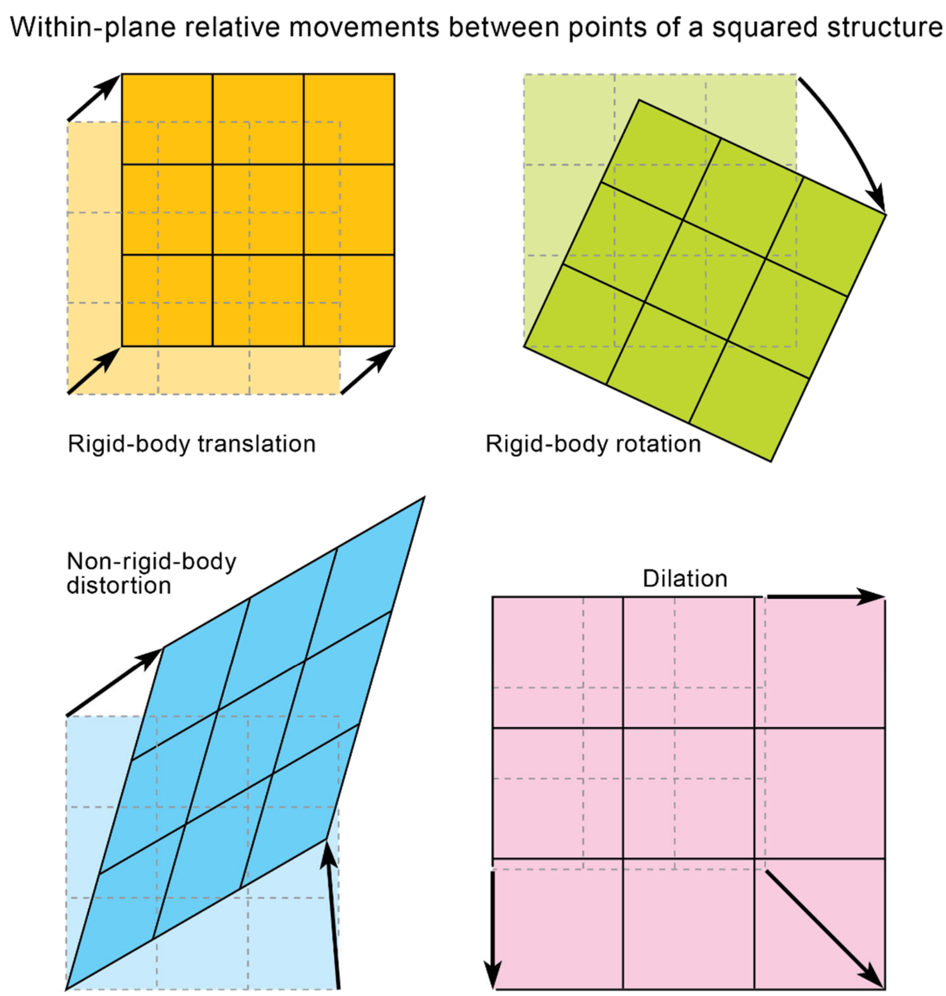
THE CAUSES: FORCES AND STRESS
MECHANICAL ASPECTS OF DEFORMATION

Mechanics deals with the effects of **forces** on **bodies**.
Forces are the demiurge behind all forms of deformation. On the Earth, plate tectonics dominate the natural forces, while other forces are often minor or temporary, leaving minimal lasting effects. We will focus on the key forces shaping our world, such as gravity and the dynamic movements of large rock masses in the Earth's crust and mantle.
This chapter will explore the fundamental concepts of stress and motion equations, which have real-world applications, particularly in the design and safety of structures such as buildings and bridges. This knowledge is essential for designing structures capable of withstanding various forces. Prepare to explore vector calculus as we investigate the key principles for analyzing and predicting material behavior under different conditions.

Physical definitions

Reminder from the general introduction



When external forces act on sturdy objects, they respond dynamically by changing their position, displacement, and sometimes their shape. For instance, a solid object may slide or rotate when pushed or pulled with sufficient force; however, its core structure remains unchanged. This phenomenon is called **rigid body deformation**, where an object translates or rotates while preserving its original size and shape (Fig2-1).
Conversely, an object that resists the force endures stress. If the forces are strong enough, they can cause particle displacements, changing the object's shape. This stress results in **strain**, manifesting as a change in shape known as **non-rigid body deformation**.

Newton's axioms: Laws of motion

Envision the universal connection among nature's elements, motion, and force, all intertwined in a graceful choreography described by Newton's enduring laws. These laws unlock the mysteries of how objects interact with their surroundings. Consider a reality where every action triggers an equal and opposite reaction and where the amount of material packed into an object, its **mass**, dictates its behavior in the presence of external quantities, the **forces**. This **dynamic** interplay embodies the essence of Sir Isaac Newton's laws of motion. They are the guiding stars, illuminating our universe's pathways of movement and transformation.

1. **Law 1 (inertia principle)**

A body continues in its state of rest or uniform motion in a straight line unless new forces compel it to change that state.

2. **Law 2 (action principle)**

The change in motion is directly proportional to the impressed force and occurs in the same direction as the line of that force.

3. **Law 3 (reaction principle)**

To every action F, there is always an equal opposed reaction $F_R = -F$. This principle means that when two bodies interact, the forces they exert on each other are equal in magnitude but opposite in direction.
For example, as a rock falls, it exerts a force on the Earth, and simultaneously, the Earth exerts an equal force on the rock.

Force

A force F moves or influences the motion of a body.

Mathematical expression

A force has both **magnitude** and **direction**, making it a vector quantity that adheres to the principles of vector algebra. An arrow typically represents it within a specified coordinate system.
- The length of the line specifies the amount of force (e.g., how strong a push is).
- The orientation of the line defines its direction of action (which way it is exerted).
- An arrow pointing towards the direction of acceleration indicates the path of motion.

The action principle (Newton's law 2) asserts that a force acting on a body with mass m accelerates the body in the direction of the force. The acceleration $\vec{a}$ is directly proportional to the applied force and inversely proportional to the body's mass.
This relationship is also written:

$$\vec{F} = \frac{\overline{m\vec{v}}}{t} = \frac{d(\overline{m\vec{v}})}{dt}$$

The product of mass and velocity, $\overrightarrow{mv}$, is the **momentum**, while t represents time.

Imagine standing firmly on the ground, feeling the implacable pull of gravity, the force that anchors us to Earth's surface. This force, known as weight, transcends mere numbers on a scale; it symbolizes the dynamic interplay between mass, density, and volume, all under the unwavering influence of gravity's power.

In our exploration of gravitational mechanics, we discover the true nature of force, a multidimensional entity transcending a single direction. Like a master painter with a brush, force manifests through various **components**, each exerting its unique pull and push. The parallelogram rule, a remarkable principle of vector analysis, elegantly encapsulates this complex interaction. For example, any force can be **resolved** into three **components** labeled $\overrightarrow{F_x}$, $\overrightarrow{F_y}$ and $\overrightarrow{F_z}$, parallel to the coordinate axes x, y, and z, respectively. This is conveniently expressed in a column form

$$F = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$

This component expression reminds us that beneath apparent simplicity lies a complex world, where each force is but a piece of a larger puzzle waiting to be unraveled.

Orientation

In mathematics, the cross product of two vectors results in a vector perpendicular to perpendicular to both, perpendicular to the plane that contains them. The resulting vector, known as the vector product, fixes the plane's orientation. This definition is instrumental in the dynamic world of three-dimensional space. The vector product is also a unit vector that emerges as the architect of how we perceive and manipulate space. In the Cartesian coordinate system, the unit vector $\vec{i}$ along the x-axis, specifies the yz plane; the unit vector $\vec{j}$, aligned with the y-axis, specifies the xz plane, and the unit vector $\vec{k}$ along the z-axis specifies the xy plane. These principles help us fully express and manipulate force vectors in three-dimensional space. The equation describing the complete force vector is:

$$\vec{F} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = F_x.\vec{i} + F_y.\vec{j} + F_z.\vec{k} = F_x.\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + F_y.\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + F_z.\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

or

$$\vec{F} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = F_x.\vec{e}_x + F_y.\vec{e}_y + F_z.\vec{e}_z$$

with $\vec{e}_i$ the unit vector along the i-axis.

When expressing the three vector components, the unit vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$ are omitted for clarity, simplifying the coordinates to the coefficients of the equation's parts. A vector defined by three numbers (magnitudes) in a coordinate system is mathematically a **first-order tensor** akin to a **scalar**.

Forces and their components are added as vectors.

Dimension / Quantity

Three independent physical dimensions, length [L], mass [M], and time [T], express the mechanical properties of a material. The brackets [ ] denote that these quantities have specific measurements. Other dimensions, such as electrical charge [Q] and temperature [q], are considered derived dimensions.

Newton's second law defines the force as the product of mass and acceleration, expressed as $\vec{F} = m \cdot \vec{a}$. The dimension of force is represented as:

$$[F] = [M.L.T^{-2}]$$

The mass is a **scalar** quantity expressed with a single number, with its unit being the kilogram (1kg). In mathematical terms, a scalar is identified as a **zero-order tensor**, meaning it can be considered a 0-dimensional matrix.

Acceleration is defined in a coordinate system.

The Newton and the dyne are the basic units of force (1 N = force required to impart an acceleration of 1 m/s^{-2} to a body of 1 kg:

$$1N = 1kg.1m.1s^{-2} = 10^5 dynes$$

Reminder: What are we talking about regarding mathematical denominations?

Scalar: a number that specifies a physical quantity based solely on its magnitude (i.e., a real number such as mass, temperature, or time).

Vector: a physical quantity with magnitude and direction, geometrically depicted as an arrow. The arrow's direction signifies the vector's orientation and its length denotes its magnitude. Mathematically, a vector is expressed as a matrix with either one row or column.

Matrix: a grid of n x m coefficients or numbers, typically 3x3 in the Cartesian world, enclosed between brackets. These tables of m lines and n columns are designed to solve linear equations. A square matrix is characterized by having an equal number of rows and columns, where m equals n.

Tensor: a mathematical structure representing quantities with multiple directions, such as a magnitude combined with two or more directions (vectors). The number of subscripts determines the order or rank of a tensor, which also defines the array's dimensionality.

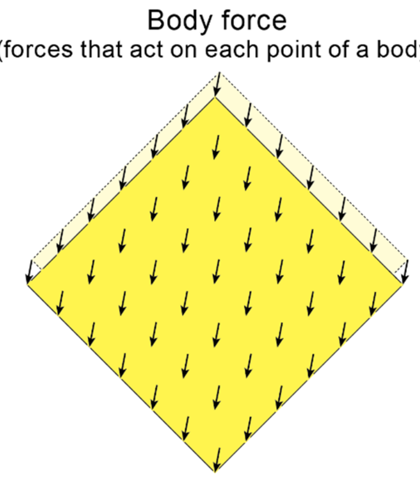
In conventional typography, scalars are represented by lowercase Greek letters. Vectors are depicted using either a lowercase, upright, boldface Latin font or a non-bold italic serif with a rightward arrow above them. Matrices and tensors are symbolized by uppercase Latin letters.

Surface - body forces

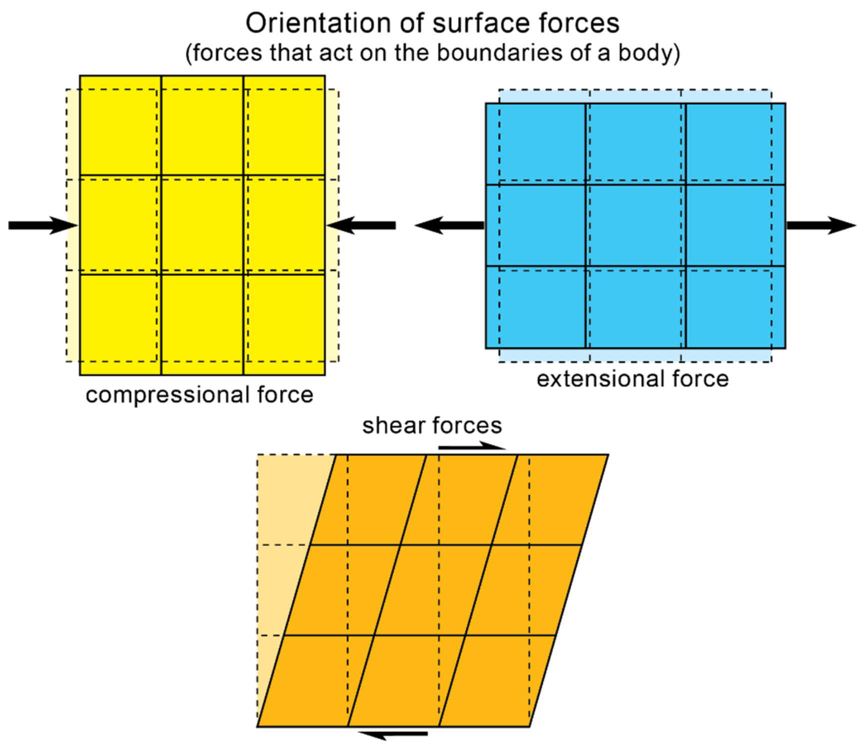
Forces: Where Gravity Holds Court

Imagine every atom in a pen feeling the tug of an invisible force, the weight of existence itself. These are body forces arising from distant external fields such as gravity and electromagnetism. They are swayed by motion from the tiniest particle to the largest mass, with their effects proportional to the volume they inhabit (Fig2-2).

In purely mechanical systems, body forces are of two kinds: those stemming from gravity and those arising from inertia. For example, a force due to inertia is exerted when pushing a heavy object. The weight of a towering column of rocks deep within the Earth is a stark reminder of gravity's unyielding grip.



Surface Forces: Where Friction Meets Action
Conversely, surface forces (or applied forces) act on an object's visible external boundaries and any internal surfaces, whether real or imagined (Fig2-3).



These forces, such as friction, are proportional to the area they affect and arise from interactions within the object, like the tension in a stretched rubber band. They often originate outside the object, serving as unseen links among all matter and transmitting energy through a continuous medium. For instance, when pushing a pen, the force travels along its length, like how seismic shifts beneath us highlight dynamic interactions. In geology, tectonic forces that reshape our planet travel through plates from their edges, demonstrating the significant impact of surface forces on our world.

The Dance of Equilibrium: Ratio of the body forces to the surface forces

Every particle seeks equilibrium in the intricate dance of forces. This delicate balance arises from the dynamic interplay between body and surface forces, leading to dynamic equilibrium. According to Newton's third law, this equilibrium is achieved when the sum of body forces is equal and opposite to the sum of surface forces. For a small body element with a characteristic length $d\ell$, the ratio of body forces to surface forces is a key factor in sustaining this balance:

$$\frac{\text{Body forces}}{\text{Surface forces}} = \mathrm{K}\frac{(d\ell)^3}{(d\ell)^2}$$

The balance between body and surface forces is essential in mechanical processes, influencing motion dynamics. When the volume element is small, body forces can often be disregarded due to their rapid (considerably for large. $d\ell$) diminishment compared to surface forces. This scaling relationship dictates the rhythm of motion within the complex network of mechanical interactions. For example, in a viscous fluid, a sphere's velocity is affected by buoyancy and friction, with buoyancy increasing more significantly as the sphere's radius grows. This principle, derived by George Gabriel Stokes in 1851, has tectonic implications, affecting phenomena like the ascent of diapirs and the descent of lithospheric plates into the mantle.

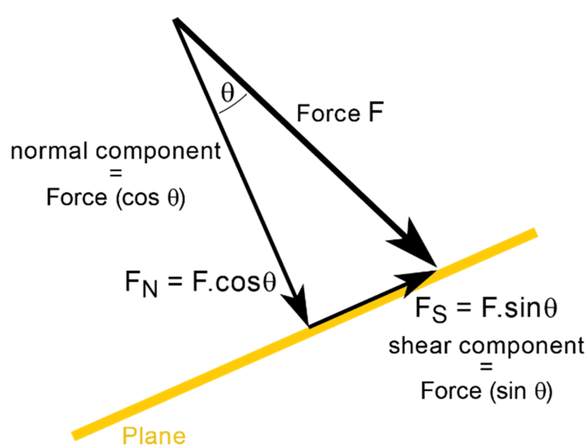
Directed forces

Directed forces are those that act in a specific direction.

- **Compression** is a force that presses bodies with its colinear reaction (Newton's law 3).
- **Tension** is the force and colinear reaction that tends to pull bodies apart.
- **Shear** is a force couple when two parallel forces act in opposite directions in the same plane but not along the same line.
- **Torsion** is twisting by two opposed force couples acting in parallel planes, like wringing out a wet cloth.

Normal and shear components

In two dimensions, a plane reduces to a line (the orange inclined line in Fig 2-4). As with any vector quantity, a force $\vec{F}$, in general, typically acts at an angle to the plane and can be decomposed into several vector components, especially those perpendicular and parallel to the line.



Two-dimensional vectorial decomposition of a force acting oblique to a plane into its normal- and shear components

$$\vec{F} = \vec{F}_N + \vec{F}_S$$

$\vec{F}_N$ and $\vec{F}_S$ are the **normal** and **shear** forces, respectively. The shear component facilitates slip along the plane, while the normal component acts perpendicularly to the surface, working to prevent slip by pressing both sides of the plane together.
In two dimensions $\vec{F}$, $\vec{F}_N$ and $\vec{F}_S$ are coplanar. The magnitude and direction of the force $\vec{F}$ is the main diagonal of the rectangular prism of the sides $\vec{F}_N$ and $\vec{F}_S$. These two perpendicular components are defined according to the right-angle trigonometry:

$$F_N = F \cos\theta$$
$$F_S = F \sin\theta$$

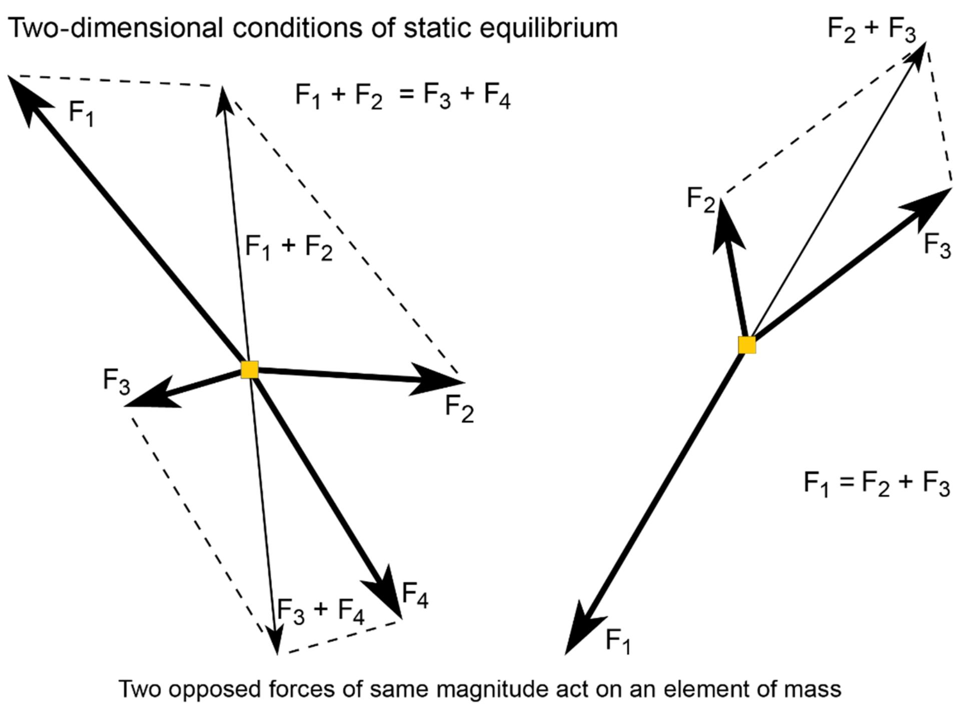
with θ the angle between the applied force and the normal to the considered plane (line in 2D). The magnitude is obtained using the Pythagoras' Theorem:

$$F^2 = F_N^2 + F_S^2$$

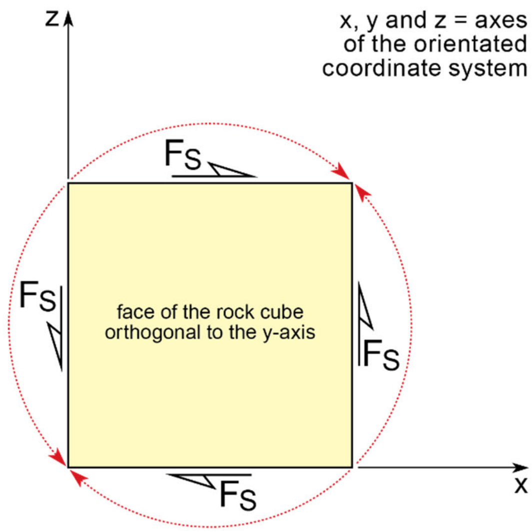
These equations show that the key to finding the component magnitudes is to know (i) the magnitude of the applied force vector and (ii) the angle it makes with the considered plane.

Action/reaction; static equilibrium

Envision a cube of rock, regarded as a continuous medium, nestled deep within the Earth's crust. Each of its six faces bears the weight of the surrounding compressing rocks. Yet, within this cube, corresponding reactions occur; a silent struggle unfolds, echoing Newton's second law: for every action, there is an equal and opposite reaction. The imaginary cube stands as a silent sentinel (Fig2-5).



The cube is in **static equilibrium**, where motion ceases, and deformation holds its breath. The system of forces is closed, and the sum of all forces in all directions converges to zero.
In this context, gravity affects every atom inside the cube, drawing them together. However, its reach extends beyond the cube, affecting every atom outside with the same gravitational pull. This interconnectedness suggests that the concept of a universal body force becomes less relevant here, as it is evenly distributed and can be reasonably disregarded in our analysis.
In **static equilibrium**, forces on opposite faces negate each other due to their equal magnitude and opposing directions. Similarly, shear forces on opposite faces must also negate each other to prevent any net couple that could cause rotation (Fig2-6). Static equilibrium is the situation treated to understand natural geological forces.



Equality of shear forces in static equilibrium
For the torque on a volume element in a continuum to be null (no angular acceleration),
the shear forces acting on opposite faces and acting in opposite directions (clockwise / anticlockwise)
must compensate each other

Stress in a continuous medium

Although stress is invisible, its effects are apparent through deformation, providing clues about its presence. Stress tends to deform a body.

Definition

Why does a larger cube appear more difficult to budge than a smaller one? The answer lies in the forces acting on its surfaces. As the cube grows, the force required to shift or change its shape rises considerably. However, size is only one factor. The varying magnitudes and directions of forces at different points on each face of the cube add complexity to the task of moving it. In geology, what is the estimated mass of Africa, minimizing significant uncertainties? Additionally, what does the African tectonic plate exert the force as it pushes against Spain and Italy?
Let’s shift our focus from abstract concepts to something tangible: when walking on various surfaces, the area of contact and the force involved play crucial roles. For instance, feet tend to sink into soft snow without snowshoes, whereas skiing allows for effortless gliding. This is because, while a person's weight stays the same, distributing that weight over a larger contact area lessens the pressure on the snow.
This winter observation reveals the relationship between materials and the forces applied to them. It is not merely the magnitude of the force that counts but how it interacts with the material, causing

it to deform. By focusing on stress instead of solely force, we can unlock the mysteries of material deformation, whether in rocks or snowy slopes beneath our feet.

So, how do we make sense of it without getting tangled in a complicated web? The key to understanding lies in a simple concept: envisioning our cube shrinking to a tiny, infinitely small point, so minuscule that its infinitesimal faces have an area of A=1. This visualization helps eliminate size distractions. We can focus solely on the forces that count by removing the size factor.

Traction

Traction T denotes the intensity of force applied to a specific surface area. When the force $\vec{F}$ is uniformly distributed across a large area, then:

$$T = F/A$$

If the force varies in direction and intensity across an area, traction should be defined only at an infinitesimal point treated as a minuscule area. Using an imaginary cubic point, traction is formally defined as the **force (F) per unit area** applied in a specific direction at a designated location on the cube. A more precise definition of traction at a point is derived from the limiting force ratio. $\Delta\vec{F}$ to the area ΔA, as the face area is permitted to shrink and approach zero by Cauchy's principle, named after Augustin Louis Cauchy.

$$\vec{T} = \lim_{\Delta A \to 0} \frac{\Delta \vec{F}}{\Delta A} = \frac{dF}{dA}$$

In this equation $\Delta\vec{F}$ is a vector quantity defined by three components:

- its magnitude;
- its orientation;
- the orientation of the plane, defined by the normal unit vector $\vec{n}$ to which it is applied:

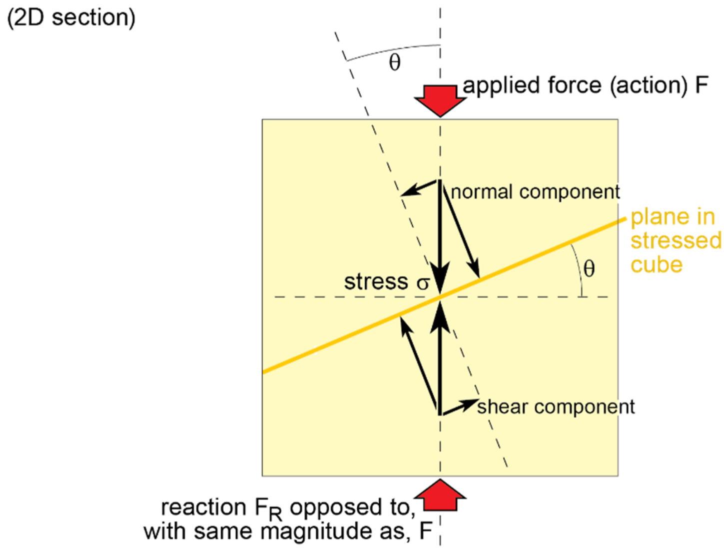
$$\vec{F} = \vec{T}.A.\vec{n}$$

This definition encompasses two directional components: one about force and the other about plane orientation. It suggests that traction is a **bound vector** that may vary from point to point on any given plane and can vary infinitely across the countless planes intersecting at any given point. Therefore, traction is always expressed with reference to a specific plane.

Stress

Newton's third law is essential for grasping the mechanics of equilibrium. It emphasizes the principle of action and reaction, fostering harmonious balance. When a traction force is applied to a body's external surface, it triggers a series of internal tractions within the body.

Now, this is where things become interesting. Internal tractions adhere to the same principles as their external counterparts. Within the body, equal and opposite forces establish a mesmerizing equilibrium. This **pair of equal and opposite forces acting on the unit area** defines **stress**. It resembles a concealed force, quietly exerting equal and opposite pressures on both sides of a cubic contact point (Fig2-7).



Stress (a pair of equal and opposite vectors) induced at a point of an inclined plane within a body subjected to uniaxial compression

Much like the tension in a spring, there are always balanced push and pull forces on the hidden faces of the cubic point, maintaining a fragile balance that ensures stability. The body enters a **state of stress**.

The concept of stress in materials involves more than just the forces exerted on them; it arises from the intricate interplay of atomic forces reminiscent of a complex tapestry being woven. When a material experiences stress, it is like participating in a cosmic tug-of-war, held together by the unseen threads of interatomic force fields. These force fields distribute stress throughout the material, revealing the remarkable complexity that underpins this fundamental concept.

When you admire a material's strength, take a moment to consider the unseen forces at work. These forces maintain a subtle balance of stress that keeps everything in place. Gaining this understanding will enhance your appreciation for the powerful collaboration of motion and reaction that ensures materials remain intact.

Dimension

Stress, like **pressure**, encompasses both the physical dimensions of force and those of the area upon which that force is exerted:

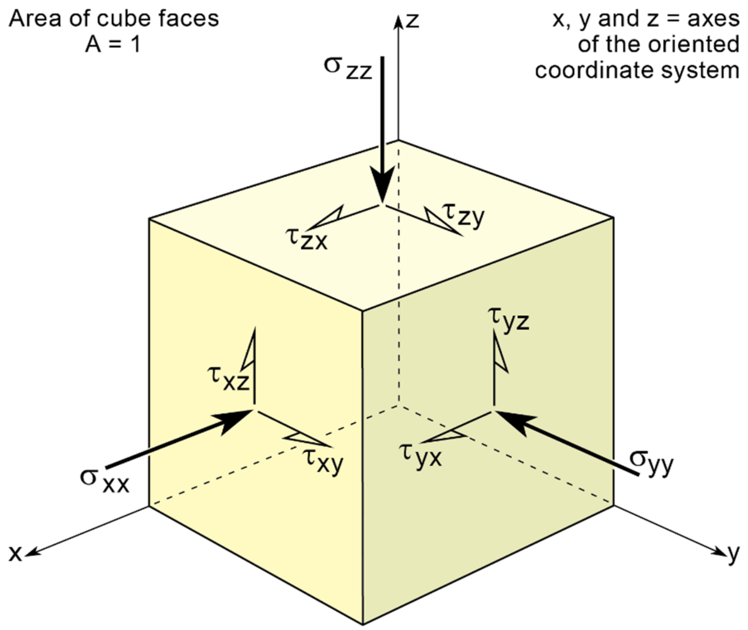
$$[M*LT^{-2}]/[L^{2}] = [\text{Mass} * \text{Length}^{-1} * \text{Time}^{-2}]$$

The unit is the Pascal (1 Pa = 1 Newton.m^{-2}, remembering that 1N = 1 kilogram meter per square second: 1 kg.$m^{-1}s^{-2}$) and Bars with 1 Bar = 1b = 10^5 Pa ~ 1 Atmosphere. Geologists more commonly use 1Megapascal = 10^6 Pa = 1 MPa. A useful number to remember for discussion with metamorphic petrologists is 1 kb = 100 MPa. 100 MPa is approximately the lithostatic pressure at the bottom of a rock column of 4 km height and with a bulk density of 2600 kg.m^{-3} (2600 kg.m^{-3} x 4000 m x 9.81 m.s^{-2} = 10^2 MPa, see the coming section "Terminology for states of stress").

Stress components

In the complex world of infinitesimal cubic points, surface forces exert a significantly greater impact than body forces. Stress is dominant in this minuscule universe. To grasp it, one must not only determine the direction of the force but also accurately evaluate the orientation of the cube's faces.

For clarity, we adopt the cube edges as the principal axes of a three-dimensional coordinate system. This strategy effectively separates the shear component into two parallel shears that align with the face edges (Fig2-8).



Cartesian components of stress at a (cubic) point
resolved into normal- (σ) and shear- (τ) stress components
Note that only stresses acting on the three visible sides of the cube are drawn.
Stresses on the opposite, hidden sides of the cube should also be represented
and all stresses should be pointing to the central point within the cube.

The forces and traction vectors acting on each face of a cube can be categorized into three components. One, known as **normal stress**, acts perpendicular to the face of the cube. The remaining two components, the **shear stresses**, operate parallel to the face and along the cube's edges. When considering cube faces with unit areas, we can decompose the stresses into three normal stresses, each acting perpendicular to one of the faces, and six shear stresses, two for each face, parallel to the coordinate directions within the face plane.

Let us symbolically recognize and appreciate these important concepts.

The normal stress, applied perpendicular to the surface, earns the symbol σ (the Greek letter sigma).

The shear stresses transmitted parallel to the surface are denoted by τ (the Greek letter tau), although you may be more accustomed to seeing them represented by the common notation σ.

__Exercise: graphic representation to be done with Excel, Mathlab, or Python__

** Draw a square ABCD and a diagonal surface on it.*
** Draw a vertical force F that acts on this surface.*
** Write equations that express the normal and shear components on this surface.*

** Show that the highest shear stress is obtained for an angle θ of 45° between the surface and F.*
** Represent graphical variations of the normal force F_N and the shear force F_S on the surface as a function of the angle θ.*

Stress at a 'point' in a continuous medium

The state of stress at a point is inherently three-dimensional, with the edges of an imaginary cubic point aligned with the Cartesian coordinates system x, y, and z (Fig2-8). We use the symbol σ_{ij} to explicitly denote the stress component acting on a pair of faces that are normal to x_i, which specifies the plane orientation and the direction of x_j, thereby defining the direction of traction. This notation effectively breaks down the stresses acting on the cube's faces into distinct components:

σ_{11} is the normal stress component acting perpendicular to the faces oriented normal to x_1 (or x).

τ_{12} and τ_{13} are the two shear components within the paired faces normal to x_1, each acting parallel to one of the other coordinates axes x_2 and x_3 (or y and z), respectively.

A corresponding face exists for every pair of faces where the inward-directed normal stress, regarded as positive, directly opposes the normal stress acting on the related face. The same procedure for faces normal to x_1 applies to the faces perpendicular to x_2 and x_3 (or y and z), resulting in nine stress components for the three pairs of faces:

Pair of faces normal to x_1:	σ_{11}	τ_{12}	τ_{13}		σ_{xx}	τ_{xy}	τ_{xz}
Pair of faces normal to x_2:	σ_{22}	τ_{21}	τ_{23}	or written as	σ_{yy}	τ_{yx}	τ_{yz}
Pair of faces normal to x_3:	σ_{33}	τ_{31}	τ_{32}		σ_{zz}	τ_{zx}	τ_{zy}

These are written in order so that:

Components in a row act on a plane perpendicular to the subscript direction, always named first.

Components in a column act in the same direction.

Using the symbol σ instead of τ yields the following ordered array:

$$\begin{matrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{matrix}$$

This geometrical arrangement represents the original set of coefficients that form the **stress matrix**:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \qquad (2\text{-}1)$$

A matrix that has the same number of rows and columns is a **square matrix**. One may collectively call this matrix of coefficients σ_{ij} in a simple form:

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

The grouping of the nine stress components constitutes the **stress tensor**. This tensor encapsulates all possible traction vectors at any location within an object, regardless of the plane's orientation (unit normal vector). It provides a complete description of the state of stress at that point. More specifically:

- The diagonal components $\sigma_{i=j}$ are normal stresses, whereas the off-diagonal components $\sigma_{i\neq j}$ indicate the shear stresses. σ_{ij} is the element positioned at the intersection of row i and column j.
- The complete stress tensor can be expressed as a matrix, typically denoted by a bold letter. Let $[\sigma_{ij}]$ be the general element of the matrix σ_{ij}. Thus, we have: $\boldsymbol{\sigma} = [\sigma_{ij}]$.
- The stress tensor is symmetric because the six off-diagonal components $\sigma_{ij} = \sigma_{ji}$ are interchangeable (subscripts for shear stresses are commutative).
- Additionally, it is a second-order because it is linked to two directional subscripts, indifferently and independently ranging from 1 to 3.

Assuming the elemental cube remains stationary, we can postulate an equilibrium condition with no body forces acting. In that case, the shear stresses on mutually perpendicular faces of the cube are equal: three of the shear components counterbalance the other three, resulting in zero rotating moments about each axis. Consequently, the **torques** measured across the diagonal of the square matrix are also zero:

$$\sigma_{12} = \sigma_{21}$$
$$\sigma_{23} = \sigma_{32}$$
$$\sigma_{31} = \sigma_{13}$$

Terminology

Torque is the product of a force vector and the perpendicular distance from the center of mass to the line of action of that force.

By leveraging the principles of symmetry, only six independent stress components are required to fully characterize the stress tensor acting on any arbitrary infinitesimal element in a stressed body:

normal stresses	σ_{11}	σ_{22}	σ_{33}
shear stresses	σ_{12}	σ_{23}	σ_{31}

In the intricate realm of mechanics, the concept of stress is both multifaceted and captivating. Imagine a set of orthogonal axes x, y, and z, each representing a distinct dimension of force and tension. To fully grasp the state of stress at any specific point, we must consider six interrelated independent quantities, creating a comprehensive depiction of equilibrium and strain.

The cube representation highlights an important difference between stress and forces. A directed force can operate in a specific direction (such as to the left), but this statement does directly apply to internal stresses. A stress component on one side of a surface element can only exist if a corresponding component of equal intensity acts in the opposite direction on the other side. This principle applies to both normal and shear stresses. Therefore, stress can certainly manifest in a vertical direction, but it cannot be defined as acting either upward or downward.

Principal stresses

Even though six independent stress magnitudes and unconstrained orientations simplify the stress tensor, the formulation remains cumbersome. Fortunately, this complexity can be reduced. In a homogeneous field of stress, it is always possible to find three mutually orthogonal planes intersecting at a given point so that all shear stresses are eliminated. Thus:

$$\tau_{12} = \tau_{23} = \tau_{31} = 0$$

In this situation, only the normal components of stress persist:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \quad \text{becomes} \quad \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix}$$

These three no-shear-stress planes are the **principal planes of stress**, intersecting at three mutually perpendicular lines known as the **principal axes of stress** at the considered point. The stresses along these axes are the **principal stresses** σ_{11}, σ_{22} , and σ_{33}, denoted σ_1, σ_2, and σ_3 to avoid repetitive subscripts, with the convention that $\sigma_1 \geq \sigma_2 \geq \sigma_3$, indicating the **maximum, intermediate**, and **minimum** principal stresses, respectively. In other words, the principal stresses are the normal stresses acting on planes where shear stresses are zero. They coincide with the principal axes of the stress ellipsoid, which will be elaborated upon later.

> *Attention! Sign convention:* In physics and engineering, understanding the concept of normal stress, which acts perpendicular to a material's surface, is a fundamental principle. When normal stress pulls material particles apart, it is considered tensile and is represented as positive. Conversely, when normal stress pushes material particles together, it is termed compressive and is described as negative.
> In geosciences, a contrary convention is often used to interpret stress, where compression is regarded as positive and tension as negative. This convention stems from the observation that natural stresses are predominantly compressional, even in regions experiencing "extension." For example, in a non-tectonic setting, the stress at any depth within the Earth is due to the overburden and induces compressive horizontal stresses. The maximum compressive stress at the Earth's surface equals the atmospheric pressure. Additionally, the anticlockwise rotation is conventionally positive when analyzing shear stresses.

Suppose the magnitudes and orientations of the three principal stresses at a particular point are known. In that case, the normal and shear stress components can be computed on any plane that intersects that point. Therefore, the state of stress at a point can be fully characterized by the values and direction of these three principal stresses. The six independent stress components are necessary only when the faces of the reference cube are misaligned with the principal planes of stress.

Terminology for states of stress

Some particular stress states are:

$\sigma_1 = \sigma_2 = 0$;	$\sigma_3 < 0$ Uniaxial tension
$\sigma_2 = \sigma_3 = 0$;	$\sigma_1 > 0$ Uniaxial compression
$\sigma_2 = 0$	Biaxial (plane) stress
$\sigma_1 > \sigma_2 > \sigma_3$	General, triaxial stress
$\sigma_1 = \sigma_2 = \sigma_3 = p$	Isotropic (also said **Hydrostatic**) state of stress: all shear stresses are zero.

If $p < 0$ (**tensile**), the stress state is **hydrostatic tension**. Hydrostatic stresses will cause volume changes without altering the shape of a material.

In earth sciences, **lithostatic pressure** is a fundamental concept that refers to the hydrostatic pressure experienced at a depth z below the ground surface. This pressure arises solely from the weight of the overlying rocks, which possess a mean density ρ. The basic formula for lithostatic pressure is given by ρgz, where g is the acceleration due to gravity. It is important to note that this equation applies only when the stress state at depth z has attained hydrostatic conditions. Such conditions are achieved only after all the shearing stresses within the rocks have been alleviated through creeping. If the stress state has not achieved hydrostaticity, we refer to the stress state resulting from the weight of a column of rocks of a certain height, which is typically denoted as:

$$\sigma_1 \approx \int_0^z \rho g . dz$$

$$\sigma_2 = \sigma_3 = [\nu/(1-\nu)]\sigma_1$$

where ν is **Poisson's ratio**.

Mean stress

The mean stress $\bar{\sigma}$ or **hydrostatic stress component** p (also called dynamic pressure) is the arithmetic average of the principal stresses:

$$\overline{\sigma} = p = (\sigma_{11} + \sigma_{22} + \sigma_{33})/3 = \sigma_{ii}/3$$

This pressure is independent of the coordinate system; it has equal magnitudes in all directions.

> *Reminder, fundamental terminology:*
>
> In mathematics, the sum of the diagonal components of a tensor remains constant regardless of the rotation of the coordinate system. This property is known as the **first invariant**.

In the Earth, the mean stress typically rises by ca. 30 MPa/km (about 3kbar/10km). This average stress thus defines the normal stress exerted on all potential fault planes, directly influencing their frictional resistance to slip. Otherwise, the mean stress may only produce a change in volume, either to reduce it if it is compressive or to expand it if it is tensile.

Deviatoric stress

Observable strain results from distortion, while measuring rock volume changes is difficult. Therefore, strain is habitually related to the deviation of stress from an isotropic state. The **deviatoric stress** quantifies this difference by subtracting the mean stress from the stress tensor. Considering that any general state of stress is the sum of the hydrostatic mean stress p and the deviatoric stress:

$$\begin{bmatrix}\sigma_{11} & \sigma_{12} & \sigma_{13}\\ \sigma_{21} & \sigma_{22} & \sigma_{23}\\ \sigma_{31} & \sigma_{32} & \sigma_{33}\end{bmatrix} = \begin{bmatrix}p & 0 & 0\\ 0 & p & 0\\ 0 & 0 & p\end{bmatrix} + \begin{bmatrix}s_1 & \sigma_{12} & \sigma_{13}\\ \sigma_{21} & s_2 & \sigma_{23}\\ \sigma_{31} & \sigma_{32} & s_3\end{bmatrix}$$

Where $s_1 + s_2 + s_3 = 0$. The second matrix on the right-hand side is the **stress deviator**, consisting of deviatoric stresses. The **principal deviatoric stresses** indicate the differences between each principal stress and the mean stress. These stresses define the effective shear stress, which quantifies the intensity of the deviator:

$$\tau_{eff} = \left(\frac{1}{2} s_{ij} s_{ij}\right)^{2} = \left[\frac{1}{2}({s_1}^2 + {s_2}^2 + {s_3}^2) + \sigma_{23}^2 + \sigma_{31}^2 + \sigma_{12}^2\right]^{\frac{1}{2}}$$

The decomposition into the deviatoric stress σ_{ij} and the volumetric stress $\delta_{ij}\bar{\sigma}$, utilizing the standard Kronecker delta, is written:

$$\sigma_{ij} = s_{ij} + \delta_{ij}\bar{\sigma}$$

and the normal stress relative to the mean stress is then described by the deviatoric stress:

$$s_{ij} = \sigma_{ij} - \delta_{ij}\bar{\sigma}$$

The Kronecker delta is a mathematical function defined as

$$\delta_{ij} \equiv \begin{cases} 0 & \text{for } i \neq j \\ 1 & \text{for } i = j \end{cases}$$

That is,

$$\delta_{11} = \delta_{22} = \delta_{33} = 1$$
$$\delta_{12} = \delta_{13} = \delta_{21} = \delta_{23} = \delta_{31} = \delta_{32} = 0$$

under another form of the matrix:

$$\begin{bmatrix}\delta_{11} & \delta_{12} & \delta_{13}\\ \delta_{21} & \delta_{22} & \delta_{23}\\ \delta_{31} & \delta_{32} & \delta_{33}\end{bmatrix}$$

is the identity matrix

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

In simpler words, where $\sigma_1 \geq \sigma_2 \geq \sigma_3$, one can think of the rock being affected by two components:

- the mean stress $\quad p = (\sigma_{11} + \sigma_{22} + \sigma_{33})/3$
- and three deviatoric stresses:

$$s_1 = \sigma_{11} - p$$
$$s_2 = \sigma_{22} - p$$
$$s_3 = \sigma_{22} - p$$

All shear stresses are deviatoric.

The main deviatoric stress, s_1, is always positive, and the smallest stress, s_3, is either negative or zero (with positive compression). Meanwhile, the intermediate deviatoric stress is nearly equal to the mean stress. The positive deviatoric stress tends to shorten the rock along its direction, while

elongation is easiest in the direction of the negative (tensional) deviator. Note that the deviatoric stress tensor always contains negative components.
Only deviatoric stresses cause permanent deformation in rocks.

Differential stress

The **differential stress** σ_d is the difference between the maximum and minimum principal stresses:

$$\sigma_d = (\sigma_1 - \sigma_3)$$

Its value and the characteristics of the deviatoric stress tensor influence the amount and type of deformation a body experiences. Note that differential stress is a scalar quantity and should not be mistaken for deviatoric stress, a tensor.

Stress acting on a specific plane.

In this demonstration, it is crucial to remember that stress is a dynamic concept defined by the interaction between forces and surfaces. The value of stress fluctuates with the orientation and magnitude of the applied force and the affected area's orientation and size. Within the world of material mechanics, we will explore the ever-changing nature of stress and its adaptability to the acting forces.

A force $\vec{F}$ acting on a real or imaginary plane, P can be resolved into two components, one perpendicular ($\vec{F}_N$) and the other parallel ($\vec{F}_S$) to the plane P. The magnitudes of these components are:

$$F_N = F \cos\theta \qquad \text{and} \qquad F_S = F \sin\theta \tag{2-2}$$

respectively.
We further consider that the cubic "point" previously used is part of the plane P.
$\vec{F}$ is designed to act normally on one face of the cube. For convenience, we consider the vertical force, $\vec{F}$, to be applied to the top face of the cubic point. This force is within the square vertical section orthogonal to P and intersects the cube. Consequently, this section transforms faces with unit area A into unit segment lengths (Fig 2-9).
Stress is defined as the concentration of force applied per unit area, often visualized as the intensity of that force. The magnitude of stress (σ) on the face of a cube can be expressed as:

Stress = Force / (Area of the cube face)

$$\sigma = F / A$$

The normal vector to plane P is inclined at an angle θ relative to force $\vec{F}$. Now, the area A_P of plane P, denoted as AP, is larger than the unit area A of the cube faces

$$P_{(Area)} = \text{Cube face}_{(Area)} / \cos\theta$$

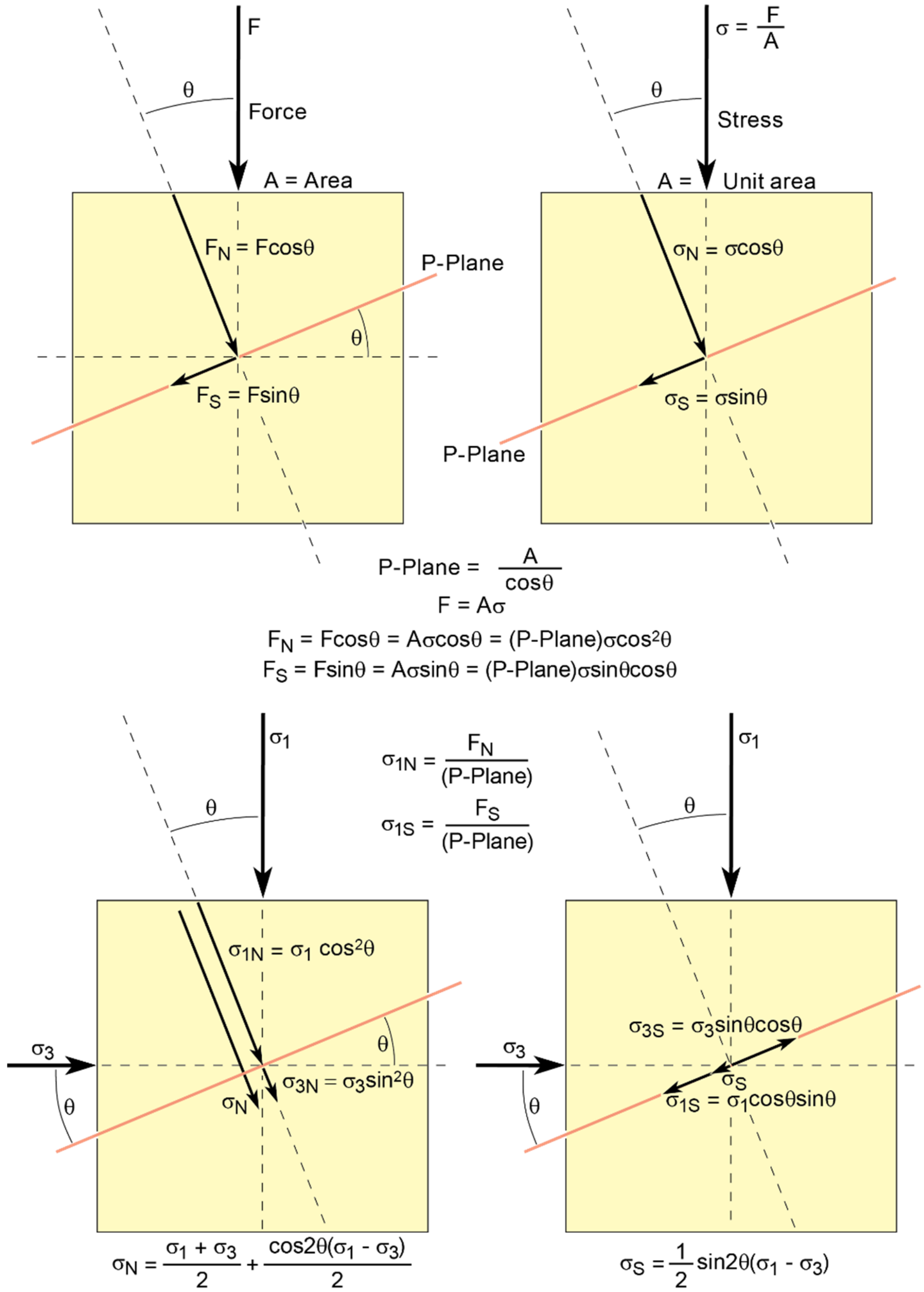
$$A_P = A/\cos\theta \tag{2-3}$$

One must recognize that A_P is the unit area to determine the magnitudes of stress's normal and shear components across plane P. Hence, the normal components of force and stress acting on plane P are:

$$F_N = F\cos\theta = A\sigma\cos\theta = A_P\sigma\cos^2\theta$$

and the shear components: (2-4)

$$F_S = F\sin\theta = A\sigma\sin\theta = A_P\sigma\sin\theta\cos\theta$$



Trigonometry states that:

$$\sin\theta\cos\theta = (\sin 2\theta)/2$$

$$\sigma_N = F_N/A_P = (F/A)\cos^2\theta = \sigma\cos^2\theta$$

And

(2-5)

$$\sigma_S = F_S/A_P = (F/A)\sin\theta\cos\theta = (\sigma/2)\sin 2\theta$$

Comparing equations (2) and (5) shows that stresses cannot be treated as vectors in the same manner as forces (Fig2-9).

Typically, any rock is under a triaxial state of stress, σ_1, σ_2 and σ_3 are the principal stresses with $\sigma_1 \geq \sigma_2 \geq \sigma_3$

> *Reminder!* In geology, the convention is to consider all positive stresses as compressive. In the non-geological literature, stresses are positive in extension!

For practical applications, one can consider an arbitrary plane P parallel to σ_2, and the horizontal x-axis of the Cartesian coordinates. The angle θ between the line orthogonal to P and the vertical σ_1 (parallel to the coordinate z-axis) is also the angle between the plane P and σ_3 (Fig2-9). We can streamline our analysis by focusing on a two-dimensional state of stress. In this case, we can concentrate exclusively on the two-dimensional principal plane (σ_1, σ_3)while disregarding σ_2, which is orthogonal to this slicing plane. This simplification is valid because it is the difference between σ_1 and σ_3 that drives deformation while σ_2 has minimal influence and can be considered negligible. Recognize that all lines in the (σ_1, σ_3) plane represent traces of planes perpendicular to it, thus parallel to σ_2.
From equations (2-5), the stress components due to σ_1 are:

$$\sigma_{1N} = \sigma_1 \cos^2 \theta$$
$$\sigma_{1S} = \sigma_1 \sin\theta \cos\theta$$

σ_3 is orthogonal to σ_1. The same trigonometric construction, which describes the relationships between the sides and angles of triangles, can be utilized to resolve stress components resulting from σ_3:

$$\sigma_{3N} = \sigma_3 \sin^2 \theta$$
$$\sigma_{3S} = \sigma_3 \sin\theta \cos\theta$$

Where the principal stresses are σ_1 and σ_3, the equations for the normal and shear stresses across a plane with its normal inclined at θ to σ_1 are

$$\sigma_N = \sigma_1 \cos^2 \theta + \sigma_3 \sin^2 \theta$$
$$\sigma_S = \sin\theta \cos\theta\, (\sigma_1 - \sigma_3)$$

where the minus sign for σ_{3S} is necessary, as the two stress directions point towards opposite directions along the plane.
From general trigonometry, one knows the double angle identities:

$$\cos^2\theta = \frac{\cos 2\theta + 1}{2}$$
$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

which one can substitute in the previous equation to write the normal stress component:

$$\sigma_N = \sigma_1 \left(\frac{\cos 2\theta + 1}{2}\right) + \sigma_3 \left(\frac{1 - \cos 2\theta}{2}\right)$$

and simplify to:

$$\sigma_N = \frac{\sigma_1 + \sigma_3}{2} + \frac{\cos 2\theta\, (\sigma_1 - \sigma_3)}{2}$$

and one can write the shear stress component as:

$$\sigma_S = \frac{\sigma_1}{2}\sin 2\,\theta - \frac{\sigma_3}{2}\sin 2\,\theta$$

$$\sigma_S = \frac{1}{2}\sin 2\,\theta\,(\sigma_1 - \sigma_3)$$

When the principal stresses are σ_1 and σ_3 the equations for the normal and shear stresses across a plane whose normal is inclined at θ to σ_1 are:

$$\sigma_N = \frac{(\sigma_2 + \sigma_3)}{2} + \frac{(\sigma_1 - \sigma_3)\cos 2\,\theta}{2}$$

(2.6)

$$\sigma_S = \frac{(\sigma_1 - \sigma_3)\sin 2\,\theta}{2}$$

respectively (Fig2-9).

Note that equation (2-6) reduces to (2-5) when σ_3 is zero. These relations are widely used in geological studies as σ_1 and σ_3 are often close to the horizontal and vertical (lithostatic) tectonic stresses.

The paired equations (2-6) demonstrate that the value of σ_S is maximum when sin2θ = 1, which corresponds to 2θ = 90°. Thus, the planes of **maximum shear stress** are oriented at an angle of 45° with respect to σ_1 and σ_3.

In all cases, where $\sigma_1 \geq \sigma_2 \geq \sigma_3$ there exist only two planes of maximum shear stress that intersect along σ_2. This predictable pattern has been observed in triaxial tests (σ_1, σ_2 and σ_3 have non-zero magnitudes) where shear fractures typically form angles close to 45° relative to the principal stress axis σ_1. The resulting paired shear fractures, called **conjugate faults,** develop synchronously in both equally favored orientations. Take as a lesson that conjugate faults intersect along a line parallel to the intermediate principal stress axis σ_2. Additionally, normal compressive stresses on these planes inhibit sliding, while shear stresses promote it.

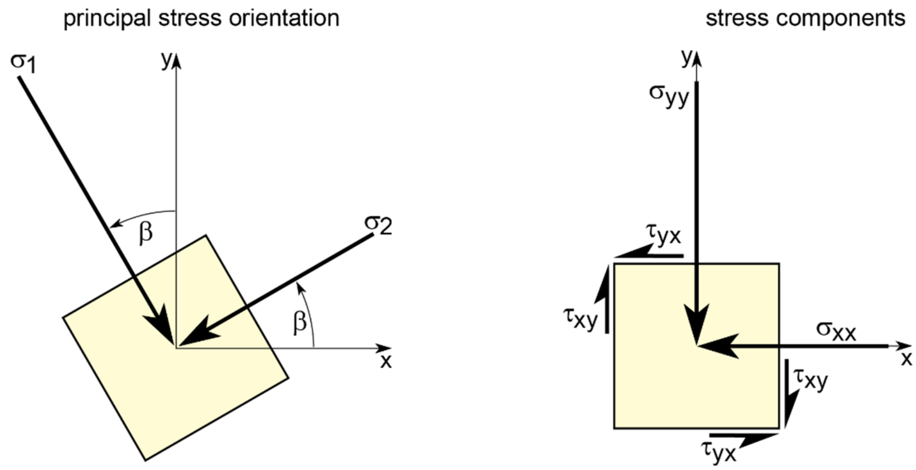
In the special situation where $\sigma_2 = \sigma_3$ or $\sigma_1 = \sigma_2$, there is an infinite number of such planes inclined at 45° to σ_1 or σ_3, respectively.

In all cases, the maximum shear stress has the value $(\sigma_1 - \sigma_3)/2$.

Equations (2-6) also imply that for any arbitrary state of stress with σ_1 and σ_3, there are surfaces on which shear forces are absent. We will use this property to establish the directions of principal stresses.

Relationship between normal stress and shear stress: Mohr circle

It is important to recognize that the principal stresses are orthogonal to three mutually perpendicular planes where shear stresses are zero. Between these specific orientations, the normal and shear stresses vary smoothly with the rotation angle θ. But how are these stress components linked in any direction β? To decode this relationship, we utilize a two-dimensional tool known as the Mohr diagram. This diagram creates an abstract coordinate system where the horizontal axis represents the normal component of a stress vector, while the vertical axis represents its tangential component (Fig2-10). Therefore, each point on this diagram corresponds to the endpoint of a constraint vector with coordinates. (σ_N,σ_S).



Stresses on a square point in the two-dimensional Cartesian coordinate system

Analytical demonstration

The Mohr construction results in a two-dimensional representation of the stress equations (2-6). Rearranging σ_N and squaring both equations, one gets:

$$\left[\sigma_N - \left(\frac{1}{2}\right)(\sigma_1+\sigma_3)\right]^2 = \left[\left(\frac{1}{2}\right)(\sigma_1-\sigma_3)\right]^2 \cos^2 2\theta$$

$$\sigma_S^2 = \left[\left(\frac{1}{2}\right)(\sigma_1-\sigma_3)\right]^2 \sin^2 2\theta \qquad (2\text{-}7)$$

One can add both equations (2-7) to write.

$$\left[\sigma_N - \left(\frac{1}{2}\right)(\sigma_1+\sigma_3)\right]^2 + \sigma_S^2 = \left[\left(\frac{1}{2}\right)(\sigma_1-\sigma_3)\right]^2 \left(\cos^2 2\theta + \sin^2 2\theta\right)$$

Since $\cos^2 + \sin^2 = 1$ for any angle:

$$\left[\sigma_N - \left(\frac{1}{2}\right)(\sigma_1+\sigma_3)\right]^2 + \sigma_S^2 = \left[\left(\frac{1}{2}\right)(\sigma_1-\sigma_3)\right]^2$$

in which one recognizes the standard equation of a circle in the coordinate plane (x,y) with center at (h,k) and radius r

$$(x - h)^2 + (y - k)^2 = r^2$$

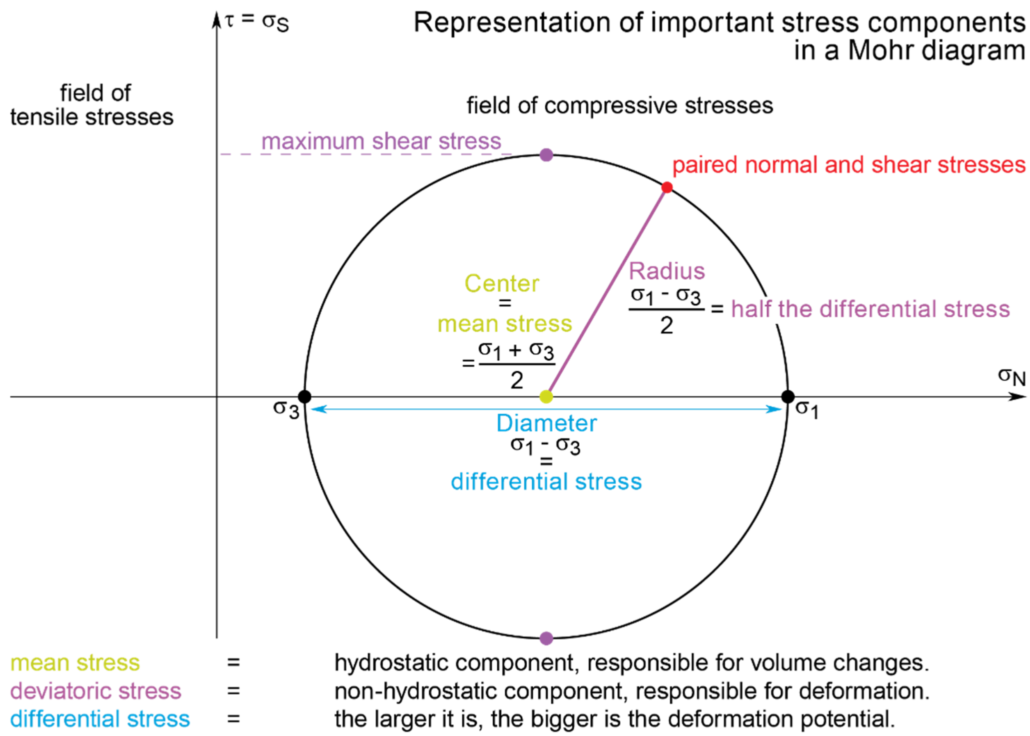
with $x = \sigma_N$, $y = \sigma_S$ and $k = 0$.

Graphical construction

The equations (2-6) describe a circular locus of paired values σ_N and σ_S (the normal and shear stresses, respectively) that operate on planes of any orientation within a body subjected to known values of σ_1 and σ_3. In other words, for a given state of stress and orientation, the paired normal and shear stresses on a plane correspond to a point of the Mohr circle (Fig2-11).

The radius of the stress circle is: $(\sigma_1 - \sigma_3/2)$

The center of the stress circle on the σ_N axis at h: $(\sigma_1 + \sigma_3/2)$



Stress in two dimensions (plane stress)

The construction of the Mohr stress circle proceeds as follows:

For the two-dimensional stress, the normal and shear stresses can be plotted along two orthogonal, scaled coordinate axes, with normal stresses σ_N along the abscissa (horizontal x-axis) and shear stress σ_S along the ordinates (or vertical y-axis).

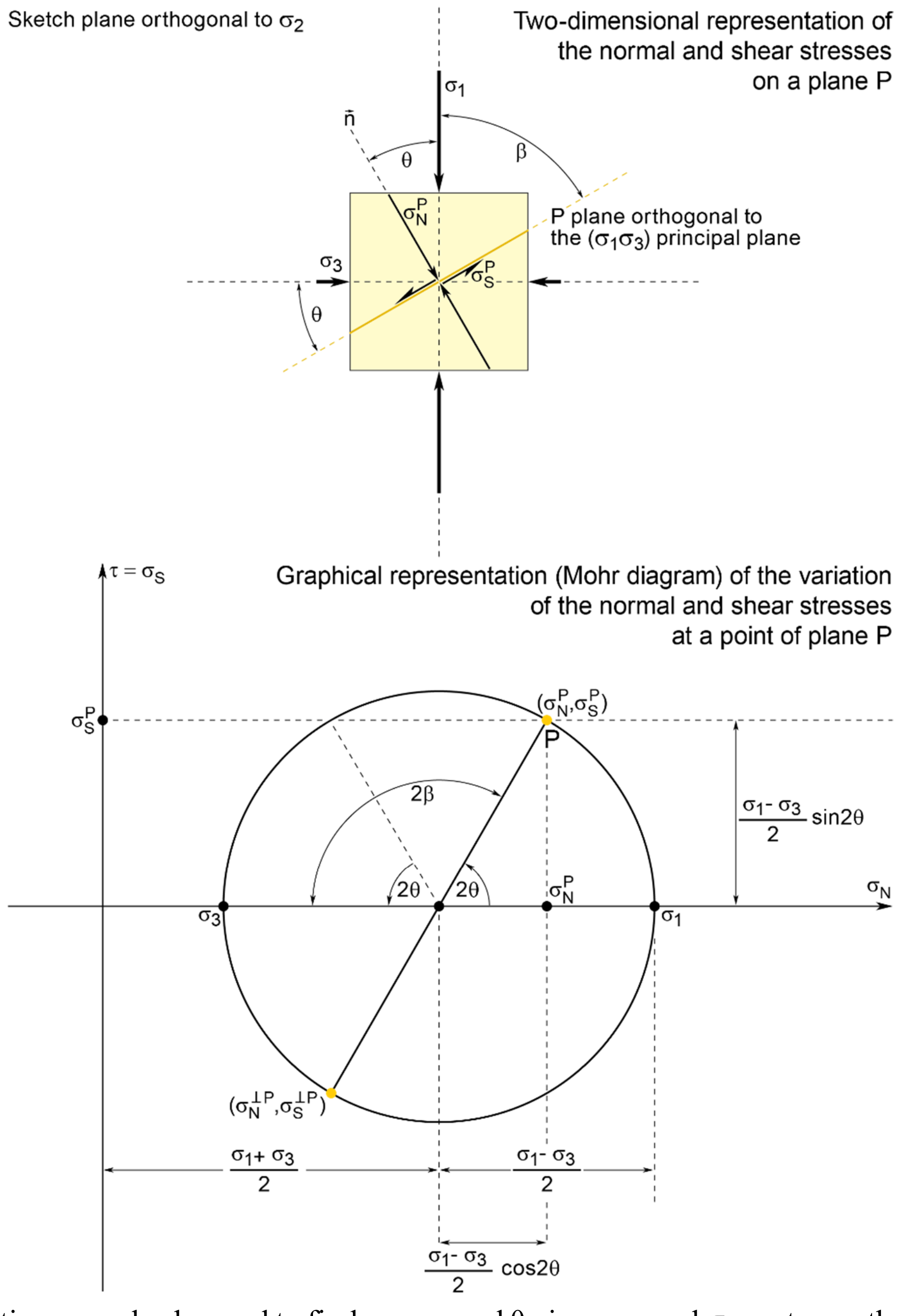
- These axes have no geographic orientation but have positive and negative directions.
- By convention, the right half of the diagram is positive for compressive normal stresses. Positive shear stresses have an anticlockwise sense (consistent with the trigonometric sense) and are plotted above the abscissa axis.

Each state of stress is defined as normal stress components, which are the principal stresses σ_1 and σ_3; they are plotted along the abscissa at a distance from the origin equal to their numerical values.

A circle is constructed with a diameter $(\sigma_1 - \sigma_3)$ and its center at $C=[(\sigma_1 + \sigma_3)/2]$. The circle passes through the points corresponding to σ_1 and σ_3. The maximum principal stress σ_1 is positioned at the right extremity of the circle, while the minimum principal stress σ_3 is placed at the left extremity of the circle.

The circumference of a Mohr circle represents the locus of all possible paired values of σ_N and σ_S. Therefore, any point P on the circle has coordinates (σ_N,σ_S) where σ_N and σ_S are given by equations (2.6); The angle 2θ between the σ_N axis and the line PC is measured in the anticlockwise (trigonometric) sense from the right-hand end of the σ_N axis. The coordinates of any point P on the circle give the normal stress σ_N (read along the abscissa) and shear stress σ_S (read along the ordinates) across a plane whose normal (ATTENTION: not the plane itself) is inclined at θ to σ_1. θ equals the angle between the fault plane and the least stress σ_3 for simple geometrical reasons.

The 2β angle measured clockwise from σ_3 to the PC radius is twice the angle between *one* and the actual fault plane (Fig2-12).



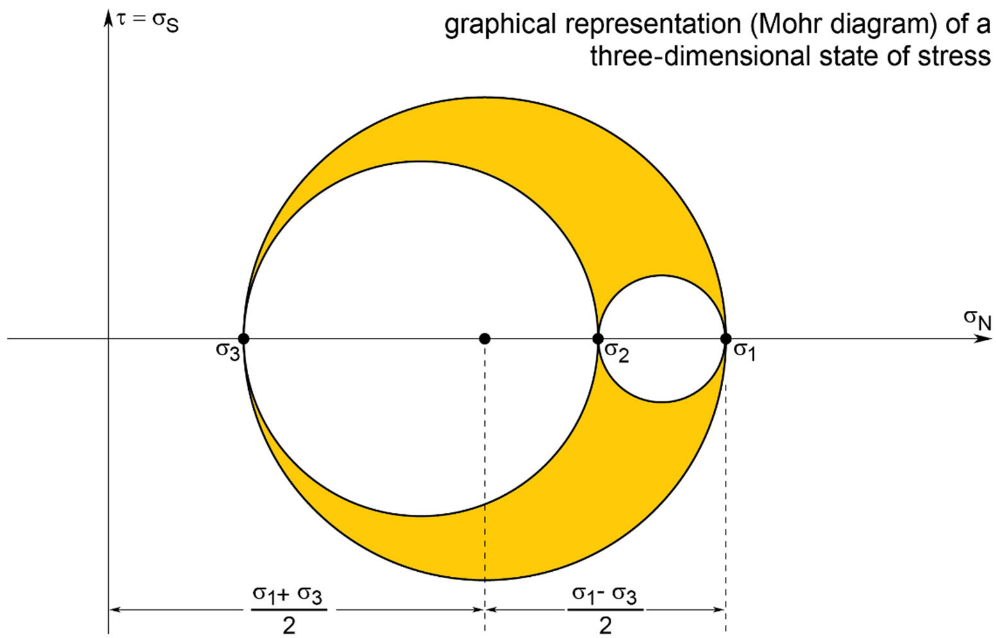
This construction may also be used to find σ_1, σ_3, and θ given σ_N and σ_S on two orthogonal planes. Since angles are doubled in this graphical format, the point representing the plane orthogonal to P is opposed to it. The Mohr circle can be constructed through these two points linked by a diameter intercepting the horizontal axis at center C.

The Mohr circle thus allows the magnitudes of normal and shear stresses on variously oriented planes to be plotted together. It neatly shows that:

- The points on the circle (hence the attitude of planes) along which shear stress σ_S is greatest correspond to values of $\theta = \pm 45°$.

- The maximum stress difference $(\sigma_1 - \sigma_3)$ determines the value of the greatest shear stress σ_S Simply because the differential stress is the diameter of the Mohr circle, which is twice the vertical radius, the maximum shear stress. Notably, the radius keeps the same magnitude regardless of the coordinate orientation. It is the second invariant of the two-dimensional stress tensor.
For reasons of symmetry, it is standard practice to depict only the upper half of the Mohr plane.

Stress in three dimensions
The cyclic interchange of the subscripts generates two other circles for the other two principal stress differences, $(\sigma_2 - \sigma_3)$ and $(\sigma_1 - \sigma_2)$. This means that the three-dimensional stress states in Mohr's construction consist of three circles: two smaller circles. (σ_1,σ_2) and (σ_2,σ_3), which are tangent at σ_2 and lay within the larger circle (σ_1,σ_3) (Fig2-13).



The three diagonal components σ_1, σ_2, and σ_3 of the stress tensor are normal stresses plotted along the horizontal axis; the non-diagonal components are shear stresses plotted along the vertical axis. All possible (σ_N,σ_S) points are plotted on the large (σ_1,σ_3) Mohr circle or between this circle and the (σ_1,σ_2) and (σ_2,σ_3) Mohr circles. The region enclosed by these three circles contains the locus of stress on planes of all orientations in three-dimensional space.
Discussions regarding the fracturing of rock masses frequently use Mohr diagrams, which can visually depict the stress variation with direction. These diagrams facilitate the identification of stresses acting on established weak planes.

<u>Exercise:</u>

Draw Mohr images of the following state of stress:
Hydrostatic, uniaxial, axial, triaxial

Effects of pore fluid pressure

Fluid pressure

Envision the vast subterranean expanses, where rocks cradle fluids within elaborate networks of intergranular spaces and fractures. These hidden depths uphold a balance of forces to the surface. The porosity of rocks, especially at depths of a few kilometers, enables a column of fluids to stand up to the surface. When the fluid reservoir is in static equilibrium, the **fluid pressure** P_f can be reliably estimated by the equation:

$$P_f = \rho_{(f)}\, gz$$

where $\rho_{(f)}$ is the fluid density, g is the acceleration of gravity, and z is the depth. This is the **hydrostatic pressure**, which differs from the **lithostatic pressure** (weight of rocks at the same depth).

Exercise:

Calculate the hydrostatic pressure gradient due to a column of pure water and the lithostatic pressure.

10 MPa/km – 23-27 MPa/km

In practice, the fluid pressures measured in geological formations are sometimes lower than, but more often exceed, the normal hydrostatic pressure. Overpressured fluids arise from various mechanisms, such as sediment compaction, diagenetic/metamorphic dehydration of minerals, and artesian circulation. Additionally, tectonic stresses in geologically active regions may increase interstitial water pressure. Carbon dioxide, released from the mantle or other sources, is a prevalent origin of abnormally pressured fluids.

Effective stress

The **total stress field** in a porous solid can be described in terms of normal and shear components across plane surfaces. The solid and its interstitial fluid combination generates the total normal stresses and shear components. The tensor of total stress, as represented in equation 2.1, expresses the total stress field:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

in which the stresses that fluids exert have also nine components:

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

in which:

$$p_{ii} = p_{jj} = p$$
$$p_{ij} = p_{ji} = 0$$

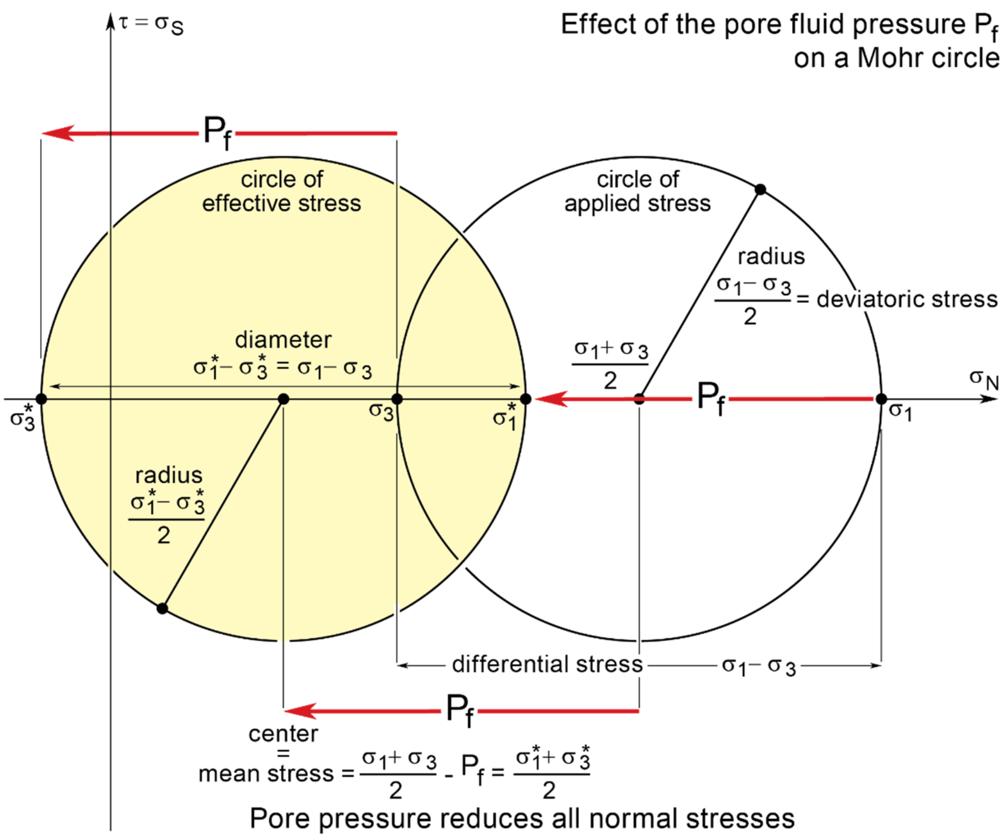
Normal (hydrostatic) pressures are equal in all directions, while the shear stresses of the pore fluids are considered negligible, as they are much smaller than those in the solid. Therefore, fluid pressure can be represented by a diagonal isotropic matrix, which is characterized by non-zero elements solely along the main diagonal:

$$\begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix}$$

The **effective stress** is the difference between the total stress and fluid pressure $P_f = p$:
The principal effective stresses σ_1^{eff}, σ_2^{eff} and σ_3^{eff} concern the solid part of the porous medium only.

Changes in confining pressure

When a material contains fluid under pressure P_f, this pressure counteracts the principal stresses due to an applied load with equal intensity in all directions. Consequently, the fluid pressure effectively diminishes the values of all normal stresses on any plane. However, the values of all shear stresses remain unchanged, demonstrating their independence from the hydrostatic component. In rocks, this phenomenon corresponds to a change in confining pressure (Fig2-14).



The **effective mean stress** is the difference between the mean stress and fluid pressure:

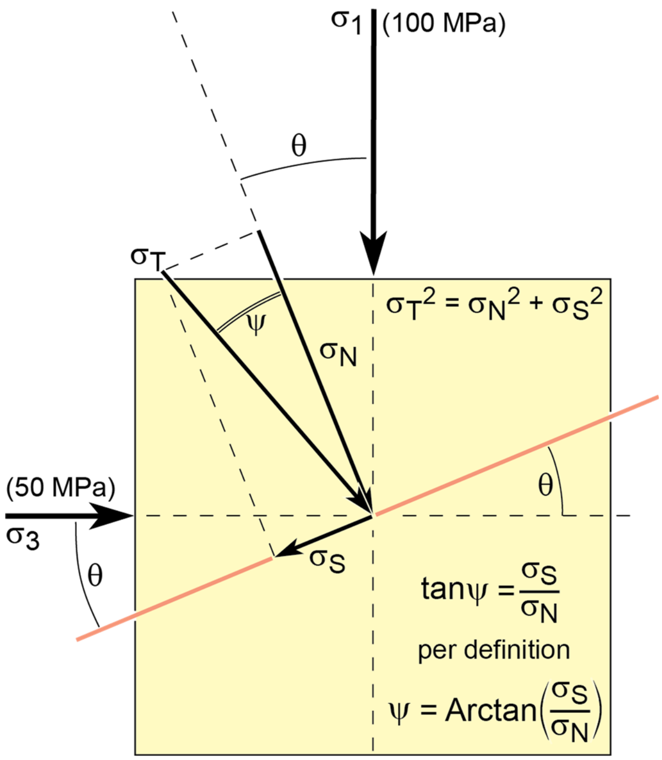
$$\sigma_{eff} = \left(\sigma_1^{eff} + \sigma_2^{eff} + \sigma_3^{eff}\right)/3 = [(\sigma_1 + \sigma_2 + \sigma_3)/3] - P_f = \bar{\sigma} - p \qquad (2.8)$$

Stress Ellipsoid

The stress ellipsoid is a graphical representation of the six parameters that comprise the stress tensor. It defines three mutually perpendicular **principal directions** and the stress intensities in these directions, called **principal stresses**.

Numerical Approach in Two Dimensions

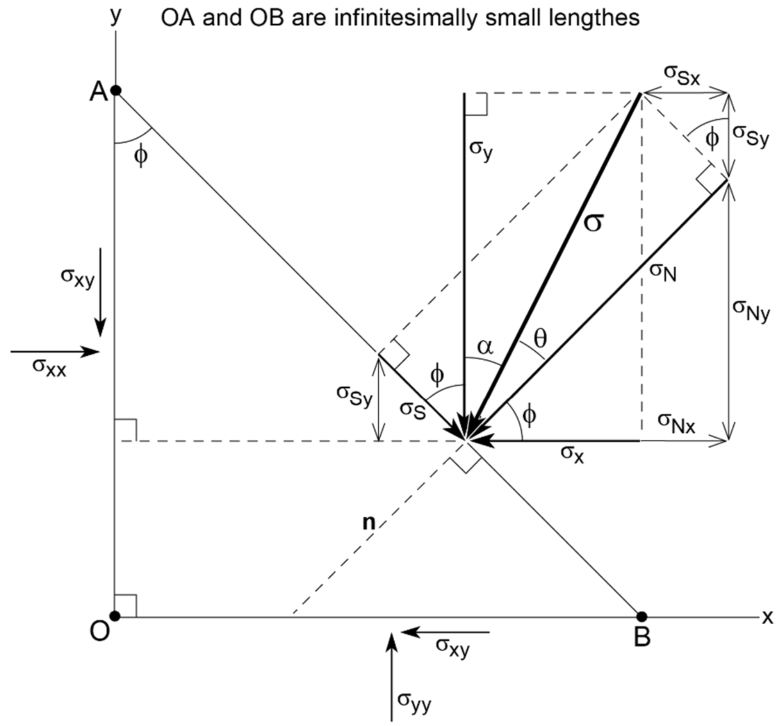
The stress system is first restricted to a two-dimensional plane (Fig2-15).



The following task requires the use of Excel, Matlab, or Python for completion:

Exercise

** Take a point O on a horizontal plane P.*
Vertical stress of 100 MPa and horizontal stress of 50 MPa act at O.
** Using a calculator and the figure below, determine the absolute values of stresses for planes inclined at 5° intervals through O* (Fig2-16).
** Make separate calculations for the normal, shear, and total stresses.*
** Describe variation of stress magnitudes as a function of orientations.*

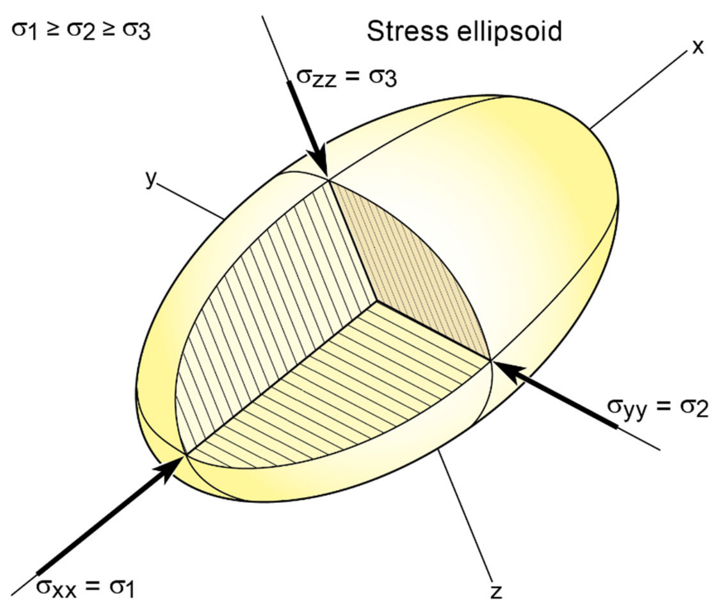


Geometrical relationship between the components of stress vectors and the components of stress tensors

An ellipse is generated for the total stress, called the **stress ellipse**. We can do the same exercise:

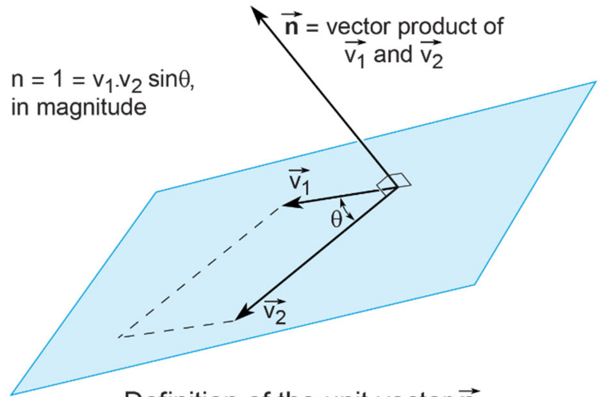
- On a vertical plane, the vertical stress is 100 MPa, and the horizontal stress has an intermediate value of 75 MPa.
- On a horizontal plane, the intensity of the two perpendicular stresses is 75 and 50 MPa, respectively.

The combination of these three stress ellipses around O generates the **stress ellipsoid**.



Analytical approach

With Oz vertical, we now consider the stress σ acting across plane P within the rectangular Cartesian coordinate directions Ox, Oy, and Oz. Let **n** be the unit normal vector defining the plane P and piercing the plane through the point P (Fig2-18).



Definition of the unit vector $\vec{n}$, perpendicular to the plane containing vectors $\vec{v_1}$ and $\vec{v_2}$

The direction of the line OP, thus the orientation of plane P, can be expressed using the spherical coordinates of **n**, which are:

$$\begin{cases} n_x = \sin\theta \, . \cos\varphi \\ n_y = \sin\theta \, . \sin\varphi \\ n_z = \cos\theta \end{cases} \qquad (2.9)$$

The spherical directions correspond to the direction of cosines $\{\cos\alpha\,;\cos\beta\,;\cos\theta\}$, directly obtained from the angles between the line normal to the plane and the coordinate axes. Given that **n** is a unit vector, these components must satisfy the unit length condition:

$$n_x^2 + n_y^2 + n_z^2 = 1 \qquad (2\text{-}10)$$

Now, one can examine the infinitely small tetrahedron bounded by plane P and the three triangular faces containing the coordinate axes. The plane intersects the Ox, Oy, and Oz axes at points X, Y, and Z. The triangle XOY is the projection of the face XYZ onto the plane xOy, viewed parallel to the Oz axis. The area of the XYZ face is:

(1/2) (base XY * height ZH)

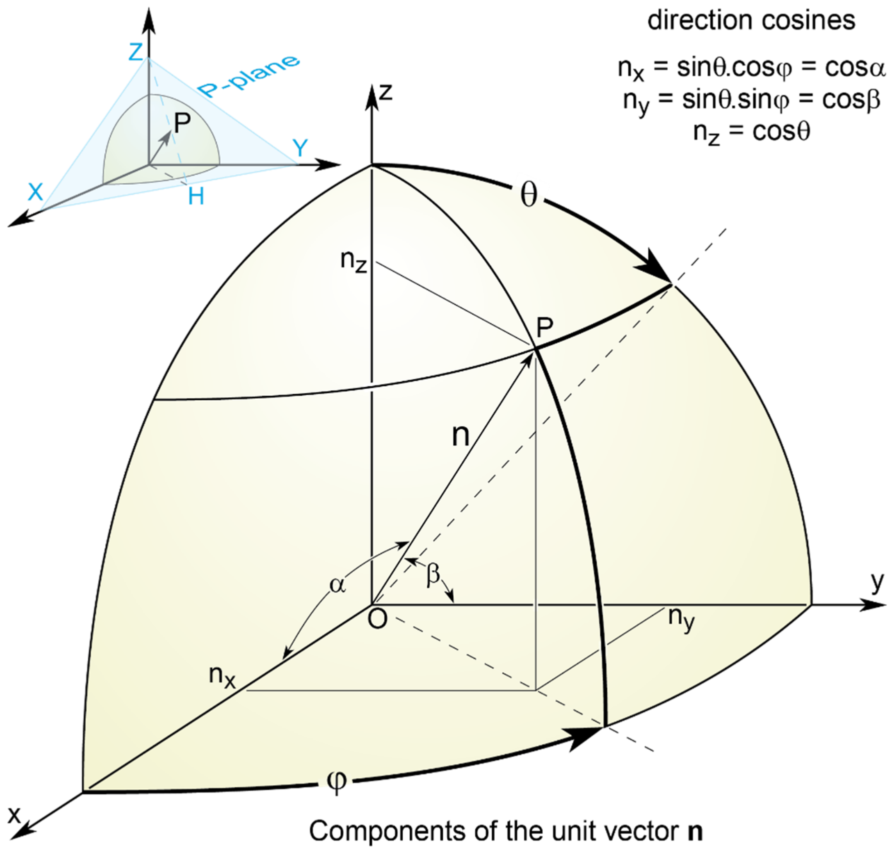
The area of the XOY face is:

(1/2) (base XY * height OH)

The ratio between these two faces is simply the ratio OH/ZH, which represents the two sides of a right triangle with the right angle at point O. A geometrical construction within the plane ZOH shows that.

$$OH/ZH = \cos\theta = n_z$$

The similar reasoning for projecting XYZ onto the other two coordinate planes shows that the proportionality factors are α (for the projection parallel to Ox) and β (for the projection parallel to Oy; Fig2-19).



Components of the unit vector **n**
orthogonal to the P-plane tangent to the unit sphere at point P

We now consider equilibrium, which refers to the balance of forces acting on the infinitely small tetrahedron under consideration. The forces applied to each face are decomposed into one normal and two shear forces.

$$\begin{matrix} \Phi_{xx} & \Phi_{xy} & \Phi_{xz} \\ \Phi_{yy} & \Phi_{yx} & \Phi_{yz} \\ \Phi_{zz} & \Phi_{zx} & \Phi_{zy} \end{matrix}$$

We take XYZ as the unit area upon which the applied force F/1 is also a stress vector **T** whose components parallel to the coordinate axes are T_x, T_y and T_z. **T** is then calculated using the simple vectorial sum, where the cosine of the angles determines the weight of the vector components:

$$\mathbf{T} = T_x \cos\alpha + T_y \cos\beta + T_z \cos\theta \qquad (2.11)$$

These three components are each balanced by force components acting in the same direction on the three other faces. For example:

$$(\text{area} = 1)T_x = \Phi_{xx} + \Phi_{yx} + \Phi_{zx} \qquad (2.12)$$

The areas of the coordinate faces with respect to the XYZ unit area have been previously calculated as $\cos\alpha$, $\cos\beta$ and $\cos\theta$. Force components Φ_{ij} are derived from the stress components σ_{ij} and τ_{ij} multiplied by the respective areas on which they act. Then, all force components can be expressed as follows:

$$\Phi_{xx} = n_x\sigma_{xx} \qquad \Phi_{xy} = n_x\tau_{xy} \qquad \Phi_{xz} = n_x\tau_{xz}$$
$$\Phi_{yy} = n_y\sigma_{yy} \qquad \Phi_{yx} = n_y\tau_{yx} \qquad \Phi_{yz} = n_y\tau_{yz}$$
$$\Phi_{zz} = n_z\sigma_{zz} \qquad \Phi_{zx} = n_z\tau_{zx} \qquad \Phi_{zy} = n_z\tau_{zy}$$

Equation (2.11) becomes:

$$T_x = n_x\sigma_{xx} + n_y\tau_{yx} + n_z\tau_{zx} \tag{2.13}$$

The absence of rotation implies that $\tau_{ij} = \tau_{ji}$ and equation (2.12) becomes:

$$T_x = n_x\sigma_{xx} + n_y\tau_{xy} + n_z\tau_{xz} \tag{2.14}$$

With similar equilibrium arguments in other directions, the coordinate axes of the stress vector **T** relative to **n** are:

$$\begin{cases} T_x = \sigma_{xx}n_x + \tau_{xy}n_y + \tau_{xz}n_z \\ T_y = \tau_{yz}n_x + \sigma_{yy}n_y + \tau_{yz}n_z \\ T_z = \tau_{zx}n_x + \tau_{zy}n_y + \sigma_{zz}n_z \end{cases}$$

These three linear equations are written in matrix notation:

$$\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \tag{2.15}$$

The 3x3 stress matrix linearly transforms every column vector as **n** into another column vector **T**, defining the second-order **stress tensor**. This tensor links any given plane with its corresponding stress vector. It is written in a condensed fashion as the Cauchy's formula:

$$\sigma_i = \sigma_{ij}\, n_j$$

The magic of this formula lies in its ability to derive the traction vector acting on a specific plane by multiplying the stress tensor, regarded as a simple matrix, with a unit vector, n_j, normal to that plane.

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Remember: If **A** and **B** are two rectangular arrays (i.e., matrices) of variables, their product **C** is defined as:

$$\mathbf{C} = \mathbf{A}.\mathbf{B}$$

where the elements c_{ij} of **C** are obtained from the i^{th} row of **A** and the j^{th} column of **B** by multiplying one by the other, element by element, and summing the product:

$$c_{ij} = \sum_k a_{ik}b_{kj}$$

AB is defined only if A's width (number of columns) equals B's height (number of rows) and, generally, **AB** ≠**BA**. Finally, **AB**=0 does not imply that **A** or **B** is a null matrix.

The parallelogram construction and equation (2.12) show that the normal stress σ acting across the plane with the normal vector **n** is given by:

$$\mathbf{T.n} = T_x n_x + T_y n_y + T_z n_z$$

$$\sigma = n_x{}^2\sigma_x + n_y{}^2\sigma_y + n_z{}^2\sigma_z + 2\left(n_y n_z \tau_{yz} + n_x n_z \tau_{zx} + n_x n_z \tau_{xy}\right) \qquad (2.16)$$

while the corresponding shear stress is:

$$\tau^2 = \mathbf{T}^2 \text{-} \ \sigma^2$$

We are seeking a geometric representation of stress variation with direction. The theory is most easily applied to a two-dimensional stress system where the angle xOP = θ. Then $n_x = \cos\theta$, $n_y = \sin\theta$ and $n_z = 0$. Equation (2.10) reduces to:

$$\sigma = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta$$

In the vertical plane xOz, where σ is parallel to the horizontal axis Ox, the tetrahedron (or prism in two dimensions) experiences two key forces in that direction: σ_x time the area of P, and σ_1 time the area of xOz. These forces must balance to achieve equilibrium within the tetrahedron

$$\sigma_x = \sigma_1 * (\text{area of xOz} / \text{area of } P)$$

Now, since:

$$\text{Area of xOz} = n_x * (\text{area of } P),$$

(areas are reduced to lines in this 2D projection) one gets:

$$\sigma_x = n_x \sigma_1$$

and, by similar arguments,

$$\sigma_y = n_y \sigma_2$$

$$\sigma_z = n_z \sigma_3$$

Substituting cosine directions from equation (2.10), it follows that:

$$(\sigma_x{}^2/\sigma_1{}^2) + \left(\sigma_y{}^2/\sigma_2{}^2\right) + (\sigma_z{}^2/\sigma_3{}^2) = 1 \qquad (2.17)$$

Equation (2.17) describes an ellipsoid centered at the origin, with its axes parallel to the coordinate axes. This ellipsoid features semiaxes corresponding to the principal stresses in direction and magnitude. This **stress ellipsoid** is a commonly used representation of stress (Fig2-17). Its principal axes indicate the **principal axes of stress**, which are the mutually perpendicular directions of zero shear stress. The direction and magnitude of a radius vector of the stress ellipsoid provide a complete depiction of the stress across the plane conjugate to that radius vector. The radius vector s is:

$$s = \sqrt{\sigma_x{}^2 + \sigma_y{}^2 + \sigma_z{}^2}$$

The ellipsoid is the loci of all s-extremities. Notice that, in general, the plane corresponding to a given radius vector is not normal to that radius vector.
Intuitively, the inherent symmetries of the ellipsoid reveal that there are always three orthogonal directions (the principal axes) for which **T** and **n** have the same direction. This alignment allows to describe the normal stress, as expressed in equation (2.15), across a plane defined by the cosine directions $\{n_x; n_y; n_z\}$ of the normal stress:

$$\sigma = {n_x}^2\sigma_1 + {n_y}^2\sigma_2 + {n_z}^2\sigma_3$$

The magnitude of the shear stress across this plane is:

$$\tau^2 = (\sigma_1\text{-}\sigma_2)^2{n_x}^2{n_y}^2 + (\sigma_2\text{-}\sigma_3)^2{n_y}^2{n_z}^2 + (\sigma_3\text{-}\sigma_1)^2{n_z}^2{n_x}^2$$

All shear components become zero when the directions align with the coordinate axes. Consequently, the stress tensor (equation 2.15) is then simplified to:

$$T = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

Stress ellipsoids –stress tensor

A numerically generated stress ellipse in a specific plane portrays a section of the stress tensor shaped like an ellipsoid in two dimensions. This ellipse captures the stress distribution in this plane within a material. The stress tensor is more than a single vector; it embodies a complete collection of stresses. It accounts for all stresses on every conceivable plane orientation that passes through a distinct point in a body, depicted by the center of the ellipsoid at a particular instant in time.

To describe the stress tensor, one must know the stress ellipsoid's orientation, size, and shape. This involves determining the orientations and lengths of its three principal axes. By defining stress tensors at every point throughout a body, one can construct a comprehensive stress field that encapsulates all the stress tensors within the body. This analytical exercise, which lies at the core of understanding the intricate relationship between stress and strain, is a fundamental tool for engineers and scientists in various fields, leaving no aspect of the stress distribution in a material unexplored.

Stress field

The Earth's lithosphere constantly experiences uneven stress distribution due to various sources of stress across its surface. In areas where external forces are most active, the magnitude of stress can be strikingly high. However, this intensity gradually diminishes with distance as the rocks deform in response to the strain energy. Such variability highlights the planet's dynamic and ever-changing nature of stress distribution. When surface forces act on a body, the stresses that develop within that body generally vary in both direction and intensity at different points. This variation creates a **stress field**, representing the distribution of stresses throughout the entire body. Visualizations such as stress ellipsoids, axes, or stress trajectories illustrate this complex field. The stress field is deemed homogeneous if the normal and shear components remain constant in magnitude and orientation across all points. However, this level of uniformity is rare in geological contexts. This rarity leads to the prevalence of **heterogeneous** stress fields, which enable earth scientists to map regional stress variations, a feat demonstrated by projects like the World Stress Map.

See the World Stress Map: http://datapub.gfz-potsdam.de/

Two or more stress fields of different origins may be superimposed to give a **combined stress field**.

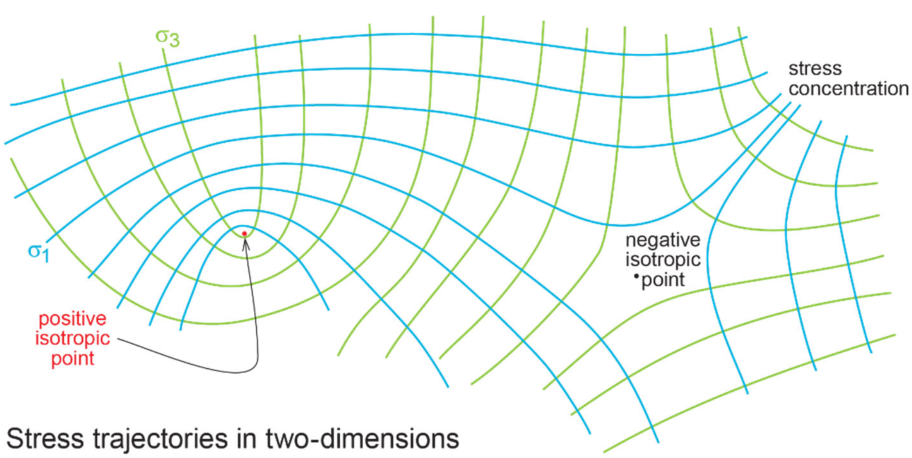
The uneven distribution of stress within the Earth has practical implications. The stress gradient quantifies the relationship between stress and space, the rate at which stress increases in a particular direction. For instance, a vertical hydrostatic gradient of 10 MPa/km and an overburden gradient of 23 MPa/km reveal the complexities of stress distribution. Curves of iso-stress-magnitude or stress contours delineate such gradients. Stresses within the lithosphere arise from forces transmitted across various points. Knowing the magnitude and the orientation of principal stresses at a particular location facilitates the calculation of normal and shear components on planes passing through that point. This knowledge sheds light on the mechanical behavior of geological materials and aids in predicting seismic activity.

Present-day tectonic stress field

Determining stress in geological contexts is challenging yet essential, often complicated by sparse data and the inherent complexity of geological systems. The results consistently indicate a stress state that is heterogeneous and unpredictable over time, influenced by various spatial and temporal dynamics. Despite these obstacles, researchers continue to extrapolate from individual measurements, although they recognize that such extrapolations can oversimplify the situation due to the impact of localized geological features. These features can introduce disturbances that further complicate stress determination. Nevertheless, comprehensive data analyses unveil the presence of strikingly uniform stress regions, referred to as 'Andersonian' stress provinces. In these provinces, two principal stresses align horizontally, while the vertical stress is contingent on the regime: it can equal σ_1 during extensional conditions, σ_2 during strike-slip, or σ_3 in compression contexts. The anthropogenic seismicity associated with reservoir filling and earthquake-triggered events points to a globally consistent frictional equilibrium within the continental crust. Overall, despite the complexities and challenges, these findings provide valuable insights into the mechanical behavior of Earth's crust and underscore the interconnected nature of geological processes on a global scale.

Stress trajectories

In two dimensions, on a surface such as a map, **stress trajectories** are virtual pathways illustrating the directions of principal stresses at every point. These trajectories connect axes of the same class of principal stresses, showing how the orientation of these stresses continuously varies in space. For example, one set of lines identifies the direction of maximum principal stress, while another set follows the direction of minimum principal stress. While individual trajectories may curve, the principal stresses must consistently maintain a right-angle relationship at every point along the trajectory (Fig2-20).



Stress trajectories in two-dimensions

When adjacent trajectories converge, it signals stress concentration, indicating areas of heightened stress intensity. Principal stresses are equal at **isotropic points**. Entwined stress trajectories bound positive isotropic points, while diverging trajectories characterize negative isotropic points. **Singular points,** where all principal stresses drop to zero, represent locations of stress equilibrium or discontinuity points.

Slip lines

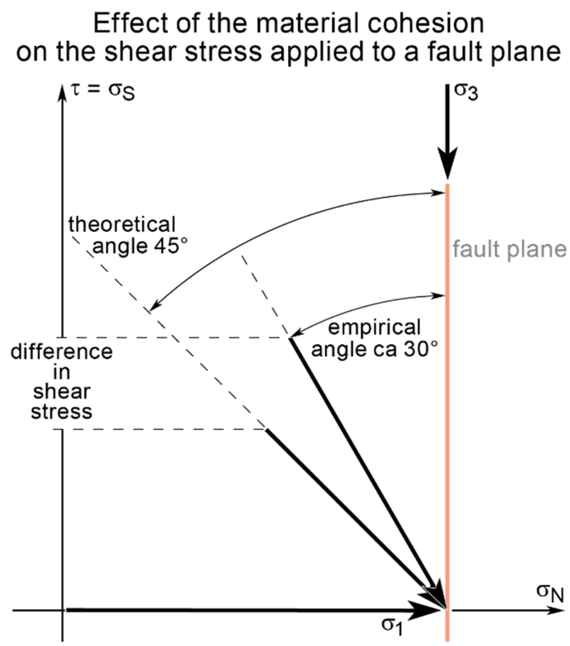
The principal stress trajectories offer practical insights into the potential shear behavior within a stress field. At any location, we can pinpoint potential shear surfaces as tangents that align with the direction of maximum shear stress. These surfaces manifest as **slip lines** on a map, indicating zones vulnerable to shear deformation. In a two-dimensional framework, these slip lines are depicted by two sets reflecting the conjugate clockwise and counterclockwise senses of shear. Remarkably, slip lines converge towards isotropic points within the stress trajectories, where the principal stresses are equal. This convergence highlights the importance of these points in influencing the distribution and dynamics of shear within the stress field.

Applications to geological shear structures

Trigonometric equations 2.6 and the Mohr construction suggest that slip lines should ideally form a network at a 45° angle to the stress field. However, geological observations of conjugate faults, identified as the planes with the highest resolved shear stress, and laboratory experiments producing these faults indicate that the actual angle is about 30° relative to the maximal compressional principal stress σ_1. Why this discrepancy?

Where reality departs from mathematical demonstration

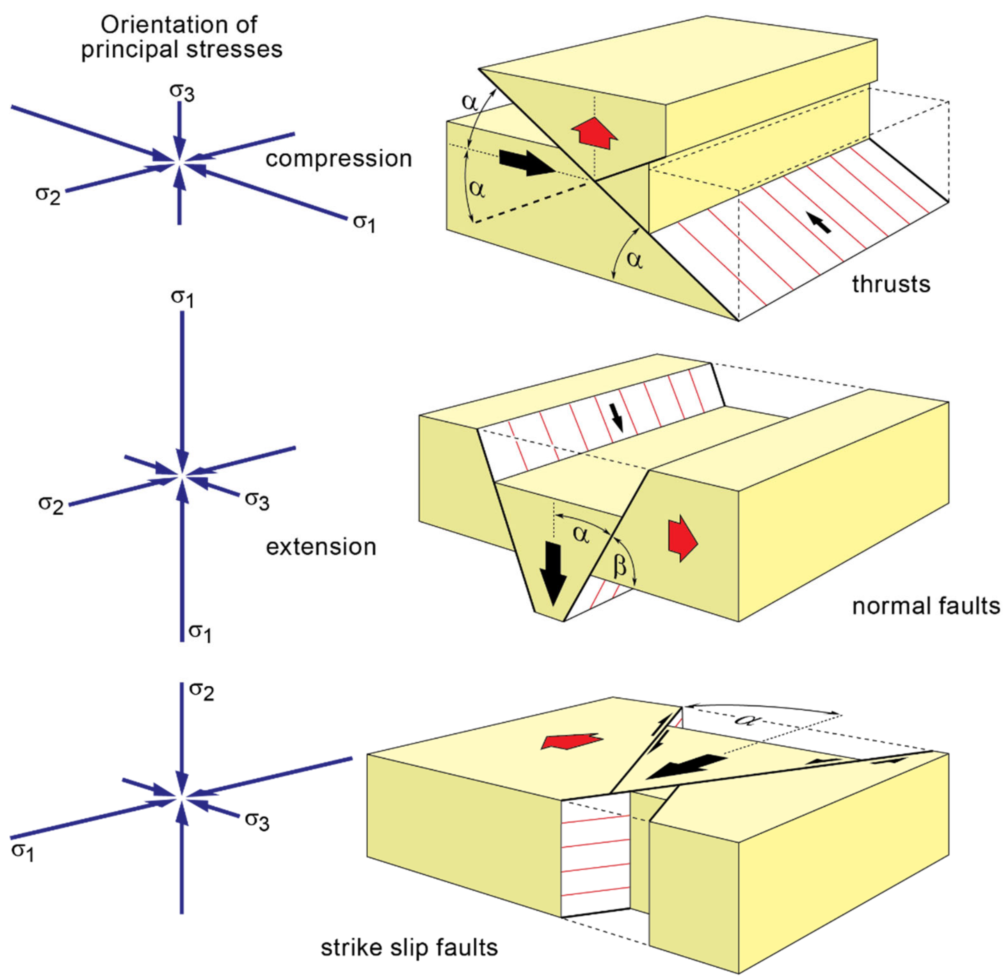
Trigonometric reasoning leads to equations (2.6), which rely on purely geometric principles. While this mathematical calculation is sound, it becomes physically inaccurate because it needs to account for essential material properties. Cohesion among the elements of a material provides internal strength, which must be surpassed for faulting and sliding to occur.



This internal strength must be integrated into geometrical calculations, implying that the actual shear stress must be greater than predicted theoretically. We must consider a larger angle θ in equations (2.6) to demonstrate this increased shear stress, necessitating an angle smaller than 45° between the fault plane and s1 (Fig2-21). The difference of about 15° (called the **angle of internal friction**) is a material requirement. It is reassuring to note that laboratory tests have consistently measured small variations according to the type of rock being tested. The stated angle of 30° serves as an acceptable average approximation.

The Geologists' lucky break

Conjugate faults often form an acute angle of approximately 60°, bisected by the compression axis, σ1. This geological quirk is a stroke of luck for geologists. How that? By measuring these conjugate directions in the field, they unlock a treasure trove of information, offering immediate insights into the regional tectonic forces at work (Fig2-22).



Fault systems

The effects of normal and shear stresses can be illustrated using two compelling geological examples of contemporary structures: If the direction of the applied force is known, one can predict the sense of fault displacement and bedding-plane slip, and conversely, knowing the displacement can help determine the force's direction. This consideration enhances our predictive capabilities and deepens our understanding of geological processes.

Where uncertainties persist

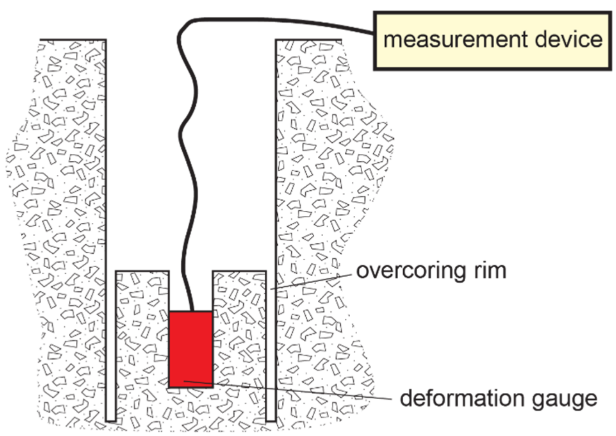
Not much is known about the stress fields in rocks during deformation, although one of the prime goals of the subject is to define these fields as closely as possible. Limiting understanding is partly due to the complexity of stress fields in deforming bodies but mostly results from a general information gap regarding the mechanical properties of rocks and rock formations.

Stress Measurement

Critical factors drive the motivation to measure stress. In geological contexts, understanding stress distributions is paramount for mitigating geological hazards, optimizing engineering designs, and identifying valuable resources. For example, rock formations initially support stresses during mining or drilling operations. As these rocks are excavated, the stresses they previously supported are promptly transferred to the surrounding geological formations. This phenomenon leads to stress concentrations, a concept well explained through elastic theory. Therefore, stress can be effectively measured by observing the response of the rocks surrounding a borehole or mine. Leveraging this knowledge, we can measure stress by examining how nearby rocks respond to these transferred stresses. Two primary methods are reliable for measuring in situ stresses utilizing stress concentration around boreholes: near-surface **overcoring** and **hydraulic fracturing**. These techniques allow researchers to glean vital assessments into the stress regime of geological formations, thus assessing rock stability, designing safe engineering structures, and optimizing resource extraction processes. Ultimately, stress measurements are fundamental to understanding and harnessing the geological forces that shape our world.

Elastic strain: Overcoring and breakout

Overcoring is a technique used in geomechanics to measure in-situ stresses in rock formations. A simplified explanation of the process is as follows (Fig2-23):



Principle of stress measurement by overcoring

1- Installation of a strain gauge on the bottom of a tubular borehole drilled into the rock formation.

2- Drilling a coaxial and annular hole around and deeper than the original borehole. Importantly, the internal radius of this second hole is larger than the radius of the first borehole, thus insulating the gauge from the surrounding stresses in the rock formation.

3- Release of stresses from the central rock cylinder containing the strain gauge: As a result, the circular inner core of rock around the strain gauge undergoes elastic deformation, transforming into an elliptical shape. This deformation occurs with the long axis of the ellipse parallel to the maximum horizontal principal stress direction. Essentially, the rock relaxes more in the most compressed direction.

4- Measurement conversion allows for determining the regional stress state within the surrounding rock formation. Given the elastic properties of the rock, the strain measured by the gauge is converted into stress magnitude.

Additionally, after drilling, a circular borehole may become elliptical (**breakout**) due to stresses in the surrounding rock. In this case, the ellipse's long axis aligns with the minimum horizontal stress direction.

Hydraulic fracturing

Hydraulic fracturing, or **fracking**, is a scientific technique to extract oil and gas. It involves the injection of fluids at high pressure into a sealed well, creating fractures in the surrounding rock strata. This method enhances recovery rates and allows us to pinpoint the magnitudes and orientations of the horizontal plane's greater and smaller principal stresses. The process is as follows:

1- Injection of fluids into a sealed well and pressurize until the rock wall fractures.

2- Fracturing occurs when the fluid pressure reaches the tensile strength of the rock, which is assumed to be equal to the minimum horizontal effective stress. The fluid pressure needed to induce a fracture is the **breakdown pressure**. When this fluid pressure equals the tensile strength σ_T of the rock, fracturing is inevitable, emphasizing the importance of comprehending both the tensile strength and the minimum horizontal effective stress $\sigma^*_{h,eff}$:

$$\sigma^*_{h,eff} = - \sigma_T$$

3- Stress conditions in rock are critical for hydraulic fracturing. This process assumes that one principal stress is vertical (near-surface condition) and is aligned with the vertical wellbore. The magnitude of this vertical stress σ_v is the weight of the overlying rocks. The objective is to find the magnitudes of the greater σ_H and smaller (σ_h) principal stresses in the horizontal plane and their orientation. Hydraulic fracturing assumes that cracks form perpendicular to the minimum horizontal stress. Hence, measuring the orientation of created hydraulic fractures and the breakdown pressure provides a clue to the stress tensor.

4- Limitations: It is worth noting that this technique does not specify the direction of principal stresses. Hydraulic fracture initiation also depends on stress regimes and the wellbore orientation. After injection ceases, the propped fractures become pathways for hydrocarbon gas or water flow from the drilled reservoir to the well, leading to increased production

Hydraulic fracturing is a particularly useful and practical stimulation technique for hydrocarbon recovery from low-permeability reservoirs where traditional extraction methods are less effective. In brief, this advanced technique is a complex process that involves understanding stress conditions, fluid dynamics, and rock mechanics to optimize oil and gas extraction from difficult reservoirs.

Conclusion

Stress induces strain, which manifests as observable structures that record Earth's physical conditions and stress history. Kinematic analysis identifies four fundamental components of deformation, each playing a pivotal role in shaping geological structures:

- Translation: The shift in rock mass position influences overall displacement.

- Rotation: The change in orientation reveals how rocks adjust under stress and contributes to their deformation patterns.
- Dilation: The expansion or contraction of materials, essential for understanding volume variations during deformation.
- Distortion: The geometric transformation of rocks, providing insight into strain patterns.

Stress, an instantaneous quantity, is defined as the force exerted per unit area at a specific point. It is expressed as vector components acting on three reference planes. Mathematically, stress is represented by aggregating traction vectors across all possible planes at a given point, forming a second-order tensor known as the **stress tensor**.
The stress tensor is denoted as σ_{ij}. It consists of nine components and requires a coordinate system for its definition. The first subscript identifies the direction of the applied force, while the second denotes the face of the reference cube on which the force acts. Unlike ordinary vectors, stresses cannot be summed through simple vector addition. Stress tensors encapsulate magnitude and direction in a manner that distinguishes them from standard vectors, emphasizing their unique mathematical properties.
The **state of stress** at a point is characterized by three principal stresses or by the normal σ_N and shear σ_S stresses acting on a plane of known orientation. This stress state can be visualized as a stress ellipsoid, with its axes corresponding to the magnitudes of the three principal stresses.
The **Mohr construction** is a practical and graphical tool for understanding the relationship between stress components on different planes. Plotting stress states on a Mohr diagram allows one to visualize how normal and shear stresses vary across orientations, providing critical insights into material behavior under stress.
Stress within the lithosphere arises from tectonic processes, such as plate motions, burial and unburial, and magma intrusion, as well as non-tectonic, localized factors like thermal expansion/contraction, meteoritic impacts, and fluid circulation. Despite this diversity, natural stress fields exhibit consistent patterns dominated by tectonic forces.
Since stress dictates deformation, understanding stress patterns is essential for describing, quantifying, and predicting rock deformation and associated tectonic processes. This knowledge is fundamental to structural geology and underpins numerous geological and geotechnical applications.

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