Cubed (demography) version 58J