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On the Non-Optimality of Information: An Analysis of the Welfare Effects of Anticipated Shocks in the New Keynesian Model*

Hans-Werner Wohltmann and Roland Winkler

Abstract: This paper compares the welfare effects of anticipated and unanticipated cost-push shocks within the canonical New Keynesian model with optimal monetary policy. We find that, for empirically plausible degrees of nominal rigidity, the anticipation of a future cost-push shock leads to a higher welfare loss than an unanticipated shock. A welfare gain from the anticipation of a future cost shock may only occur if prices are sufficiently flexible. We show analytically that this result holds although unanticipated shocks lead to higher negative impact effects on welfare than anticipated shocks.

Keywords: Anticipated Shocks, Optimal Monetary Policy, Sticky Prices, Welfare Analysis

JEL classification: E31, E32, E52

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1 Introduction

Does the *anticipation* of future shocks have a stabilizing and hence welfare-enhancing effect on the economy when compared to *unanticipated* shocks? In this paper, we attempt to answer this question by comparing the welfare effects of unanticipated and anticipated cost-push shocks within the canonical New Keynesian model with a monetary authority which minimizes a standard loss function that weights the volatility of inflation and the output gap. In particular, we analytically solve for the dynamics and the welfare in the case of optimal monetary policy under timeless perspective commitment and discretion. We distinguish the common case of unanticipated cost-push shocks and the case of future cost-push disturbances that are known in advance.

Since the real business cycle revolution, initiated by Kydland and Prescott (1982), unanticipated random disturbances are considered as the main driving force in explaining business cycles. New Keynesians add nominal rigidities to the real business cycle framework to study the role of monetary policy in aggregate fluctuations but maintain the assumption of unpredictable random shocks (see, for example, the textbooks of Walsh (2003), Woodford (2003), or Galí (2008)). An exception is the stream of literature that analyzes anticipated disinflations going back to Ball (1994) who shows that a simple variant of the New Keynesian model predicts a boom in response to an anticipated disinflation. However, the literature on the optimal design of monetary policy usually considers only unanticipated shocks (see, for example, Clarida, Galí, and Gertler (1999), Svensson (1999), King, Khan, and Wolman (2003), or Woodford (2003)).

Recently, a number of macroeconometric studies emphasized the role of anticipated shocks as sources of macroeconomic fluctuations. Beaudry and Portier (2006) find that more than one-half of business cycle fluctuations are caused by news about future technological opportunities. Davis (2007) and Fujiwara, Hirose, and Shintani (2008) analyze the importance of anticipated shocks in medium-scale New Keynesian DSGE models and find that these disturbances are important components of aggregate fluctuations. Schmitt-Grohé and Uribe (2008) conduct a Bayesian estimation of a real-business cycle model and find that anticipated shocks are the most important source of aggregate fluctuations.

In particular, they show that anticipated shocks explain two thirds of the volatility in consumption, output, investment, and employment.

Theoretical studies about the role of anticipations regarding business cycle fluctuations include Beaudry and Portier (2004, 2007), Beaudry, Collard, and Portier (2006), Jaimovich and Rebelo (2006, 2008), Den Haan and Kaltenbrunner (2007), or Christiano, Ilut, Motto, and Rostagno (2008).

However, none of these studies considered the welfare effects of the anticipation of future shocks. In this paper, we derive a solution of welfare as a function of the time span between the anticipation and the realization of the shock which allows us to discover the dependency of welfare on the length of the anticipation period. Furthermore, we contribute to the literature by systematically investigating the role of nominal rigidities for the welfare impacts of anticipations.

The main results of this paper are as follows: For empirically plausible degrees of nominal rigidity, the anticipation of a future cost-push shock leads to a higher welfare loss than an analogous unanticipated shock. A welfare gain from the anticipation of a future cost shock may only occur if prices are sufficiently flexible. This result is consistent with the findings of Schmitt-Grohé and Uribe (2008) who show that the anticipation of future shocks has a stabilizing effect on an economy without nominal rigidities. We point out that precisely this degree of nominal rigidity plays an important role for the evaluation of the welfare effects of anticipations.

Our results are driven by two opposing effects. On the one hand, we obtain the well-known result that the anticipation of a future shock dampens its impact effect. On the other hand, we show that the anticipation of future cost-push shocks increases the persistence of output and inflation and therefore increases the welfare loss. This persistence effect, in turn, is amplified by the degree of price stickiness.

Nevertheless, at a first glance, our finding seems to be puzzling since it suggests that the information about the occurrence of future shocks is in general welfare-reducing. But this prompts the question, why rational agents do not ignore their knowledge about future disturbances. In the remainder of this paper, we will seek to shed more light on this question.

Our paper is organized as follows. Section 2 presents the canonical New Keynesian model and its solution under the policy regimes of timeless perspective commitment and of discretion. In Section 3, we report and discuss our main findings. Furthermore, we provide analytical proofs and, for the purpose of illustration, numerical simulations. Section 4 summarizes and provides a conclusion. Attached to this paper is a mathematical appendix.

2 The Framework

The canonical New Keynesian model serves as an analytical framework. It consists of an optimizing IS-type relationship of the form

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) \qquad (\sigma > 0) \tag{1}$$

and a price adjustment equation of the Calvo-Rotemberg type, often referred to as New Keynesian Phillips Curve (NKPC)

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + k_t \qquad (0 < \beta < 1,\ \kappa > 0) \tag{2}$$

x_t denotes the output gap, π_t is inflation, and i_t is the nominal interest rate. E_t is the expectations operator conditional on information up to date t. The parameter β is the discount factor and $1/\sigma$ denotes the intertemporal elasticity of substitution. It is well-known that under the assumptions of Calvo (1983) price setting, a constant returns to scale production function with labor as its single input, and perfect labor markets, the slope parameter κ is given by $\kappa = (\eta + \sigma)(1 - \omega)(1 - \beta\omega)/\omega$, where η is the inverse of the labor supply elasticity.[1] Obviously, κ is negatively correlated with the degree of price rigidity ω. According to the Calvo price adjustment mechanism, a fraction $1 - \omega$ of firms can adjust their price in period t. Simultaneously, ω is the probability that a single price – which is reoptimized in period t – also holds in the next period $t+1$. The Calvo parameter

[1] See, e.g., Walsh (2003) for a derivation of the NKPC under Calvo pricing.

ω is therefore a measure of the degree of price rigidity in the goods markets.

In the NKPC, k_t represents a temporary cost-push shock that is assumed to be autoregressive of order one with AR parameter $\varphi \in [0, 1)$ and a one-unit cost shock ε_t

$$k_t = \varphi k_{t-1} + \varepsilon_t \qquad (t \geq T > 0) \tag{3}$$

Since we consider anticipated cost-push shocks, the one-unit cost shock ε_t is not white noise, but known to the public before the shock actually occurs.[2] Assume that at time $t = 0$ the public anticipates the cost-push shock to take place at some future time $T > 0$. Then,

$$\varepsilon_t = \begin{cases} 1 & \text{for } t = T > 0 \\ 0 & \text{for } t \neq T \end{cases} \tag{4}$$

The adjustment dynamics induced by the anticipated shock are comprised of two phases, the time span between the anticipation and the realization of the shock ($0 \leq t < T$) and the time span after the occurrence of the shock ($T \leq t \leq \infty$). The lead time T up to the realization of the shock is equal to the length of the anticipation phase $0 \leq t < T$. An implication of our definition of anticipated shocks is that rational expectations are equivalent to perfect foresight so that we can omit the expectations operator.

The policy maker's objective at the time of anticipation, $t = 0$, is to minimize the intertemporal loss function

$$V = E_0 \sum_{t=0}^{\infty} \beta^t (\alpha_1 \pi_t^2 + \alpha_2 x_t^2) \qquad (\alpha_1 > \alpha_2 > 0,\ 0 < \beta \leq 1) \tag{5}$$

which reflects the objective of flexible inflation targeting (see Svensson (1999)). Rotemberg and Woodford (1999) and Woodford (2003) show that, under the assumptions made in our study, a quadratic loss function in inflation and the output gap is the correct approximation to the representative agent's utility function.[3]

[2] Schmitt-Grohé and Uribe (2007) study the impacts of anticipated cost shocks on the pass-through to prices.
[3] Note that α_1 and α_2 are then functions of the underlying structural parameters of the model and given by

The first-order conditions for the policy problem under timeless perspective precommitment monetary policy as well as under discretion are well known and need not to be derived here (see, for example, Walsh (2003)). Under the optimal timeless perspective precommitment policy, inflation satisfies the targeting rule

$$\pi_t = -\frac{\alpha_2}{\alpha_1 \kappa}(x_t - x_{t-1}) \tag{6}$$

while the output gap is described by the second-order difference equation

$$\left(1 + \beta + \frac{\alpha_1 \kappa^2}{\alpha_2}\right) x_t - x_{t-1} - \beta E_t x_{t+1} = -\frac{\alpha_1 \kappa}{\alpha_2} k_t \tag{7}$$

where the expectations operator can be omitted in the case of anticipated shocks.

To solve the difference equation for x_t, write equation (7) as

$$\begin{pmatrix} x_{t+1} \\ w_{t+1} \end{pmatrix} = C \begin{pmatrix} x_t \\ w_t \end{pmatrix} + \begin{pmatrix} \frac{\alpha_1 \kappa}{\alpha_2 \beta} \\ 0 \end{pmatrix} k_t \tag{8}$$

where $w_t = x_{t-1}$ and

$$C = \begin{pmatrix} \frac{1}{\beta}\left(1 + \beta + \frac{\alpha_1 \kappa^2}{\alpha_2}\right) & -\frac{1}{\beta} \\ 1 & 0 \end{pmatrix} \tag{9}$$

The auxiliary variable w_t is backward-looking (with the initial value $w_0 = 0$), while the output gap x_t is forward-looking. The system matrix C has two real eigenvalues r_1 and r_2 with $r_1 > 1 > r_2 > 0$ so that the Blanchard and Kahn (1980) saddle path stability condition is satisfied.

The solution for the output gap over the anticipation phase is given by

$$x_t = -\frac{1}{r_1 - \varphi} \frac{1}{r_1 - r_2} \frac{\alpha_1 \kappa}{\alpha_2 \beta} r_1^{-T} (r_1^{t+1} - r_2^{t+1}) \quad \text{for} \quad t < T \tag{10}$$

$\alpha_1 = \varepsilon\omega/((1-\omega)(1-\beta\omega))$ and $\alpha_2 = \sigma + \eta$, where ε denotes the elasticity of substitution between different varieties of goods.

with the initial values

$$x_0 = -\frac{1}{r_1 - \varphi}\frac{\alpha_1\kappa}{\alpha_2\beta}r_1^{-T}\ , \qquad x_{-1} = 0 \tag{11}$$

while the solution for $t \geq T$ is defined as

$$x_t = \frac{\alpha_1\kappa}{\alpha_2\beta}\frac{1}{(r_1 - \varphi)(r_2 - \varphi)}\left[\varphi^{t+1-T} - \frac{(r_1 - \varphi)r_2^{-T} - (r_2 - \varphi)r_1^{-T}}{r_1 - r_2}r_2^{t+1}\right] \text{ for } t \geq T \tag{12}$$

In the limiting case of unanticipated shocks ($T = 0$), the term in brackets in equation (12) simplifies to $\varphi^{t+1} - r_2^{t+1}$. Note that the solution formula (10) also holds in the shock period $t = T$.

Using (6), the solution time path of the inflation rate follows

$$\pi_t = \frac{1}{\beta}\frac{1}{r_1 - \varphi}\frac{1}{r_1 - r_2}r_1^{-T}\left[(r_1 - 1)r_1^t - (r_2 - 1)r_2^t\right] \quad \text{for } t \leq T \tag{13}$$

with the initial value

$$\pi_0 = \frac{1}{\beta}\frac{1}{r_1 - \varphi}r_1^{-T} \tag{14}$$

and

$$\pi_t = \frac{1}{\beta}\frac{1}{r_1 - \varphi}\frac{1}{r_2 - \varphi}\left[(1 - \varphi)\varphi^{t-T} - \frac{(r_1 - \varphi)r_2^{-T} - (r_2 - \varphi)r_1^{-T}}{r_1 - r_2}(1 - r_2)r_2^t\right] \text{ for } t \geq T \tag{15}$$

In the limiting case $T = 0$, the term in the brackets simplifies to $(1 - \varphi)\varphi^t - (1 - r_2)r_2^t$.

To determine the welfare loss under the optimal precommitment policy, we write the loss function as $V = V_1 + V_2$, where

$$V_1 = E_0\sum_{t=0}^{T-1}\beta^t\left(\alpha_1\pi_t^2 + \alpha_2 x_t^2\right) \tag{16}$$

is the loss in the anticipation period and

$$V_2 = E_0 \sum_{t=T}^{\infty} \beta^t \left(\alpha_1 \pi_t^2 + \alpha_2 x_t^2\right) \tag{17}$$

is the loss caused by the realization of the shock.

By substituting the solution for x_t and π_t, the loss V_1 can be rewritten as

$$V_1 = \alpha_1 \lambda^2 r_1^{-2T} \left(r_1^T - r_2^T\right) \left(\frac{r_1 - 1}{r_2^T} + \frac{1 - r_2}{r_1^T}\right) \tag{18}$$

where

$$\lambda = \frac{1}{\beta} \frac{1}{r_1 - \varphi} \frac{1}{r_1 - r_2} \quad . \tag{19}$$

Accordingly, the loss V_2 can be rewritten as

$$V_2 = \frac{\alpha_1 \beta^T}{\beta^2 (r_1 - \varphi)^2} \left\{ \frac{\left(r_2^T - r_1^T\right)^2 (1 - r_2)}{(r_1 - r_2)^2 r_1^{2T}} + \frac{r_1}{r_1 r_2 - \varphi^2} \right\} \tag{20}$$

The total loss V is then simply given by $V = V_1 + V_2$.

Under the policy regime discretion (D), the central bank is unable to make a commitment to future policies. Then, private expectations are given for the central bank and the reduced form of the first-order conditions reads as

$$\pi_t = -\frac{\alpha_2}{\alpha_1 \kappa} x_t \tag{21}$$

$$E_t x_{t+1} = \frac{1}{\beta} \left[1 + \frac{\alpha_1 \kappa^2}{\alpha_2}\right] x_t + \frac{\alpha_1 \kappa}{\alpha_2 \beta} k_t \tag{22}$$

with $E_t x_{t+1} = x_{t+1}$ in the case of anticipated shocks. The difference equation in x_t has an unstable eigenvalue r_D, defined as

$$r_D = \frac{1}{\beta} \left[1 + \frac{\alpha_1 \kappa^2}{\alpha_2}\right] = \frac{1}{\alpha_2 \beta} \left[\alpha_2 + \alpha_1 \kappa^2\right] > 1 \tag{23}$$

and the forward solution

$$x_t = -\sum_{s=0}^{\infty} r_D^{-s} \frac{1}{r_D} \frac{\alpha_1 \kappa}{\alpha_2 \beta} k_{t+s} \tag{24}$$

Since

$$k_{t+s} = \begin{cases} \varphi^{t+s-T} & \text{for } t+s \geq T \\ 0 & \text{for } t+s < T \end{cases} \tag{25}$$

we obtain for $t \geq T$

$$x_t = -\frac{\alpha_1 \kappa}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} \varphi^{t-T} \tag{26}$$

and for $t < T$

$$x_t = -\frac{\alpha_1 \kappa}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} r_D^{t-T} \tag{27}$$

Due to $r_D^{t-T} = 1$ for $t = T$, the solution formula (27) also holds in the shock period $t = T$. For $t = 0$ we obtain

$$x_0 = -\frac{\alpha_1 \kappa}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} r_D^{-T} \tag{28}$$

so that the the size of the initial jump of x_t decreases with increasing T.

For the inflation rate π_t we obtain the solution time path

$$\pi_t = \begin{cases} \dfrac{\alpha_2}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} r_D^{t-T} & \text{if} \quad 0 \leq t \leq T \\ \dfrac{\alpha_2}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} \varphi^{t-T} & \text{if} \quad t \geq T \end{cases} \tag{29}$$

Note that the limiting case $\varphi = 0$ implies $\pi_t = x_t = 0$ for $t > T$.

It is well-known that the loss under discretion, V_D, is greater than the total loss under the optimal precommitment policy. By inserting the solution time paths for π_t and x_t in the loss function, we obtain

$$
\begin{aligned}
V_D = V_1^D + V_2^D &= \sum_{t=0}^{T-1} \beta^t \left[\frac{\alpha_2^2}{\alpha_1 \kappa^2} + \alpha_2 \right] x_t^2 + \sum_{t=T}^{\infty} \beta^t \left[\frac{\alpha_2^2}{\alpha_1 \kappa^2} + \alpha_2 \right] x_t^2 \\
&= \frac{\alpha_1 \alpha_2 [\alpha_2 + \alpha_1 \kappa^2]}{[\alpha_2(1-\beta\varphi) + \alpha_1 \kappa^2]^2} \left(\frac{r_D^{-2T} - \beta^T}{1 - \beta r_D^2} + \frac{\beta^T}{1 - \beta \varphi^2} \right) \\
&= \frac{\alpha_1 \alpha_2 [\alpha_2 + \alpha_1 \kappa^2]}{[\alpha_2(1-\beta\varphi) + \alpha_1 \kappa^2]^2} \frac{1}{1 - \beta r_D^2} \left(r_D^{-2T} - \frac{\beta (r_D^2 - \varphi^2)}{1 - \beta \varphi^2} \beta^T \right)
\end{aligned}
\tag{30}
$$

where

$$
\frac{1}{1 - \beta r_D^2} = \frac{\alpha_2^2 \beta}{\alpha_2^2 \beta - (\alpha_2 + \alpha_1 \kappa^2)^2} < 0 \tag{31}
$$

3 Main Results

In this section, we compare the welfare loss induced by anticipated shocks ($T > 0$) to the corresponding loss if the same deterministic shock is not anticipated in advance ($T = 0$). In particular, we investigate the properties of the welfare loss V considered as function of the lead time T.

Since the size of the initial jumps of the forward-looking variables x_t and π_t are negatively correlated with the lead time T, we can conjecture that the loss function $V = V(T)$ is a decreasing function in T. In the following, we will demonstrate that this conjecture is, in general, false. It will be shown that it is only true, if the degree of price flexibility is very high.

Our main results can be summarized in the form of four propositions.

Proposition 1. *Without discounting (i.e. $\beta = 1$), the welfare loss induced by an anticipated cost-push shock is greater than the corresponding loss in the case of an unanticipated shock. This result is independent of the length of the lead time T and the degree of price* *rigidity* ω*:*

$$\textit{If } \beta = 1, \quad \textit{then } V(0) < V(T) \quad \textit{for all} \quad T > 0 \qquad (32)$$
$$\textit{and all} \quad \omega > 0\,.$$

A similar result holds with discounting ($\beta < 1$), provided that the degree of price rigidity ω is sufficiently high and the time span between anticipation and realization of the shock is not too large.

Proposition 2. *If* β *is less than unity and the degree of price flexibility,* $1 - \omega$*, is low, then there exists a positive upper bound* T_c^* *for the lead time* T *which depends positively on* ω*, such that*

$$V(0) < V(T) \quad \textit{for all} \quad 0 < T < T_c^*\,. \qquad (33)$$

Proposition 3. *If the degree of price flexibility is very high (i.e.* ω *is very small) then* $T_c^* = 0$*, so that*

$$V(T) < V(0) \quad \textit{for all} \quad T > 0\,. \qquad (34)$$

Only in this case (which seems empirically not very realistic), the welfare loss under anticipated cost-push shocks is always smaller than under unanticipated shocks.

Proposition 4. *Propositions 1, 2, and 3 hold under the optimal monetary policy regimes timeless perspective commitment and discretion. They also hold under Taylor-type (optimal) simple rules.*

Sketch of the Proof of Propositions 1, 2, and 3. Consider the partial loss function V_1, given by equation (18), as a function of T (the time span between the anticipation and realization of the cost-push shock).

The function $V_1 = V_1(T)$ has the following properties:

$$V_1(0) = 0, \qquad \lim_{T\to\infty} V_1(T) = \begin{cases} 0 & \text{for } \beta < 1 \\ \overline{V}_1 > 0 & \text{for } \beta = 1 \end{cases} \tag{35}$$

where

$$\overline{V}_1 = \frac{\alpha_1(r_1 - 1)}{(r_1 - \varphi)^2(r_1 - r_2)^2} \tag{36}$$

The derivative of V_1 with respect to T

$$\frac{dV_1}{dT} = \alpha_1\lambda^2 \Bigg\{ 2\ln r_1 \cdot r_1^{-2T}[r_1 + r_2 - 2] - (r_1 - 1)\ln(r_1 r_2)\cdot(r_1 r_2)^{-T} - (1 - r_2)\ln\left(\frac{r_2}{r_1^3}\right)\cdot\left(\frac{r_2}{r_1^3}\right)^T \Bigg\} \tag{37}$$

is positive at time $T = 0$:

$$\left.\frac{dV_1}{dT}\right|_{T=0} = \alpha_1 \frac{1}{\beta^2}\frac{1}{(r_1 - \varphi)^2}\frac{1}{r_1 - r_2}[\ln r_1 - \ln r_2] > 0 \tag{38}$$

Therefore, $V_1(T)$ starts to rise with increasing T (although the size of the initial jumps of x_t and π_t is decreasing in T). For $\beta < 1$, the limit value $\lim_{T\to\infty} V_1(T)$ is equal to zero. Therefore, $V_1(T)$ must decrease when T is sufficiently large.

The loss function $V_2 = V_2(T)$, given by equation (20), has the following properties:

$$V_2(0) = \frac{\alpha_1}{\beta^2(r_1 - \varphi)^2}\frac{r_1}{r_1 r_2 - \varphi^2} > 0 \tag{39}$$

$$\lim_{T\to\infty} V_2(T) = \begin{cases} 0 & \text{if } \beta < 1 \\ \overline{V}_2 > V_2(0)\Big|_{\beta = 1} = \dfrac{\alpha_1 r_1}{(r_1 - \varphi)^2(1 - \varphi^2)} & \text{if } \beta = 1 \end{cases} \tag{40}$$

where

$$\overline{V}_2 = \frac{\alpha_1}{(r_1 - \varphi)^2}\left\{\frac{1 - r_2}{(r_1 - r_2)^2} + \frac{r_1}{1 - \varphi^2}\right\} \tag{41}$$

The first derivative of V_2 with respect to T

$$\frac{dV_2}{dT} = \frac{\alpha_1}{\beta^2(r_1-\varphi)^2}\beta^T\left\{\frac{r_1}{r_1r_2-\varphi^2}\ln\beta\right. \qquad (42)$$
$$\left.+\frac{1-r_2}{(r_1-r_2)^2}\left[(\ln r_2 - 3\ln r_1)\left(\frac{r_2}{r_1}\right)^{2T} + 4\ln r_1\left(\frac{r_2}{r_1}\right)^T + \ln\beta\right]\right\}$$

implies for $\beta < 1$ and $T = 0$

$$\left.\frac{dV_2}{dT}\right|_{T=0} = \frac{\alpha_1}{\beta^2(r_1-\varphi)^2}\frac{r_1}{r_1r_2-\varphi^2}\ln\beta < 0 \qquad (43)$$

since $\beta = 1/(r_1r_2)$. For $\beta < 1$, the derivative dV_2/dT is also negative if T is sufficiently large. In the limiting case $\beta = 1$, the loss function $V_2(T)$ is an increasing function in T with a limit value $\overline{V}_2 > V_2(0)$.

We can now investigate how the total loss $V = V_1 + V_2$ depends on the lead time T. In the limiting case $\beta = 1$, the total loss $V(T)$ is an overall increasing function in T with $V(0) = V_2(0) > 0$ and

$$\lim_{T\to\infty} V(T) = \frac{\alpha_1}{(r_1-\varphi)^2}\left\{\frac{1}{r_1-r_2} + \frac{r_1}{1-\varphi^2}\right\} > V_2(0)\Big|_{\beta=1} > 0 \qquad (44)$$

If $\beta = 1$, we can write $V(T)$ as $V_1(T) + V_2(T)$, where

$$V_1(T) = \frac{\alpha_1}{(r_1-\varphi)^2(r_1-r_2)^2}\left[(r_1-1) + (2-r_1-r_2)r_1^{-2T} - (1-r_2)\left(\frac{r_2}{r_1^3}\right)^T\right] \qquad (45)$$

$$V_2(T) = \frac{\alpha_1}{(r_1-\varphi)^2}\left\{\frac{1-r_2}{(r_1-r_2)^2}\left[1-\left(\frac{r_2}{r_1}\right)^T\right]^2 + \frac{r_1}{1-\varphi^2}\right\} \qquad (46)$$

Then

$$\frac{dV_1}{dT} = \frac{\alpha_1}{(r_1-\varphi)^2(r_1-r_2)^2}\left\{2[r_1+r_2-2]\ln r_1\right. \qquad (47)$$
$$\left.+[3\ln r_1 - \ln r_2](1-r_2)\left(\frac{r_2}{r_1}\right)^T\right\}r_1^{-2T} > 0 \quad \text{for all} \quad T \geq 0$$

(due to $r_1 + r_2 = \text{tr}\ C > 2$ and $\ln r_2 < 0$) and

$$\frac{dV_2}{dT} = \frac{\alpha_1}{(r_1-\varphi)^2}\frac{1-r_2}{(r_1-r_2)^2}\left\{-2\left(1-\left(\frac{r_2}{r_1}\right)^T\right)\ln\left(\frac{r_2}{r_1}\right)\right\}\left(\frac{r_2}{r_1}\right)^T \underset{(=)}{>} 0 \text{ if } T \underset{(=)}{>} 0 \tag{48}$$

(because $0 < r_2 < 1 < r_1$). Therefore, $dV/dT > 0$ for all $T \geq 0$ so that V is a monotonically increasing function in T. This result holds independently of the degree of price rigidity ω.

For $\beta < 1$, $V(0) = V_2(0) > 0$ (with $V_2(0)$ defined in (39)) and $\lim_{T\to\infty} V(T) = 0$. For small values of ω, i.e. a high degree of price flexibility, the total loss V is a decreasing function in T implying $V(T) < V(0)$ for all $T > 0$. With high price flexibility, the welfare loss under anticipated shocks is smaller than under unanticipated shocks.

For the derivative dV/dT at time $T = 0$ we obtain

$$\left.\frac{dV}{dT}\right|_{T=0} = \frac{\alpha_1}{\beta^2(r_1-\varphi)^2}\left\{\left[\frac{1}{r_1-r_2} - \frac{r_1}{r_1r_2-\varphi^2}\right]\ln r_1 - \left[\frac{1}{r_1-r_2} + \frac{r_1}{r_1r_2-\varphi^2}\right]\ln r_2\right\} \tag{49}$$

By using $\ln r_2 = -(\ln r_1 + \ln\beta)$, it turns out that

$$\left.\frac{dV}{dT}\right|_{T=0} > 0 \;\Leftrightarrow\; 2\left(\frac{1}{\beta} - \varphi^2\right)\ln r_1 + \left(r_1^2 - \varphi^2\right)\ln\beta > 0 \tag{50}$$

A rising ω induces a fall in the unstable eigenvalue r_1 because $d\kappa/d\omega < 0$. Since the fall in r_1^2 is stronger than the decrease in $\ln r_1$, and $1/\beta - \varphi^2 > 0$, inequality (50) is fulfilled if the degree of price rigidity ω is sufficiently large. In this case, $V(T)$ starts to rise and due to $\lim_{T\to\infty} V(T) = 0$ its development must be hump-shaped implying the existence of an upper bound $T_c^* > 0$ such that $V(T) > V(0) > 0$ for all $T < T_c^*$.

The value of the upper bound T_c^* is the positive solution to the equation $V(T) = V(0)$, where $V(0) = V_2(0)$ is given by (39). This leads to the equation

$$1 - \left(\frac{r_2}{r_1}\right)^T = \left[(r_1r_2)^T - 1\right]\frac{r_1(r_1-r_2)}{r_1r_2-\varphi^2} \tag{51}$$

Equation (51) can be written as

$$\beta^T r_1^{2T}\left[\beta r_1^2\left(1-\frac{1}{\beta^T}\right)+\frac{1}{\beta^T}-\beta\varphi^2\right]=1-\beta\varphi^2 \quad \Leftrightarrow \tag{52}$$

$$r_1^{2T}\left[\beta^{T+1}\left(r_1^2-\varphi^2\right)+\left(1-\beta r_1^2\right)\right]=1-\beta\varphi^2 \tag{53}$$

so that T_c^* is also the positive solution of (52) and (53). The value of T_c^* is dependent on ω and β. A rising ω (a higher degree of price rigidity) lowers the unstable eigenvalue r_1 so that the left-hand side of equation (52) is decreased, while the right-hand side remains unchanged. Since $\beta^T r_1^{2T} = (r_1/r_2)^T$ is increasing in T, equation (52) implies that the solution value T_c^* must increase if ω rises. Conversely, a higher degree of price flexibility induces a fall in T_c^*. For sufficiently small values of ω, the only solution to (53) is $T_c^* = 0$ (so that $V(T) < V(0)$ for all $T > 0$). If a positive solution T_c^* to (53) exists, then it is also an increasing function in the discount factor β with $T_c^* = \infty$ if $\beta = 1$. □

Sketch of the Proof of Proposition 4. Consider V_D (given by (30)) as function in T. Then

$$V_D(0)=\frac{\alpha_1\alpha_2[\alpha_2+\alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi)+\alpha_1\kappa^2]^2}\frac{1}{1-\beta\varphi^2}>0 \tag{54}$$

and

$$\lim_{T\to\infty}V_D(T)=\begin{cases}0 & \text{if } \beta<1\\ \frac{\alpha_1\alpha_2[\alpha_2+\alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi)+\alpha_1\kappa^2]^2}\left(\frac{1}{r_D^2-1}+\frac{1}{1-\varphi^2}\right)>V_D(0)>0 & \text{if } \beta=1\end{cases} \tag{55}$$

The partial loss function

$$V_2^D(T)=\frac{\alpha_1\alpha_2[\alpha_2+\alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi)+\alpha_1\kappa^2]^2}\frac{\beta^T}{1-\beta\varphi^2} \tag{56}$$

has the properties

$$V_2^D(0)=V_D(0) \tag{57}$$

$$\lim_{T\to\infty} V_2^D(T) = 0 \quad \text{if} \quad \beta < 1 \tag{58}$$

$$\frac{dV_2^D}{dT} = (\ln\beta)V_2^D(T) < 0 \quad \text{if} \quad \beta < 1 \quad \text{for all} \quad 0 \leq T < \infty \tag{59}$$

For $\beta = 1$, the function $V_2^D(T)$ is constant (independent of T).

The partial loss function $V_1^D(T)$, given by

$$V_1^D(T) = \frac{\alpha_1\alpha_2[\alpha_2 + \alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi) + \alpha_1\kappa^2]^2} \frac{r_D^{-2T} - \beta^T}{1 - \beta r_D^2} \tag{60}$$

has similar properties as the corresponding function $V_1(T)$ under the policy regime timeless perspective commitment:

$$V_1^D(0) = 0 \tag{61}$$

$$\lim_{T\to\infty} V_1^D(T) = \begin{cases} 0 & \text{if } \beta < 1 \\ \dfrac{\alpha_1\alpha_2[\alpha_2 + \alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi) + \alpha_1\kappa^2]^2} \dfrac{1}{r_D^2 - 1} > 0 & \text{if } \beta = 1 \end{cases} \tag{62}$$

The first derivative with respect to T

$$\frac{dV_1^D(T)}{dT} = \frac{\alpha_1\alpha_2[\alpha_2 + \alpha_1\kappa^2]}{[\alpha_2(1-\beta\varphi) + \alpha_1\kappa^2]^2} \frac{1}{1 - \beta r_D^2} \left[-2(\ln r_D)r_D^{-2T} - (\ln\beta)\beta^T\right] \tag{63}$$

is positive at time $T = 0$, since $1 - \beta r_D^2 < 0$ and $-2\ln r_D - \ln\beta < 0$ due to $r_D > 1 \geq \beta$.

In the case $\beta < 1$, the evolution of $V_1^D(T)$ is hump-shaped with a maximum value at time T_d^* which is the solution to the equation

$$2(\ln r_D)r_D^{-2T} + (\ln\beta)\beta^T = 0 \tag{64}$$

Equation (64) is equivalent to

$$-\frac{2\ln r_D}{\ln\beta} = (\beta r_D^2)^T \tag{65}$$

with the solution

$$T_d^* = \frac{\ln\left[-\frac{2\ln r_D}{\ln\beta}\right]}{\ln(\beta r_D^2)} > 0 \tag{66}$$

The total loss function $V_D(T) = V_1^D(T) + V_2^D(T)$ has a similar shape as the corresponding function $V(T)$ under timeless perspective commitment. In the limiting case $\beta = 1$, it is overall increasing. For $\beta < 1$, it is hump-shaped, if the degree of price flexibility is not too large, while it is monotonically decreasing in T if the value of ω is small. For small values of ω, the derivative of V_D at time $T = 0$ is negative, while it is positive if ω is sufficiently large. For the sake of brevity, the proof for the case of simple (optimal) Taylor rules is presented in the mathematical appendix. □

The Propositions 1 to 3 follow from two opposing effects on the welfare loss which head in opposite directions with increasing lead time T. On the one hand, the size of the initial jumps of the forward-looking variables x_t and π_t, taking place at the time of anticipation, is inversely related to the time span between anticipation and realization of the cost-push shock. The longer the lead time T, the smaller the responses of output and inflation are on impact. Hence, the contribution of this *anticipation effect* to the welfare loss V *decreases* with increasing T. On the other hand, the *persistence effect* of the cost-push shock on the target variables x_t and π_t is *increasing* in T. Thereby, we depart from the usual approach of measuring persistence by the speed of dying out. Instead – and in the spirit of the measure of quantitative inertia proposed by Merkl and Snower (2009) – we measure persistence as the total variation of a variable over time, i.e. its intertemporal deviation from the respective initial steady state. For example, the persistence of the price level p_t is given by $\sum_{t=0}^{\infty} |p_t - \overline{p}_0|$ where the initial steady state can be normalized to zero.

In the appendix, we derive the persistence of p_t, x_t, and π_t under the optimal monetary policy regimes commitment and discretion. We then show that persistence is smaller in the case of unanticipated shocks than in the case of anticipated shocks.

For the purpose of illustration, we numerically simulated our solutions by using a standard calibration. The time unit is one quarter. The discount rate is equal to $\beta = 0.99$, implying an annual steady state real interest rate of approximately 4 percent. The inverse of the intertemporal elasticity of substitution is set to $\sigma = 2$. We set $\eta = 1$ implying a quadratic disutility of labor. The Calvo parameter ω is set to either 0.25 implying an average duration of price contracts of four months or to 0.75 implying an average duration of price contracts of one year. The weights in the loss function are set to $\alpha_1 = 1$ and $\alpha_2 = 0.5$, reflecting the objective of flexible inflation targeting.[4] Finally, we assume the cost-push shock to be persistent and choose $\varphi = 0.5$.

Figure 1 depicts the impulse response functions of inflation, output gap, and price level in the case of low ($\omega = 0.25$) and high ($\omega = 0.75$) price rigidity under the optimal monetary policy with timeless perspective commitment on the left and right column, respectively. Solid lines with stars denote responses to a cost-push shock that unexpectedly emerged in period $t = 0$, while solid lines with circles denote responses to a cost-push shock whose realization in period $T = 2$ is anticipated in period $t = 0$.

– Figure 1 about here –

We first consider the empirically plausible case of *high price rigidity*. In the case of an unanticipated cost-shock, both the price level and the inflation rate rise, whereas output falls in response to the realization of an increase in the costs of production.[5] Subsequently, all variables converge in a hump-shaped fashion to their respective steady state values.

Anticipated cost shocks have two effects, namely the anticipation effect which reflects the change in x_t, π_t, and p_t in response to the anticipation of a future change in costs and the realization effect which occurs when the anticipated change in costs actually takes place. Under the optimal monetary policy with commitment, output starts to decline and prices begin to rise in response to the anticipation of a future increase in the production costs. Both variables respond in a hump-shaped fashion, peaking at the date of realization. The increase in prices causes inflation to jump at the time of anticipation, peaking at the date of realization and then returning in a hump-shaped fashion to its initial steady state

[4] Note that our results hold for all $\alpha_1 > 0$ and $\alpha_2 > 0$. We choose these often used, but not fully microfounded values, only for the purpose of illustration.

[5] We could think about this cost-push shock as an exogenous rise in wage mark-ups (see, for example, Galí (2008)).

level.

In the case of *low price rigidity*, an unanticipated cost shock causes an instantaneous rise in prices and drop in output. Subsequently, both variables converge monotonically to their initial steady state levels. After the initial jump, inflation falls sharply and converges from below to its pre-shock level. The announcement of a future rise in costs has negligible anticipation effects when prices are highly flexible. The reason is that the price setting problem of firms becomes more of an atemporal (static) nature when the Calvo parameter ω is decreased. In this case firms know that they will be able – with a high degree of probability – to raise their price when the anticipated shock eventually materializes in period T. Thus both, output and prices change only slightly in response to an announcement or anticipation of future cost-push shocks.

Regardless of the degree of price rigidity, Figure 1 illustrates that the initial jumps of inflation, output gap and price level are greater in the case of unanticipated ($T = 0$) than in the case of anticipated shocks ($T = 2$). On the other hand, anticipated shocks amplify the persistence of p_t, x_t, and π_t when compared to unanticipated shocks.[6]

Figure 2 illustrates the welfare loss $V = V(T)$ in the case $\beta = 1$. Without time discounting in the intertemporal loss function, the persistence effect always dominates the anticipation effect so that Proposition 1 holds. In Figure 2, the total loss $V = V(T)$ is an overall increasing function in T if $\beta = 1$.

– Figure 2 about here –

If future deviations of the state variables from their initial steady state levels are discounted, the contribution of the initial jumps of output and inflation for the determination of the total loss becomes more important. The same holds for an increasing degree of price flexibility $1-\omega$, since the persistence of prices, output and inflation is a decreasing function of $1-\omega$. If the degree of price flexibility is high, the value of the total loss is almost completely determined by the size of the initial jumps of x_t and π_t which in turn is inversely proportional to the lead time T. With a sufficiently high degree of price flexibility, the

[6]This result also holds in the special case $\varphi = 0$, i.e. if the shock exhibits no serial correlation. It is well-known that even in this case the optimal precommitment policy introduces inertia in the impulse response functions.

total loss under unanticipated cost-push shocks is greater than the loss under anticipated shocks so that Proposition 3 holds. This result is also illustrated in Figure 3, where $V(T)$ is a monotonically decreasing function in the lead time T if the degree of price rigidity ω is very small.

– Figure 3 about here –

From an empirical point of view, the parameter ω is not that small so that the development of the impulse response functions shows inertia or strong serial correlation. Then, if the time span between the anticipation and the implementation of the cost-push shock is not too large, the persistence effect dominates and the value of the total loss $V(T)$ is greater than $V(0)$. This is illustrated in Figure 3, where the loss function $V(T)$ evolves in a hump-shaped manner and is monotonically increasing for small values of T.

Propositions 1 to 3 are independent of the chosen optimal monetary policy regime. They hold under timeless perspective commitment as well as under discretion (see Figure 4 and 5 for a numerical visualization). They also hold under simple monetary policy rules (such as Taylor-type rules or a money growth peg).

– Figure 4 and 5 about here –

In order to check whether the welfare-reducing effects of anticipations hold for empirically plausible degrees of nominal rigidity, we compute the critical anticipation values T_c^* (commitment) and T_d^* (discretion). Table 1 depicts the values of T_c^* and T_d^* for a persistent cost-push shock ($\varphi = 0.5$) and a one-off cost-push shock ($\varphi = 0$).

– Table 1 about here –

Table 1 shows that the anticipation of cost-push shocks dampens the welfare loss induced by such shocks but only for empirically unrealistic degrees of nominal rigidity. For the widely used values of $\omega = 0.75$ or $\omega = 0.66$, the anticipation period or lead time T must be extremely large to obtain a welfare gain from anticipation. Under commitment and a value for $\omega = 0.75$, the loss under an anticipated shock is smaller than the loss under an unanticipated shock of same magnitude when the shock is anticipated to take place in $T_c^* = 54$ (for $\varphi = 0.5$) or $T_c^* = 66$ (for $\varphi = 0$) quarters. Even larger values are obtained under optimal discretionary policy. A Calvo parameter of 0.5 represents the lower bound in the range of values that are reported in the literature. In this case and under the monetary policy regime commitment, the anticipation of future cost shocks has a welfare-enhancing effect if the lead time is larger or equal to two quarters for persistent and three quarters for one-off shocks, respectively. Under discretionary monetary policy, these critical values are three and four quarters.

Our simulations illustrate that for a wide range of empirically realistic degrees of nominal rigidities (i.e., $\omega \geq 0.5$) in conjunction with a plausible length for the anticipation period, the welfare loss of anticipated cost shocks exceeds the welfare loss of unanticipated cost shocks.

4 Conclusion

In this paper we investigated the welfare effects resulting from the anticipation of future shocks. In particular, we analyzed the welfare loss for different lengths of the time span between the anticipation and the realization of cost-push shocks. This includes the frequently used case of unanticipated cost-push shocks. Our analysis is based on the canonical New Keynesian model with a monetary authority that seeks to minimize a quadratic loss function in inflation and the output gap.

We emphasize the role of nominal rigidities for the welfare effects of anticipations. We have shown that for empirically plausible degrees of nominal rigidity, anticipated cost shocks entail higher welfare losses than unanticipated cost shocks. The anticipation of a future cost-push shock dampens the volatility of output and inflation only under the assumption that prices are highly flexible. These results hold independently of the monetary policy regime (timeless perspective commitment, discretion, or (optimal) simple rules).

Our results imply that the knowledge about the realization of future cost shocks is in general welfare-reducing. The question remains why rational agents do not simply ignore this information. However, this would be inconsistent with the profit-maximizing behavior of individual firms and the utility-maximizing behavior of individual households on which our model is based. In fact, the firm's optimality condition necessitates an increase in the prices in response to the anticipation of a future rise in costs. By simply ignoring this information, the firm would make a loss.

Hence, our results reveal a contradiction between the optimal behavior of individuals and the optimum from a social point of view.

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Mathematical Appendix

Optimal timeless perspective precommitment policy

The solution time path of the price level p_t under the optimal timeless perspective precommitment policy can be derived from the solution of π_t due to

$$p_t = \sum_{k=0}^{t} \pi_k \tag{A1}$$

We then obtain for $t \leq T$:

$$\begin{aligned} p_t =& \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_1-r_2} r_1^{-T} \sum_{k=0}^{t} \left[(r_1-1)r_1^k - (r_2-1)r_2^k\right] \\ =& \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_1-r_2} r_1^{-T} \left[r_1^{t+1} - r_2^{t+1}\right] \end{aligned} \tag{A2}$$

and for $t \geq T$

$$\begin{aligned} p_t &= \sum_{k=0}^{T-1} \pi_k + \sum_{k=T}^{t} \pi_k \\ &= \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_1-r_2} r_1^{-T} \left[r_1^T - r_2^T\right] + \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_2-\varphi} \cdot \\ &\quad \cdot \sum_{k=T}^{t} \left\{(1-\varphi)\varphi^{k-T} - \frac{(r_1-\varphi)r_2^{-T} - (r_2-\varphi)r_1^{-T}}{r_1-r_2}(1-r_2)r_2^k\right\} \\ &= \frac{1}{\beta}\frac{1}{r_2-\varphi}\frac{1}{r_1-r_2} r_2^{t+1-T} - \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_1-r_2} r_1^{-T} r_2^{t+1} - \frac{1}{\beta}\frac{1}{r_1-\varphi}\frac{1}{r_2-\varphi}\varphi^{t+1-T} \end{aligned} \tag{A3}$$

Obviously,

$$\lim_{t\to\infty} p_t = 0 \quad \text{for all} \quad T \geq 0 \tag{A4}$$

and

$$p_0 = \frac{1}{\beta}\frac{1}{r_1-\varphi} r_1^{-T} = \pi_0 > 0 \tag{A5}$$

so that the size of the initial jump in p is inversely proportional to the lead time T.

Similar results hold for the state variables x_t and π_t. Since

$$\sum_{k=0}^{t}(x_k - x_{k-1}) = x_t \tag{A6}$$

equation (6) implies

$$p_t = \sum_{k=0}^{t}\pi_k = -\frac{\alpha_2}{\alpha_1\kappa}\sum_{k=0}^{t}(x_k - x_{k-1}) = -\frac{\alpha_2}{\alpha_1\kappa}x_t \tag{A7}$$

so that $p_t > 0$ if and only if $x_t < 0$. The optimal policy under timeless perspective implies $p_t > 0$ for all $0 \leq t < \infty$ so that $x_t < 0$ for all $t < \infty$.

In the following, we show that the *persistence* or *total variation* of p_t is positive correlated with T, i.e.

$$\sum_{t=0}^{\infty} p_t\Big|_{T=0} < \sum_{t=0}^{\infty} p_t\Big|_{T>0} \quad \text{for all} \quad T > 0 \tag{A8}$$

where the infinite sum $\sum_{t=0}^{\infty} p_t\Big|_{T>0}$ is an increasing function in T.

The persistence measure used here is based on the deviation of p_t from its initial steady state level $\overline{p}_0$, where the deviation $|p_t - \overline{p}_0|$ is calculated both for $t < T$ and $t \geq T$. Thereafter the differences $|p_t - \overline{p}_0|$ are summed up. Since $\overline{p}_0 = 0$ and $p_t > 0$ for all t we must determine the infinite sum $\sum_{t=0}^{\infty} p_t$.

Inequality (A8) holds although the initial jump of p_t is a negative function in T. To prove the inequality note that

$$\sum_{t=0}^{\infty} p_t\Big|_{T=0} = \frac{1}{\beta(r_1-\varphi)(1-r_2)(1-\varphi)} \tag{A9}$$

$$\sum_{t=0}^{T} p_t\Big|_{T>0} = \frac{1}{\beta(r_1-\varphi)(r_1-r_2)}r_1^{-T}\left[r_1\frac{1-r_1^{T+1}}{1-r_1} - r_2\frac{1-r_2^{T+1}}{1-r_2}\right] \tag{A10}$$

and

$$\sum_{t=T+1}^{\infty} p_t\Big|_{T>0} = \frac{1}{\beta(r_2-\varphi)(r_1-r_2)} r_2^{1-T} \frac{r_2^{T+1}}{1-r_2} - \frac{1}{\beta(r_1-\varphi)(r_1-r_2)} r_1^{-T} r_2 \frac{r_2^{T+1}}{1-r_2} - \frac{1}{\beta(r_1-\varphi)(r_2-\varphi)} \varphi^{1-T} \frac{\varphi^{T+1}}{1-\varphi} \tag{A11}$$

so that

$$\sum_{t=0}^{\infty} p_t\Big|_{T>0} = \frac{1}{\beta(r_1-\varphi)(r_1-r_2)} \left[\frac{r_1}{1-r_1}\left(r_1^{-T} - r_1\right) - \frac{r_2 r_1^{-T}}{1-r_2} \right] + \frac{1}{\beta(r_2-\varphi)} \left[\frac{1}{r_1-r_2}\frac{r_2^2}{1-r_2} - \frac{1}{r_1-\varphi}\frac{\varphi^2}{1-\varphi} \right] \tag{A12}$$

After some tedious manipulations, we obtain

$$\sum_{t=0}^{\infty} p_t\Big|_{T>0} > \sum_{t=0}^{\infty} p_t\Big|_{T=0} \quad \Leftrightarrow \quad 1 - r_1^{-T} > 0 \tag{A13}$$

Since $r_1 > 1$, the last inequality is fulfilled. Note that the total variation of p_t, i.e. $\sum_{t=0}^{\infty} p_t\Big|_{T>0}$ is an increasing function in T. This follows from equation (A12), since the derivative of $\frac{r_1}{1-r_1} r_1^{-T} - \frac{r_2}{1-r_2} r_1^{-T}$ with respect to T is positive. An implication of inequality (A13) is

$$\sum_{t=0}^{\infty} |x_t|\Big|_{T=0} < \sum_{t=0}^{\infty} |x_t|\Big|_{T>0} \tag{A14}$$

since

$$|x_t| = \frac{\alpha_1 \kappa}{\alpha_2} p_t \tag{A15}$$

The persistence of output in the case of anticipated cost-push shocks is therefore stronger than in the case of unanticipated shocks.

A similar result can be shown for the inflation rate π_t if the limiting case $\varphi = 0$ is considered. We then obtain for $T = 0$

$$\pi_t = \begin{cases} 1 - (1 - r_2) = r_2 & \text{if} \quad t = 0 \\ -(1 - r_2) r_2^t < 0 & \text{if} \quad t > 0 \end{cases} \tag{A16}$$

implying

$$\begin{aligned} \sum_{t=0}^{\infty} \pi_t &= \pi_0 + \sum_{t=1}^{\infty} \pi_t = r_2 - (1 - r_2) \sum_{t=1}^{\infty} r_2^t \\ &= r_2 - (1 - r_2) \left[\frac{1}{1 - r_2} - 1 \right] = r_2 - r_2 = 0 \end{aligned} \tag{A17}$$

and

$$\sum_{t=0}^{\infty} |\pi_t| \Big|_{T=\varphi=0} = r_2 + (1 - r_2) \sum_{t=1}^{\infty} r_2^t = 2 r_2 \tag{A18}$$

In the case $T > 0$ and $\varphi = 0$, we obtain

- for $t \leq T$:

$$\pi_t = \frac{r_2}{r_1 - r_2} r_1^{-T} \left[(r_1 - 1) r_1^t - (r_2 - 1) r_2^t \right] > 0 \tag{A19}$$

- for $t > T$:

$$\pi_t = -\frac{r_1 r_2^{-T} - r_2 r_1^{-T}}{r_1 - r_2} (1 - r_2) r_2^t < 0 \tag{A20}$$

Then

$$\sum_{t=0}^{T} \pi_t = \frac{r_2}{r_1 - r_2} r_1^{-T} \sum_{t=0}^{T} \left[(r_1 - 1) r_1^t - (r_2 - 1) r_2^t \right] = \frac{r_2}{r_1 - r_2} r_1^{-T} \left[r_1^{T+1} - r_2^{T+1} \right] \tag{A21}$$

and

$$\sum_{t=T+1}^{\infty} \pi_t = -\frac{1 - r_2}{r_1 - r_2} \left[r_1 r_2^{-T} - r_2 r_1^{-T} \right] \frac{r_2^{T+1}}{1 - r_2} = -\frac{r_2}{r_1 - r_2} r_1^{-T} \left[r_1^{T+1} - r_2^{T+1} \right] \tag{A22}$$

so that

$$\sum_{t=0}^{\infty} \pi_t = 0 \tag{A23}$$

and

$$\sum_{t=0}^{\infty} |\pi_t| \Big|_{\substack{T>0 \\ \varphi=0}} = 2 \frac{r_2}{r_1 - r_2} r_1^{-T} \left[r_1^{T+1} - r_2^{T+1} \right] \tag{A24}$$

Now

$$\frac{r_2}{r_1 - r_2} r_1^{-T} \left[r_1^{T+1} - r_2^{T+1} \right] > r_2 \quad \Leftrightarrow \quad r_2 \left[r_1^T - r_2^T \right] > 0 \tag{A25}$$

Due to $r_1 > 1 > r_2 > 0$ the last inequality is met so that

$$\sum_{t=0}^{\infty} |\pi_t| \Big|_{T=\varphi=0} < \sum_{t=0}^{\infty} |\pi_t| \Big|_{\substack{T>0 \\ \varphi=0}} \tag{A26}$$

The case $\varphi > 0$ is more difficult to analyze since π_t can take both positive and negative values for $t > T > 0$. If $T = 0$, π_t changes sign immediately after the initial jump. Since

$$\pi_t = \frac{1}{\beta(r_1 - \varphi)(r_2 - \varphi)} \left[(1-\varphi)\varphi^t - (1 - r_2) r_2^t \right] \quad (\text{if } T = 0) \tag{A27}$$

we obtain

$$\pi_0 = \frac{1}{\beta(r_1 - \varphi)} > 0 \tag{A28}$$

and

$$\begin{aligned} \sum_{t=1}^{\infty} \pi_t \Big|_{T=0} &= \frac{1}{\beta(r_1 - \varphi)(r_2 - \varphi)} \left[(1-\varphi) \sum_{t=1}^{\infty} \varphi^t - (1 - r_2) \sum_{t=1}^{\infty} r_2^t \right] \\ &= \frac{1}{\beta(r_1 - \varphi)(r_2 - \varphi)} (\varphi - r_2) = -\frac{1}{\beta(r_1 - \varphi)} = -\pi_0 \end{aligned} \tag{A29}$$

so that

$$\sum_{t=0}^{\infty} |\pi_t| \Big|_{T=0} = 2 \frac{1}{\beta(r_1 - \varphi)} \tag{A30}$$

In the case $T > 0$, π_t is positive for $0 \leq t \leq T$ and due to (13) we obtain

$$\begin{aligned} \sum_{t=0}^{T} \pi_t &= \frac{1}{\beta(r_1 - \varphi)(r_1 - r_2)} r_1^{-T} \left[(r_1 - 1) \frac{1 - r_1^{T+1}}{1 - r_1} - (r_2 - 1) \frac{1 - r_2^{T+1}}{1 - r_2} \right] \\ &= \frac{r_1}{\beta(r_1 - \varphi)(r_1 - r_2)} \left[1 - \left(\frac{r_2}{r_1} \right)^{T+1} \right] > 0 \end{aligned} \tag{A31}$$

(since $r_1 > 1 > r_2 > 0$). If $t > T$, π_t is negative for sufficiently large values of t. For small values of $t > T$, π_t may be positive. Due to

$$\lim_{t \to \infty} p_t = 0 \quad \text{and} \quad p_t = \sum_{k=0}^{t} \pi_k \tag{A32}$$

we must have

$$\sum_{t=0}^{\infty} \pi_t = 0 \tag{A33}$$

so that

$$\sum_{t=T+1}^{\infty} \pi_t = - \sum_{t=0}^{T} \pi_t < 0 \tag{A34}$$

The last equation also follows from (15). With

$$\psi = - \frac{(r_1 - \varphi) r_2^{-T} - (r_2 - \varphi) r_1^{-T}}{r_1 - r_2} \tag{A35}$$

we obtain

$$\sum_{t=T+1}^{\infty} \pi_t = \frac{1}{\beta(r_1-\varphi)(r_2-\varphi)} \left[(1-\varphi)\varphi^{-T} \sum_{t=T+1}^{\infty} \varphi^t + \psi(1-r_2) \sum_{t=T+1}^{\infty} r_2^t \right] \tag{A36}$$
$$= \frac{1}{\beta(r_1-\varphi)(r_2-\varphi)} \left[(1-\varphi)\varphi^{-T} \frac{\varphi^{T+1}}{1-\varphi} + \psi(1-r_2) \frac{r_2^{T+1}}{1-r_2} \right]$$
$$= -\frac{r_1}{\beta(r_1-\varphi)(r_1-r_2)} \left[1 - \left(\frac{r_2}{r_1} \right)^{T+1} \right] = -\sum_{t=0}^{T} \pi_t < 0$$

Therefore,

$$\sum_{t=0}^{T} \pi_t \Big|_{T>0} - \sum_{t=T+1}^{\infty} \pi_t \Big|_{T>0} = 2 \sum_{t=0}^{T} \pi_t \Big|_{T>0} > \sum_{t=0}^{\infty} |\pi_t| \Big|_{T=0} = 2\pi_0 \Big|_{T=0} \quad \Leftrightarrow \tag{A37}$$
$$\sum_{t=0}^{T} \pi_t \Big|_{T>0} > \pi_0 \Big|_{T=0} \quad \Leftrightarrow$$
$$r_1^T > r_2^T$$

The last inequality is met due to $r_1 > 1 > r_2 > 0$. Since

$$-\sum_{t=T+1}^{\infty} \pi_t \Big|_{T>0} \leq \sum_{t=T+1}^{\infty} |\pi_t| \Big|_{T>0} \tag{A38}$$

the stronger persistence in the case of anticipated shocks follows:

$$\sum_{t=0}^{\infty} |\pi_t| \Big|_{T>0} = \sum_{t=0}^{T} \pi_t + \sum_{t=T+1}^{\infty} |\pi_t| \geq \sum_{t=0}^{T} \pi_t - \sum_{t=T+1}^{\infty} \pi_t > \sum_{t=0}^{\infty} |\pi_t| \Big|_{T=0} \tag{A39}$$

Note that for arbitrary $T > 0$

$$\pi_0 \Big|_{T=0} < \sum_{t=0}^{T} \pi_t \Big|_{T>0} \tag{A40}$$

but

$$\pi_t \Big|_{T>0} < \pi_0 \Big|_{T=0} \quad \text{for all} \quad 0 \leq t \leq T \tag{A41}$$

In particular,

$$\pi_T\Big|_{T>0} < \pi_0\Big|_{T=0} \tag{A42}$$

since

$$\frac{1}{\beta(r_1-\varphi)(r_1-r_2)}\left[(r_1-1)-(r_2-1)\left(\frac{r_2}{r_1}\right)^T\right] < \frac{1}{\beta(r_1-\varphi)} \quad \Leftrightarrow \tag{A43}$$
$$\left(\frac{r_2}{r_1}\right)^T < 1$$

Since the last equation holds, the value of the inflation rate at the time of implementation of the cost-push shock is smaller in the case of anticipated compared to unanticipated shocks.[7]

Optimal policy under discretion

For all $0 \leq \varphi < 1$, the adjustment processes of x_t and π_t in the case of anticipated cost-push shocks show a stronger persistence than in the case $T = 0$. With the abbreviation

$$\tilde{\phi} = \frac{\alpha_2}{\alpha_2 + \alpha_1\kappa^2 - \alpha_2\beta\varphi} > 0 \tag{A44}$$

we have

$$\sum_{t=0}^{\infty} |\pi_t|\Big|_{T=0} = \tilde{\phi}\sum_{t=0}^{\infty}\varphi^t = \frac{\tilde{\phi}}{1-\varphi} \tag{A45}$$

[7] This result holds under the optimal timeless perspective precommitment policy. Under the policy regime discretion we have (cf. (29))

$$\pi_0\Big|_{T=0} = \pi_T\Big|_{T>0} = \frac{\alpha_2}{\alpha_2 + \alpha_1\kappa^2 - \alpha_2\beta\varphi}$$

and

$$\sum_{t=0}^{\infty} |\pi_t| \Big|_{T>0} = \sum_{t=0}^{T-1} \pi_t \Big|_{T>0} + \sum_{t=T}^{\infty} \pi_t \Big|_{T>0} \tag{A46}$$
$$= \tilde{\phi} \frac{1}{1-\varphi} + \tilde{\phi} \frac{1 - r_D^{-T}}{r_D - 1} > \tilde{\phi} \frac{1}{1-\varphi} = \sum_{t=0}^{\infty} |\pi_t| \Big|_{T=0}$$

since $r_D > 1$ and $0 < r_D^{-T} < 1$ if $T > 0$. An analogous result holds for x_t.

The policy regime discretion implies

$$\sum_{t=0}^{\infty} \pi_t \Big|_{T=0} = \sum_{t=T}^{\infty} \pi_t \Big|_{T>0} \tag{A47}$$

and

$$\sum_{t=0}^{\infty} x_t \Big|_{T=0} = \sum_{t=T}^{\infty} x_t \Big|_{T>0} \tag{A48}$$

so that the stronger persistence of π_t and x_t in the case $T > 0$ is due to the anticipation effects $\sum_{t=0}^{T-1} \pi_t > 0$ and $\sum_{t=0}^{T-1} x_t < 0$.

The solution time path for the price level p_t results from

$$p_t = \sum_{k=0}^{t} \pi_k \tag{A49}$$

For $0 \leq t \leq T$ we obtain

$$p_t = \frac{\alpha_2}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} \sum_{k=0}^{t} r_D^{t-T} = \frac{\alpha_2}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} r_D^{-T} \frac{1 - r_D^{t+1}}{1 - r_D} \tag{A50}$$

and for $t \geq T$

$$p_t = \sum_{k=0}^{T-1} \pi_k + \sum_{k=T}^{\infty} \pi_k = \frac{\alpha_2}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} \left[r_D^{-T} \frac{1 - r_D^{T}}{1 - r_D} - \varphi^{-T} \frac{\varphi^{t+1} - \varphi^{T}}{1 - \varphi} \right] \tag{A51}$$

with

$$\lim_{t\to\infty} p_t = \frac{\alpha_2}{\alpha_2 + \alpha_1\kappa^2 - \alpha_2\beta\varphi}\left[\frac{1 - r_D^{-T}}{r_D - 1} + \frac{1}{1-\varphi}\right] > 0 \tag{A52}$$

Note that the limit value of p_t is a positive function in T. It is well-known that a temporary cost-push shock yields a permanent rise in the price level under the policy regime discretion. By contrast, under the optimal timeless perspective precommitment policy there is only a temporary rise in the price level.

Total loss under a simple rule

We can also determine the total loss under an ad hoc Taylor rule

$$i_t = \delta_\pi \pi_t + \delta_x x_t \tag{A53}$$

with exogenously given coefficients δ_π and δ_x. It is well-known that under the condition $\delta_\pi > 1$ and $\delta_x \geq 0$ the baseline New Keynesian model satisfies the Blanchard/Kahn (1980) saddlepath condition. The state equations

$$A\begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{pmatrix} = B\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} k_t \tag{A54}$$

with

$$A = \begin{pmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{pmatrix}, \qquad B = \begin{pmatrix} 1 + \frac{\delta_x}{\sigma} & \frac{\delta_\pi}{\sigma} \\ -\kappa & 1 \end{pmatrix} \tag{A55}$$

have two unstable eigenvalues belonging to the state matrix $A^{-1}B$. Solving the state equations forward we obtain with

$$v_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}, \quad P = B^{-1}A, \quad q = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{A56}$$

the solution time paths in the case of anticipated cost-push shocks:

- For $t \geq T$

$$v_t = -\left(\sum_{s=0}^{\infty} \varphi^s P^s\right) B^{-1} q \varphi^{t-T} = -[B - \varphi A]^{-1} q \varphi^{t-T} \tag{A57}$$

- For $t < T$

$$v_t = -\left(\sum_{s=T-t}^{\infty} \varphi^s P^s\right) B^{-1} q \varphi^{t-T} = -[I_{2\times 2} - \varphi P]^{-1} P^{T-t}\, B^{-1} q \tag{A58}$$

The solution formula for $t < T$ also holds in $t = T$ since

$$v_T = -[B - \varphi A]^{-1} q = -[I_{2\times 2} - \varphi P]^{-1} B^{-1} \tag{A59}$$

The total loss under the simple Taylor rule (V_{STR}) can be written as

$$V_{STR} = \sum_{t=0}^{\infty} \beta^t v_t' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} v_t = V_1^{STR} + V_2^{STR} \tag{A60}$$

where

$$V_1^{STR} = \sum_{t=0}^{T-1} \beta^t v_t' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} v_t \tag{A61}$$

and

$$V_2^{STR} = \sum_{t=T}^{\infty} \beta^t v_t' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} v_t \tag{A62}$$

Define

$$M = (B - \varphi A)^{-1} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \tag{A63}$$

Then

$$
\begin{aligned}
V_2^{STR} &= \sum_{t=T}^{\infty} \beta^t q' M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M q \varphi^{2(t-T)} \\
&= q' M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M q \varphi^{-2T} \left(\sum_{t=T}^{\infty} \beta^t \varphi^{2t} \right) \\
&= \frac{(\beta\varphi^2)^T}{1-\beta\varphi^2} \varphi^{-2T} q' M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M q \\
&= \frac{\beta^T}{1-\beta\varphi^2} \operatorname{tr} \left(M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M q q' \right)
\end{aligned} \tag{A64}
$$

where

$$
M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M = \begin{pmatrix} \alpha_2 m_{11}^2 + \alpha_1 m_{21}^2 & \alpha_2 m_{11} m_{12} + \alpha_1 m_{21} m_{22} \\ \alpha_2 m_{11} m_{12} + \alpha_1 m_{21} m_{22} & \alpha_2 m_{12}^2 + \alpha_1 m_{22}^2 \end{pmatrix} \tag{A65}
$$

Since

$$
q q' = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{A66}
$$

we obtain

$$
\operatorname{tr} \left(M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M q q' \right) = \alpha_2 m_{12}^2 + \alpha_1 m_{22}^2 \tag{A67}
$$

The definition of the matrices A and B implies

$$
B - \varphi A = \begin{pmatrix} 1 + \frac{\delta_x}{\sigma} - \varphi & \frac{\delta_\pi}{\sigma} - \frac{\varphi}{\sigma} \\ -\kappa & 1 - \varphi\beta \end{pmatrix} \tag{A68}
$$

$$
\Delta = |B - \varphi A| = \left(1 + \frac{\delta}{\sigma} - \varphi \right) (1 - \varphi\beta) + \kappa \left(\frac{\delta_\pi}{\sigma} - \frac{\varphi}{\sigma} \right) = \frac{1}{\sigma} b \tag{A69}
$$

where

$$b = (1-\varphi)(1-\varphi\beta)\sigma + \delta_x(1-\varphi\beta) + \kappa(\delta_\pi - \varphi) > 0 \quad \text{if } \delta_\pi > 1 \text{ and } \delta_x > 0 \tag{A70}$$

Then

$$M = (B - \varphi A)^{-1} = \frac{1}{b}\begin{pmatrix} \sigma(1-\varphi\beta) & -(\delta_\pi - \varphi) \\ \sigma\kappa & \sigma(1-\varphi) + \delta_x \end{pmatrix} \tag{A71}$$

so that

$$m_{12} = -\frac{1}{b}(\delta_\pi - \varphi), \qquad m_{22} = \frac{1}{b}[\sigma(1-\varphi) + \delta_x] \tag{A72}$$

and

$$V_2^{STR} = \frac{\beta^T}{1-\beta\varphi^2}\frac{1}{b^2}\left[\alpha_2(\delta_\pi - \varphi)^2 + \alpha_1(\sigma(1-\varphi) + \delta_x)^2\right] \tag{A73}$$

The loss function $V_2^{STR} = V_2^{STR}(T)$ hat the same properties as the corresponding function under discretion ($V_2^D(T)$).

To calculate the loss V_1^{STR}, set

$$Q = [I_{2\times 2} - \varphi P]^{-1} \qquad (\text{where } P = B^{-1}A) \tag{A74}$$

and

$$\tilde{q} = B^{-1}q \tag{A75}$$

Then

$$v_t = -QP^{T-t}\tilde{q} \qquad \text{for} \quad t \leq T \tag{A76}$$

and

$$V_1^{STR} = \tilde{q}' \left(\sum_{t=0}^{T-1} \beta^t (P^{T-t})' Q' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} Q P^{T-t} \right) \tilde{q} \tag{A77}$$

$$= \tilde{q}' \left(\sum_{k=1}^{T} \beta^{T-k} (P^{k})' Q' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} Q P^{k} \right) \tilde{q}$$

$$= \beta^T \tilde{q}' \tilde{W} \tilde{q} = \beta^T \mathrm{tr}(\tilde{W} \tilde{q} \tilde{q}')$$

where

$$\tilde{q}\tilde{q}' = B^{-1} q q' (B^{-1})' = B^{-1} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} (B^{-1})' \tag{A78}$$

$$= \frac{1}{(\sigma + \delta_x + \kappa\delta_\pi)^2} \begin{pmatrix} \delta_\pi^2 & -\delta_\pi(\sigma + \delta_x) \\ -\delta_\pi(\sigma + \delta_x) & (\sigma + \delta_x)^2 \end{pmatrix}$$

and

$$\tilde{W} = \sum_{k=1}^{T} \beta^{-k} (P^k)' Q' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} Q P^k \tag{A79}$$

$\tilde{W}$ satisfies the following matrix equation. Let

$$\tilde{D} = Q' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} Q \tag{A80}$$

The definition of $\tilde{W}$ then implies

$$\tilde{W} = \beta^{-1} P' \tilde{D} P + \sum_{k=2}^{T} \beta^{-k} (P^k)' \tilde{D} P^k \tag{A81}$$

$$= \beta^{-1} P' \tilde{D} P + \sum_{k=1}^{T-1} \beta^{-(k+1)} (P^{k+1})' \tilde{D} P^{k+1}$$

$$= \beta^{-1} P' \tilde{D} P - \beta^{-(T+1)} (P^{T+1})' \tilde{D} P^{T+1} + \beta^{-1} P' \tilde{W} P$$

or in compact representation

$$\tilde{W} = \tilde{H} + \beta^{-1} P' \tilde{W} P \tag{A82}$$

where

$$\tilde{H} = \beta^{-1} P' \tilde{D} P - \beta^{-(T+1)} (P^{T+1})' \tilde{D} P^{T+1} \tag{A83}$$

To solve for $\tilde{W}$, use the vectorization of a matrix and the Kronecker product of matrices. Since

$$\mathrm{vec}\,(\beta^{-1} P' \tilde{W} P) = [\beta^{-1} P' \otimes P'] \mathrm{vec}\,\tilde{W} \tag{A84}$$

we obtain

$$\mathrm{vec}\,\tilde{W} - [\beta^{-1} P' \otimes P'] \mathrm{vec}\,\tilde{W} = \mathrm{vec}\,\tilde{H} \tag{A85}$$

with the solution

$$\mathrm{vec}\,\tilde{W} = [I_{4\times 4} - \beta^{-1} P' \otimes P']^{-1} \mathrm{vec}\,\tilde{H} \tag{A86}$$

where

$$\begin{aligned} \mathrm{vec}\,\tilde{H} &= \mathrm{vec}\,(\beta^{-1} P' \tilde{D} P) - \mathrm{vec}\,(\beta^{-(T+1)} (P^{T+1})' \tilde{D} P^{T+1}) \\ &= \Big([\beta^{-1} P' \otimes P'] - [\beta^{-(T+1)} (P^{T+1})' \otimes (P^{T+1})'] \Big) \mathrm{vec}\,\tilde{D} \end{aligned} \tag{A87}$$

and

$$\begin{aligned} \mathrm{vec}\,\tilde{D} &= Q' \otimes Q' \mathrm{vec} \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} \\ &= ([I_{2\times 2} - \varphi P]^{-1})' \otimes ([I_{2\times 2} - \varphi P]^{-1})' \begin{pmatrix} \alpha_2 & 0 & 0 & \alpha_1 \end{pmatrix}' \end{aligned} \tag{A88}$$

Note that $\operatorname{vec} \tilde{D}$ equals $\operatorname{vec} \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix}$ in the special case $\varphi = 0$. The development of V_1^{STR} as function in T is analogous to the loss function $V_1^D(T)$. Therefore, the total loss function $V^{STR}(T) = V_1^{STR}(T) + V_2^{STR}(T)$ has the same properties as the total loss under discretion.

Figures and Tables

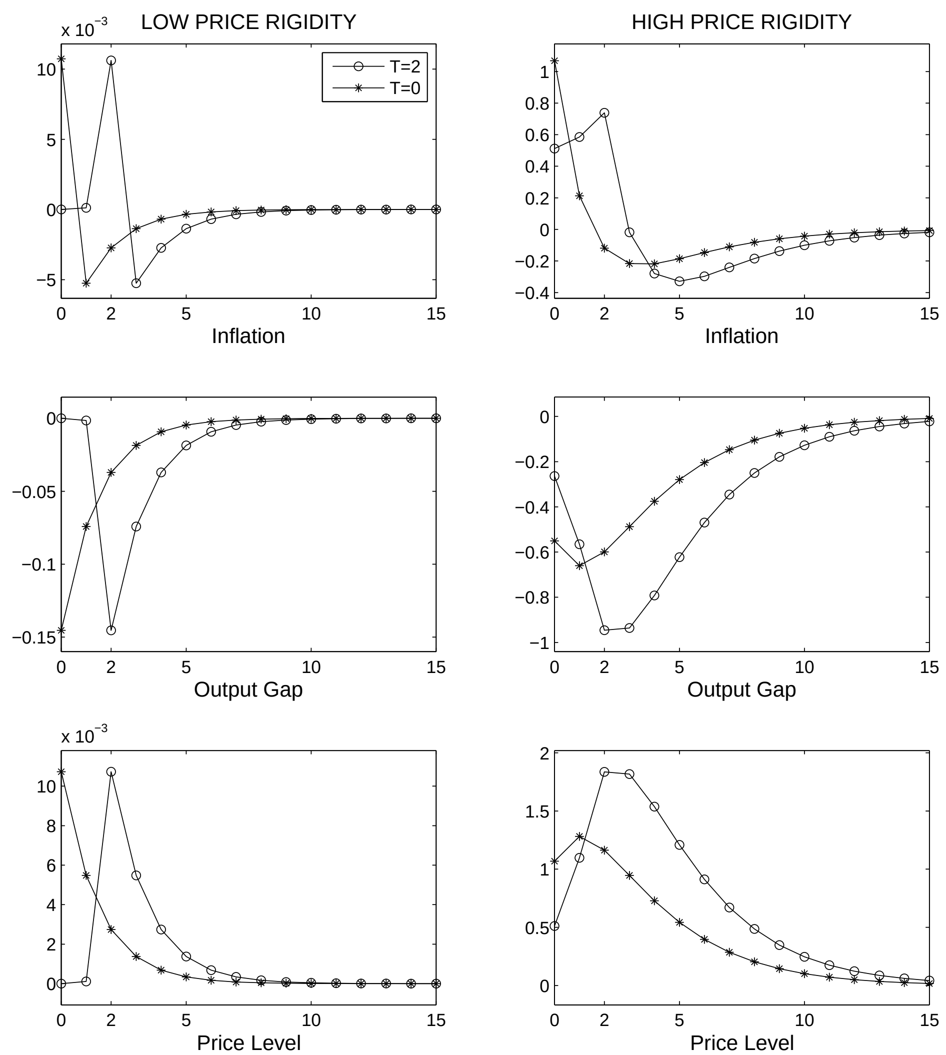

Figure 1: Impulse response functions under optimal policy with timeless perspective commitment.

Notes: Solid lines with stars denote responses to an unanticipated cost-push shock, solid lines with circles denote responses to an anticipated cost-push shock. In the case of low price rigidity, the Calvo parameter ω is set to 0.25; in the case of high price rigidity, ω is set to 0.75.

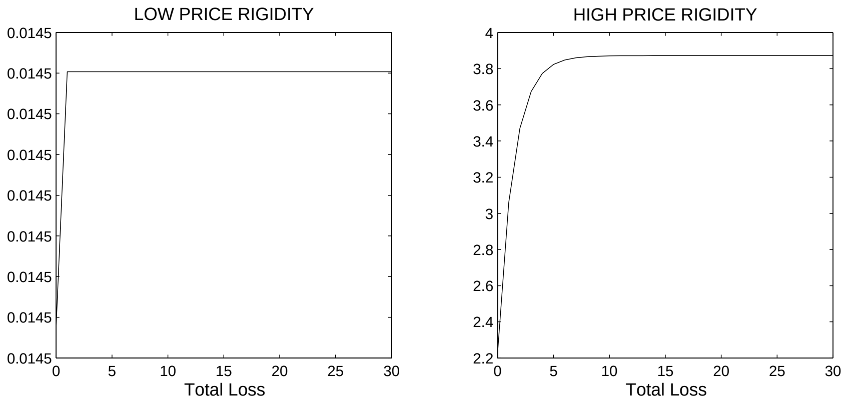

Figure 2: Welfare loss for different lengths of the anticipation period under optimal timeless perspective commitment policy in the case $\beta = 1$.

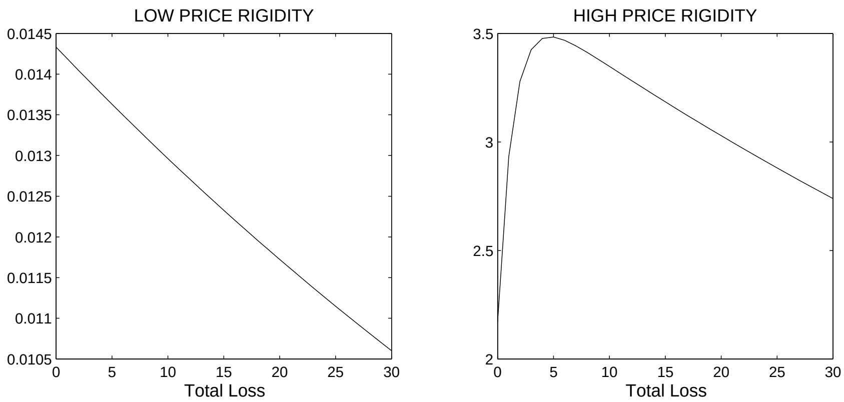

Figure 3: Welfare loss for different lengths of the anticipation period under optimal timeless perspective commitment policy in the case $\beta = 0.99$.

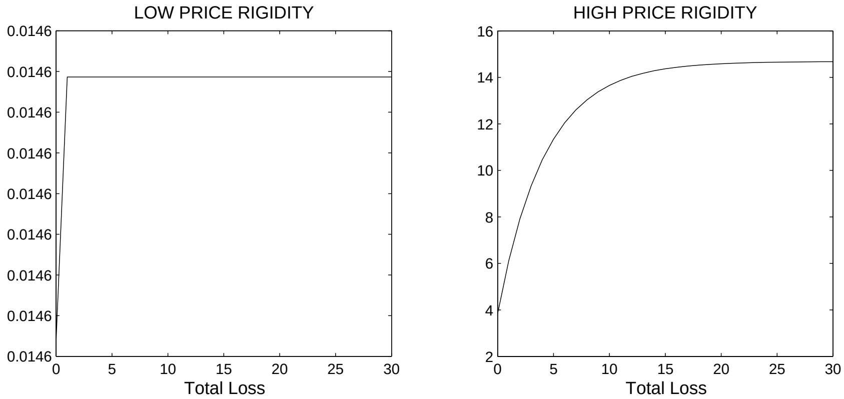

Figure 4: Welfare loss for different lengths of the anticipation period under the optimal discretionary policy in the case $\beta = 1$.

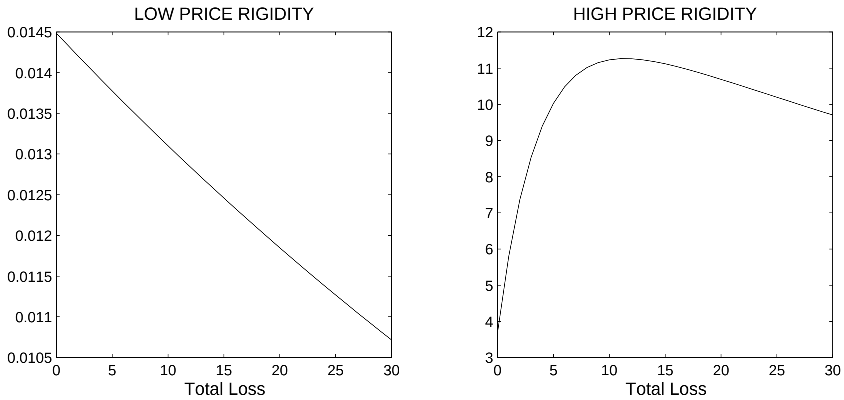

Figure 5: Welfare loss for different lengths of the anticipation period under the optimal discretionary policy in case $\beta = 0.99$.

Table 1: Values of the critical lead time T_c^* and T_d^*

	Degree of price rigidity ω							
Monetary policy	0.75	0.66	0.60	0.55	0.50	0.45	0.40	0.25
With $\varphi = 0.5$								
Commitment	53.09	19.82	9.00	4.23	1.82	0.69	0.16	0
Discretion	125.90	40.41	15.61	6.37	2.42	0	0	0
With $\varphi = 0$								
Commitment	65.78	25.57	11.79	5.59	2.41	0.95	0.28	0
Discretion	146.99	50.77	20.25	8.38	3.20	0	0	0

Note: For an anticipation period $0 < T < T_i^*$ it is true that $V|_T > V|_{T=0}$, for $T > T_i^*$ it is true that $V|_T < V|_{T=0}$ where $i = c, d$.